TAU PHYSICS*

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Lectures presented at the
20th SLAC Summer Institute on Particle Physics,
Stanford, California, July 13–24, 1992

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* This work was supported by the Department of Energy, contract DE–AC03–76SF00515.

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H. ACKNOWLEDGEMENT

A. INTRODUCTION

... they are ill discoverers that think there is no land
when they can see nothing but sea.
Francis Bacon

This is the written version of three lectures on tau lepton physics which I presented at the 1992 SLAC Summer Institute on Particle Physics. Since the discovery of the τ seventeen years ago, there have been hundreds of published measurements in τ physics and several hundred papers on theory and speculation in τ physics. I have chosen to emphasize three themes out of these many, many works: an overview of tau physics including future prospects; a general introduction to theory but without details or proofs; and a summary of present experimental knowledge of the properties of the τ and ν_τ. And since these lectures were informal, I shall give my opinion on two continuing issues in τ physics: the comparison of the leptonic branching fractions, B_e and B_μ, with the τ lifetime, T_τ; and the comparison of the sum of one-charged particle branching fractions, $\sum_i B_{1i}$, with the one-charged particle topological branching fraction, B_1.

I refer the reader to recent reviews of τ physics (Perl (1992), Pich (1990a, 1990b), Kiesling (1988), Barish and Stroynowski (1988), Gan and Perl (1988), Burchat (1988)), for details on the theory, on experimental results and on experimental techniques. Three volumes of proceedings are also useful: Proceedings of the Workshop on Tau Lepton Physics (Davier and Jean-Marie 1991), Proceedings of the Meeting on the Tau-Charm Factory Detector and Machine (Kirkby and Quesada 1992), and Proceedings of the Tau-Charm Factory Workshop (Beers, 1989).

In the lectures I use three kinds of world averages of τ branching fractions: my own averages which show the changes in these averages from 1990 to the present, those of Galik (1992), and those of the Particle Data Group (Aguilar-Benitez *et al.* 1992). The data compilation used by Aguilar-Benitez *et al.* (1992) does not include data presented in talks or published in middle or late 1992, but that data is used by Galik (1992) and by me. Hence the world averages of Aguilar-Benitez *et al.* are sometimes different from Galik's averages and my averages. On the other hand, the averages of Aguilar-Benitez *et al.* are most authoritative due to the detached work of Hayes (1992a).

I am very grateful to my old friends and colleagues, Kenneth Hayes of Hillsdale College and Keith Riles of the University of Michigan, for their papers and comments on τ branching ratios and average values (Riles 1992, and Hayes 1992a, 1992b). I am also very grateful to two other old friends, Michel Davier (Davier 1992) and Richard Galik (Galik 1992) for their extensive work on tau branching fractions.

In these talks I devoted part of one lecture to the tau-charm factory, a proposed high luminosity, two-ring, electron-positron collider and detector with the following properties:

- Range of total energy = 3.0 to 5.0 GeV ,
- Design luminosity $\geq 10^{33}$ cm^{-2} s^{-1} ,
- High resolution, large acceptance detector specially designed for tau and charm physics.

There is now an extensive literature on the design and physics potential of a tau-charm factory. The original descriptions are by Kirkby (1987, 1989) and Jowett (1987, 1988, 1989). Two international workshops have been devoted to the tau-charm factory (Beers 1989, Kirkby and Quesada 1992). Studies of the design of the collider have been done by Brown *et al.* (1989), Gonichon *et al.* (1990), Barish *et al.* (1990), Danilov *et al.* (1990) and Baconnier *et al.* (1990). Papers on the physics and on detector designs include Schindler (1989, 1990a, 1990b), Perl (1991), Wermes (1992), and Davier (1991). A recent review of the concept and potential physics of a tau-charm factory has been written by Kirkby and Rubio (1992).

I will not review in these written lectures the work on the tau-charm factory concept and physics. But I will, from time to time, point out the advantages of using experiments at a tau-charm factory to explore tau physics.

The quotation from Francis Bacon which heads this section describes the Standard Model of particle physics, a uniform and endless sea which seems to surround us. Perhaps the tau will provide the island, the new land, which will enable us to climb out of that sea.

B. τ PRODUCTION AND RELATED τ PROPERTIES

He had brought a large map representing the sea,
Without the least vestige of land:
And the crew were much pleased when they found it to be
A map they could all understand.
Lewis Carroll, *The Hunting of the Snail*

This quotation was suggested to me by Myron Bander. The map is the Standard Model, we can all understand it, but it does not tell us where to look for the land outside the Standard Model. To use the τ as a possible guide to that land we must make τ's and experiment with them. This section describes the way we have made τ's, through $e^+ + e^- \to \tau^+ + \tau^-$ and through particle decays, it also describes possible future methods.

B.1 $e^+ + e^- \to \tau^+ + \tau^-$

Figure 1 shows the six energy regions for τ pair production through

$$e^+ + e^- \to \tau^+ + \tau^- \quad . \tag{B.1}$$

B.1.a Threshold Region

At threshold the total pair production cross section is

$$\sigma_\tau = \frac{4\pi\alpha^2}{3s}\,\frac{\beta(3-\beta^2)}{2}\,F_c \tag{B.2a}$$

$$F_c = \frac{\pi\alpha/\beta}{1 - exp(-\pi\alpha/\beta)} \tag{B.2b}$$

where F_c is caused by the coulomb attraction between the τ^+ and τ^- as shown in Fig. 2 (Landau and Lifshitz 1958). At threshold $s = 4m_\tau^2$ and $\beta = 0$

$$\sigma_\tau\ (\text{threshold}) = \frac{\pi^2\alpha^3}{2m_\tau^2} = 0.23\ \text{nb} \tag{B.2c}$$

Khoze (1992) has pointed out that Eq. B.2 does not include the effect of initial state radiation, that is radiation of γ's by the e^+ and e^-. This will decrease σ_τ (threshold). Figure 3 shows the behavior of σ_τ near threshold ignoring the effect of initial state radiation.

The non-zero σ_τ at threshold is important for τ studies which will be done at a Tau-Charm Factory very close to threshold (Gomez-Cadenas *et al.* 1989).

The classic way to measure the τ mass, m_τ, is to find the threshold energy, $E_{threshold} = 2m_\tau$, using the first part of the cross section curve in Fig. 1. Until this year we used

$$m_\tau = 1784.3^{+2.7}_{-3.6}\ \text{MeV/c}^2$$

based mostly on the 1978 measurement by (Bacino *et al.* 1978) using the DELCO experiment. Just this year there was a new and more precise threshold measurement using the BEPC e^+e^- collider in Beijing (Bai *et al.* 1992). In this paper I use their value of

$$m_\tau = 1776.9^{+0.4}_{-0.5} \pm 0.2\ \text{MeV/c}^2 \tag{B.3a}$$

sometimes rounding it off to 1777 MeV/c^2 when I don't need the error.

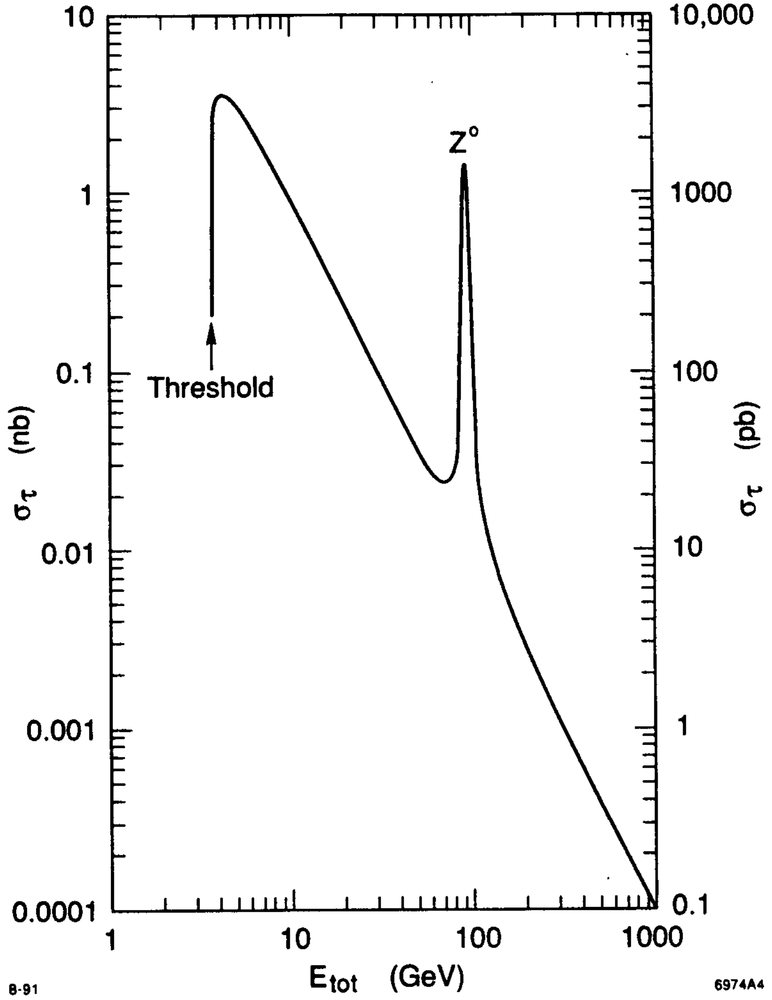

Figure 1.

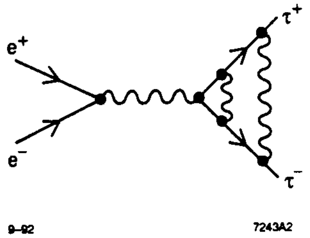

Figure 2.

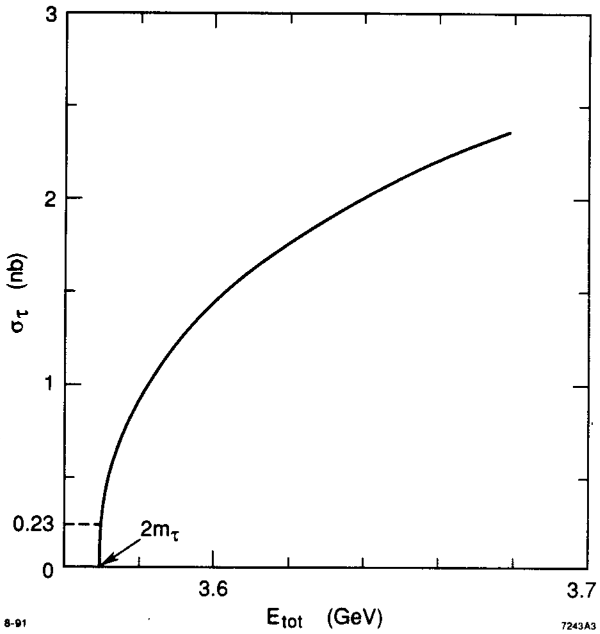

Figure 3.

There have been two other new measurements of m_τ. Albrecht *et al.* (1992a) used the spectrum of the invariant mass of the 3π's in

$$\tau^- \rightarrow \pi^- + \pi^+ + \pi^- + \nu_\tau$$

to find

$$m_\tau = 1776.3 \pm 2.4 \pm 1.4 \text{ MeV/c}^2 \ . \qquad (B.3b)$$

The CLEO experimenters as reported by Marsiske (1992) used the $\pi^- + n\pi^0$ invariant mass spectrum in

$$\tau^- \rightarrow \pi^- + n\pi^0 + \nu_\tau \ , \ 0 \leq n \leq 2$$

to find

$$m_\tau = 1777.6 \pm 0.9 \pm 1.5 \text{ MeV/c}^2 \qquad (B.3c)$$

Marsiske (1992) gives the average value for m_τ based on the three new measurement in Eq. B.3 as

$$m_\tau = 1777.1 \pm 0.5 \text{ MeV/c}^2 \ .$$

B.1.b Above Threshold to About 10 GeV

In this energy range τ pair production is dominated by the γ exchange diagram in Fig. 4a and

$$\sigma_\tau = \frac{4\pi\alpha^2}{3s}\,\frac{\beta(3-\beta^2)}{2} = \frac{86.8}{s}\,\frac{\beta(3-\beta^2)}{2}\ \text{nb} \qquad (B.4)$$

where s is in GeV^2 in the rightmost formula. This is the energy region where the τ was discovered and where a great many studies of τ physics have been carried out at the SPEAR, DORIS, CESR and BEPC e^+e^- colliders. This will continue to be an important region for τ studies at CESR, BEPC and DORIS II, and further along at Tau-Charm Factories and B-Factories. As shown in Fig. 1, σ_τ has its maximum value in this energy region.

B.1.c Above 10 GeV to Below Z^0 Resonance

In this energy region the Z^0 exchange amplitude, Fig. 4b, contributes through interference with the γ exchange amplitude. In the past this energy region provided a vast amount of data on the τ experiments at the PETRA, PEP, and TRISTAN e^+e^- colliders. At present only the TRISTAN collider is still operating.

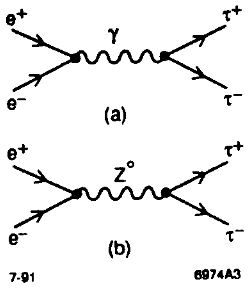

Figure 4.

The data on the total and differential cross sections, σ_τ and $d\sigma_\tau/d\Omega$, for

$$e^+ + e^- \rightarrow \tau^+ + \tau^- \quad , \qquad (B.5)$$

this energy region was used extensively for searches for deviations from the conventional theory for the process in Eq. B.5. As discussed in Perl (1992), two different models were used to parameterize deviations. One model, an old one (Feynman 1949, Drell 1958), allows for modifications of the photon propagator or $\tau - \gamma - \tau$ vertex in the diagram of Fig. 4a such as

$$\sigma_\tau \text{ (modified)} = \sigma_\tau \, F^2_\pm(s) \qquad (B.6a)$$

where

$$F_\pm(s) = 1 \mp \frac{s}{s - \Lambda^2_\pm} \quad . \qquad (B.6b)$$

The other newer model (Eichten, Lane and Peskin 1983) assumes that the τ and e are composite particles and introduces an effective Lagrangian for a contact interaction between the constituent particles. Thus for a vector-vector interaction

$$L_{eff} = \pm \frac{g^2}{2\Lambda^{c\,2}_\pm} \, \bar{\psi}_2 \, \gamma^\mu \, \psi_2 \, \bar{\psi}_1 \, \nu_\mu \, \psi_1 \qquad (B.7)$$

with $g^2/4\pi$ set equal to 1 to define Λ^c.

No deviations have been found, hence there are only lower limits on the parameters $\Lambda_\pm$ and $\Lambda^c_\pm$. Examples of 95% C.L. lower limits on $\Lambda_\pm$ and $\Lambda^c_\pm$ are given in Eq. B.8. The $\Lambda^c_\pm$ limits are for the vector-vector interaction in Eq. B.7.

Reference	Λ_+(GeV)	Λ_-(GeV)	Λ^c_+(TeV)	Λ^c_-(TeV)
Bartel *et al.* (1986)	285	210	4.1	5.7
Adeva *et al.* (1986)	235	205		
Behrend *et al.* (1989)	318	231		

. (B.8)

However, these deviation models give a false sense of the precision of such tests. Suppose there is a new particle χ^0 which contributes to $e^+ + e^- \rightarrow \tau^+ + \tau^-$ through the diagram in Fig. 5, and suppose the χ^0 mass is small or zero. This would not have been detected if it contributes less than about 5% to σ_τ or $d\sigma_\tau/d\Omega$. Since the contribution would be through interference with γ-exchange, the new process would not have been detected if

$$\frac{g_{ee\chi^0} \, g_{\tau\tau\chi^0}}{g_{ee\gamma} \, g_{\tau\tau\gamma}} \lesssim 5\% \quad . \qquad (B.9)$$

Of course there are constraints on $g_{ee\chi^0}$ from other studies of the $ee\gamma$ vertex.

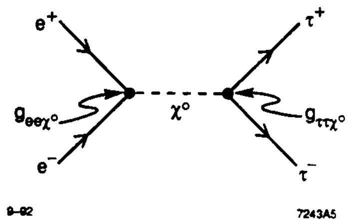

Figure 5.

B.1.d Z^0 *Resonance*

At the Z^0 resonance, Fig. 1, the dominant process is Z^0–exchange in Fig. 4b. Ignoring γ–exchange and radiative corrections, the resonance is given by

$$\sigma_\tau \approx \frac{\Gamma_{z\tau\tau}}{\Gamma_z} \frac{\Gamma_z^2/m_z^2}{[s/m_z^2 - 1]^2 + \Gamma_z^2/m_z^2} \sigma_z \, (s = m_z^2) \qquad (B.10)$$

using

$$\Gamma_{z\tau\tau}/\Gamma_z \approx 0.034 \ , \ \sigma_z(s = m_z^2) \approx 59 \text{ nb} \qquad (B.11)$$

$$\sigma_\tau \text{ (no rad. corr.,} \ \ s = m_z^2) \approx 2.0 \text{ nb} \qquad (B.12)$$

with radiative correction

$$\sigma_\tau(s = m_z^2) \approx 1.4 \text{ nb} \ \ . \qquad (B.13)$$

The four experiments at the LEP e^+e^- collider have provided and will continue to provide a large amount of data on the τ. For example, their studies of

$$e^+ + e^- \to Z^0 \to \tau^+ + \tau^- \qquad (B.14)$$

show that the $Z^0\tau^+\tau^-$ vertex obeys $e - \mu - \tau$ universality within experimental error, Table 1. In this table

$$R_\ell = \Gamma_{z\to hadrons}/\Gamma_{z\ell\ell} \ . \qquad (B.15)$$

Experiments at the SLC e^+e^- linear collider have and are also contributing to τ studies at the Z^0.

Table 1. Average LEP line shape parameters. Values for χ^2 of the weighted average are quoted for those parameters given directly by the four LEP experiments. From LEP Collaborations (1992).

Parameter	Average Value	χ^2
M_Z (GeV)	91.175 ± 0.021	3.4
Γ_Z (GeV)	2.487 ± 0.010	2.0
σ_h^0 (nb)	41.33 ± 0.23	2.3
R_e	20.91 ± 0.22	
R_μ	20.88 ± 0.18	
R_τ	21.02 ± 0.23	
Γ_e (MeV)	83.20 ± 0.55	1.0
Γ_μ (MeV)	83.35 ± 0.86	5.1
Γ_τ (MeV)	82.76 ± 1.02	0.3
$Br(Z^0 \to e^+e^-)$ (%)	3.345 ± 0.020	
$Br(Z^0 \to \mu^+\mu^-)$ (%)	3.351 ± 0.034	
$Br(Z^0 \to \tau^+\tau^-)$ (%)	3.328 ± 0.040	

B.1.e Above the Z^0 Resonance

Until this section, I have described energy regions which have been achieved and used for τ studies. In thinking about the energy region above the Z^0, Fig. 1, we must rely on the conventional theory of the processes in Fig. 4. If there are no higher mass resonances or other new physics in

$$e^+ + e^- \rightarrow \tau^+ + \tau^- \quad ,$$

then far above the Z^0 resonance

$$\sigma_\tau = \frac{4\pi\alpha^2}{3s}\,[1 + 0.14] \sim \frac{0.1}{s}\ \mathrm{pb} \qquad (B.16)$$

where s is in TeV^2. In the square bracket in Eq. B.16, the 1 is from γ–exchange and the 0.14 is for Z^0–exchange. Thus far alone the Z^0 resonance γ–exchange once again dominates.

If the cross section, σ_τ, is as small as the conventional theory predicts in Eq. B.16, then this energy range will not be useful for τ decay studies. This energy region will be useful to look for compositeness in the τ as in Eq. B.7 or to look for other new physics in the $e^+ + e^- \rightarrow \tau^+ + \tau^-$.

In the near future, the LEP e^+e^- collider will be increased in energy to about 200 GeV total energy and thus experiments will begin to enter this region. But linear e^+e^- colliders in the total energy range of 0.5 to 1 TeV offer the main future in this energy range.

B.1.f Some τ studies related to $e^+ + e^- \rightarrow \tau^+ + \tau^-$

In the course of studying $e^+ + e^- \rightarrow \tau^+ + \tau^-$ experimenters have looked in vain for non-conservation of the τ lepton number

$$e^+ + e^- \rightarrow \tau^\pm + e^\mp \qquad (B.17)$$

$$e^+ + e^- \rightarrow \tau^\pm + \mu^\mp \quad . \qquad (B.18)$$

At $\sqrt{s} = 29$ GeV Gomez-Cadenas *et al.* (1991) found the 95% confidence level upper limits

$$\sigma(e^+e^- \rightarrow \tau^\pm e^\mp)/\sigma_\tau \leq 1.2 \times 10^{-3}$$

$$\sigma(e^+e^- \rightarrow \tau^\pm \mu^\mp)/\sigma_\tau \leq 4.1 \times 10^{-3}$$

and at the Z^0 (Akrawy *et al.* 1991) the 95% confidence level upper limits are

$$B(Z^0 \rightarrow \tau^\pm e^\mp)/B(Z^0 \rightarrow \ell^+\ell^-) \leq 2.2 \times 10^{-3} \qquad (B.19)$$

$$B(Z^0 \rightarrow \tau^\pm \mu^\mp)/B(Z^0 \rightarrow \ell^+\ell^-) \leq 11. \times 10^{-3} \quad . \qquad (B.20)$$

Here $\ell = e,\ \mu$ or τ.

Since the beginning of $e^+ + e^- \rightarrow \tau^+ + \tau^-$ studies there have been searches for the hypothetical excited τ, τ^*, defined by

$$\tau^{*\pm} \rightarrow \tau^\pm + \gamma \qquad (B.21)$$

being the dominant decay. Searches at the Z^0 (Akrawy *et al.* 1990, Adeva *et al.* 1990, Decamp *et al.* 1990) provide the most stringent lower mass limits on m_{τ^*}. For the τ^* pair process

$$e^+ + e^- \rightarrow Z^0 \rightarrow \tau^{*+} + \tau^{*-} \rightarrow \tau^+ + \tau^- + \gamma + \gamma \qquad (B.22)$$

the lower limit on m_{τ^*} is

$$m_{\tau^*} \gtrsim 45\ \mathrm{GeV/c^2} \quad . \qquad (B.23)$$

The process

$$e^+ + e^- \rightarrow Z^0 \rightarrow \tau^{*\pm} + \tau^\mp \rightarrow \tau^+ + \tau^- + \gamma \qquad (B.24)$$

depends not only upon the existence of the τ^*, but also upon the strength of the $Z^0\tau^*\tau$ coupling. The searches using this process find

$$m_{\tau^*} \gtrsim 89\ \mathrm{GeV/c^2} \quad . \qquad (B.25)$$

B.2 Photoproduction: $\gamma + N \rightarrow \tau^+ + \tau^- + N'$

Tsai (1979) has discussed the photoproduction of τ pairs, Fig. 6a,

$$\gamma + N \rightarrow \tau^+ + \tau^- + N' \qquad (B.26)$$

where N is a target proton or nucleus and N' represents the final hadronic state. The behavior of the cross section, $\sigma_{\tau,photo}$, is sketched in Fig. 7. This method of producing τ's has not yet been used for experiments because it seems much more difficult to use than $e^+ + e^- \rightarrow \tau^+ + \tau^-$. However, it may have special uses, thus Tsai (1992a) has pointed out that it is a means of producing a ν_τ, $\bar{\nu}_\tau$ beam through decay of the τ's in Eq. B.26.

Incidentally, electroproduction, Fig. 6b,

$$e^- + N \rightarrow e^- + \tau^+ + \tau^- + N' \qquad (B.27)$$

might also be used.

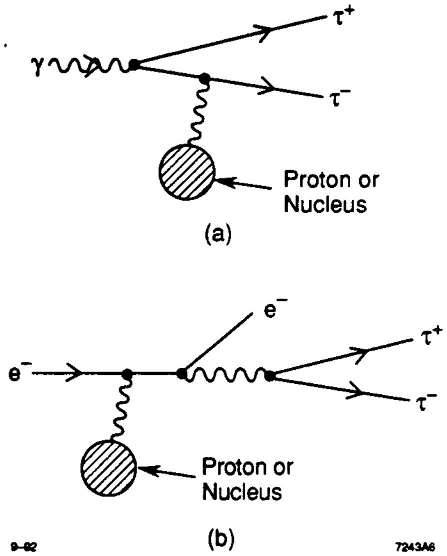

Figure 6.

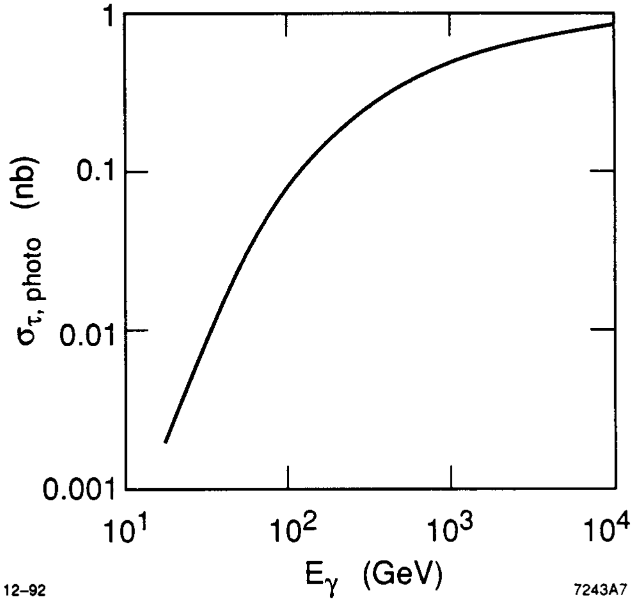

Figure 7.

B.3 Particle Decays to τ and ν_τ

B.3.a $W^+ \to \tau^+ + \nu_\tau$

The decays

$$W^+ \to \tau^+ + \nu_\tau \ , \ W^- \to \tau^- + \bar{\nu}_\tau \qquad (B.28)$$

have been used for two purposes. One purpose is to identify W's (Savoy-Navarro 1991).

The other purpose is the study of the $W\tau\nu_\tau$ vertex, Fig. 8, at $\sqrt{s} = m_W$. This is in contrast to the $W\tau\nu_\tau$ vertex in τ decays where $\sqrt{s} \leq m_\tau$. The basic question is whether the coupling constant g_τ at the $W\tau\nu_\tau$ vertex obeys e, μ, τ universality. Within the experimental errors, universality is obeyed as shown below:

g_τ/g_e	Reference	
0.97 ± 0.07	Abe *et al.* 1992	
1.01 ± 0.10	Albajar *et al.* 1987 .	$(B.29)$
1.00 ± 0.07	Alitti *et al.* 1991	

B.3.b D Decays to τ and ν_τ

None of the pure leptonic decays of the $D^\pm$ and $D_s^\pm$

$$\left.\begin{array}{l} D^+ \to \ell^+ + \nu_\ell \\ D_s^+ \to \ell^+ + \nu_\ell \end{array}\right\} \quad \ell = e,\ \mu,\ \tau \qquad (B.30)$$

have been observed, whether the ℓ is an e, a μ or a τ. The decay width is

$$\begin{aligned} \Gamma\left(D^+, D_s^+ \to \ell^+ \nu_\ell\right) &= \frac{G_F^2}{8\pi}\, f_{D,D_s}^2\, m_{D,D_s}\, m_\ell^2 \\ &\times |V_{cd,cs}|^2\, [1 - m_\ell^2/m_{D,D_s}^2]^2 \qquad (B.31) \end{aligned}$$

Here f_{D,D_s} are the so-called weak decay constants of the D and D_s and take into account the strong interaction dynamics of cd and cs annihilation inside the meson. Theory estimates their size to be 150 to 250 MeV, but they must be measured through these decay processes. The m_ℓ^2 term in Eq. B.31 leads to the τ mode having the largest Γ. Using

$$V_{cd} \approx 0.22 \ , \ V_{cs} \approx 0.97 \qquad (B.32)$$

$$f_D \approx 200 \text{ MeV} \ , \ f_{D_s} \approx 200 \text{ MeV} \qquad (B.33)$$

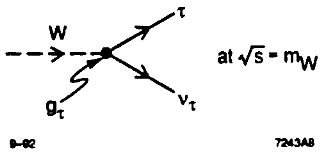

Figure 8.

I calculate the D, D_s branching fractions

$$B(D^+ \to \tau^+ \nu_\tau) \approx 0.8 \times 10^{-3} \qquad (B.34a)$$

$$B(D_s^+ \to \tau^+ \nu_\tau) \approx 3 \times 10^{-2} \ . \qquad (B.34b)$$

Thus the decays

$$D^+ \to \tau^+ + \nu_\tau \qquad (B.35a)$$

$$D_s^+ \to \tau^+ + \nu_\tau \qquad (B.35b)$$

provide the best way to measure f_D and f_{D_S}.

Tsai (1992b) has pointed out that the decay processes $D^+, D_s^+ \to \tau^+ \nu_\tau$ provide polarized τ's, and given enough such events the τ decays can be used for special studies of the $\tau - W - \nu_\tau$ vertex.

The decays

$$\begin{aligned} D_s^+ &\to \tau^+ + \nu_\tau \\ D_s^- &\to \tau^- + \bar{\nu}_\tau \end{aligned} \qquad (B.36)$$

are crucial for fixed target production of ν_τ and $\bar{\nu}_\tau$ beams through the sequence

$$p + N \to D_s + \ldots \qquad (B.37)$$

$$\begin{array}{ll} D_s^+ \to \tau^+ + \nu_\tau \ , & D_s^- \to \tau^- + \bar{\nu}_\tau \\ \quad\ \llcorner\!\!\to \bar{\nu}_\tau \ldots & \quad\ \llcorner\!\!\to \nu_\tau + \ldots \end{array} \qquad (B.38)$$

where N is a nucleon or nucleus (Sec.G.7).

Since

$$m_D - m_\tau = 92 \ \mathrm{MeV/c^2} \qquad (B.39)$$

there are no semi-leptonic decays of the D to τ. But, the larger mass of the D_s allows the semileptonic decay

$$D_s^+ \to \tau^+ + \nu_\tau + \pi^0 \quad , \qquad (B.40)$$

not yet observed.

B.3.c B Decays to τ and ν_τ

The theory of leptonic decays of the B mesons

$$\begin{aligned} B^+ &\to \tau^+ + \nu_\tau \\ B_c^+ &\to \tau^+ + \nu_\tau \end{aligned} \qquad (B.41)$$

is analogous to that for D decays, but the smaller values of V_{ub} and V_{cb} reduce the decay widths.

The semileptonic decays of B to τ have substantial widths due to the large $m_B - m_\tau$ difference. References and some details are given in Sec. 4.3 of Perl (1992). The total semileptonic branching fraction is

$$B(B \to \tau + \nu_\tau + \ldots) = 0.04 \qquad (B.42)$$

as measured by Buskulic *et al.* (1992).

B.4 τ Production in Hadron Collisions

B.4.a τ Production in $p + N$ Collisions

As described in connection with Eqs. B.37 and B.38, τ's can be produced in $p + N$ collisions where N is a number or nucleus. In addition to the route through D_S production and leptonic decay, there is the route through B production and semi-leptonic decay

$$B \to \tau^+ + \nu_\tau + \ldots \qquad (B.43)$$

The $p + N$ collisions can be from an external proton beam on a fixed target, from a circulating proton beam on a gas jet target, or from a proton-proton collider.

Excluding the production of ν_τ, $\bar{\nu}_\tau$ beams, I have not seen any arguments for studying τ physics this way rather than through $e^+ + e^- \to \tau^+ + \tau^-$ production; there are tremendous background problems when τ's are produced through hadron collisions. But there may be special uses.

B.4.b $\tau^+\tau^-$ Production in Heavy Ion Collisions

Figure 9 shows how the virtual photons emitted in the collision of a pair of heavy ions can produce a $\tau^+\tau^-$ pair when the ions are at energies much greater than the τ mass.

$$\mathrm{Ion} + \mathrm{Ion} \to \tau^+ + \tau^- + \ldots \qquad (B.44)$$

At 100 GeV/nucleon for Au Bottcher and Strayer (1990) find $\sigma \approx 3 \ \mu b$. The Pb + Pb case has been discussed by del Aguila *et al.* (1991) who emphasize that the production cross section depends on the two $\tau\gamma\tau$ vertices in Eq. B.44.

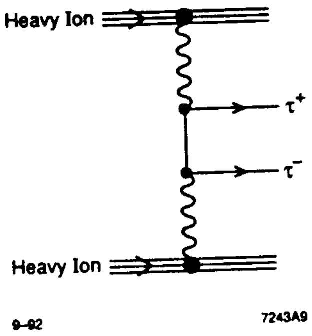

Figure 9.

As with τ production in $p+N$ collisions, there are huge backgrounds to τ production by heavy ions. Amaglobeli *et al.* (1991) have pointed out a possible use for the large rate of τ production; look for the unconventional decay

$$\tau^- \rightarrow \mu^- + \mu^+ + \mu^- \tag{B.45}$$

which violates the conservation of lepton number (Sec. C.4.a). It might be possible to pick τ's out of the background since the invariant mass of the three μ's in Eq. B.45 must equal the τ mass.

I urge the reader to be open minded about the prosecution of τ physics through $p+N$ or Ion + Ion collisions; the exclusive use of $e^+ + e^- \rightarrow \tau^+ + \tau^-$ in the past may have blinded us to seeing the future.

C. GENERAL DISCUSSION OF τ DECAYS

Dans les champs de l'observation le hasard ne
favorise que les esprits préparés.
Where observation is concerned, chance favours
only the prepared mind.
Louis Pasteur

C.1 Overview of τ Decay

The conventional theory of τ decays is that they occur through the process, Fig. 10,

$$\begin{aligned} &\tau^- \rightarrow \nu_\tau + W^-_{virtual} \\ &W^-_{virtual} \rightarrow \text{final particles} \end{aligned} \tag{C.1}$$

with lepton number separately conserved at each vertex. With the possible exception of the comparison of $\sum_i B_{1i}$ with B_1, the one-charged particle decay modes problem discussed in Sec. E.5, all experimental results in τ physics are compatible with this conventional theory.

C.2 Overview of Branching Fractions

Table 2 gives an overview of present knowledge of the major decay branching fractions and some other branching fractions of the τ. The particle category

$$\begin{aligned} h^- &= \pi^- \text{ or } K^- \\ h^+ &= \pi^+ \text{ or } K^+ \end{aligned} \tag{C.2}$$

is discussed in Sec. E.1.

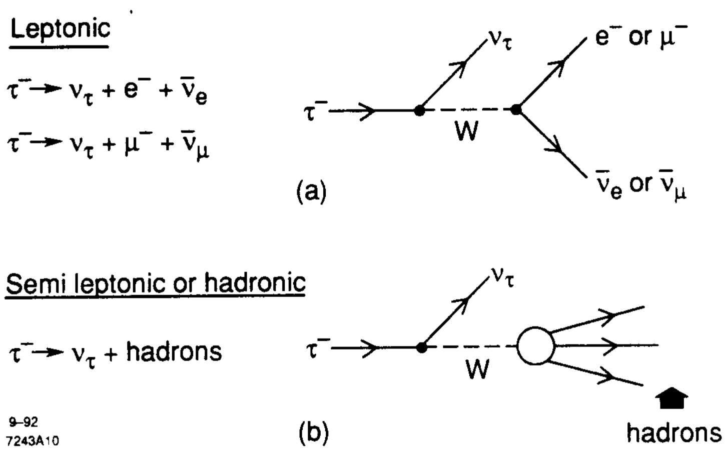

Figure 10.

Table 2. Branching fractions for major τ decay modes and two modes containing K's. The former are taken from or deduced from Galik (1992). The latter are from Aguilar-Benitez *et al.* (1992). The symbol h^- means π^- or k^-.

Mode	Branching Fraction in %
$e^-\bar{\nu}_e\nu_\tau$	17.75 ± 0.15
$\mu^-\bar{\nu}_\mu\nu_\tau$	17.39 ± 0.17
$\pi^-\nu_\tau$	11.73 ± 0.35
$\rho^-\nu_\tau$	23.82 ± 0.25
$h^-2\pi^0\nu_\tau$	8.76 ± 0.33
$2h^-h^+\nu_\tau$	8.62 ± 0.19
$2h^-h^+ \geq 1\pi^0$	5.45 ± 0.22
$K^-\nu_\tau$	0.7 ± 0.2
K^{*-} $(892)\nu_\tau$	1.4 ± 0.2

Table 3. Topological branching fractions of the τ. B_1, B_3, and B_5 are from Galik (1992); B_7 is from Aguilar-Benitez et al. (1992).

Mode	Branching Fraction in %
B_1	85.26 ± 0.18
B_3	14.63 ± 0.18
B_5	0.13 ± 0.03
B_7	< 0.019

Some remarks. The large branching fraction modes are the leptonic modes, the modes with one π or three π's, and the ρ mode. The relatively small mass of the τ favors these modes over modes with more pions or more massive resonances. The modes with a K or K^* (890) are suppressed by the Cabibbo factor $\sin^2\theta_c = 0.049$ relative to the corresponding π or ρ mode.

C.3 Topological Branching Fractions

Although they have no precise physical significance the topological branching fractions in Table 3 are important in the methods for selecting and studying τ events produced in e^+e^- annihilation, as described in Sec. 5.2 of Perl (1992). The notation B_n means n charged particles are produced directly in the τ decay. For example,

$$\tau^- \rightarrow \pi^- + K^0 + \nu_\tau \qquad (C.3)$$

with subsequent decay of the $K^0 \rightarrow \pi^+ + \pi^-$ is counted as a one-charged particle decay.

Table 4. B_1 in %. The errors are ±.

WORLD AVERAGE VALUES

B	ERROR	WORLD AVERAGE
86.13	0.33	1990 Particle Data Group Average
84.97	0.22	1990-1992 Average
85.33	0.18	My 1992 Average
85.94	0.23	1992 Particle Data Group Average
85.26	0.18	Galik(1992) Average

INDIVIDUAL 1990-1992 EXPERIMENTS

B	ERROR	WEIGHT	EXPER	REFERENCE
85.09	0.37	0.36	ALEPH	M. Davier; Proc. Second Workshop Tau Lepton Phys.;1992
84.08	0.74	0.09	DELPHI	P. Vaz; Proc. Second Workshop Tau Lepton Phys.;1992
85.60	0.67	0.11	L3	B. Adeva Phys. Lett. B265;451;1991
84.59	0.36	0.37	OPAL	J. Banks; Proc. Second Workshop Tau Lepton Phys.;1992
86.60	0.85	0.07	ARGUS	H. Albrecht et al. Z. Phys. C53;367;1992

Table 4 shows the changes in B_1 between the 1990 Particle Data Group values (Aguilar-Benitez *et al.* 1990) and my 1992 average values. In this table and all analogous later tables I show the new measurements published since 1990, average those new measurements, and then combine them with the 1990 values to get my 1992 values. The averages are done by weighting each value by the inverse of the square of the associated error. The tables also give the 1992 values of the Particle Data Group (Aguilar-Benitez *et al.* 1992) and of Galik (1992).

C.4 Unconventional Decays

C.4.a No ν_τ

The class of unconventional τ decay modes usually discussed has no ν_τ in the mode, the particles occurring in the mode all being conventional. Examples of such hypothetical modes are

$$\begin{aligned} &\tau^- \rightarrow e^- + \gamma \\ &\tau^- \rightarrow \mu^- + \gamma \\ &\tau^- \rightarrow e^- + \pi^0 \\ &\tau^- \rightarrow e^+ + e^- + e^+ \\ &\tau^- \rightarrow \mu^+ + \mu^- + \mu^+ \quad . \end{aligned} \qquad (C.4)$$

If such modes exist they violate τ lepton number conservation and either e or μ lepton number conservation. If $\bar{p}$ is substituted for an e or μ, the hypothetical modes

$$\begin{aligned} &\tau^- \rightarrow \bar{p} + \gamma \\ &\tau^- \rightarrow \bar{p} + \pi^0 \end{aligned} \qquad (C.5)$$

violate τ lepton number conservation and baryon number conservation. If one wants to test just τ lepton number conservations then the non-conservation of total spin must be allowed; for example,

$$\begin{aligned} &\tau^- \rightarrow \pi^- + \gamma \\ &\tau^- \rightarrow \pi^- + \pi^0 \quad . \end{aligned} \qquad (C.6)$$

None of these no ν_τ modes have been found, the upper limits on the branching fractions are given in Table 5. Incidentally, it is easy to look for these modes since all the particles in the final state can be detected and their invariant mass must equal the τ mass

$$\left[\left(\sum_n E_n\right)^2 - \left(\sum_n \vec{p}_n\right)^2\right]^{\frac{1}{2}} = m_\tau \quad . \qquad (C.7)$$

Here the sum is over all the particles in the final state.

Table 5. Upper limits on unconventional branching fractions from Albrecht *et al.* (1992b).

		Upper Limits $[10^{-5}]$ (90% CL)				
Nr.	Decay Channel	MARK II	ARGUS 86	Crystal Ball	CLEO	ARGUS 91
1.	$\tau^- \rightarrow e^- e^+ e^-$	40	3.8		2.7	1.3
2.	$\tau^- \rightarrow e^- \mu^+ \mu^-$	33	3.3		2.7	1.9
3.	$\tau^- \rightarrow e^+ \mu^- \mu^-$				1.6	1.8
4.	$\tau^- \rightarrow \mu^- e^+ e^-$	44	3.3		2.7	1.4
5.	$\tau^- \rightarrow \mu^+ e^- e^-$				1.6	1.4
6.	$\tau^- \rightarrow \mu^- \mu^+ \mu^-$	49	2.9		1.7	1.9
7.	$\tau^- \rightarrow e^- \pi^+ \pi^-$		4.2		6.0	2.7
8.	$\tau^- \rightarrow e^+ \pi^- \pi^-$				1.7	1.8
9.	$\tau^- \rightarrow \mu^- \pi^+ \pi^-$		4.0		3.9	3.6
10.	$\tau^- \rightarrow \mu^+ \pi^- \pi^-$				3.9	6.3
11.	$\tau^- \rightarrow e^- \rho^0$	37	3.9			1.9
12.	$\tau^- \rightarrow \mu^- \rho^0$	44	3.8			2.9
13.	$\tau^- \rightarrow e^- \pi^+ K^-$		4.2		5.8	2.9
14.	$\tau^- \rightarrow e^+ \pi^- K^-$				4.9	2.0
15.	$\tau^- \rightarrow \mu^- \pi^+ K^-$		12		7.7	11
16.	$\tau^- \rightarrow \mu^+ \pi^- K^-$				4.0	5.8
17.	$\tau^- \rightarrow e^- K^{*0}$	130	5.4			3.8
18.	$\tau^- \rightarrow \mu^- K^{*0}$	100	5.9			4.5
19.	$\tau^- \rightarrow e^- \gamma$	64		20		12
20.	$\tau^- \rightarrow e^- \pi^0$	210		14		17
21.	$\tau^- \rightarrow \mu^- \gamma$	55				3.4
22.	$\tau^- \rightarrow \mu^- \pi^0$	82				4.4
23.	$\tau^- \rightarrow e^- \eta$			24		6.3
24.	$\tau^- \rightarrow \mu^- \eta$					7.3
25.	$\tau^- \rightarrow \bar{p} \gamma$					29.0
26.	$\tau^- \rightarrow \bar{p} \pi^0$					65.5
27.	$\tau^- \rightarrow \pi^- \gamma$					28
28.	$\tau^- \rightarrow \pi^- \pi^0$					37
29.	$\tau^- \rightarrow \bar{p} \eta$					129

The attainable lower limits on the branching fractions for these modes are set by the number of identified τ pairs in a data sample and by misidentification of normal τ decays. Misidentification of normal τ decays as no ν_τ modes will occur if the neutrinos carry off so little energy that Eq. C.7 is satisfied within experimental error. Thus the radiative decay

$$\tau^- \rightarrow e^- + \gamma + \nu_\tau + \bar{\nu}_e \qquad (C.8a)$$

could be misidentified as

$$\tau^- \rightarrow e^- + \gamma \ . \qquad (C.8b)$$

C.4.b With X^0

A class of unconventional decays which is much more difficult to study supposes that there is a small mass, weakly interacting boson X^0 which allows lepton number violation between τ and e or τ and μ. Then the unconventional modes

$$\begin{aligned} \tau^- &\rightarrow e^- + X^0 \\ \tau^- &\rightarrow \mu^- + X^0 \end{aligned} \qquad (C.9)$$

and perhaps the modes

$$\begin{aligned} \tau^- &\rightarrow e^- + \text{hadrons } + X^0 \\ \tau^- &\rightarrow \mu^- + \text{hadrons } + X^0 \end{aligned} \qquad (C.10)$$

could occur.

Such modes are very difficult to find because, unlike the modes in Eq. C.4, the τ mass cannot be reconstructed. The problem of misidentification of normal modes is severe. For example, an event

$$\tau \rightarrow e^- + X^0 \qquad (C.11a)$$

might actually be

$$\tau \rightarrow \pi^- + \nu_\tau \qquad (C.11b)$$

where the π^- is misidentified as an e^-. Or, it might actually be

$$\tau^- \rightarrow e^- + \bar{\nu}_e + \nu_\tau \qquad (C.11c)$$

with $\bar{\nu}_e + \nu_\tau$ taken as a single particle X^0. The only search for this class which has been made (Baltrusaitis *et al.* 1985) was for

$$\begin{aligned} \tau^- &\rightarrow e^- + G \\ \tau^- &\rightarrow \mu^- + G \end{aligned} \qquad (C.12)$$

where G is a Goldstone boson.

A tau-charm factory operated near the τ pair threshold is necessary for searches in this class.

C.4.c Non−W Exchange

A third class of unconventional decays involves non−W exchange. For example, in Fig. 11 an unknown particle, U, which couples only to leptons is involved in τ leptonic decays. As discussed by Tsai (1989a, 1989b) U might be a special kind of Higgs particle. The presence of this type of unconventional decay process cannot be detected by the presence of an unconventional decay mode, it can only be detected by a change in the properties of a conventional decay mode, for example, by a deviation in the kinematic distributions from those predicted by conventional theory.

D. LEPTONIC DECAYS

> *The aim of science is to seek the simplest explanation of complex facts. We are apt to fall into the error of thinking that the facts are simple because simplicity is the goal of our quest. The guiding motto in the life of every natural philosopher should be "Seek simplicity and distrust it."*
> *Alfred North Whitehead*

D.1 Overview of Decay Widths and Branching Fractions

I begin with some notations and definitions:

Mode	Decay Width	Branching Fraction	
$\tau^- \rightarrow e^- + \bar{\nu}_e + \nu_\tau$	Γ_e	B_e	
$\tau^- \rightarrow \mu^- + \bar{\nu}_\mu + \nu_\tau$	Γ_μ	B_μ	$(D.1)$
$\tau^- \rightarrow$ hadrons $+ \nu_\tau$	Γ_{had}	B_{had}	

The total width is

$$\Gamma = \Gamma_e + \Gamma_\mu + \Gamma_{had} \tag{D.2}$$

and

$$B_e = \Gamma_e/\Gamma \ , \ B_\mu = \Gamma_\mu/\Gamma \ , \ B_{had} = \Gamma_{had}/\Gamma \ . \tag{D.3}$$

At present we can precisely calculate Γ_e and Γ_μ (Sec. D.3) but there is no way to precisely calculate Γ_{had}, hence at present we cannot calculate precisely any B_i. However, as described in Sec. E, from theory and other data we can calculate

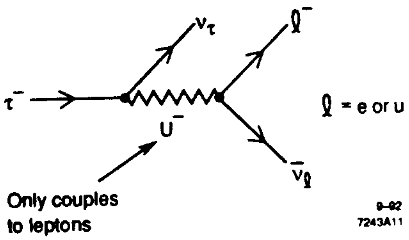

Figure 11.

precise decay widths for some hadronic modes

$$\begin{aligned} \tau^- &\to \pi^- + \nu_\tau \ : \ \Gamma_\pi \\ \tau^- &\to K^- + \nu_\tau \ : \ \Gamma_K \\ \tau^- &\to \rho^- + \nu_\tau \ : \ \Gamma_\rho \ . \end{aligned} \qquad (D.4)$$

Since

$$B_i/B_j = \Gamma_i/\Gamma_j \qquad (D.5)$$

we can predict precise value for ratios of branching fractions such as:

$$B_\mu/B_e, \ B_\pi/B_e, \ B_K/B_\pi, \ B_\rho/B_e \ . \qquad (D.6)$$

D.2 Crude Calculation of B_e, B_μ, and B_{had}

A crude calculation of B_e, B_μ, and B_{had} can be made using the diagram in Fig. 12, setting to 0 the masses of the e, μ and all quarks, taking all ν masses as 0, and ignoring the effects of the strong interaction on the conversion of quarks to hadrons. Then

$$B_e = B_\mu = \frac{1}{5} = 20\% \qquad (D.7)$$

$$B_{had} = 1 - B_e - B_\mu = 60\% \ . \qquad (D.8)$$

I have ignored the Cabibbo-suppressed channel $\bar{u} + s$.

It is surprising that these crude calculations give B's close to present average measured values, Table 2:

$$B_e = (17.8 \pm 0.2)\% \qquad (D.9)$$

$$B_\mu = (17.4 \pm 0.2)\% \qquad (D.10)$$

$$B_{had} = (64.8 \pm 0.3)\% \ . \qquad (D.11)$$

Surprising, because this calculation uses quark counting in an energy region where half of Γ_{had} is due to two resonances, the π and the ρ.

It is instructive to carry out the same calculation for the decay of a real W. Then there are the additional decay channels

$$\begin{aligned} \tau, \ \bar{\nu}_\tau &\to 1 \\ \bar{c}, s &\to 3 \end{aligned} \qquad (D.12)$$

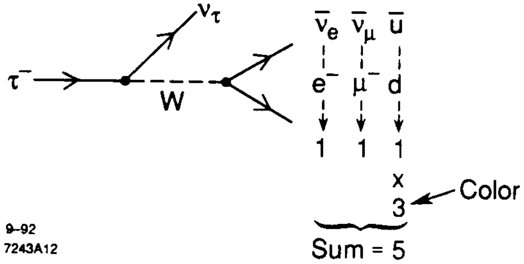

Figure 12.

and

$$B_e = B_\mu = B_\mu = \frac{1}{9} = 11\% \qquad (D.13)$$
$$\Gamma_{had} = 1 - B_e - B_\mu - B_\tau = 67\% \ .$$

D.3 Precise Calculation of Γ_e and Γ_μ

$$\Gamma_\ell = \frac{G_F^2 m_\tau^5}{192\pi^3} F_\ell(y)\, F_W\, F_{rad} \qquad (D.14)$$

where

$$G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2} \ , \qquad (D.15)$$
$$m_\tau = 1776.9 \pm 0.5 \ . \qquad (D.16)$$

The function

$$F_\ell(y) = 1 - 8y + 8y^3 - y^4 - 12y^2 \ell n y \qquad (D.17a)$$

is the correction for non-zero ℓ mass (Tsai 1971) and

$$y = m_\ell^2/m_\tau^2 \ . \qquad (D.17b)$$

Specifically

$$F_e = 1.000 \ , \ F_\mu = 0.973 \ . \qquad (D.17c)$$

Furthermore, in Eq. D.14

$$F_W = 1 + \frac{3}{5}\frac{m_\tau^2}{m_{W^2}} = 1.0003 \qquad (D.18)$$

is the correction for m_W being finite, and

$$F_{rad} = 1 - \frac{\alpha_\tau}{2\pi}\left(\pi^2 - \frac{25}{4}\right) = 0.9957 \qquad (D.19)$$

is the electromagnetic radiative correction (Marciano and Sirlin 1988).

The Γ_ℓ in Eq. D.14 includes the basic decay

$$\tau^- \to \ell^- + \bar{\nu}_\ell + \nu_\tau \ , \qquad (D.20a)$$

the radiative decay into γ's

$$\tau^- \to \ell^- + \bar{\nu}_\ell + \nu_\tau + n\gamma \ , \ n \geq 1 \ , \qquad (D.20b)$$

and the radiative decay into e^+e^- pairs

$$\tau^- \to \ell^- + \bar{\nu}_\ell + \nu_\tau + e^+ + e^- \qquad (D.20c)$$

(Sec. D.4).

Using Eqs. D.14-D.19, conventional theory predicts

$$\Gamma_e = 4.029 \times 10^{-13}\ \mathrm{GeV} \qquad (D.21)$$
$$\Gamma_\mu = 3.920 \times 10^{-13}\ \mathrm{GeV} \ .$$

The fraction error is $\pm 1.4 \times 10^{-3}$ due to the uncertainty in m_τ in D.16.

D.4 Aside on Radiative Decays

Figure 13 shows the processes which lead to a radiative leptonic decay with one γ, the dominant process is radiation from the e or μ since these have the smallest masses (Wu 1990a, Marciano and Sirlin 1988). From the work of Kinoshita and Sirlin (1959) on radiative decay of the muon, for photons with energy

$$E_\gamma \lesssim m_\tau/2$$
$$\frac{d\Gamma(\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma)}{dy} \approx \Gamma(\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau) \times \frac{1-y}{y}\left[\frac{\alpha}{\pi}\left(2\,\ell n \frac{m_\tau}{m_\ell} - \frac{17}{6}\right)\right] \qquad (D.22)$$

where

$$y = 2\,E_\gamma/m_\tau \ . \qquad (D.23)$$

The factor in the square bracket is 0.031 for $\ell = e$ and 0.0065 for $\ell = \mu$.

Returning to Eq. D.20 recall that Eq. D.14 gives the total width for all these processes. If we make the Γ for the radiative decay in Eq. D.20b larger by going to smaller y in Eq. D.22, then the Γ for the non-radiative decay in Eq. D.20a becomes smaller.

There are only two studies of τ radiative decays, Wu *et al.* (1990b) measured

$$\tau^- \to \mu^- + \bar{\nu}_\mu + \nu_\tau + \gamma \ ; \qquad (D.24)$$

and the CLEO experimenters (Mistry 1992) indirectly studied

$$\tau^- \to e^- + \bar{\nu}_e + \nu_\tau + \gamma \ . \qquad (D.25)$$

A great deal of work remains to be done on τ radiative decays, not only to test conventional theory, but also to explore hadronic radiative decays such as

$$\tau^- \to \pi^- + \nu_\tau + \gamma \qquad (D.26)$$
$$\tau^- \to \rho^- + \nu_\tau + \gamma \ . \qquad (D.27)$$

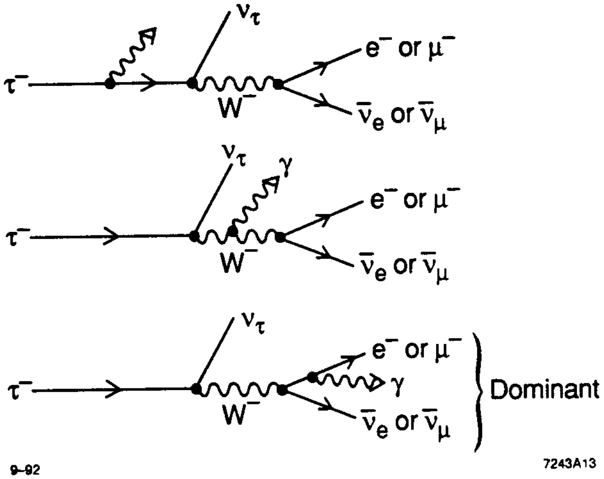

Figure 13.

D.5 Comparison of B_e, B_μ, and T_τ Measurements

We expect

$$B_\mu / B_e = 0.973 \tag{D.28}$$

from Eq. D.17c. Using the average values of Table 2

$$\begin{aligned} B_\mu &= (17.39 \pm 0.17)\% \\ B_e &= (17.75 \pm 0.15)\% \end{aligned} \tag{D.29}$$

gives

$$B_\mu / B_e \,(\text{measured}) = 0.980 \pm 0.013 \tag{D.30}$$

which agrees with Eq. D.28.

Table 6. $B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$ in %. The errors are ±.

WORLD AVERAGE VALUES

B	ERROR	WORLD AVERAGE
17.90	0.40	1990 Particle Data Group Average
17.77	0.16	1990-1992 Average
17.79	0.15	My 1992 Average
17.85	0.29	1992 Particle Data Group Average
17.75	0.15	Galik (1992) Average

INDIVIDUAL 1990-1992 EXPERIMENTS

B	ERROR	WEIGHT	EXPER.	REFERENCE
18.20	0.35	0.22	ALEPH	S. Snow; Proc. Second Workshop Tau Lepton Phys.;1992
18.60	1.00	0.03	DELPHI	P. Abreu et al.; CERN-PPe/92-060;1992
17.62	0.69	0.06	DELPHI	P. Vaz; Proc. Second Workshop Tau Lepton Phys.;1992
17.90	0.57	0.08	L3	N. Colino; Proc. Second Workshop Tau Lepton Phys.;1992
17.50	0.42	0.15	OPAL	J. Hobbs; Proc. Second Workshop Tau Lepton Phys.;1992
17.30	0.64	0.06	ARGUS	H. Albrecht et al. Z. Phys. C53;367;1992
19.20	0.72	0.05	CLEO	R. Ammar et al. Phys. Rev. D45;3976;1992
17.42	0.27	0.35	CLEO	N. Mistry; Proc. Second Workshop Tau Lepton Phys.;1992

Table 7. $B(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)$ in %. The errors are ±.

WORLD AVERAGE VALUES

B	ERROR	WORLD AVERAGE
17.80	0.40	1990 Particle Data Group Average
17.76	0.20	1990-1992 Average
17.77	0.18	My 1992 Average
17.45	0.27	1992 Particle Data Group Average
17.39	0.17	Galik (1992)

INDIVIDUAL 1990-1992 EXPERIMENTS

B	ERROR	WEIGHT	EXPER	REFERENCE
18.61	0.33	0.38	ALEPH	S. Snow; Proc. Second Workshop Tau Lepton Phys.;1992
17.40	0.92	0.05	DELPHI	P. Abreu et al.; CERN-PPe/92-060;1992
17.73	0.62	0.11	DELPHI	P. Vaz; Proc. Second Workshop Tau Lepton Phys.;1992
17.60	0.57	0.13	L3	N. Colino; Proc. Second Workshop Tau Lepton Phys.;1992
16.80	0.42	0.23	OPAL	J. Hobbs; Proc. Second Workshop Tau Lepton Phys.;1992
17.20	0.64	0.10	ARGUS	H. Albrecht et al. Z. Phys. C53;367;1992

It is interesting to look at the measured values of B_e and B_μ published in the last two years, Tables 6 and 7, to get a feeling for the individual measurements and their spread. My world averages are:

$$\begin{aligned} B_\mu &= (17.77 \pm 0.18)\% \\ B_e &= (17.79 \pm 0.15)\% \ , \end{aligned} \qquad (D.31)$$

slightly different from those of Galik (1992) in Eq. D.29 and indicating the type of uncertainties in world average value calculations.

Conventional theory predicts

$$\begin{aligned} T_\tau &= \hbar B_e/\Gamma_e \\ T_\tau &= \hbar B_\mu/\Gamma_\mu \end{aligned} \qquad (D.32)$$

where Γ_e and Γ_μ have been calculated as in Eq. D.21 while B_e and B_μ must be measured. Using the values in Eq. D.29

$$\begin{aligned} T_\tau \text{ (from } B_e) &= 290.0 \pm 2.4 \text{ fs} \\ T_\tau \text{ (from } B_\mu) &= 291.9 \pm 2.9 \text{ fs} \end{aligned} \qquad (D.33)$$

I remind you that $1\,\text{fs} = 10^{-15}$ s.

Now I compare these values with directly measured values of T_τ. A recent compilation by Trischuk (1992) gives

$$T_\tau = 295.7 \pm 3.2 \text{ fs} \ . \qquad (D.34)$$

The average measured value of T_τ in Eq. D.34 is larger than the predicted values in Eq. D.33 by about 1.5 standard deviations. This type of difference, the measured lifetime being larger than the predicted lifetime, has been present for years in τ physics; but the difference has never had strong statistical significance and still doesn't. If the difference is taken as real, the usual speculation is that G_F in Eq. D.14 is smaller for τ decays than the universal constant G_F given in Eq. D.15. Sometimes this decrease in G_F is obtained by assuming the existence of a fourth neutrino, ν_4, which couples to the τ but has $m_{\nu_4} > m_\tau$. Then in Eq. D.14 G_F^2 is replaced by $G_F^2 \cos^2\theta_\tau$. At this time it is impossible to know if the $T_\tau\,(\text{measured}) - T_\tau\,(\text{predicted})$ difference is significant or is due to an experimental problem in measuring T_τ.

D.6 Momentum Spectra in Leptonic Decays

There is a great deal that can be learned about τ decays from modes which have three or more particles: momentum spectra, angular distributions, polarization information. See, for example, Secs. 6.5 and 9.7 in Perl (1992) for references. Here I give the simplest example, the momentum spectrum of the electron in

$$\tau^- \to e^- + \bar{\nu}_e + \nu_\tau \ .$$

If we suppose the $\tau W \nu_\tau$ vertex in Fig. 10a is not exactly V-A we can look for new physics in the matrix element

$$M = \frac{G}{\sqrt{2}} \left[\bar{u}_e\, \gamma^\mu (1-\gamma_5) v_{\bar{\nu}_e}\right] \times \left[\bar{u}_{\nu_\tau}\, \gamma_u\, (v_\tau + a_\tau \gamma_5)\, u_\tau\right] \ . \qquad (D.35)$$

Then defining

$$x = 2E_e/m_\tau \qquad (D.36)$$

and setting

$$m_e = m_{\nu_e} = m_{\nu_\tau} = 0 \qquad (D.37)$$

the momentum spectrum in the τ rest frame is given by

$$\frac{d\Gamma_e}{\Gamma_e dx} = \left[12(x^2 - x^3)\right] + \left[\frac{8\rho_\tau}{3}\,(4x^3 - 3x^2)\right] \qquad (D.38a)$$

$$\rho_\tau = \frac{3}{4}\,\frac{(v_\tau - a_\tau)^2}{(v_\tau - a_\tau)^2 + (v_\tau + a_\tau)^2} \ . \qquad (D.38b)$$

In the Standard Model for τ, $v_\tau = +1$, $a_\tau = -1$ and

$$\rho_\tau\,(\text{Standard Model}) = \frac{3}{4} \ . \qquad (D.39)$$

From measurements (Aguilar-Benitez *et al.* 1992)

$$\rho_\tau\,(\text{measured}) = 0.727 \pm 0.033 \qquad (D.40)$$

which agrees with Eq. D.39. The error in Eq. D.40 is about 5%. For the μ, ρ has been measured more precisely

$$\rho_\mu = 0.7518 \pm 0.0026 \ , \qquad (D.41)$$

an error of about 0.4%. We would certainly like to measure ρ_τ as precisely.

To see how this can be done return to Eq. D.38a and call the second square bracket factor the ρ part of the momentum spectrum. Figure 14a shows $d\Gamma_e/\Gamma_e dx$ and the contribution of the ρ part when the τ is at rest in the laboratory frame, Fig. 14b shows the same quantities when the τ has high energy in the laboratory frame, $E_\tau \gg m_\tau$. The ρ part contributes most and is most precisely measured in Fig. 14a, the rest system. By studying $\tau^- \to e^- + \bar{\nu}_e + \nu_\tau$ and also $\tau^- \to \mu^- + \bar{\nu}_\mu + \nu_\tau$ at an energy close to the τ pair threshold, the τ is close to rest in the laboratory frame and Fig. 14a applies. This can only be done with sufficient statistics at a tau-charm factory.

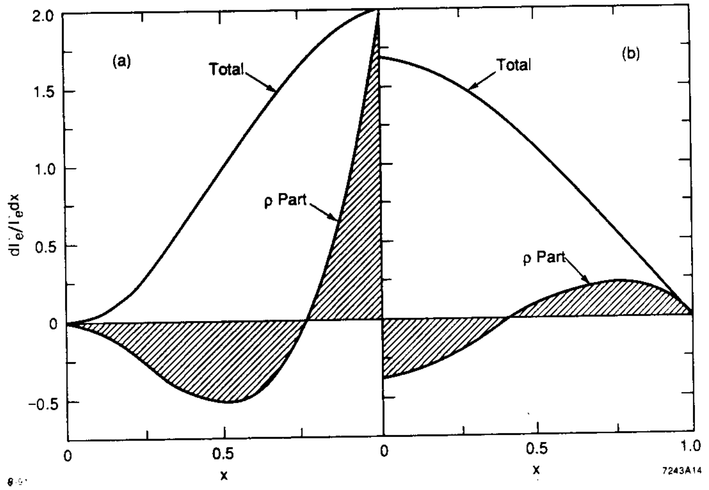

Figure 14.

E. HADRONIC DECAYS

> *False facts are highly injurious to the progress of science, for they often endure long; but false views, if supported by some evidence, do little harm, for everyone takes a salutary pleasure in proving their falseness; and when this is done, one path towards error is closed and the road to truth is often at the same time opened.*
>
> *Charles Darwin*

E.1 $\tau^- \to \pi^- + \nu_\tau$, $\tau^- \to K^- + \nu_\tau$

As already discussed there is no general and precise method for calculating the dynamics and B_i of the general hadronic decay

$$\tau^- \to (\text{hadrons})^-_i + \nu_\tau \tag{E.1}$$

because in the energy range

$$\sqrt{s} < m_\tau$$

the vertex in Fig. 15 is too complicated. We must use special methods which depend on other data. In this section I show the special methods for

$$\tau^- \to \pi^- + \nu_\tau \tag{E.2}$$

$$\tau^- \to K^- + \nu_\tau \ . \tag{E.3}$$

Figure 16a shows the diagram for the τ decay in Eq. E.2. We cannot calculate the strength of the $W\pi$ vertex, but it is exactly the same vertex as in π decay, Fig. 16b.

$$\pi^- \to \mu^- + \bar{\nu}_\mu \ . \tag{E.4}$$

For Eq. E.2

$$\Gamma(\tau^- \to \pi^- \nu_\tau) = \frac{G_F^2 m_\tau^3 f_\pi^2 \cos^2\theta_c}{16\pi}\left[1 - \frac{m_\pi^2}{m_\tau^2}\right]^2 \tag{E.5}$$

and for Eq. E.4

$$\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu) = \frac{G_F^2 m_\pi m_\mu^2 f_\pi^2 \cos^2\theta_c}{8\pi}\left[1 - \frac{m_\mu^2}{m_\pi^2}\right]^2 \ . \tag{E.6}$$

In these equations f_π summarizes what we cannot calculate precisely about the $W\pi$ vertex. Radiative corrections which are of order α/π are ignored in these equations.

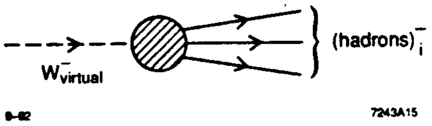

Figure 15.

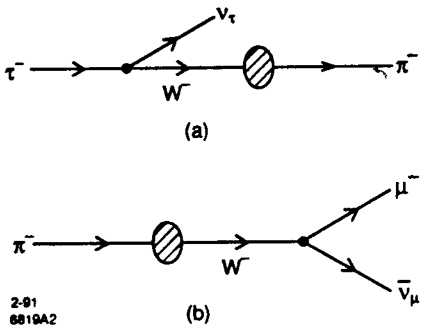

Figure 16.

Then one of the branching fraction ratios of Eq. D.6, B_π/B_e, is given by

$$\frac{B_\pi}{B_e} = \frac{12\,\pi^2 f_\pi^2 \cos^2\theta_c}{m_\tau^2}\left[1-\frac{m_\pi^2}{m_\tau^2}\right]^2 \qquad (E.7)$$

using Eqs. E.5 and D.14, and again ignoring radiative corrections. Using Eq. E.6 and the π lifetime

$$f_\pi = 132 \text{ MeV} \quad , \qquad (E.8)$$

the calculation in Eq. E.7 gives

$$B_\pi/B_e \text{ (predicted) } = 0.61 \quad . \qquad (E.9)$$

Before I discuss the measured value of B_π/B_e, I must discuss the present average measured value of B_π. As first noted in Eq. C.2, τ decay mode studies at LEP and in some other present studies, do not allow separation of π^- from K^- or π^+ from K^+. Therefore, in the last two years we have had new measurements, not of $B(\tau^- \to \pi^-\nu_\tau)$, but of

$$B(\tau^- \to \pi^-\nu_\tau) + B(\tau^- \to K^-\nu_\tau) = B(\tau^- \to h^-\nu_\tau) \qquad (E.10)$$

here h^- means π^- or K^- and h^+ means π^+ or K^+.

Table 8 gives recent measurements and averages for $B(\tau^- \to h^-\nu_\tau)$. The Particle Data Group value and the recent values of Galik (1992) are respectively

$$B(\tau^- \to h^-\nu_\tau) = (12.47 \pm 0.35)\% \qquad (E.11a)$$
$$B(\tau^- \to h^-\nu_\tau) = (12.40 \pm 0.26)\% \quad . \qquad (E.11b)$$

To find $B(\tau^- \to \pi^-\nu_\tau)$ we must know $B(\tau^- K^-\nu_\tau)$; unfortunately the measurements are old (Aguilar-Benitez *et al.* 1992).

$$B(\tau^- \to K^-\nu_\tau) = (0.67 \pm 0.23)\% \quad . \qquad (E.12)$$

Combining with Eq. 11b yields

$$B(\tau^- \to \pi^-\nu_\tau) = (11.73 \pm 0.35)\% \quad . \qquad (E.13)$$

As we go through this section on hadronic decays, the problem of not being able to separate π's and K's will become increasingly important. First, the problem in general tends to negate the value of increasing precision in branching fraction measurements. Second, the problem obscures our understanding of the comparison of $\sum_i B_{1i}$ with B_1, the one-charged particle decay modes issue.

Returning to Eq. E.9, and using Eqs. D.29 and E.13,

$$B_\pi/B_e \text{ (measured) } = 0.66 \pm 0.02 \qquad (E.14)$$

which is in fair agreement with

$$B_\pi/B_e \text{ (predicted)} = 0.61 \quad .$$

But more measurement precision and a consideration of the radiative correction are required.

Next we consider

$$B_K/B_\pi = B(\tau^- \to K^-\nu_\tau)/B(\tau^- \to \pi\nu_\tau) \qquad (E.15)$$

to test the effect of Cabibbo suppression. In analogy to Eq. E.5

$$\Gamma(\tau^- \to K^-\nu_\tau) \;=\; \frac{G_F^2 m_\tau^3 f_K^2 \sin^2\theta_c}{16\pi}\left[1-\frac{m_K^2}{m_\tau^2}\right]^2 \qquad (E.16)$$

and from the lifetime for

$$K^- \to \mu^- + \bar{\nu}_\mu \qquad (E.17)$$
$$f_K = 161 \text{ MeV} \quad . \qquad (E.18)$$

Combining Eqs. E.5 and E.16, the prediction is

$$B_K/B_\pi = \tan^2\theta_c \left(\frac{f_K}{f_\pi}\right)^2 \left[\frac{m_\tau^2 - m_K^2}{m_\tau^2 - m_\pi^2}\right]^2 \;=\; 0.071 \quad . \qquad (E.19)$$

The measured value of B_K/B_π from Eqs. E.12 and E.13 is

$$B_K/B_\pi \text{ (measured)} = 0.057 \pm 0.020 \qquad (E.20)$$

which agrees with Eq. E.19.

E.2 Application of Quantum Number Conservation in Non-Strange Hadronic Decays

The rules from quantum number conservation which control non-strange hadronic decays of the τ have been frequently derived and discussed since the original work of Tsai (1971). I will not repeat the discussion here but simply quote the conclusions from Perl (1992).

The weak charged current in τ decay has the following properties:

$$\begin{aligned} \text{Isospin} : & \ I = 1 \text{ for vector and axial vector currents} \\ G-\text{parity} : & \ G = +1 \text{ for vector current} \\ & \ G = -1 \text{ for axial vector current} \\ \text{Spin-parity} : & \ J^P = 1^- \text{ for vector current} \\ & \ J^P = 0^-,\ 1^+ \text{ for axial vector current} . \end{aligned} \tag{E.21}$$

The G-parity assignment opposite to that in Eq. E.21 corresponds to a so-called second class current, the decay width is then suppressed by a factor of 10^{-4} to 10^{-6} as discussed below.

It is straightforward to apply the G and J^P requirements to the non-strange hadrons which are produced in τ decay:

$$\begin{aligned} \pi : & \ G = -1 ,\ J^P = 0^- \\ \eta : & \ G = +1 ,\ J^P = 0^- \\ \rho : & \ G = +1 ,\ J^P = 1^- \\ \omega : & \ G = -1 ,\ J^P = 1^- \end{aligned} \tag{E.22}$$

and so forth. For example in $\tau^- \to \nu_\tau \pi^-$ the π with $G = -1$, $J^P = 0^-$ is produced through the axial vector current decay. Conversely, the decay $\tau^- \to \nu_\tau \rho^-$ occurs through the vector current since $G = +1$. However the decay

$$\tau^- \to \nu_\tau + \pi^- + \eta \tag{E.23}$$

is forbidden since $G(\pi\eta) = -1$ requires an axial vector current with $J^P = 0^-$ or 1^+. But for $J = 0$ $P(\pi\eta) = +1$ and for $J = 1$ $P(\pi\eta) = -1$.

In a decay with n π's

$$\tau^- \to \nu_\tau + (n\ \pi)^- \tag{E.24}$$

$G = (-1)^n$. Hence the vector current produces states with an even number of π's, the axial vector current produces states with an odd number of π's.

Isospin conservation is also used to derive inequalities between different hadronic decay modes with the same I (Gilman and Rhie 1985). Consider, for example, the 3π modes

$$\begin{aligned} \tau^- &\to \nu_\tau + \pi^- + \pi^0 + \pi^0 \\ \tau^- &\to \nu_\tau + \pi^- + \pi^+ + \pi^- \end{aligned} \tag{E.25}$$

with I=1. Gilman and Rhie (1985) show

$$\frac{\Gamma(\tau^- \to \nu_\tau \pi^- \pi^0 \pi^0)}{\Gamma(\tau^- \to \nu_\tau \pi^- \pi^0 \pi^0) + \Gamma(\tau^- \to \nu_\tau \pi^- \pi^+ \pi^-)} \leq \frac{1}{2} . \tag{E.26}$$

Hence

$$B(\tau^- \to \nu_\tau \pi^- \pi^0 \pi^0) \leq B(\tau^- \nu_\tau \pi^- \pi^+ \pi^-) . \tag{E.27}$$

The G-parity rule in Eq. E.21 depends upon ignoring the effect of the unequal masses of the u and d quarks, $m_\mu \neq m_d$, and ignoring the effect of electromagnetism. Once these effects are taken into account the τ decay can occur through the so-called second-class current. For second-class current decays

$$\begin{aligned} \text{Vector:}\ & G = -1 \ ,\ J^P = 1^- \\ \text{Axial vector:}\ & G = +1 \ ,\ J^P = 0^-, 1^+ . \end{aligned} \tag{E.28}$$

But the decay widths and hence the branching fractions are reduced by

$$\left(\frac{m_d - m_\mu}{m_\pi} \right)^2 \sim 10^{-4} \tag{E.29}$$

or

$$\alpha^2 \sim 10^{-4} \tag{E.30}$$

or even more (Leroy and Pestieau 1978, Pich 1987, Zachos and Meurice 1987). Quoting again from Perl (1992), there are two interests in observing and studying second-class current decays. First, what is the strength of a second-class current decay due to the electromagnetic correction, that is, a decay within the Standard Model? Second, are there second-class current decays whose properties cannot be explained by the Standard Model? Interesting discussions are given by Berger and Lipkin (1987) and by Bramon *et al.* (1987).

E.3 $\tau^- \to \rho^- + \nu_\tau$ and Other Vector Decay Modes

The decay width of a τ vector decay mode can be calculated from the cross section for e^+e^- annihilation to a related final state (Tsai 1971, Gilman and Rhie 1985, Perl 1992), but the e^+e^- annihilation section must be measured. The calculation can be done because the unknown W−hadron vertex in the decay process is connected by the conserved vector current hypothesis to the unknown γ−hadron vertex in the annihilation process, Fig. 17.

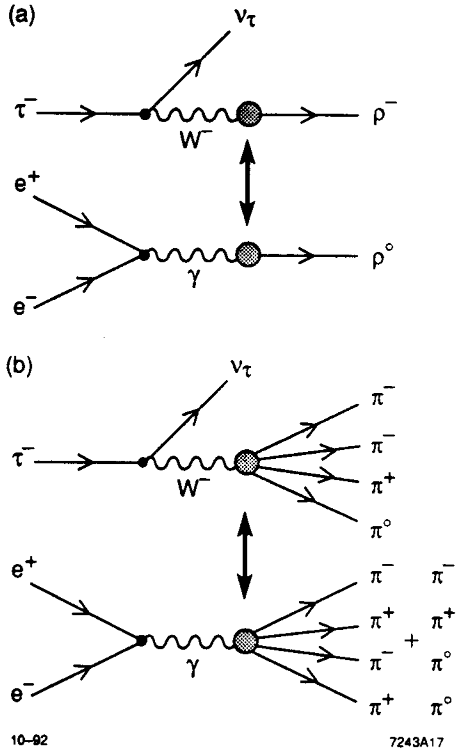

Figure 17.

For example, the decay width, Γ, for

$$\tau^- \to \rho^- + \nu_\tau \tag{E.31}$$

is related to the cross section, σ, for

$$e^+ + e^- \to \rho^0 \tag{E.32}$$

through

$$\begin{aligned}\Gamma(\tau^- \to \rho^- \nu_\tau) \; = \; & \frac{G_F^2 \cos^2 \theta_c m_\tau^3}{384 \pi^5 \alpha^2} \\ \times \int_0^{m_\tau^2} q^2 d q^2 \left(1 - \frac{q^2}{m_\tau^2}\right)^2 & \left(1 + 2\frac{q^2}{m_\tau^2}\right) \sigma_{I=1} \left(e^+ e^- \to \rho^0, \, q^2\right) \quad .\end{aligned} \tag{E.33}$$

Using Eq. 33, Kühn and Salamaria (1990) use $\sigma(e^+e^- \to \rho^0)$ measurements of Barkov *et al.* (1985) to predict

$$B_\rho / B_e(\text{predicted}) \; = 1.32 \pm 0.05 \quad . \tag{E.34}$$

From B_e in Table 2

$$B_\rho(\text{predicted}) = (23.4 \pm 0.9)\% \tag{E.35}$$

which is in good agreement with

$$B_\rho \;(\text{measured}) = (23.82 \pm 0.25)\% \tag{E.36}$$

from Table 2.

E.4 Measurements of Major Hadronic Branching Fractions

Tables 8 through 13 summarize measurements on the major hadronic branching fractions giving: the world averages in 1990, the measurements in 1990–1992, and the 1992 world averages of myself, of Aguilar-Benitez *et al.* (1992) and of Galik (1992). My 1992 world averages and those of Galik (1992) include measurements too recent for inclusion in Aguilar-Benitez *et al.* (1992).

I remind the reader that these measurements include modes containing K's. Figure 18 is a rough estimate of the K mode content of the 1-charged decay modes using the summary of Aguilar-Benitez *et al.* (1992). The measurements have large errors and the total of $(4.0 \pm 0.5)\%$ may be overestimated.

Table 8. $B(\tau^- \rightarrow h^- \nu_\tau)$ in %. The errors are ±.

WORLD AVERAGE VALUES

B	ERROR	WORLD AVERAGE
12.00	0.60	1990 Particle Data Group Average
12.50	0.26	1990-1992 Average
12.42	0.24	My 1992 Average
12.47	0.35	1992 Particle Data Group Average
12.40	0.26	Galik (1992) Average

INDIVIDUAL 1990-1992 EXPERIMENTS

B	ERROR	WEIGHT	EXPER	REFERENCE
12.81	0.34	0.59	ALEPH	S. Snow; Proc. Second Workshop Tau Lepton Phys.;1992
11.90	0.99	0.07	DELPHI	P. Abreu et al.; CERN-PPE/92-060;1992
12.20	0.50	0.27	OPAL	M. Sasaki; Proc. Second Workshop Tau Lepton Phys.;1992
11.70	1.00	0.07	ARGUS	H. Albrecht et al. Z. Phys. C53;367;1992

Table 9. $B(\tau^- \rightarrow h^- \pi^0 \nu_\tau)$ in %. The errors are ±.

WORLD AVERAGE VALUES

B	ERROR	WORLD AVERAGE
22.20	1.00	1990 Particle Data Group Average
24.12	0.31	1990-1992 Average
23.95	0.29	My 1992 Average
23.40	0.60	1992 Particle Data Group Average
24.29	0.24	Galik (1992) Average

INDIVIDUAL 1990-1992 EXPERIMENTS

B	ERROR	WEIGHT	EXPER	REFERENCE
25.04	0.55	0.31	ALEPH	S. Snow; Proc. Second Workshop Tau Lepton Phys.;1992
22.40	1.53	0.04	DELPHI	P. Abreu et al.; CERN-PPE/92-060;1992
23.57	0.92	0.11	DELPHI	P. Vaz; Proc. Second Workshop Tau Lepton Phys.;1992
23.80	0.92	0.11	OPAL	M. Sasaki; Proc. Second Workshop Tau Lepton Phys.;1992
22.60	0.98	0.10	ARGUS	D. Toepfer Proc. Second Workshop Tau Lepton Phys.;1992
24.35	0.55	0.31	CLEO	A. Weinstein; Proc. Second Worksho Tau Lepton Phys.;1992
22.00	2.06	0.02	C. BALL	D. Antreasyan et al. Phys. Lett. B259;216;1991

Table 10. $B(\tau^- \rightarrow h^- 2\pi^0 \nu_\tau)$ in %. The errors are ±.

WORLD AVERAGE VALUES

B	ERROR	WORLD AVERAGE
7.50	0.90	1990 Particle Data Group Average
9.00	0.35	1990-1992 Average
8.80	0.33	My 1992 Average
9.00	0.60	1992 Particle Data Group Average
8.76	0.33	Galik (1992) Average

INDIVIDUAL 1990-1992 EXPERIMENTS

B	ERROR	WEIGHT	EXPER	REFERENCE
9.98	0.56	0.39	ALEPH	S. Snow; Proc. Second Workshop Tau Lepton Phys.;1992
8.64	0.47	0.55	CLEO	J. Urheim; Proc. Second Workshop Tau Lepton Phys.;1992
5.70	1.49	0.06	C. BALL	D. Antreasyan et al. Phys. Lett. B259;216;1991

Table 11. $B(\tau^- \rightarrow h^- \geq 3\pi^0 \nu_\tau)$ in %. The errors are ±.

WORLD AVERAGE VALUES

B	ERROR	WORLD AVERAGE
3.00	2.70	1990 Particle Data Group Average
1.24	0.14	1990-1992 Average
1.24	0.14	My 1992 Average
1.80	0.60	1992 Particle Data Group Average
1.26	0.13	Galik (1992)

INDIVIDUAL 1990-1992 EXPERIMENTS

B	ERROR	WEIGHT	EXPER	REFERENCE
1.46	0.36	0.15	ALEPH	S. Snow; Proc. Second Workshop Tau Lepton Phys.;1992
1.20	0.15	0.85	CLEO	J. Urheim; Proc. Second Workshop Tau Lepton Phys.;1992

Table 12. $B(\tau^- \rightarrow 2h^-h^+\nu_\tau)$ in %. The errors are ±.

WORLD AVERAGE VALUES

B	ERROR	WORLD AVERAGE
6.70	0.60	1990 Particle Data Group Average
8.49	0.23	1990-1992 Average
8.27	0.21	My 1992 Average
8.00	0.30	1992 Particle Data Group Average
8.62	0.19	Galik (1992)

INDIVIDUAL 1990-1992 EXPERIMENTS

B	ERROR	WEIGHT	EXPER	REFERENCE
9.56	0.32	0.50	ALEPH	S. Snow; Proc. Second Workshop Tau Lepton Phys.;1992
7.82	0.41	0.30	DELPHI	P. Vaz; Proc. Second Workshop Tau Lepton Phys.;1992
6.80	0.51	0.20	ARGUS	D. MacFarlane; Proc. Second Workshop Tau Lepton Phys.;1992

Table 13. $B(\tau^- \rightarrow 2h^-h^+ \geq 1\pi^0\nu_\tau)$ in %. The errors are ±.

WORLD AVERAGE VALUES

B	ERROR	WORLD AVERAGE
4.60	1.00	1990 Particle Data Group Average
5.50	0.27	1990-1992 Average
5.44	0.26	My 1992 Average
5.20	0.40	1992 Particle Data Group Average
5.45	0.22	Galik (1992)

INDIVIDUAL 1990-1992 EXPERIMENTS

B	ERROR	WEIGHT	EXPER	REFERENCE
5.52	0.30	0.82	ALEPH	S. Snow; Proc. Second Workshop Tau Lepton Phys.;1992
5.40	0.64	0.18	ARGUS	D. Wegener; Proc. 1991 Photon Lepton Conf.

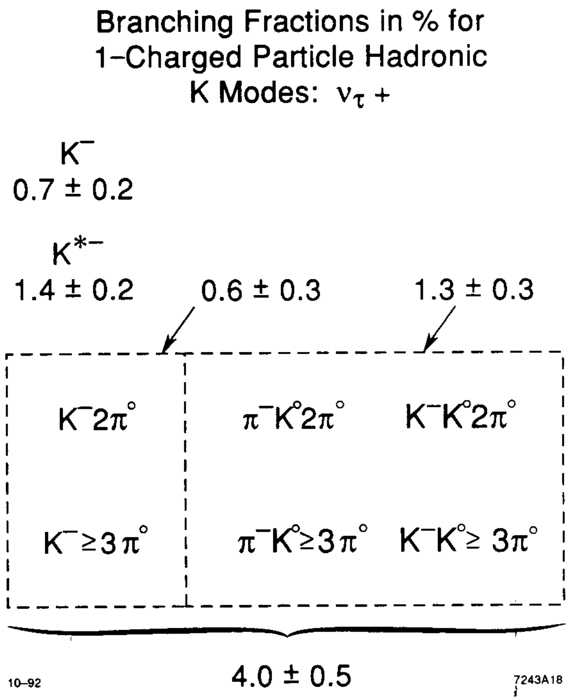

Figure 18.

E.5 Comparison of B_1 and B_3 with $\sum_i B_i$

Since the work of Gilman and Rhie (1985) and Truong (1984) the world of τ research has been faced with the question: Can we find and identify all the decay modes of the τ with branching fractions

$$B_i \gtrsim \text{few } \times 0.1\% \qquad (E.37)$$

such that

$$\sum_i B_i = 100\% \quad ? \qquad (E.38)$$

On the face of it Eq. E.38 is an identity; the fundamental question is: Are there some unknown and unconventional τ decays such that

$$\sum_i B_i \text{ (known and measured)} < 100\% \ ? \qquad (E.39)$$

Historically the question was first asked about decay modes with 1-charged particle, B_{1i}, since these made up most τ decays. The topological 1- and 3-charged particle branching fractions according to the Particle Data Group (Aguilar-Benitez *et al.* 1992) are

$$\begin{aligned} B_1 &= (85.52 \pm 0.25)\% \\ B_3 &= (14.06 \pm 0.25)\% \quad ; \end{aligned} \qquad (E.40a)$$

and in a more recent computation by Galik (1992)

$$\begin{aligned} B_1 &= (85.26 \pm 0.18)\% \\ B_3 &= (14.63 \pm 0.18)\% \quad ; \end{aligned} \qquad (E.40b)$$

We usually break up the question in Eq. E.39 into two questions. Does

$$\sum_i B_{1i} \text{ (known and measured) } = B_1 \qquad (E.41a)$$

and does

$$\sum_i B_{3i} \text{ (known and measured) } = B_3 \ ? \qquad (E.41b)$$

Table 14. World average values of τ branching fractions in % from Aguilar-Benitez *et al.* (1992).

Decay Mode	B(%)
$e^-\bar{\nu}_e\nu_\tau$	17.85 ± 0.29
$\mu^-\bar{\nu}_\mu\nu_\tau$	17.45 ± 0.27
$h^-\nu_\tau$	12.47 ± 0.35
$h^-\pi^0\nu_\tau$	23.4 ± 0.6
$h^-2\pi^0\nu_\tau$	9.0 ± 0.6
$h^- \geq 3\pi^0\nu_\tau$	1.8 ± 0.6
$\sum_i B_{1i}$	82.0 ± 1.2
B_1	85.94 ± 0.23
$\Delta_1 = B_1 - \sum_i B_{1i}$	3.9 ± 1.2
$2h^-h^+\nu_\tau$	8.0 ± 0.3
$2h^-h^+ \geq 1\pi^0\nu_\tau$	5.2 ± 0.4
$\sum_i B_{3i}$	13.2 ± 0.5
B_3	14.06 ± 0.20
$\Delta_3 = B_3 - \sum_i B_{3i}$	0.9 ± 0.5
B_5	0.11 ± 0.03
$\sum_i B_i$	95.3 ± 1.3

Turning to the data, Tables 14 and 15 show two recent compilations from the Particle Data Group (Aguilar-Benitez *et al.* 1992) and from Galik (1992). Table 15 contains very recent data from the LEP experiments and from the CLEO II experiment as well as much of the data used in Table 14. The numbers in the tables are averages of measurements from several or even many experiments. To try to answer the questions in Eqs. E.35 I give

$$\Delta_1 = B_1 - \sum_i B_{1i} \text{ (known and measured)} \qquad (E.42a)$$

$$\Delta_3 = B_3 - \sum_i B_{3i} \text{ (known and measured)} \qquad (E.42b)$$

in Tables 14 and 15. Remember, these compilations have many data sets in common, they are not statistically independent. Hayes (1992a) has given an important discussion of the problems in compiling such tables.

Understanding the true errors in these average values is very difficult, as I remarked in the previous section. Are the systematic errors underestimated? Have

Table 15. World average values of τ branching fractions in % from Galik (1992).

Decay Mode	B(%)
$e^-\bar{\nu}_e\nu_\tau$	17.75 ± 0.15
$\mu^-\bar{\nu}_\mu\nu_\tau$	17.39 ± 0.17
$h^-\nu_\tau$	12.40 ± 0.26
$h^-\pi^0\nu_\tau$	24.29 ± 0.24
$h^-2\pi^0\nu_\tau$	8.76 ± 0.33
$h^- \geq 3\pi^0\nu_\tau$	1.26 ± 0.13
$\pi^-\pi^0\eta\nu_\tau$	0.08 ± 0.03
$\sum_i B_{1i}$	81.93 ± 0.55
B_1	85.26 ± 0.18
$\Delta_1 = B_1 - \sum_i B_{1i}$	3.3 ± 0.6
$2h^-h^+\nu_\tau$	8.62 ± 0.19
$2h^-h^+ \geq 1\pi^0\nu_\tau$	5.45 ± 0.22
$\sum_i B_{3i}$	14.07 ± 0.29
B_3	14.63 ± 0.18
$\Delta_3 = B_3 - \sum_i B_{3i}$	0.6 ± 0.3
B_5	0.13 ± 0.03
$\sum_i B_i$	96.13 ± 0.62

the proper corrections been made for modes which have a K_L^0? In a particular decay mode, do almost all experiments have the same bias, a bias which is not corrected? Therefore, at this time it is probably best to take the errors in $\sum B_{1i}$ and $\sum B_{3i}$ to be of the order of 1%, and to recognize that at present we do not know if there are missing decay modes, modes which are unconventional and hence not detected and not measured. Davier (1992), Galik (1992), and Drell (1992) have discussed these issues.

It has long been recognized that these questions would be best answered by a single experiment in which every B_{1i} and B_{3i} has high statistics. We do not yet have such an experiment with sufficient statistics to reduce errors to a few tenths of a per cent, the size of errors we would like. The closest we come at present to such an experiment is that carried out by the ALEPH experimenters at LEP (Davier 1992) as summarized in Table 16 from Snow (1992).

Returning to Tables 14 and 15 we see

$$\Delta_1 = (3.9 \pm 1.2)\% \qquad (E.43a)$$

$$\Delta_1 = (3.3 \pm 0.6)\% \qquad (E.43b)$$

respectively. There are four classes of explanations for $\Delta_1 \neq 0$:

Table 16. A complete set of branching fraction measurements from the ALEPH experimenters (Snow 1992). The third error on $\sum B_i$ is the normalization uncertainty.

Decay Mode	B(%)
$e^-\bar{\nu}_e\nu_\tau$	$18.23 \pm 0.30 \pm 0.22$
$\mu^-\bar{\nu}_\mu\nu_\tau$	$17.70 \pm 0.29 \pm 0.21$
$h^-\nu_\tau$	$12.63 \pm 0.28 \pm 0.24$
$h^-\pi^0\nu_\tau$	$26.04 \pm 0.57 \pm 0.63$
$h^-2\pi^0\nu_\tau$	$8.69 \pm 0.61 \pm 0.52$
$h^- \geq 3\pi^0\nu_\tau$	$1.65 \pm 0.41 \pm 0.41$
$2h^-h^+\nu_\tau$	$9.57 \pm 0.24 \pm 0.22$
$2h^-h^+ \geq 1\pi^0\nu_\tau$	$5.42 \pm 0.26 \pm 0.34$
$\sum B_i$	$99.93 \pm 0.83 \pm 0.72 \pm 0.67$

- The measured values of B_1 are too large due to experimental error. I don't believe this explanation because B_1 is relatively easy to measure.
- Some of the individual B_{1i}'s are too small. This can occur because no experiment is capable of measuring any B_{1i} without making corrections to get from the observed value of that B_{1i} to the true value. Corrections must be made for less than 100% acceptance, for mismeasurements which cause an event to fall outside selection criteria, for misidentification of a mode, and for contamination from other modes on non$-\tau$ events. This explanation claims that on the average some or all of the B_{1i}'s are too small when corrected from their observed values.
- The stated errors on $\sum B_{1i}$ are too small, the errors are actually larger and there is no significance to $\Delta_1 \neq 0$ in Eq. E.43. This could occur because the error calculations combine statistical and systematic errors in quadrature, and the systematic errors may be too small. Hayes and Perl (1988) and Hayes *et al.* (1989) have discussed this.
- The τ has one or more unconventional, one-charged particle decay modes. This mode or these modes would not be found when an experimenter selects any of the conventional modes in a τ event and hence the unconventional

modes would not contribute to $\sum_i B_{1i}$ (known and measured) in Eq. E.42a. But the unconventional modes would contribute to B_1.

My dream is that the last explanation is correct, that the τ has some unexpected and undetected decay modes. This could mean that τ decay physics contains new physics which is not in the Standard Model.

But at present the hope for resolving this one-charged particle decay mode puzzle is clouded by two problems. First, the experimental situation is very complicated and the second or third explanations may be right. Second, there is no comfortable model or even speculation for the origin or nature of unconventional modes. The question is: What sort of one-charged particle decay mode would be counted in the topological B_1 but not in any individual, conventional B_{1i}?

F. THE τ IN ATOMIC PHYSICS

Everything is made of atoms. That is the key hypothesis.

Richard Feynman

F.1 The $\tau^+ + \tau^-$ Atom

F.1.a Energy Levels

In this section I discuss the $\tau^+\tau^-$ atom, an entity which would be analogous to the e^+e^- atom called positronium. I have avoided calling the $\tau^+\tau^-$ atom tauonium, as some authors do, because the name muonium means the μ^+e^- atom, not the $\mu^+\mu^-$ atom, and the name tauonium might be interpreted as the τ^+e^- atom. The $\tau^+\tau^-$ atom can be made in e^+e^- annihilation just below τ pair threshold

$$e^+ + e^- \to \tau^+\tau^- \text{ atom} \qquad (F.1)$$

and has been discussed by Moffat (1975), Avilez *et al.* (1978) and Avilez *et al.* (1979).

The energy levels of the $\tau^+\tau^-$ atom are shown in Fig. 19 where the atomic spectroscopy notation

$$n^{2S+1}\,L_J \qquad (F.2)$$

is used. Here n is the principle quantum number; S is the total spin quantum number and is 0 or 1, L is the orbital angular momentum quantum number with L = S, P, D ... for L = 0, 1, 2 ..., and J is the total angular momentum quantum

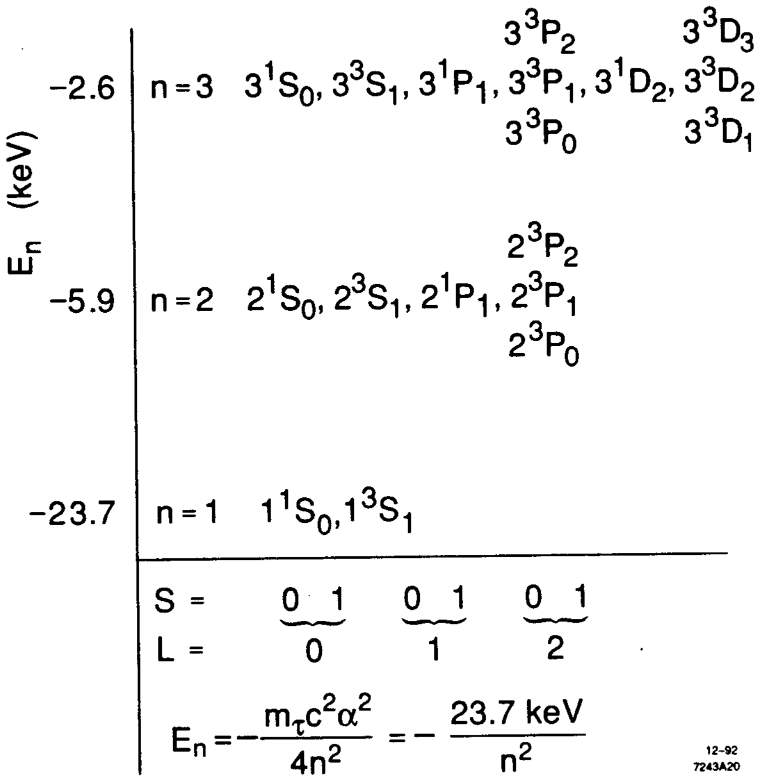

Figure 19.

number. Ignoring fine structure, the energy levels are given by

$$E_n = -\frac{m_\tau c^2 \alpha^2}{4n^2} = -\frac{23.7 \text{ keV}}{n^2} \quad . \tag{F.3}$$

F.1.b Charge Conjugation Rules for Production and Decay

Charge conjugation, C, imposes selection rules on the production and decay of the $\tau^+\tau^-$ atom

$$C\psi(\tau^+\tau^- \text{ atom}, \ n, S, L) = (-1)^{S+L}\psi(\tau^+\tau^- \text{ atom}, \ n, S, L) \tag{F.4}$$

and for a state of N photons

$$C \ \psi(N \text{ photons}) = (-1)^N \ \psi(N \text{ photon}) \quad . \tag{F.5}$$

Therefore in production

$$e^+ + e^- \rightarrow \gamma_{virtual} \rightarrow \tau^+\tau^- \text{ atom} \quad , \tag{F.6}$$

the atom must be produced in a state with

$$S + L = \text{ odd number} \quad . \tag{F.7}$$

The decay

$$\tau^+\tau^- \text{ atom} \rightarrow \gamma + \gamma \tag{F.8a}$$

requires

$$S + L = \text{ even number} \quad , \tag{F.8b}$$

and the decay

$$\tau^+\tau^- \text{ atom} \rightarrow \gamma + \gamma + \gamma \tag{F.9a}$$

requires

$$S + L = \text{ odd number} \quad . \tag{F.9b}$$

F.1.c Decay Channels of the $\tau^+\tau^-$ Atom

Next I discuss the decay of the $\tau^+\tau^-$ atom. There are two classes of decay channel. In the first class the τ^+ or τ^- decay through the weak interaction in the normal way and the atomic state disappears. The decay width is

$$\Gamma(\text{atom}, \ \tau \text{ decay}) = 2\hbar/\tau_{lifetime} = 4.4 \times 10^{-3} \text{ eV} \tag{F.10}$$

where the 2 occurs because the decay of either τ breaks up the atomic state.

In the second class of decay channels the τ^+ and τ^- annihilate. The annihilation requires that the atomic wave function $\psi(\mathrm{r})$ be unequal to 0 at $\mathrm{r} = 0$

$$\psi(0) \neq 0 \quad . \tag{F.11}$$

Here r is the distance between the τ^+ and τ^-. Therefore in lowest order annihilation only occurs in L = 0 states, that is, S states. There are five annihilation channels.

The annihilation channel

$$\tau^+\tau^- \text{ atom} \rightarrow \gamma + \gamma \tag{F.12a}$$

is even under charge conjugation, therefore

$$\text{atomic state } = n \ {}^1S_0 \quad . \tag{F.12b}$$

The decay width is

$$\begin{aligned} \Gamma(\text{atom} \rightarrow 2\gamma) &= \frac{\alpha^5 m_\tau c^2}{2n^3} \\ &= \frac{1.8 \times 10^{-2} \text{ eV}}{n^3} \quad . \end{aligned} \tag{F.12c}$$

The four other annihilation channels have odd charge conjugation, therefore

$$\text{atomic state} = n \ {}^3S_1 \quad . \tag{F.13}$$

The channel

$$\tau^+\tau^- \text{atom} \rightarrow \gamma + \gamma + \gamma \quad , \tag{F.14a}$$

has the width

$$\begin{aligned} \Gamma \text{ (atom)} \rightarrow 3\gamma) &= \frac{2(\pi^2 - 9)\alpha^6 m_\tau c^2}{9\pi n^3} \\ &= \frac{1.7 \times 10^{-5} \text{ eV}}{n^3} \quad . \end{aligned} \tag{F.14b}$$

The two channels, Fig. 20,

$$\tau^+\tau^- \text{ atom} \rightarrow e^+ + e^- \tag{F.15a}$$

$$\tau^+\tau^- \text{ atom} \rightarrow \mu^+ + \mu^- \tag{F.15b}$$

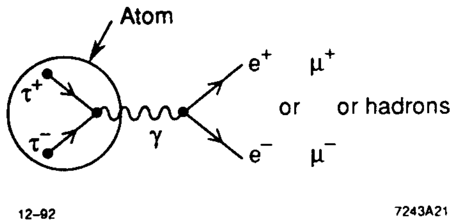

Figure 20.

have the same width

$$\Gamma(\text{atom} \to e^+e^-) = \Gamma(\text{atom} \to \mu^+\mu^-) = \frac{\alpha^5 m_\tau c^2}{6n^3} = \frac{6.1 \times 10^{-3}\ \text{eV}}{n^3} \qquad (F.15c)$$

when we neglect the masses of the e and μ. Finally there is the channel, Fig. 20,

$$\tau^+\tau^- \text{ atom} \to \text{hadrons} \ . \qquad (F.16a)$$

The width cannot be calculated from first principles, however from colliding beams e^+e^- annihilation data at $E_{tot} \sim 2\,m_\tau$ we know

$$\sigma(e^+ + e^- \to \text{hadrons}) \approx 2\sigma(e^+ + e^- \to \mu^+ + \mu^-) \ . \qquad (F.16b)$$

Therefore,

$$\Gamma(\text{atom} \to \text{hadrons}) \approx 2\ \Gamma_{\mu\mu} \ . \qquad (F.16c)$$

Collecting all this together, for $n\ ^1S_0$ states

$$\begin{aligned}\Gamma_{tot}(n\ ^1S_0) &= \Gamma(\text{atom},\ \tau \text{ decay}) + \Gamma(\text{atom} \to 2\gamma) \\ &= \left(4.4 \times 10^{-3} + \frac{3.7 \times 10^{-2}}{n^3}\right) \text{eV} \ .\end{aligned} \qquad (F.17)$$

For the $n\ ^3S_1$ states we can neglect $\Gamma(\text{atom} \to 3\gamma)$, Eq. F.14b, and set

$$\begin{aligned}\Gamma_{tot}(n\ ^3S_1) &\approx \Gamma(\text{atom}, \tau \text{ decay}) + 4\Gamma(\text{atom} \to e^+e^-) \\ &\approx \left(4.4 \times 10^{-3} + \frac{2.44 \times 10^{-2}}{n^3}\right) \text{eV} \ .\end{aligned} \qquad (F.18)$$

I remind the reader that in addition to the decays which destroy the $\tau^+\tau^-$ atom there are electromagnetic decays within the atom from an upper level to a lower level

$$\psi(\tau^+\tau^- \text{ atom},\ n') \to \psi(\tau^+\tau^- \text{ atom},\ n) + \gamma \ , \quad n' > n \ . \qquad (F.19)$$

F.1.d Production of the $\tau^+\tau^-$ Atom

As noted in Sec. F.1.b, the production process

$$e^+ + e^- \to \gamma_{virtual} \to \tau^+\tau^- \text{ atom} \qquad (F.20)$$

requires S + L = odd number. Furthermore, the produced state must have

$\psi(0) \neq 0$ and hence L = 0. Therefore, S = 1 and the produced state must be $n\,^3S_1$.

The production cross section for the process in Eq. F.20 is

$$\sigma(e^+e^- \to \tau^+\tau^- \text{ atom}) = \frac{3\pi(\hbar c)^2}{4m_\tau^2} \frac{\Gamma_{ee}\Gamma_{tot}}{(E_{tot} - 2m_\tau)^2 + \Gamma_{tot}^2/4} \quad . \qquad (F.21)$$

Here Γ_{ee} means $\Gamma(\text{atom} \to e^+e^-)$ and is given by Eq. F.15c. Γ_{tot} is given by Eq. F.18. Thus the production cross section is given by the Breit-Wigner equation with full width at half-height of Γ_{tot} and peak cross section

$$\sigma(e^+e^- \to \tau^+\tau^- \text{ atom, peak}) = \frac{3\pi(\hbar c)^2}{m_\tau^2} \frac{\Gamma_{ee}}{\Gamma_{tot}} \quad . \qquad (F.22)$$

As an example consider $\tau^+\tau^-$ atom production into the ground state 1^3S_1. Then

$$\begin{aligned} \Gamma_{ee} &= 6.1 \times 10^{-3} \text{ eV} \\ \Gamma_{tot} &\approx 2.9 \times 10^{-2} \text{ eV} \quad , \end{aligned} \qquad (F.23)$$

and

$$\sigma(e^+e^- \to \tau^+\tau^- \text{ atom, peak}) \approx 2.4 \times 10^{-28} \text{ cm}^2 \quad . \qquad (F.24)$$

This is a large cross section, <u>but</u> the energy spread of the e^+ and e^- beams, ΔE, is much larger than Γ_{tot}. Thus in a tau-charm factory we expect

$$\Delta E \sim 1 \text{ MeV} \qquad (F.25)$$

and the effective cross section is

$$\begin{aligned} &\sigma(e^+e^- \to \tau^+\tau^- \text{ atom, effective}) \sim \\ &\qquad 2.4 \times 10^{-28} \text{ cm}^2 \times \frac{2.9 \times 10^{-2}}{10^6} \sim 10^{-35} \text{ cm}^2 \quad . \end{aligned} \qquad (F.26)$$

Therefore for a tau-charm factory luminosity of 10^{33} cm^{-2} s^{-1} we expect

$$\tau^+\tau^- \text{ atoms produced per sec.} \sim 10^{-2} \quad . \qquad (F.27)$$

There are two crucial unanswered questions about the $\tau^+\tau^-$ atom:

- How can the production of $\tau^+\tau^-$ atoms be detected?
- Can we make sufficiently precise studies of the properties of $\tau^+\tau^-$ atoms so that we can learn more about the τ itself.

F.2 τ^--Nucleus Atoms

In analogy to the μ^--nucleus atom, there is the τ^--nucleus atom. Its possible production and expected properties have been discussed by Strobel and Wells (1983) and by Ching and Oset (1991). There are three unresolved questions about the τ^--nucleus atom.

- How can a τ-nucleus atom be made? Figure 21 from Morley (1992) shows one possibility where a τ^- enters material very close to the τ^- production point and is then captured before it decays.
- How can the τ^--nucleus atom be detected?
- Can we make sufficiently precise studies of the properties of the τ^--nucleus atoms so that we can learn more about the τ itself.

G. THE τ NEUTRINO: ν_τ

This is my letter to the world,
That never wrote to me, -
The simple news that Nature told,
With simple majesty.

Emily Dickinson

Is the tau neutrino a simple, massless, stable, Dirac particle which obeys perfectly the conventional theory of weak interactions? Or is the ν_τ a complicated particle with non-zero mass, perhaps with mixing properties, perhaps with decays? All confirmed experimental results agree with the first alternative. In this section I summarize that data, but I also outline some speculations on the ν_τ being a complicated particle.

G.1 ν_τ Mass Limits

Present upper limits from terrestial experiments on the ν_τ mass, m_{ν_τ} are derived from the decay modes

$$\tau^- \to \nu_\tau + 3\pi^- + 2\pi^+ \qquad (G.1)$$

$$\tau^- \to \nu_\tau + 2\pi^- + \pi^+ + 2\pi^0 \quad . \qquad (G.2)$$

For each event the invariant mass of the five pions, $m_{5\pi}$ is calculated and then the spectrum of $m_{5\pi}$ is plotted. Ignoring errors and statistics

$$m_{\nu_\tau} = m_\tau - m_{5\pi} \text{ (maximum)} \quad . \qquad (G.3)$$

The classic measurement, Eq. G.1, Albrecht *et al.* (1988) corrected for a new m_τ

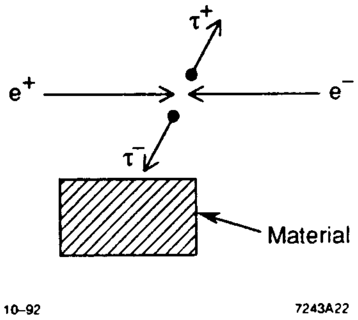

Figure 21.

of 1777 MeV/c^2 (Britton 1992) is

$$m_{\nu_\tau} < 31 \text{ MeV/c}^2, 95\% CL \quad . \tag{G.4}$$

Recently the CLEO experimenters (Cowan 1992) used events from both Eq. G.1 and Eq. G.2 to find

$$m_{\nu_\tau} < 37 \text{ MeV/c}^2, 95\% CL \quad . \tag{G.5}$$

Improvements in this method require large statistics and data obtained close to the τ pair threshold. It is possible to probe m_{ν_τ} masses at or below 3 MeV/c^2 (Gomez-Cadenas *et al.* 1990a), a tau-charm factory is required.

The decay mode (Gomez-Cadenas *et al.* 1990b)

$$\tau^- \to \nu_\tau + K^- + K^+ + \pi^- \tag{G.6}$$

and the decay mode (Gomez-Cadenas and Gonzales-Garcia 1989, Mendel *et al.* 1986)

$$\tau^- \to \nu_\tau + e^- + \bar{\nu}_e \tag{G.7}$$

can also be used to probe m_{ν_τ}, but are probably less sensitive.

Thus the upper limits on the masses of the three neutrinos are (Aguilar-Benitez *et al.* 1992):

$$\begin{aligned} m_{\nu_\tau} &< 31. \text{ MeV/c}^2 \ , \quad 95\% \ CL \\ m_{\nu_\mu} &< 0.27 \text{ MeV/c}^2, \quad 90\% \ CL \\ m_{\nu_e} &< 10 \text{ eV/c}^2 \quad , \quad 95\% \ CL \ . \end{aligned} \tag{G.8}$$

To compare these limits people sometimes use the assumption

$$\frac{m_{\nu_1}}{m_{\nu_2}} = \frac{m_1^2}{m_2^2} \quad . \tag{G.9}$$

Using Eq. G.9

$$m_{\nu_\mu} \left(\frac{m_e}{m_\mu}\right)^2 < 6.3 \text{ eV/c}^2 \tag{G.10}$$

$$m_{\nu_\tau} \left(\frac{m_e}{m_\tau}\right)^2 < 2.6 \text{ eV/c}^2 \tag{G.11}$$

to be compared to $m_{\nu_e} \lesssim 10$ eV/c^2.

There are also astrophysical and cosmological limits on m_{ν_τ} (Kolb and Turner 1990, Harari and Nir 1987, Grifols and Massó 1990, Gaemers *et al.* 1989, Turner 1992). For example, with some assumptions, including $m_{\nu_\tau} \lesssim 1$ MeV/c^2

$$m_{\nu_\tau} \lesssim 100 \text{ eV/c}^2 \quad . \qquad (G.12)$$

G.2 ν_τ as Dark Matter?

There have been many papers considering the possibility that the ν_τ is the hypothetical dark matter of the universe (Harari 1989, Bergström and Rubinstein 1991, McKay and Ralston 1988, Langacker 1988, Giudice 1990, Giudice 1991). For example, Harari (1989) has discussed the possibility that m_{ν_τ} lies in the range of 15–65 eV/c^2, and the use of $\nu_\mu - \nu_\tau$ oscillations to detect such a mass. Sciama (1992) has recently shown how a ν_τ mass of about 30 eV/c^2 would solve a number of problems in astrophysics. Also, Ellis *et al.* (1992) has suggested $m_{\nu_\tau} \sim 10$ eV/c^2.

G.3 ν_τ Lifetime Limits

There is <u>no</u> evidence that the ν_τ is unstable. However, if $m_{\nu_\tau} > 0$, then ν_τ might decay in a variety of ways:

$$\nu_\tau \to \gamma + \nu_x \qquad (G.13a)$$
$$\nu_\tau \to e^+ + e^- + \nu_x \qquad (G.13b)$$
$$\nu_\tau \to \nu_x + \bar{\nu}_x + \nu_y \qquad (G.13c)$$
$$\nu_\tau \to b^0 + \nu_x \quad . \qquad (G.13d)$$

In Eq. G.13d b^0 would be a boson. If the ν_τ decayed through the processes in Eqs. G.13a or G.13b, then with a sufficiently short ν_τ lifetime, T_{ν_τ}, these decays would have been seen in $e^+e^- \to \tau^+\tau^-$ events. None have been reported and I estimate this leads to a lower limit

$$T_{\nu_\tau}/m_{\nu_\tau} \gtrsim 1 \text{ sec/eV} \quad . \qquad (G.14)$$

There are much more stringent lower limits from astrophysical and cosmological consideration as summarized by Aguilar-Benitez *et al.* (1992) and by Kolb and Turner (1990). These lower limits depend upon assumptions for m_{ν_τ}. Lower limits of the order of $T_{\nu_\tau}/m_{\nu_\tau} \gtrsim 10^{15}$ sec/eV have been calculated.

The subject of possible instability of the ν_τ remains speculative.

G.4 ν_τ Weak Interactions

In earlier sections I have discussed the $\tau - W - \nu_\tau$ vertex and pointed out that all evidence, except possibly for the comparison of B_1 with $\sum_i B_{1i}$ (Sec. F.5), agrees with that vertex being conventional.

Precise studies of the invisible width in Z^0 decays (LEP Collaborations 1992) give the number of light neutrinos as

$$N_\nu = 3.00 \pm 0.05 \quad , \qquad (G.15)$$

assuming that the ν_e and ν_μ couplings to the Z are conventional, from Eq. G.15.

$$g_{\nu_\tau z \nu_\tau}/g_{\nu_e z \nu_e} \approx 1.00 \pm 0.025 \quad . \qquad (G.16)$$

Hence within present errors the $\nu_\tau - Z - \nu_\tau$ vertex is conventional.

G.5 The ν_τ and Neutrino Mixing

In this and the next section I reproduce the discussion from Perl (1992) with a few additions. At present there is no confirmed evidence for the mixing of the τ neutrino with any other neutrino (Vannucci 1992a, 1992b) The theory of neutrino mixing and oscillation is recounted well by Böehm and Vogel (1987).

The present upper limits on $\nu_e \to \nu_\tau$ and $\nu_\mu \to \nu_\tau$ mixing come from the oscillation search experiment of Ushida *et al.* (1986), Fig. 22. A general review has been given by Eichler (1987). There are proposals (Vannucci 1992a, 1992b) to FNAL and to CERN for more sensitive searches for $\nu_e \to \nu_\tau$ and $\nu_\mu - \nu_\tau$ oscillations: Kodama *et al.* (1990), Armenise *et al.* (1990), and Astier *et al.* (1991). An interesting discussion has been given by Frekers (1991) on searching for $\nu_\mu \to \nu_\tau$ oscillations using the KAON 30 GeV proton accelerator proposed for the TRIUMF laboratory.

The τ neutrino may be connected with the possible existence of a neutrino with a mass of about 17 keV/c^2, which I designate here by ν_{17}. Starting with the work of Simpson (1985) there has been some indications that the ν_{17} is produced in about 1% of the beta decays of the nuclei ^3H, ^{14}C, ^{35}S, and perhaps other nuclei. (Hime and Jelley 1991, Sur *et al.* 1991). However, at present there are also contradictory experiments which do not observe the ν_{17}. For example, Kawakami *et al.* (1992) have recently published strong evidence against the existence of the ν_{17} with a $\nu_e - \nu_{17}$ mixing probability greater than about 0.1%. Jaros (1992) has recently reviewed the question of the existence of the ν_{17} and I reproduce here his conclusions:

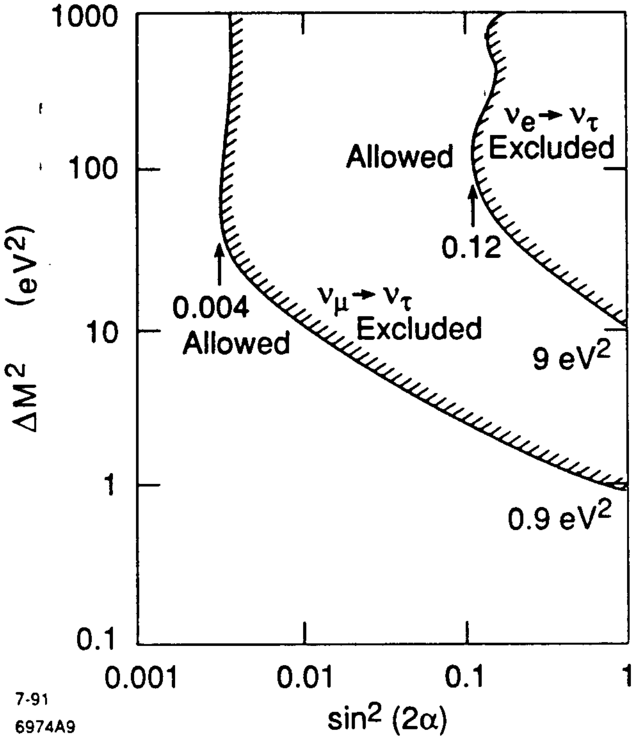

Figure 22.

"1. Massive $\bar{\nu}_{17}$ not confirmed.

2. No magnetic spectrometer experiments, including the very impressive INS-Tokyo study, shows any indication of the 17 keV neutrino.

3. Several carefully executed solid state detector experiments show unexplained special distortions.

4. Believable interpretation of these distortions as due to a 17 keV $\bar{\nu}_{17}$ awaits

   (i) demonstration that shape correction is understood

   or

   (ii) new and better experiments.

5. Many 17 keV experiments are in progress."

If the ν_{17} exists there are three hypotheses. The ν_{17} might be the ν_μ; the ν_{17} might be the ν_τ; or the ν_{17} might be a neutrino which has unconventionally small coupling to the Z^0 and hence does not contribute significantly to the invisible width of the Z^0. The limits on $\nu_e - \nu_\mu$ oscillations give an upper limit on $\nu_e - \nu_\mu$ mixing considerably below the roughly 1% mixing of $\nu_e - \nu_\tau$ given by Hime and Jelley (1991) and by Sur *et al.* (1991). Thus if the ν_{17} exists, it is the ν_τ and the ν_τ has a mass about 17 keV/c^2; or the ν_{17} does not couple like a conventional neutrino to the Z^0. In addition, if the ν_{17} is the ν_τ, $\nu_e - \nu_\tau$ oscillations should eventually be detected with approximately 1% mixing. All this depends upon whether or not the existence of the ν_{17} is confirmed.

G.6 ν_τ–Nucleon Interactions

As yet there are no experiments on the interaction of the ν_τ with matter. The study of ν_τ interactions would be directed first to the weak charged current reaction

$$\nu_\tau + N \rightarrow \tau^- + \text{hadrons} \qquad (G.17)$$

where N is a nucleon. Eventually the weak neutral current reaction

$$\nu_\tau + N \rightarrow \nu_\tau + \text{hadrons} \qquad (G.18)$$

and the weak leptonic reaction

$$\nu_\tau + e^- \rightarrow \nu_\tau + e^- \qquad (G.19)$$

might be studied. However, at present just studying Eq. G.17 is very difficult because: (a) it is necessary to produce a neutrino beam with sufficient ν_τ intensity and (b) it is difficult to identify the $\nu_\tau - N$ interaction.

The best known method for producing a neutrino beam containing ν_τ's begins with the reactions

$$\begin{aligned} p + N &\rightarrow D_s^{\pm} + \text{hadrons} \\ p + N &\rightarrow B^{\pm 0} + \text{hadrons} \quad . \end{aligned} \tag{G.20}$$

Here N means p, n or nucleus. These reactions are followed by the meson decays

$$\begin{aligned} D_s^- &\rightarrow \tau^- + \bar{\nu}_\tau \\ D_s^+ &\rightarrow \tau^+ + \nu_\tau \end{aligned} \tag{G.21a}$$

$$\begin{aligned} B^{\pm 0} &\rightarrow \tau^- + \bar{\nu}_\tau + \text{ hadrons} \\ B^{\pm 0} &\rightarrow \tau^+ + \nu_\tau + \text{ hadrons} \end{aligned} \tag{G.21b}$$

and then the τ decays

$$\begin{aligned} \tau^- &\rightarrow \nu_\tau + \text{other particles} \\ \tau^+ &\rightarrow \bar{\nu}_\tau + \text{other particles} \quad . \end{aligned} \tag{G.22}$$

This beam of ν_τ's and $\bar{\nu}_\tau$'s would also contain the other neutrinos: ν_e, $\bar{\nu}_e$, ν_μ, $\bar{\nu}_\mu$. Indeed there would be as many or more non-τ neutrinos than τ neutrinos.

The reactions

$$\begin{aligned} \nu_\tau + N &\rightarrow \tau^- + \text{ hadrons} \\ \bar{\nu}_\tau + N &\rightarrow \tau^+ + \text{ hadrons} \end{aligned} \tag{G.23}$$

would then be studied using a neutrino interaction detector with properties which allowed separation of Eq. G.23 from non-ν_τ reactions such as

$$\begin{aligned} \nu_e + N &\rightarrow e^- + \text{ hadrons} \\ \nu_e + N &\rightarrow \nu_e + \text{ hadrons} \end{aligned} \tag{G.24}$$

and so forth.

One bubble chamber experiment (Talebzadeh *et al.* 1987) used this method with 400 GeV protons interacting in a Cu target and beam dump. No ν_τ or $\bar{\nu}_\tau$ interactions were found, but the upper limit was consistent with the expected rate of such interactions assuming conventional weak interaction theory.

There have been studies for ν_τ interaction experiments using external proton beams from the Fermilab Tevatron (Hafen *et al.* 1980, Asratyan *et al.* 1980) and from the CERN SPS (Myatt 1983). But there have not been any experiments.

As discussed by De Rújula and Rückl (1984), Isaev and Tsarev (1989), Winter *et al.* (1989), Foverre (1990), and De Rújula *et al.* (1992), the higher energies of future proton accelerators and proton-proton colliders bring two substantial benefits. First the cross section for the D_s and B production reactions (Eq. G.21) increase with energy. Second, the principle proposed method for detecting

$$\nu_\tau + N \rightarrow \tau^- + \text{ hadrons}$$

and

$$\bar{\nu}_\tau + N \rightarrow \tau^+ + \text{hadrons}$$

uses the spatial separation between the primary ν_τ or $\bar{\nu}_\tau$ interaction vertex and the secondary decay vertex of the τ^- or τ^+. The larger the initial proton energy in Eq. G.20 the larger the average ν_τ and $\bar{\nu}_\tau$ energies, and hence the larger the separation between the vertices. The authors referenced at the beginning of this paragraph discuss proposed ν_τ interaction experiments, calculating expected event rates. There are two methods for accomplishing the ν_τ and $\bar{\nu}_\tau$ production (Eqs. G.20–G.22): an external proton beam interacting with nucleons in a beam dump or proton-proton collisions in a collider. Three future accelerators are considered: the Accelerator and Storage Complex at Serpukhov (UNK), the Superconducting Super Collider (SSC) and the Large Hadron Collider (LHC).

H. ACKNOWLEDGEMENT

I am very grateful to my many colleagues in tau physics research for extensive help in preparing these lectures. A great deal has been done in tau physics, and there remains even more to do.

References

Abe, K. *et al.*, 1992, *Phys. Rev. Lett.* **68**, 3398.

Adeva, B. *et al.*, 1986, *Phys. Lett.* **B179**, 177.

Adeva, B. *et al.*, 1990, *Phys. Lett.* **B250**, 205.

Aguilar-Benitez, M. *et al.*, 1990, *Phys. Lett.* **B239**, 1.

Aguilar-Benitez, M. *et al.*, 1992, *Review of Particle Properties*, in *Phys. Rev.* **D45**, Part 2.

Akrawy, M. Z. *et al.*, 1990, *Phys. Lett.* **B244**, 135.

Akrawy, M. Z. *et al.*, 1991, *Phys. Lett.* **B254**, 293.

Albajar, C. *et al.*, 1987, *Phys. Lett.* **B185**, 233.

Albrecht, H. *et al.*, 1988, *Phys. Lett.* **202B**, 149.

Albrecht, H. *et al.*, 1992a, *Phys. Lett.* **B292**, 221.

Albrecht, H. *et al.*, 1992b, *Z. Phys.* **C55**, 25.

Alitti, J. *et al.*, 1991, *Z. Phys.* **C52**, 209.

Amaglobeli, N. S. *et al.*, 1991, *Massive Neutrinos, Tests of Fundamental Symmetries* (Editions Frontiéres, Gif-sur-Yvette), ed. J. Tran Thanh Van, p. 335.

Armenise, N. *et al.*, 1990, CERN Proposal CERN-SPSC/90-42, unpublished.

Asratyan, A. *et al.*, 1980, FNAL Proposal E-646 (Fermi Nat. Accel. Lab., Batavia), unpublished.

Astier, P. *et al.*, 1991, CERN Proposal CERN-SPSC/91-21, unpublished.

Avilez, C. *et al.*, 1978, *Lett. Nuovo Cimento* **21**, 301.

Avilez, C. *et al.*, 1979, *Phys. Rev.* **D19**, 2214.

Bacino, W. *et al.*, 1978, *Phys. Rev. Lett.* **42**, 749.

Baconnier, Y. *et al.*, 1990, CERN/AC/90-07, unpublished.

Bai, J. Z. *et al.*, 1992, BEPC-EP-92-01, SLAC-PUB-5878, submitted to *Phys. Rev. Lett.*

Baltrusaitis, R. M. *et al.*, 1985, *Phys. Rev. Lett.* **55**, 1842.

Barish, B. C. and Stroynowski, R., 1988, *Phys. Reports* **157**, 1.

Barish, B. C. *et al.*, 1990, SLAC-PUB-5180, unpublished.

Barkov, L. M. *et al.*, 1985, Nucl. Phys. **B256**, 365.

Bartel, W. *et al.*, 1986, *Z. Phys.* **C30**, 371.

Beers, L., ed., 1989, *Proc. Tau-Charm Factory Workshop*, SLAC-REPORT-343 (SLAC, Stanford).

Behrend, H. J. *et al.*, 1989, *Phys. Lett.* **B222**, 163.

Berger, E. L. and Lipkin, H. J., 1987, *Phys. Lett.* **B189**, 226.

Bergström, L. and Rubinstein, H. R., 1991, *Phys. Lett.* **B253**, 168.

Boehm, F. and Vogel, P., 1987, *Physics of Massive Neutrinos* (Cambridge: Cambridge Univ. Press).

Bottcher, C. and Strayer, M. R., 1990, *J. Phys. G: Nucl. Part. Phys.* **16**, 975.

Bramon, A. *et al.*, 1987, *Phys. Lett.* **B196**, 543.

Britton, D., 1992, *Proc. Second Workshop on Tau Lepton Physics* (The Ohio State Univ., Columbus), ed. K. K. Gan, to be published.

Brown, K. L. *et al.*, 1989, *Proc. Tau-Charm Factory Workshop*, ed. L. Beers, SLAC-REPORT-343 (SLAC, Stanford), p. 244.

Burchat, P. R., 1988, *Proc. SIN Spring School on Heavy Flavour Physics* (Zuoz, Switzerland), p. 125.

Buskulic, D. *et al.*, 1992, CERN-PPE/92-184, submitted to *Phys. Lett. B.*

Ching, C. H. and Oset, E., 1991, *Phys. Lett.* **B259**, 239.

Cowan, D., 1992, *Proc. Second Workshop on Tau Lepton Physics* (The Ohio State Univ., Columbus), ed. K. K. Gan, to be published.

Danilov, M. V. *et al.*, 1990, ITEP-90-67, unpublished.

Davier, M., 1991, *Les Rencontres de Physique de la Vallée d'Aoste*, ed. M. Greco (Editions Frontiéres, Gif-sur-Yvette), p. 595.

Davier, M. and Jean-Marie, B., 1991, *Proc. Workshop on Tau Lepton Physics* (Editions Frontiéres, Gif-sur-Yvette).

Davier, M., 1992, *Proc. Second Workshop on Tau Lepton Physics* (The Ohio State Univ., Columbus), ed. K. K. Gan, to be published.

Decamp, D. *et al.*, 1990, *Phys. Lett.* **B236**, 501.

del Aguila, F. *et al.*, 1991, *Phys. Lett.* **B271**, 256.

De Rújula, A. and Rückl, R., 1984, *Proc. Large Hadron Collider in the LEP Tunnel*, CERN 84-10, Vol. II, ed. M. Jacob (CERN, Geneva), p. 573.

De Rújula, A. *et al.*, 1992, CERN-TH-6452, IFAE-92/001.

Drell, P., 1992, *Proc. 26th Int. Conf. High Energy Physics*, to be published.

Drell, S. D., 1958, Ann. Phys. **4**, 75.

Eichler, R., 1987, *Proc. 1987 Int. Symp. Lepton and Photon Interactions at High Energies* (Amsterdam: North Holland), eds. W. Bartel and R. Rückl, p. 389.

Eichten, E., Lane, K. and Peskin, M., 1983, *Phys. Rev. Lett.* **50**, 811.

Ellis, J. *et al.*, 1992, CERN-TH-6569/92, CTP-TAMU-53/92.

Feynman, R. P., 1949, *Phys. Rev.* **76**, 769.

Foverre, P. F., 1990, *Phys. Rev.* **D41**, 322.

Frekers, D., 1991, *Proc. Lake Louise Sixth Winter Inst. Particle Physics - The Factory Era*, eds. B. A. Campbell, A. N. Kamal, P. Kitching, F. C. Khanna (Singapore: World Scientific), p. 361.

Gaemers, K. J. F. *et al.*, 1989, *Phys. Rev.* **D40**, 309.

Galik, R., 1992, *Proc. Second Workshop on Tau Lepton Physics* (The Ohio State Univ., Columbus), ed. K. K. Gan, to be published.

Gan, K. K. and Perl, M. L., 1988, *Int. J. Mod. Phys.* **A3**, 351.

Gilman, F. J. and Rhie, S. H., 1985, *Phys. Rev.* **D31**, 1066.

Giudice, G. F., 1990, *Phys. Lett.* **B251**, 460.

Giudice, G. F., 1991, *Mod. Phys. Lett.* **A6**, 851.

Gomez-Cadenas, J. J., Heusch, C. A. and Seiden, A., 1989, *Particle World* **1**, 10.

Gomez-Cadenas, J. J. and Gonzales-Garcia, M. C., 1989, *Phys. Rev.* **D39**, 1370.

Gomez-Cadenas, J. J. *et al.*, 1990a, *Phys. Rev.* **D41**, 2179.

Gomez-Cadenas, J. J. *et al.*, 1990b, *Phys. Rev.* **D42**, 3093.

Gomez-Cadenas, J. J. *et al.*, 1991, *Phys. Rev. Lett.* **66**, 1007.

Gonichon, J. *et al.*, 1990, LAL/RT 90-02, unpublished.

Grifols, J. A. and Massó, E., 1990, *Phys. Lett.* **B242**, 77.

Hafen, E. S. *et al.*, 1980, FNAL Proposal E-636 (Fermi Nat. Accel. Lab., Batavia), unpublished.

Harari, H. and Nir, Y., 1987, *Nucl. Phys.* **B292**, 251.

Harari, H., 1989, *Phys. Lett.* **B216**, 413.

Hayes, K. G. and Perl, M. L., 1988, *Phys. Rev.* **D38**, 3351.

Hayes, K. G. *et al.*, 1989, *Phys. Rev.* **D39**, 274.

Hayes, K. G., 1992a, *Proc. Second Workshop on Tau Lepton Physics* (The Ohio State Univ., Columbus), ed. K. K. Gan, to be published.

Hayes, K. G., 1992b, *Phys. Rev.* **D45**, Part 2, p. VI.19.

Hime, A. and Jelley, N. A., 1991, *Phys. Lett.* **B257**, 441.

Isaev, P. S. and Tsarev, V. A., 1989, *Sov. J. Part. Nucl.* **20**, 419.

Jaros, J., 1992, *Proc. Trieste Workshop on Search for New Elementary Particles: Status and Prospects* (Singapore: World Scientific), eds. L. Beers, G. Herten, M. L. Perl, to be published.

Jowett, J. M., 1987, CERN LEP-TH/87-56, unpublished.

Jowett, J. M., 1988, *Proc. European Particle Accelerator Conf.*, ed. S. Tazzari (Singapore: World Scientific), p. 177.

Jowett, J. M., 1989, *Proc. Tau-Charm Factory Workshop*, ed. L. Beers, SLAC-REPORT-343 (SLAC, Stanford), p. 7.

Kawakami, H. *et al.*, 1992, *Phys. Lett.* **B287**, 45.

Khoze V., 1992, private communication.

Kiesling, C., 1988, *High Energy Electron-Positron Physics*, eds. A. Ali and P. Söding (Singapore: World Scientific), p. 177.

Kinoshita, T. and Sirlin, A., 1959, *Phys. Rev. Lett.* **2**, 177.

Kirkby, J., 1987, *Spectroscopy of Light and Heavy Quarks*, eds. U. Gastaldi, R. Klapisch and F. Close (New York: Plenum), p. 401.

Kirkby, J., 1989, *Particle World* **1**, 27.

Kirkby, J. and Quesada, M., 1992, *Proc. Mtg. on Tau-Charm Factory Detector and Machine* (Universidad de Sevilla, Andalucia).

Kirkby, J. and Rubio J. A, 1992, *Particle World* **3**, 77.

Kodama, K. *et al.*, 1990, Fermilab Proposal P803, unpublished.

Kolb, E. W. and Turner, M. S., 1990, *The Early Universe* (Reading: Addison Wesley).

Kühn, J. H. and Santamaria, A., 1990, *Z. Phys.* **C48**, 445.

Landau, L. D. and Lifshitz, E. M., 1958, *Quantum Mechanics - Non-Relativistic Theory* (Reading: Addison-Wesley), Section 113.

Langacker, P., 1988, *New Directions in Neutrino Physics at Fermilab* (Fermi Nat. Accel. Lab., Batavia), p. 95.

LEP Collaborations, 1992, *Phys. Lett.* **B276**, 247.

Leroy, C. and Pestieau, J., 1978, *Phys. Lett.* **B72**, 398.

Marciano, W. J. and Sirlin, A., 1988, *Phys. Rev. Lett.* **61**, 1815.

Marsiske, H., 1992, *Proc. Second Workshop on Tau Lepton Physics* (The Ohio State Univ., Columbus), ed. K. K. Gan, to be published.

McKay, D. W. and Ralston, J. P., 1988, *Neutrino 88*, eds. J. Schneps, T. Kafka, W. A. Mann and P. Nath (Singapore: World Scientific), p. 717.

Mendel, R. R. *et al.*, 1986, *Z. Phys.* **C32**, 517.

Mistry, N., 1992, *Proc. Second Workshop on Tau Lepton Physics* (The Ohio State Univ., Columbus), ed. K. K. Gan, to be published.

Moffat, J. W., 1975, *Phys. Rev. Lett.* **35**, 1605.

Morley, P. D., 1992, Preprint-92-0080, Dept. of Physics, Univ. of Texas-Austin.

Myatt, G., 1983, *Proc. Workshop on SPS Fixed-Target Physics in the Years, 1984-1989* CERN 83-02, Vol. II, ed. I. Mannelli (Geneva, CERN), p. 121.

Perl, M. L., 1991, *Proc. Lake Louise Sixth Winter Inst. on Particle Physics - The Factory Era*, eds. B. A. Campbell, A. N. Kamal, P. Kitching and F. C. Khanna (Singapore: World Scientific), p. 812.

Perl, M. L., 1992, *Reports Progress Phys.* **55**, 653.

Pich, A., 1987, *Phys. Lett.* **B196**, 561.

Pich, A., 1990a, *Proc. 17th Int. Meeting Fundamental Physics*, eds. M. Aguilar-Benitez and M. Cerrada (Singapore: World Scientific), p. 323.

Pich, A., 1990b, *Mod. Phys. Lett.* **A5**, 1995.

Riles, K., 1992, UM-HE-92-15, to be published in *Int. J. Mod. Phys. A.*

Savoy-Navarro, A., 1991, *Proc. Workshop on Tau Lepton Physics* (Editions Frontiéres, Gif-sur-Yvette), eds. M. Davier and B. Jean-Marie.

Schindler, R. H., 1989, *Les Rencontres de Physique de la Vallée d'Aoste*, ed. M Greco (Editions Frontiéres, Gif-sur-Yvette), p. 89.

Schindler, R. H., 1990a, *12th Int. Workshop on Weak Interactions and Neutrinos*, eds. P. Singer and G. Eilam (Amsterdam: North Holland), p. 89.

Schindler, R. H., 1990b, *Proc. Lake Louise Winter Inst. on the Standard Model and Beyond*, ed. A. Astbury *et al.*, (Teaneck: World Scientific), p. 217.

Sciama, D., 1992, private communication.

Simpson, J. J., 1985, *Phys. Rev. Lett.* **54**, 1891.

Snow, S., 1992, *Proc. Second Workshop on Tau Lepton Physics* (The Ohio State Univ., Columbus), ed. K. K. Gan, to be published.

Strobel, G. L. and Wills, E. L., 1983, Phys. Rev. **D28**, 2191.

Sur, B. *et al.*, 1991, *Phys. Rev. Lett.* **66**, 2444.

Talebzadeh, M. *et al.*, 1987, *Nucl. Phys.* **B291**, 503.

Trischuk, W., 1992, *Proc. Second Workshop on Tau Lepton Physics* (The Ohio State Univ., Columbus), ed. K. K. Gan, to be published.

Truong, T. N., 1984, *Phys. Rev.* **D30**, 1509.

Tsai, Y. S., 1971, *Phys. Rev.* **D4**, 2821.

Tsai, Y. S., 1979, SLAC-PUB-2356.

Tsai, Y. S., 1989a, *Proc. Tau-Charm Factory Workshop*, ed. L. Beers, SLAC-REPORT-343 (SLAC, Stanford), p. 387.

Tsai,Y. S., 1989b, *Proc. Tau-Charm Factory Workshop*, ed. L. Beers, SLAC-REPORT-343 (SLAC, Stanford), p. 394.

Tsai, Y. S., 1992a, private communication.

Tsai, Y. S., 1992b, private communication.

Turner, M. S., 1992, *Proc. Second Workshop on Tau Lepton Physics* (The Ohio State Univ., Columbus), ed. K. K. Gan, to be published.

Ushida, N. *et al.*, 1986, *Phys. Rev. Lett.* **57**, 2897.

Vannucci, F., 1992a, *Proc. Trieste Workshop on Search for New Elementary Particles: Status and Prospects* (Singapore: World Scientific), eds. L. Beers, G. Herten, M. L. Perl, to be published.

Vannucci, F., 1992b, *Proc. Second Workshop on Tau Lepton Physics* (The Ohio State Univ., Columbus), ed. K. K. Gan, to be published.

Wermes, N., 1992, *Sov. J. Nucl. Phys.* **55**, 878.

Winter, K. *et al.*, 1989, Protvino UNK Workshop.

Wu, D. Y., 1990a, Ph.D. Thesis, California Inst. Tech. CALT-68-1638, unpublished.

Wu, D. Y. *et al.*, 1990b, *Phys. Rev.* **D41**, 2339.

Zachos, C. K. and Meurice, Y., 1987, *Mod. Phys. Lett.* **A2**, 247.