THE SPIN STRUCTURE FUNCTION \( g_2 \)

Stephen Rock for the Real Photon Collaboration

University of Mass, Amherst MA 01003

Abstract. We have measured the spin structure functions \( g_2 \) over the kinematic range \( 0.02 \leq x \leq 0.8 \) and \( 0.7 \leq Q^2 \leq 20 \text{ GeV}^2 \) by scattering 29.1 and 32.3 GeV longitudinally polarized electrons from transversely polarized NH\(_3\) and \(^6\)LiD targets. Our measured \( g_2 \) approximately follows the twist-2 Wandzura-Wilczek calculation. The twist-3 reduced matrix elements \( d_2^p \) and \( d_2^n \) are less than two standard deviations from zero. The data are inconsistent with the Burkhardt-Cottingham sum rule. The Efremov-Leader-Teryaev integral is consistent with zero within our measured kinematic range.

The deep-inelastic spin structure functions of the nucleons, \( g_1(x, Q^2) \) and \( g_2(x, Q^2) \), depend on the spin distribution of the partons and their correlations. The function \( g_1 \) can be primarily understood in terms of the quark parton model (QPM) and perturbative QCD with higher twist terms at low \( Q^2 \). The function \( g_2 \) is of particular interest since it has contributions from quark-gluon correlations and other higher twist terms at leading order in \( Q^2 \) which cannot be described perturbatively. By interpreting \( g_2 \) using the operator product expansion (OPE) [1, 2], it is possible to study contributions to the nucleon spin structure beyond the simple QPM.

The structure function \( g_2 \) can be written [3]:

\[
g_2(x, Q^2) = g_{2W}(x, Q^2) + \bar{g}_2(x, Q^2)
\]

where

\[
g_{2W}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(y, Q^2)}{y} \, dy,
\]

\[
\bar{g}_2(x, Q^2) = -\int_x^1 \frac{\partial}{\partial y} \left( \frac{m}{M} h_T(y, Q^2) + \xi(y, Q^2) \right) \frac{dy}{y},
\]

\( x \) is the Bjorken scaling variable and \( Q^2 \) is the absolute value of the virtual photon four-momentum squared. The twist-2 term \( g_{2W} \) was derived by Wandzura and Wilczek [4] and depends only on \( g_1 \). The function \( h_T(x, Q^2) \) is an additional twist-2 contribution [3, 5] that depends on the transverse polarization density. The \( h_T \) contribution to \( \bar{g}_2 \) is suppressed by the ratio of the quark to nucleon masses \( m/M \) [5] and its effect is thus small for up and down quarks. The twist-3 part (\( \xi \)) comes from quark-gluon correlations. Low-precision measurements of \( g_2 \) exist for the proton and deuteron [6, 7, 8], as well as for the neutron [9, 10]. Here, we report new, precise measurements of \( g_2 \) for the proton and deuteron.
Electron beams with energies of 29.1 and 32.3 GeV and longitudinal polarization $P_b = (83.2 \pm 3.0)\%$ struck approximately transversely polarized NH$_3$ [12] (average polarization $< P_t > = 0.70$) or $^6$LiD ($< P_t > = 0.22$) targets. The beam helicity was randomly chosen pulse by pulse. Scattered electrons were detected in three independent spectrometers centered at 2.75°, 5.5°, and 10.5°. The two small-angle spectrometers were the same as in SLAC E155 [11], while the large-angle spectrometer had additional hodoscopes and a more efficient pre-radiator shower counter. Further information on the experimental apparatus can be found in references [11, 12, 13]. The approximately equal amounts of data taken with the two beam energies and opposites signs of target polarization gave consistent results.

The measured asymmetry, $\tilde{A}_\perp$, differs from $A_\perp$ because the target polarizations were not exactly perpendicular to the beam line. We determined $\tilde{A}_\perp$ using:

$$\tilde{A}_\perp = \frac{1}{f_{RC}} \left[ C_1 f_{P_t} \left( \frac{A_{raw}}{P_b} - A_{EW} \right) + C_2 \sigma_p A_p \right] + A_{RC}$$

where $A_{raw}$ is the measured counting rate asymmetry from the two beam helicities, including small corrections for pion and charge symmetric backgrounds, dead-time and tracking efficiency, and $A_{EW}$ is the electroweak asymmetry. The target dilution factor, $f$, is the fraction of free polarizable protons ($\approx 0.13$) or deuterons ($\approx 0.18$). $C_1$ and $C_2$ are nuclear corrections. The quantities $f_{RC}$ and $A_{RC}$ are radiative corrections determined using a method similar to E143 [12]. The detailed results for $A_{\perp}$ are shown in Ref. [14].

The multiplicative uncertainties due to target and beam polarization and dilution factor combined are 5.1% (proton) and 6.2% (deuteron). are small compared to the statistical errors. We determined $g_2(x, Q^2)$ from $\tilde{A}_\perp$ (dominant contribution) and the previously measured $g_1$.

The data cover the kinematic range $0.02 \leq x \leq 0.8$ and $0.7 \leq Q^2 \leq 20$ GeV$^2$ with an average $Q^2$ of 5 GeV$^2$. Tables of the complete results are in Ref. [14]. Figure 1 (left) shows the values of $xg_2$ as a function of $Q^2$ for several values of $x$ along with results from E143 [12] and E155 [8]. The systematic error on $xg_2$ is much smaller than the statistical error. The former includes the systematic errors on $A_\perp$, the 5% normalization uncertainty of $g_1$, the 2% uncertainty of $F_2$, and the systematic errors of $R$. The data approximately follow the $Q^2$ dependence of $g_{WW}^2$ (solid curve), although for the proton, the data points are slightly lower than $g_{WW}^2$ at low and intermediate $x$, and higher at high $x$. The predictions of Stratmann [15] are closer to the data.

We obtained values at the average $Q^2$ for each $x$ bin by using the $Q^2$-dependence of $g_{WW}^2$. Figure 1 (right) shows the averaged $xg_2$ of this experiment. The figure also has $xg_{WW}^2$ calculated using our parameterization of $g_1$. The combined new data for $p$ disagree with $g_{WW}^2$ with a $\chi^2$/dof of 3.1 for 10 degrees of freedom. For $d$ the new data agree with $g_{WW}^2$ with a $\chi^2$/dof of 1.2 for 10 dof. The data for $g_2^p$ are inconsistent with zero ($\chi^2$/dof=15.5) while $g_2^d$ differs from zero only at $x \sim 0.4$. Also shown in Fig. 1 (right) is the bag model calculation of Stratmann [15] which is in good agreement with the data, chiral soliton models calculations [16, 17] which are too negative at $x \sim 0.4$, and the bag model calculation of Song [5] which is in clear disagreement with the data.
FIGURE 1. LEFT) $x g_2^p$ and $x g_2^d$ as a function of $Q^2$ for selected values of $x$ from this experiment (solid), E143 [12] (open diamond) and E155 [8] (open square). Errors are statistical, the systematic errors are small. The curves show $x g_{WW}^p$ (solid) and the bag model of Stratmann [15] (dash-dot).

RIGHT) The $Q^2$-averaged structure function $x g_2$ from this experiment (solid circle), E143 [7] (open diamond) and E155 [8] (open square). The errors are statistical; systematic errors are shown as the width of the bar at the bottom. Also shown is our twist-2 $g_{WW}^p$ at the average $Q^2$ of this experiment at each value of $x$ (solid line), the bag model calculations of Stratmann [15] (dash-dot-dot) and Song [5] (dot) and the chiral soliton models of Weigel and Gamberg [16] (dash dot) and Wakamatsu [17] (dash).

The OPE allows us to write the hadronic matrix element in deep-inelastic scattering in terms of a series of renormalized operators of increasing twist [1, 2]. The moments of $g_1$ and $g_2$ for even $n \geq 2$ at fixed $Q^2$ can be related to twist-3 reduced matrix elements, $d_n$, and higher-twist terms which are suppressed by powers of $1/Q$. Neglecting quark mass terms:

$$d_n = 2 \frac{n + 1}{n} \int_0^1 dx x^n \langle \bar{g}_2(x, Q^2) \rangle.$$ 

The matrix element $d_n$ measures deviations of $g_2$ from the twist-2 $g_{WW}^p$ term. Note that some authors [2, 18] define $d_n$ with an additional factor of two. We calculated $d_2$ with the assumption that $\bar{g}_2$ is independent of $Q^2$ in the measured region. This is not unreasonable since $d_2$ depends only logarithmically on $Q^2$ [1]. The part of the integral for $x$ below the measured region was assumed to be zero because of the $x^2$ suppression. For $x \geq 0.8$ we used $\bar{g}_2 \propto (1 - x)^m$ where $m=2$ or 3, normalized to the data for $x \geq 0.5$. Because $\bar{g}_2$ is small at high $x$, the contribution was negligible for both cases. We obtained values of $d_2^p = 0.0025 \pm 0.0016 \pm 0.0010$ and $d_2^d = 0.0054 \pm 0.0023 \pm 0.0005$ at an average $Q^2$ of 5 GeV$^2$. We combined these results with those from SLAC experiments on the neutron (E142 [9] and E154 [10]) and proton and deuteron (E143 [12] and E155 [8]).
to obtained average values $d_2^p = 0.0032 \pm 0.0017$ and $d_2^n = 0.0079 \pm 0.0048$. These are consistent with zero (no twist-3) to within two standard deviations. The values of the 2nd moments alone are: \( \int_0^1 dx x^2 g_2(x, Q^2) = -0.0072 \pm 0.0005 \pm 0.0003 \) (p) and $-0.0019 \pm 0.0007 \pm 0.0001$ (d).

Figure 2 shows the experimental values of $d_2^p$ and $d_2^n$ plotted along with theoretical models from left to right: bag models [5, 15, 19], QCD Sum Rules [20, 21, 22], Lattice QCD [18] and chiral soliton models [16, 17]. The Burkhardt-Cottingham (BC) sum rule [24] for $g_2$ at large $Q^2$, \( \int_0^1 g_2(x) dx = 0 \), was derived from virtual Compton scattering dispersion relations. It does not follow from the OPE since $n = 0$. Its validity depends on the lack of singularities for $g_2$ at $x = 0$, and a dramatic rise of $g_2$ at low $x$ could invalidate the sum rule. We evaluated the BC integral in the measured region of $0.02 \leq x \leq 0.8$ at $Q^2 = 5$ GeV$^2$. The results for the proton and deuteron are $-0.044 \pm 0.008 \pm 0.003$ and $-0.008 \pm 0.012 \pm 0.002$ respectively. Averaging with the E143 and E155 results which cover a slightly more restrictive $x$ range gives $-0.042 \pm 0.008$ and $-0.006 \pm 0.011$. This does not represent a conclusive test of the

FIGURE 2. The twist-3 matrix element $d_2$ for the proton and neutron from the combined data from this and other SLAC experiments (E142 [9], E143 [12], E154 [10] and E155 [8]) (DATA). The region between the dashed lines indicates the experimental errors. Also shown are theoretical model values from left to right: bag models [5, 15, 19], QCD Sum Rules [20, 21, 22], Lattice QCD [18] and chiral soliton models [16, 17].
sum rule because the behavior of $g_2$ as $x \to 0$ is not known. However, if we assume that $g_2 = g_2^{WW}$ for $x < 0.02$, and use the relation $\int_0^x g_2^{WW}(y)dy = x[g_2^{WW}(x) + g_1(x)]$, there is an additional contribution of 0.020 (p) and 0.004 (d). This leaves a $\sim 2.8\sigma$ deviation from zero for the proton.

The Efremov-Leader-Teryaev (ELT) sum rule [25] involves the valence quark contributions to $g_1$ and $g_2$: $\int_0^1 x[g_1^p(x) + 2g_2^p(x)]dx = 0$. If the sea quarks are the same in protons and neutron this becomes $\int_0^1 x[g_1^n(x) + 2g_2^n(x)]dx = 0$. We evaluated this ELT integral in the measured region using the fit to $g_1$. The result at $Q^2 = 5$ GeV$^2$ is $-0.013 \pm 0.008 \pm 0.002$, which is consistent with the expected value of zero. Including the data of E143 [12] and E155 [8] leads to $-0.011 \pm 0.008$. The extrapolation to $x=0$ is not known, but is suppressed by a factor of $x$.

The values of the 1st moments at $Q^2 = 5$ GeV$^2$ are: $\int_0^1 dx xg_2(x, Q^2) = -0.0157 \pm 0.0012 \pm 0.0005$ (p) and $-0.0037 \pm 0.0016 \pm 0.0002$ (d).

In summary, our results for $g_2$ follow approximately the twist-2 $g_2^{WW}$ shape, but deviate significantly at some values of $x$. The twist-3 matrix elements $d_2$ are less than two standard deviations from zero. The data over the measured range are inconsistent with the BC sum rule and consistent with the ELT integral.

REFERENCES