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Supersymmetry: the Next Spectroscopy

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ABSTRACT

I describe the picture by which supersymmetry—the possible symmetry of Nature that converts fermions to bosons and vice versa—accounts for the next stage of physics beyond the Standard Model. I then survey the future experimental program implied by this theory, in which the spectrum of particles associated with supersymmetry will be determined with precision.

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1 Introduction

This lecture is a contribution to the celebration of the centenary of Werner Heisenberg. Heisenberg was one of the greatest physicists of the twentieth century, the man responsible for the crucial breakthrough that led to the final formulation of quantum mechanics. The organizers of this Symposium have asked me to look ahead to the physics of the twenty-first century in the spirit of Heisenberg.

This is a daunting assignment, and not just for the obvious reasons. The current period in our understanding of microphysics could not be more different from the period of ferment which led to the breakthrough of 1925. Today, we have a 'Standard Model' of strong, weak, and electromagnetic interactions that describes the major facts about elementary particle interactions with great precision. The Standard Model has major problems, but these are mainly conceptual. This contrasts markedly with the great periods of revolution in physics, when concrete experimental data presented phenomena that could not be explained by the classical theory of the time or by its simple variants.

Nothing illustrates this better than the achievement of Werner Heisenberg. In 1925, classical atomic theory was beset by conceptual difficulties. Neither classical mechanics nor its direct modification by Einstein and Bohr could explain why the atom was stable against radiation and collapse, or what actually happened to an electron in the process of making a quantum transition. Heisenberg was concerned with these issues, but his main energies went to problems of a very different kind. He wanted to find the mathematical description of concrete new phenomena that were emerging from the study of atomic spectra—the anomalous Zeeman effect, the dispersion of light in media and its association with atomic resonances. It is an odd and striking fact that in the fall of 1925, when Heisenberg had already made the breakthrough of defining and solving the quantum-mechanical harmonic oscillator but did not yet appreciate the generality of his new theory, he lectured at Cambridge not on his new mechanics but instead on the subject 'Termzoologie und Zeemanbotanik' [1]. This zoological classification of the details of atomic spectra had been Heisenberg's main preoccupation since the beginning of his undergraduate studies. After the structure of quantum mechanics had become clear, Heisenberg put the theory to the test against these same problems and found its success in clarifying details of spectroscopy that were otherwise inexplicable, most notably, the spectra of ortho- and para-Helium [2]. It was out of this struggle to find patterns in spectroscopy that Heisenberg's quantum theory was born.

Today, some physicists talk about finding a 'theory of everything' that will unite the interactions of microphysics with gravity and explain the various types of elementary particles found in Nature. The approach is intriguing, but I am skeptical about

it. We have a long way to go toward this ultimate theory. It is likely that it lies on the other side of another era of experimental confusion, of crisis and resolution. Instead of asking about final unification, we should be asking a different question: Where will the next crisis in fundamental physics come from, and how can we help it come more rapidly?

This question is increasingly pressing as we move into the twenty-first century. We have left behind long ago the era in which it is possible to probe new domains of physics with a tungsten wire and a Bunsen burner. Today, probes beyond the known realms of physics require giant accelerators, huge telescopes, massive detectors. We ask governments and the public to pay for these endeavors, at the level of billions of dollars or euros. They, in turn, ask for an increasingly concrete picture of what we intend to explore and what insights we will bring back.

In this lecture, I would like to describe a path we might take to the next corpus of data that could overturn our current physical pictures. Any such story is to some extent speculative, or else completely uninteresting. But despite some speculative jumps, I hope you will find this story plausible and even compelling. I believe that there is a path to an era when we will be challenged by data to make a revolution in physics, perhaps even one as profound as Heisenberg's. The crucial element in this path is the appearance of *supersymmetry* in high-energy physics.

2 Triumphs and problems of the Standard Model

Before explaining why supersymmetry is important, or even what it is, I would like to recall the status of our current understanding of elementary particle physics. In 1925, there were only three elementary particles known, the electron, the proton, and the photon. By the last decade of Heisenberg's life, the three interactions of subatomic physics—the strong, weak, and electromagnetic interactions—were clearly delineated. However, the first two of these were still mysterious. For the strong interactions, bubble chamber experiments were turning up hundred of new particles that needed classification. For the weak interactions, the property of parity violation had been discovered but its ultimate origin remained unknown.

Today, the situation has been clarified almost completely. The hundreds of strongly interacting particles are now understood to be bound states of more elementary fermions, called 'quarks'. Three varieties of fermions with charge -1 are known, the electron, muon, and tau, each accompanied by a species of neutrino. These 'leptons' share with the quarks a very simple structure of couplings to heavy spin-1 bosons that accounts for their weak interactions. All three interactions of elementary particle physics, in fact, are known to be mediated by spin-1 particles. The equations

of motion for these particles are known to have the form of generalized Maxwell equations with couplings representing the actions of a fundamental group of symmetries. This set of equations is called a 'Yang-Mills theory' [3]; the spin-1 particles described are called 'Yang-Mills bosons' or 'gauge bosons'. For the strong interactions, the Yang-Mills symmetry group is $SU(3)$; for the weak and electromagnetic interactions, which appear in a unified structure, the group is $SU(2) \times U(1)$. The resulting structure of interacting quarks, leptons, and gauge bosons is called, in a somewhat self-deprecating way, the 'Standard Model' (SM) [4].

The most important result of high-energy physics experiments in the 1990's was the detailed confirmation of the predictions of the Standard Model for all three of the interactions of elementary particle physics. Experiments at the CERN collider LEP provided the centerpiece of this program, with important contributions coming also from SLAC, Fermilab, and elsewhere. Rather than give a complete review of this program, I would like to present just one illustrative result. The SM predicts that one of the Yang-Mills bosons mediating the weak interaction is a heavy particle called the Z^0 boson. The Z^0 is a neutral particle with a mass of about 91 GeV that can appear as a resonance in e^+e^- annihilation. The resonance is a striking one: the annihilation cross section increases by a factor of about 1000. The SM predicts the width of the resonance in terms of the mass of the Z^0, the Fermi constant G_F, and the fine structure constant α. The prediction is a sum over all species into which the Z^0 can decay, that is, over all quark and lepton species with mass less than $m_Z/2$. In this way, the prediction invokes the basic structure of the weak interactions. When quarks are produced, the decay width is enhanced by a factor 3, the number of quantum states of the strong interaction group $SU(3)$, and then by an extra 4% from strong interaction dynamics in the decay process. Finally, the emission of photons by the electron and positron that create the Z^0 distorts the resonance from a simple Breit-Wigner line-shape, causing the resonance to be somewhat reduced in height and more weighted to high energies. Thus, the complete theory of the line-shape involves detailed properties of all three of the basic interactions of microphysics. In Fig. 1, I show the comparison of this theory to the experimental data of the OPAL experiment at LEP. The agreement is extraordinary. The residual difference between theory and experiment in the extracted Z^0 lifetime is at the level of parts per mil [5,6].

The success of the SM in explaining this and similar data makes a strong case for the idea that the $SU(3) \times SU(2) \times U(1)$ symmetry of the SM is an exact symmetry of the laws of Nature. First of all, we see this symmetry experimentally in the relations among the couplings of quarks and leptons to the gauge bosons which lead to the predictions such as that of Fig. 1. Second, from a theoretical viewpoint, the Yang-Mills equations of motion rely on their basic symmetry being exact; otherwise, they are actually inconsistent, leading to violations of unitarity and other severe problems.

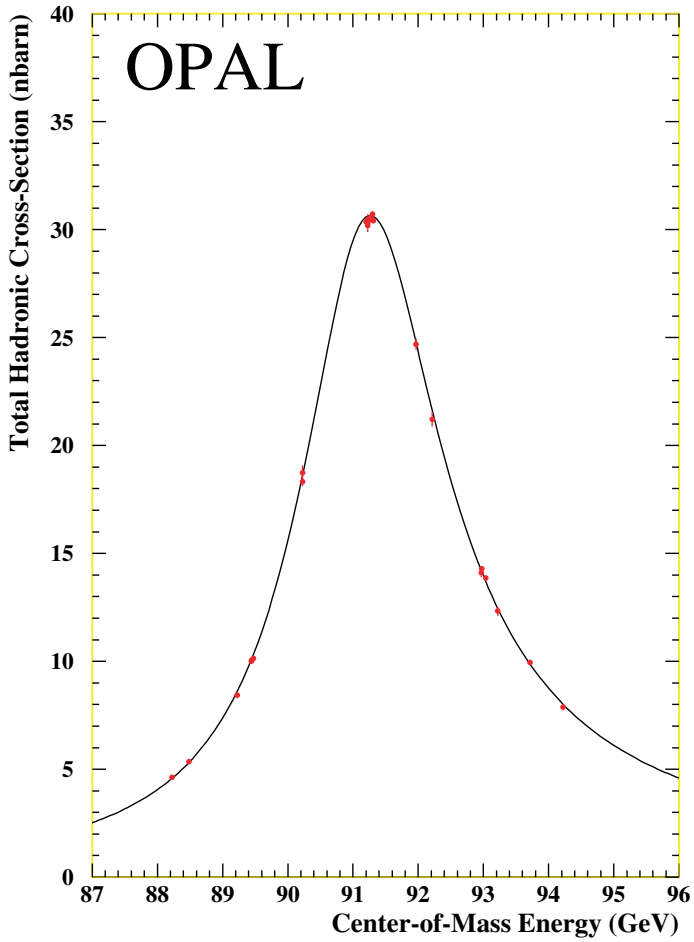

Figure 1: Comparison of theory and experiment for the line-shape of the Z^0 resonance in e^+e^- annihilation, with data from the OPAL experiment [5].

However, for the case of the weak interaction group $SU(2) \times U(1)$, the symmetry is not at all manifest in the masses of elementary particles. The Yang-Mills symmetry requires that the weak interaction bosons $W^\pm$ and Z^0 should be massless like the photon. In addition, this symmetry group assigns different quantum numbers to the left-handed and right-handed spin states of quarks and leptons. This property is actually attractive and required when applied to the couplings; it accounts for the manner in which the weak interactions violate parity. But it also forbids the appearance of quark and lepton masses.

There is a way in which symmetries of Nature can be exact and also appear broken. It is possible that the Hamiltonian can have an exact symmetry but that the ground state of this Hamiltonian might not respect this symmetry. As an example, consider

a magnet; the Hamiltonian describing the spins of electrons is rotationally invariant, but in the ground state the spins all orient in a certain direction. This situation is called 'spontaneous symmetry breaking'. Many condensed matter physics systems exhibit spontaneous symmetry breaking, including magnets, binary alloys (for which the symmetry is the lattice translation), and superfluids and superconductors (for the symmetry is the phase rotation symmetry of the atomic or electron wavefunction). In each case, some aspect of the atomic interactions causes a macroscopic degree of freedom to pick a direction with respect to the symmetry operation and sit down in such as way as to hold that orientation uniformly throughout the material.

We could imagine that the Yang-Mills symmetry of the weak interactions is spontaneously broken. But then there is a question: What entity and what physics are responsible for choosing the orientation uniformly throughout space. In the simplest realization of the SM, we postulate a new scalar field, called the 'Higgs field' φ, and give it the responsiblity for this spontaneous symmetry breaking. Very little is known about the Higgs field from experiment. The success of the SM brings this question into tight focus: What is this Higgs field? Why does it appear in Nature? Why does its energetics favor symmetry-breaking and orientation?

The mystery of the nature of the Higgs field is the most compelling single problem in elementary particle physics today. It is not unreasonable to create a model of new interactions of elementary particles simply to address this question. But there are other mysterious aspects of the SM and microphysics, and it would be good if a model that explains the Higgs field also has something to say about these. For me, the most interesting of these properties are the following:

- The heaviest particle of the SM is the top quark, with a mass much heavier than the W boson: $m_t/m_W = 2.1$ [7].
- The Higgs boson must not only exist, but it is required by the constraint of the precision electroweak data to be light [8]

$$m_h < 193 \ \text{ GeV} . \qquad (95\% \text{ CL}) \tag{1}$$

  It is possible that the Higgs boson was observed in the last year of operation of LEP, at a mass of 115 GeV [9].
- The precision experiments give quite definite values for the three gauge coupling constants of the SM. Writing $\alpha_i = g_i^2/4\pi$ with $g_i = g_1'$ for $U(1)$, g_2 for $SU(2)$, g_3 for $SU(3)$, we have found that

$$\alpha_1' = 1/98.4 \ , \qquad \alpha_2 = 1/29.6 \ , \qquad \alpha_3 = 1/8.5 \ , \tag{2}$$

  with errors of 2% for the strong interaction coupling α_3 and of 0.1% for the electroweak couplings [10].

- As explained in Michael Turner's lecture at this symposium, ordinary matter is far from being the dominant form of energy in the universe. In units where the energy density in a flat universe is $\Omega_0 \sim 3$ GeV/m^3, about 30% is composed of 'dark matter', a heavy, non-luminous, non-baryonic form of matter. And almost 70% is composed of 'dark energy', energy of the vacuum or of a new field which obtains a vacuum expectation value [11].

A theory that supercedes the SM should have a place for these phenomena.

3 Supersymmetry

The search for a framework in which to build a theory beyond the SM brings us to supersymmetry. Supersymmetry is a mathematical idea of a means to generalize quantum field theory. It was introduced in the early 1970's by Golfand and Likhtman [12], Volkov and Akulov [13] and Wess and Zumino [14]. The last of these papers, which introduced the linear representations of the symmetry on fields, opened a floodgate to theoretical developments. In this lecture, I will explain in the simplest terms what supersymmetry is, and then I will pursue its implications in a way that will link with the questions of the previous section. Broader reviews of supersymmetry can be found in many articles and books, including [15–17].

Formally, a supersymmetry is a symmetry of a quantum system that converts fermions to boson and bosons to fermions.

$$[Q_\alpha, H] = 0 \qquad Q_\alpha \left|b\right\rangle = \left|f\right\rangle \qquad Q_\alpha \left|f\right\rangle = \left|b\right\rangle \ . \tag{3}$$

In relativistic quantum field theory, bosons carry integer spin and fermions carry half-integer spin, so Q_α must have half-integer spin. The simplest case is spin-$\frac{1}{2}$. The assumption that there exists a spin-$\frac{1}{2}$ charge that commutes with H seems innocuous, but it is not.

To see this, consider the object $\{Q_\alpha, Q_\alpha^\dagger\}$. This quantity commutes with H. It carries two spinor indices; under the Lorentz group, it is a component of a four-vector. And, it is positive if Q_α is nontrivial. To see this, note that

$$\left\langle\psi\right| \{Q_\alpha, Q_\alpha^\dagger\} \left|\psi\right\rangle = \|Q_\alpha \left|\psi\right\rangle\|^2 + \|Q_\alpha^\dagger \left|\psi\right\rangle\|^2 \tag{4}$$

The presence of a supersymmetry thus implies the presence of a conserved vector charge. But this is a problem. Lorentz invariance and energy-momentum conservation already severely restricts the form of two-particle scattering amplitudes. The scattering amplitude for a fixed initial state is a function of only one continuous variable, the center-of-mass scattering angle. If there is an additional conserved charge

that transforms as a vector under Lorentz transformations, there are too many conditions for the scattering amplitude to be nonzero except at some discrete angles. In quantum field theory, the scattering amplitude must be analytic in the momentum transfer, so in such a case it can only be zero at all angles. A rigorous proof of this statement, applicable also to any conserved charge of (integer) higher spin, has been given by Coleman and Mandula [18].

Only one possibility evades the theorem: We must identify the conserved vector charge with the known conserved energy-momentum. That is,

$$\{Q_\alpha, Q^\dagger_\beta\} = 2\gamma^\mu_{\alpha\beta} P_\mu \ . \tag{5}$$

Let me put it more bluntly: If a nontrivial relativistic quantum field theory contains a supersymmetry charge Q_α, the square of this charge is *the energy-momentum of everything*. If Q_α is to be an exact symmetry of Nature, it cannot be restricted to some small part of the equations of motion. Q_α must act on every particle.

It follows from this that, in a supersymmetric theory, every particle must have a partner of same rest energy or mass and the opposite statistics. If there is a photon with spin 1, there must be a 'photino' ($\widetilde{\gamma}$) with spin $\frac{1}{2}$. If there is a W^+ boson, there must be a spin-$\frac{1}{2}$ $\widetilde{w}^+$. We have already noted that, in the SM, the left- and right-handed components of quark and lepton fields have different $SU(2) \times U(1)$ quantum numbers. This means that the basic fields of a supersymmetry SM should include separate spin-0 fields $\widetilde{e}_L$, $\widetilde{e}_R$, for example, or $\widetilde{u}_L$, $\widetilde{u}_R$. In the following, I will follow the common terminology by referring to the partners of Yang-Mills bosons as 'gauginos'—'photino', 'wino', 'zino', 'gluino'—and to the partners of quarks and leptons as 'sfermions'–'squarks', 'sleptons', 'selectrons', *etc.*

One known fact about sfermions is that they do not exist with masses equal to the masses of their partners. There is no scalar particle of charge -1 with the mass of the electron, and their is no scalar particle coupling to the $SU(3)$ gauge bosons with the mass of the u quark. Such particles might exist with higher masses, but this would require that supersymmetry is not an exact symmetry. It is possible, however, that supersymmetry, like the $SU(2) \times U(1)$ symmetry of the SM, is a spontaneously broken symmetry, an exact symmetry of the equations of motion that does not lead to a symmetrical vacuum configuration. In that case, the supersymmetry partners of the quarks, leptons, and gauge bosons could well be heavier than the familiar SM particles, but they must exist at mass values that we might eventually reach in our experiments.

If supersymmetry acts on all fields in Nature, it must also act on the gravitational field. Indeed, a supersymmetric theory that contains gravity must also contain a spin-$\frac{3}{2}$ partner of the graviton. Beginning with an apparently innocent assumption, we have learned that we must change the basic structural equations of space-time.

Figure 2: A string is a particle which is also a one-dimension quantum system.

There is another way of understanding the universal character of supersymmetry that opens another set of connections. Supersymmetry was originally discovered as a property of *string theory*, an idea that generalizes quantum field theory by modellng particles as one-dimensional extended objects embedded in space-time. The embedding is represented by a set of functions $X^\mu(\sigma)$, where σ is a coordinate along the string. Neveu, Schwarz, and Ramond [19,20] found that certain difficulties of this theory are ameliorated by adding to the string Hamiltonian a set of fermionic coordinates $\Psi^\mu(\sigma)$. (See Fig. 2.) The resulting quantum theory of fields on the string has a supersymmetry, and the theory also naturally leads to a supersymmetric theory of particles in space-time [21]. The mathematical structure is that of a string moving in a 'superspace' with both bosonic and fermionic coordinates. This structure becomes a part of the description of space-time and influences all particles that move in it. String theory is described in some detail in Joseph Polchinski's lecture at this symposium [22].

String theory is often described as the 'theory of everything'. While that statement lacks definite experimental support, string theory is a mathematical framework that successfully incorporates gravity into relativistic quantum theory. It is, in fact, the only known framework in which the weak-coupling perturbation theory for gravity is well-defined to all orders. String theory also contains interesting ideas for how gravity fits together with the elementary microscopic interactions. We will find some inspiration from these ideas at a later point in the lecture.

4 Supersymmetry as the Successor to the Standard Model

I have described supersymmetry as a mathematical refinement of quantum field theory. From this point of view, it is surprising that supersymmetry can address the questions about microscopic physics that we posed in Section 2. In fact, a construction based on adding supersymmetry straightforwardly to the SM is dramatically successful in resolving those questions. This is not the only possible picture, but it is, at this moment, the one which is most complete and compelling. In this lecture, I will describe only the approach to the questions of the SM based on supersymmetry. For a look at the variety of other proposed models of $SU(2) \times U(1)$ symmetry breaking, see [23–25]. In only a few years—at the latest, when the Large Hadron Collider

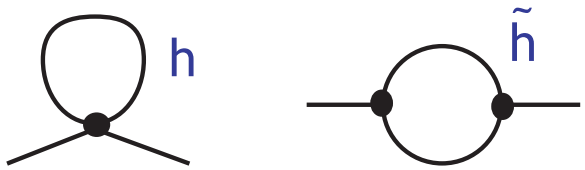

Figure 3: Feynman diagrams contributing to the first loop correction to the Higgs boson mass.

(LHC) begins operation at CERN—we will know whether this model or one of its competitors is correct.

Consider, then, the supersymmetric extension of the SM. For each boson field in the model, we add a fermion with the same quantum numbers. For each fermion, we add a boson. The interactions of these new fields are dictated by supersymmetry. To this, we must add mass terms that make the new particles heavy and other interactions that might be induced by spontaneous supersymmetry breaking. (These mass terms will have only a minor effect in this section, but they will become significant later.) Let us see what consequences this model has for the problems discussed in Section 2.

4.1 Higgs field

Consider first the question of the nature of the Higgs field, its origin and the reason for its instability to spontaneous symmetry breaking. Within the Standard Model, the Higgs field is anomalous. It is the only scalar particle and the only particle that can acquire a mass without spontaneous symmetry breaking.

At a deeper level, these curiosities of the Higgs boson turn into serious conceptual problems. The Feynman diagrams that give higher-order corrections to the Higgs boson mass are ultraviolet-divergent. As an example, consider the first diagram in Fig. 3, in which the Higgs boson interacts with its own quantum fluctuations through its nonlinear interaction. Evaluating this contribution for momenta of the virtual Higgs boson running up to a scale Λ, we find

$$m_h^2 = m_h^2(\text{bare}) + \frac{\lambda}{8\pi}\Lambda^2 + \cdots \ , \tag{6}$$

where λ is the Higgs field nonlinear coupling. If the SM is valid up to the scale where quantum gravity effects become important, this equation should be the correct first approximation to the Higgs boson mass for the value $\Lambda \sim 10^{19}$ GeV. We have already noted that m_h itself is of order 100 GeV. Thus, in the SM, the bare Higgs mass parameter and the higher-order corrections must cancel in the first 36 decimal places.

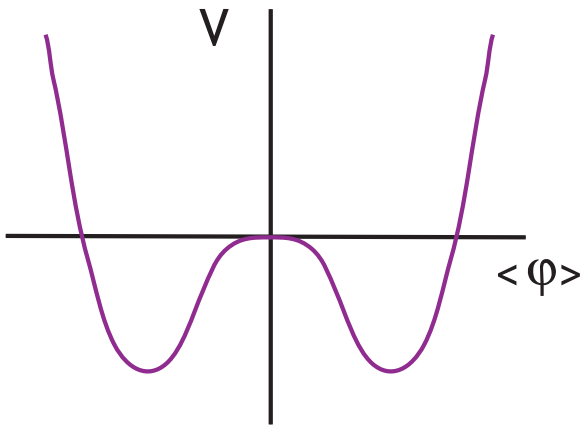

Figure 4: General form of a Higgs potential unstable to symmetry breaking.

This type of delicate cancellation is familiar from the theory of second-order phase transitions in condensed matter systems. Anyone who has experimented on a liquid-gas critical point knows that the temperature and pressure must be delicately adjusted to see the characteristic phenomena of the critical point, for example, the critical opalescence that results from density fluctuations on a scale much larger than the atomic size. In a fundamental theory of Nature, we would like this delicate adjustment to happen automatically, not as some whim of the underlying parameters.

Further, if m_h^2 is the result of such a cancellation, it is an accident that the parameter should be negative rather than positive, giving an unstable potential such as that shown in Fig. 4. But if we cannot predict the sign of m_h^2, we cannot explain why the electroweak gauge symmetry should be broken.

Supersymmetry repairs these problems one after another. First of all, supersymmetry gives a *raison d'etre* for the appearance of a scalar field. In a supersymmetric generalization of the SM, there are many scalar fields, since every quark and lepton must have a spin-0 partner. Potentially, any of these fields could acquire a vacuum expectation value and break the symmetries of the model. So we must ask why only the Higgs field has an instability. I will address this problem in a moment.

Next, we should analyze the problem of large higher-order corrections to the Higgs boson mass. In the supersymmetric SM, the calculation of m_h^2 has additional contributions. One of these is shown as the second diagram in Fig. 3: In addition to loop diagrams containing Higgs bosons, supersymmetry requires diagrams containing the spin-$\frac{1}{2}$ partners of Higgs bosons. In a theory with unbroken supersymmetry, the terms in these diagrams proportional to Λ^2 precisely cancel. This is a natural consequence of supersymmetry: In quantum field theory, chiral symmetry requires that

the higher-order corrections to the mass m_f of a fermion are of the form

$$m_f = m_f(bare) + a_f \frac{\lambda}{4\pi} m_f \log \frac{\Lambda^2}{m_f^2} \ , \tag{7}$$

where a is a numerical constant. The radiative correction to the electron mass in quantum electrodynamics, for example, has this form. By supersymmetry, the bosonic partner of this fermion must have the same mass corrections. In a theory with spontaneous supersymmetry breaking, the boson and fermion mass corrections need not be equal. However, since spontaneous symmetry breaking is a property of the lowest-energy state of the theory, it cannot affect the structure deep in the ultraviolet. Then the boson mass is still corrected only by terms of the form

$$m^2 = m^2(bare) + a \frac{\lambda}{4\pi} m^2 \log \frac{\Lambda^2}{m^2} \ , \tag{8}$$

Having established the validity of the form (8), we might next ask what is the value of the coefficient a. This question is more significant than it might appear at first sight. If a is negative, the corrected m^2 is negative if the bare value of m^2 is sufficiently smaller than Λ^2. If a is negative and the bare value of m^2 is computable from a theory of spontaneous supersymmetry breaking, we can build a quantitative theory of $SU(2) \times U(1)$ symmetry-breaking. In the supersymmetric generalization of the SM, there are a variety of contributions to a coming from the various quarks, leptons, and gauge bosons that can contribute to loop corrections to the Higgs potential. However, if the top quark is heavy, it must couple especially strongly to the Higgs field. Then this contribution to (8)—the contribution with top quarks and their scalar partners in the loop—is the dominant one. That contribution is negative, by explicit calculation, and drives the instability of the Higgs potential to spontaneous symmetry breaking. It turns out also that, for a large region of the parameter space, the Higgs is the only unstable mode among the many scalar fields of the theory.

Thus, supersymmetry gives an origin for the Higgs field. It also explains its instability to spontaneous symmetry breaking by relating this to the observed large mass of the top quark.

4.2 Coupling constants

In (2), I have reported the values of the three elementary coupling constants of the SM as determined by the recent precision experiments. Supersymmetry gives the relation among these values.

In quantum field theory, coupling constants are not absolute. They vary as a function of the distance scale on which they are measured, according to the properties of

the interaction. Again, the behavour of quantum electrodynamics (QED) provides a reference point. In QED, electron-positron pairs can appear and disappear in the vacuum as quantum fluctuations. These evanescent pairs give the vacuum state of QED dielectric properties. As one approaches a charged particle very closely, coming inside the polarization cloud, one sees a stronger charge. Since electron-positron production in the vacuum occurs on all length scales (smaller than the electron Compton wavelength), the strength of a charge in QED appears to increase systematically on a logarithmic scale of distance. More precisely, the values of $\alpha = e^2/4\pi$ at two large mass scales are related by

$$\alpha^{-1}(M) = \alpha^{-1}(M_*) - \frac{b}{2\pi} \log \frac{M}{M_*} + \cdots \ , \tag{9}$$

where b is a constant that can be straightforwardly computed using Feynman diagrams. The sign $b < 0$ corresponds to charge screening by vacuum polarization.

Similar considerations apply to the three coupling constants of the SM. All three couplings change slowly, as a logarithmic function of the mass or distance scale. In a non-Abelian gauge theory, there is a new physical effect that allows the coefficient b to be positive, so that the value of g or α decreases at very short distances or large momenta. In general, the value of b is a sum over the contributions of all particles that couple to the bosons of the gauge theory, including quarks, leptons, Higgs bosons, and, in the non-Abelian case, the gauge bosons themselves.

It is attractive to speculate that all three of the intereractions of the SM arise from a single, unified, non-Abelian gauge symmetry, called the 'grand unification' symmetry group. The splitting of the three interactions would result from the spontaneous breaking of the grand unification group to the SM gauge group $SU(3) \times SU(2) \times U(1)$. The values of the three coupling constants must be equal at the mass scale of this symmetry-breaking, but then, by the effects just explained, they will differ at larger distance scales. The coupling constant of the $U(1)$ factor, α_1, will be the smallest; the coupling of the largest non-Abelian group, the $SU(3)$ coupling α_3, will be the largest. This is just the pattern actually seen in (2).

We must now investigate whether this picture gives a quantitative explanation of the magnitudes of the three couplings. Before we begin, there is one subtlety to take care of. The normalization of the coupling constant of a non-Abelian group is unambiguous, but, for an Abelian group, this normalization is a matter of convention. The coupling

$$\alpha_1 = \frac{5}{3}\alpha_1' \tag{10}$$

is correctly normalized so that it equals α_2 and α_3 at the scale of grand unification symmetry breaking in the case of grand unification groups $SU(5)$, $SO(10)$, and E_6, the groups that are attractive candidates for the unification symmetry because their

simplest representations reproduce the quantum numbers of the SM quarks and leptons. The value of this coupling at the energies of the Z^0 experiments is $\alpha_1 = 1/59.0$.

With this convention, the hypothesis of grand unification implies that the three couplings α_1, α_2, α_3 have values at the mass scale of m_Z given in terms of a unification mass scale M_U and a corresponding unification coupling value α_U by the relation

$$\alpha_i^{-1}(m_Z) = \alpha_U^{-1} - \frac{b_i}{2\pi} \log \frac{m_Z}{M_U} + \cdots \ . \tag{11}$$

with

$$b_1 = -\frac{41}{10} \qquad b_2 = \frac{19}{6} \qquad b_3 = 7 \tag{12}$$

We can test this relation in two ways. First, we can use (11) and the precisely known values of α_1 and α_2 to compute α_U and M_U, and then use these values to compute α_3. The result is $\alpha_3 \approx 0.07$, in serious disagreement with (2). Second, we can eliminate α_U and M_U among the three relations (11), to obtain the prediction

$$\begin{aligned} B = \frac{b_3 - b_2}{b_2 - b_1} &= \frac{\alpha_3^{-1} - \alpha_2^{-1}}{\alpha_2^{-1} - \alpha_1^{-1}} \\ &= 0.717 \pm 0.008 \pm 0.03 \ , \end{aligned} \tag{13}$$

where the first error is due to the experimental determination of the values of the α_i and the second is my estimate of the theoretical error from neglect of higher-order corrections in (11) [26]. The coefficients (12) give $B = 0.528$, again, in poor agreement with the data.

The determination of α_3 from α_1 and α_2 is shown graphically as the lower set of curves in Fig. 5. A significant aspect of the calculation is that the grand unification scale turns out to be more than 10 orders of magnitude higher than the highest energy currently explored at accelerators. If new particles appear at higher energy, their contributions will change the values of the b_i. If the SM is extended by the addition of supersymmetry, and if supersymmetry partners have masses within about an order of magnitude of m_Z, the appropriate values of the b_i to use in computing the predictions of grand unification are those including the contributions from the supersymmetry partners of quarks, leptons, gauge bosons, and Higgs bosons:

$$b_1 = -\frac{33}{5} \qquad b_2 = -1 \qquad b_3 = 3 \tag{14}$$

These values give

$$B = \frac{5}{7} = 0.714 \ , \tag{15}$$

in remarkable agreement with (13). The new evaluation of α_3 is shown in Fig. 5 as the upper set of curves. The grand unification scale in this calculation is $M_U = 2 \times 10^{16}$

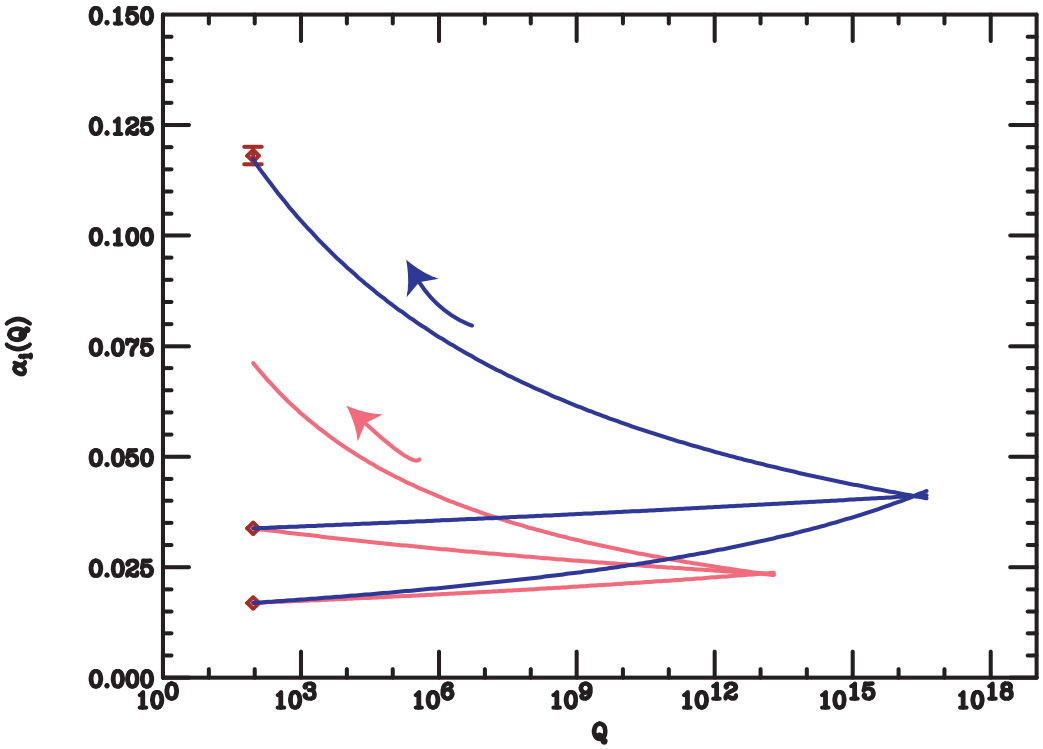

Figure 5: Determination of $\alpha_3(m_Z)$ from $\alpha_1(m_Z)$ and $\alpha_2(m_Z)$ using the grand unification of couplings in the SM and in its supersymmetric extension. The lower set of three curves uses the b_i values from the SM, the upper set those of its supersymmetric extension.

GeV, a value that is not so different (at least on a log scale) from the mass scale of quantum gravity.

The hypothesis of grand unification has implications for the properties of the Higgs boson. Like the gauge couplings, the parameters that determine the mass of the Higgs boson vary as functions of the mass scale as the result of quantum field theory corrections. The effect of the corrections is always to lower the prediction for the Higgs boson mass as the length of the extrapolation from the grand unification scale to the Z scale is increased. In a supersymmetric grand unified theory with the value of M_U just computed, it is difficult to arrange for a Higgs boson mass larger than 150 GeV. Even extensive searches have turned up no such theory in which the Higgs boson mass is larger 208 GeV [27]. This purely theoretical constraint on the Higgs boson mass corresponds nicely to the experimental constraint (1) discussed in Section 2.

4.3 Dark matter and dark energy

As I have already discussed, probes of the cosmological mass and energy distribution indicate that the energy content of the universe is close to its critical value Ω_0. About 30% of this energy is composed of nonrelativistic particles of non-baryonic matter. About 70% comes from the energy of the vacuum, or from some entity that behaves like vacuum energy on the time scales of cosmological observations.

Supersymmetry gives a natural candidate for the identity of the dark matter and a mechanism for the survival of dark matter particles from the Big Bang. Consider the quantity

$$R = (-1)^{3B-L+2J} \ . \tag{16}$$

where B is baryon number ($3B$ is quark number), L is lepton number, and J is spin. This object is constructed in such a way that all ordinary particles—leptons, baryons, mesons, gauge bosons, and even Higgs bosons—have $R = +1$. The superpartners of these particles, however, have $R = -1$. It is observed that B and L are quite good symmetries, so it is not difficult to arrange that R is conserved. Then the lightest supersymmetry partner will be absolutely stable. If this stable particle is the partner of the photon, or of the $U(1)$ gauge boson of $SU(2) \times U(1)$, it has all the properties required of a dark matter particle, being neutral, heavy, and weakly interacting.

The origin of the dark energy is more mysterious. It is difficult in any current theoretical framework to understand why the energy density of the vacuum is so small. The spontaneous breaking of $SU(2) \times U(1)$ changes the energy density of the vacuum by an amount of order $\Delta\rho \sim m_h^4$. However, the observed energy density is

$$\rho_\Lambda \sim (2 \times 10^{-14} m_h)^4 \ . \tag{17}$$

Without supersymmetry, however, no one even knows how to begin. In a non-supersymmetric theory, the energy of the vacuum is shifted by quantum corrections in an arbitrary and uncontrolled way. With supersymmetry, there is at least a natural zero of the energy. It follows from (5) that

$$H = \frac{1}{4} \mathrm{tr} \left\{ Q_\alpha, Q_\alpha^\dagger \right\} \tag{18}$$

By (4), the energy is positive, and it is zero in a state $|0\rangle$ annihilated by Q and $Q^\dagger$. If supersymmetry is spontaneously broken, the vacuum energy becomes nonzero, but at least we know in principle where the zero is.

4.4 Hints and anomalies

At any given time, the data of elementary particle physics shows some small deviations from the predictions of the SM that may or may not materialize in the

future into a real discrepancy. I would like to highlight two current anomalies that might be hints of the presence of supersymmetry.

In the last few months of the operation of LEP, events accumulated that seemed to be inconsistent with SM background and consistent with the production of a Higgs boson of mass about 115 GeV. This was a marked contrast to previous experience at LEP, in which the observed event distributions had been in excellent agreement with SM calculations. However, the final significance of the observation was only about 2 σ, statistically unconvincing [9]. (Compare, for example, [28] and [29].) I have already explained that supersymmetry typically implies a low mass for the Higgs boson. But this result is especially tantalizing because there is a stronger upper bound on the Higgs boson mass in the 'minimal' supersymmetric extension of the SM, the model with the minimum number of Higgs fields. In this model, supersymmetry constrains the Higgs field potential in such a way that the mass of the Higgs boson must be comparable to that of the Z^0. The Higgs boson mass must be less than 135 GeV, and for typical parameters the value is between 90 and 120 GeV.

The Brookhaven Muon g-2 experiment has reported a discrepancy from the SM of about 4 parts per billion [30]. In a theory in which the supersymmetry partners of the leptons and the W boson are both about 200 GeV, this is roughly the expectation for the new contribution to the muon g-2 from radiative corrections containing these supersymmetric particles. However, the status of this anomaly is still in question, because parts of the SM contribution to the muon g-2, the hadronic vacuum polarization and hadronic light-by-light scattering diagrams, are not under control at the level of parts-per-billion contributions [31,32]. As a result of this uncertainty, we can only say that the significance of the anomaly is somewhere between 1 and 3 σ.

It will be interesting to see whether these anomalies are confirmed in the next few years.

5 Beyond the Supersymmetric Standard Model

We have now seen that the addition of supersymmetry to the SM addresses many of the major questions about that model that I have posed in Section 2. For this reason, I consider it likely that supersymmetric partners of the SM particle really do exist, and that they will be discovered at accelerators before the end of the decade. But this will only be the beginning of the path to the next revolution in physics. Let us now look at what lies further down this road.

I have already noted that, to describe Nature, supersymmetry must be a spontaneously broken symmetry. Many aspects of the arguments given in the previous

section that supersymmetry is relevant to particle physics depend not only on the presence of the new symmetry but also on the values of the superpartner masses. In the arguments given above, it is actually the scale of the supersymmetry-breaking mass parameters that determines the size of the Higgs mass and vacuum expectation value, and also the mass of the particles of cosmological dark matter.

It is therefore important to investigate the mechanism of the spontaneous breaking of supersymmetry. The first place to look for this mechanism is in the dynamics of the supersymmetric extension of the Standard Model. However, this leads to a dead end. Not only is there no obvious mechanism to be found, but there are good reasons why supersymmetry breaking cannot come from physics directly connected to the Standard Model particles. For example, if an extension of the Standard Model contained a tree-level potential that gave supersymmetry-breaking, the fermion and boson masses generated by this model would obey the constraint

$$\mathrm{tr}(m_f^2 - m_b^2) = 0 \tag{19}$$

This constraint would hold, not only for the whole spectrum, but also separately for each charge sector. Then, for example, there would need to be very light squarks. More general constraints come from the strong bounds on the supersymmetric contributions to quark mixing processes such as the K^0 or B^0 mixing amplitudes. The superparticle mass spectrum must take a special form to avoid these contributions. For example, it must be almost degenerate among squarks of the three generations. It is not clear how dynamics in which the quark masses or other species-dependent couplings play an important role can lead to such degeneracy.

Successful models of the supersymmetry spectrum start with a different strategy, assuming that supersymmetry breaking arises in a 'hidden sector' that is only weakly coupled to the Standard Model particles. The hidden sector is assumed to couple through gauge bosons and gauginos, through supergravity, or through other particles whose couplings can be sufficiently isolated from the physics that leads to quark and lepton masses.

Where did this 'hidden sector' come from? What requires it? Doesn't this constitute an unnecessary multiplication of hypotheses?

The answer to this question comes from string theory. As I have discussed above, I do not insist that string theory is correct, but I am impressed that it does give an example of a theory that could, in principle, contain all of the interactions of Nature. So it is worth taking seriously what string theory has to say about the formulation of a 'theory of everything'.

In fact, unified theories of Nature within string theory require a large superstructure. String theory specifies the number of space-time dimensions to be eleven. The

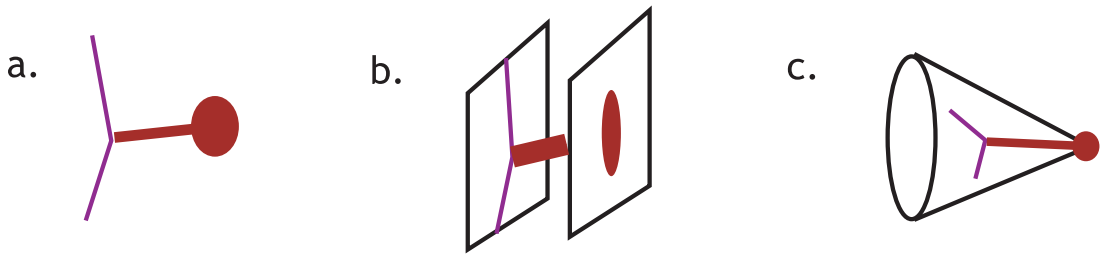

Figure 6: Some pictures of the generation of masses for supersymmetric particles by coupling to a 'hidden sector' with spontaneous supersymmetry breaking: a. from a gauge interaction outside the Standard Model; b. from a brane displaced into an extra space dimension; c. from a sector of particles bound to a singularity in the compact manifold of extra dimensions.

familiar four dimensions of space fill out part of this structure. Part is taken up by curved space dimensions. These form compact manifolds whose symmetries are the symmetries of the Standard Model gauge group and which, by virtue of this, give rise to the Standard Model gauge bosons. But there is room for more. Typical models of Nature built from string theory contain additional gauge interactions from a variety of sources. These can arise from additional symmetries of compactified extra dimensions. They can also arise in more subtle ways. For example, string theories contain as classical solutions hypersurfaces (called 'branes') with associated gauge bosons. Branes can float freely in the extra dimensions or wrap around singularities or topological cycles of the compact manifolds that these directions form. A new non-Abelian gauge sector outside the Standard Model is potentially a source of new interactions that could break supersymmetry. Since all parts of the model are linked by string interactions and gravity, a new sector of this type would be a hidden sector in the sense of used earlier in this section. In Fig. 6, I show some examples of hidden sectors in extra dimensions whose weak coupling to the Standard Model fields can be understood geometrically.

The geometrical relations seen in Fig. 6 determine the pattern of the soft supersymmetry-breaking parameters induced among the Standard Model superpartners. Some relatively simple schemes that generate simple but nontrivial patterns in the spectrum are described in [33–35]. More complicated—and perhaps more realistic—patterns due to the geometry of supersymmetry breaking remain to be discovered. Conversely, the evidence of this geometry, or of some more subtle picture of supersymmetry breaking, is present in the patterns that can be observed in the superpartner mass spectrum. These traces of physics at extremely small distances are waiting there for us to tease them out.

6 Interpretation of the SUSY-breaking parameters

In the previous two sections, I have argued that supersymmetric particles must be light—light enough to be discovered at the next generation of particle accelerators. I have also argued that their mass spectrum will be interesting to study, because its regularities encode information about the geometry of space at very short distances. However, there is a complication in obtaining this information that should be discussed. The observed masses do not fall simply into the pattern of the underlying SUSY-breaking parameters. Rather, they are modified by quantum field theory effects that we must disentangle.

In Section 4.2, I explained that the Standard Model coupling constants, which appear to be unequal by large factors, actually have the same value at the scale of grand unification. The couplings are then modified by different amounts when we analyze their influence on measurements at length scales much larger than the grand unification scale. After measuring these couplings with precision, however, we can perform the analysis shown in Fig. 5 and discover the regularity. The supersymmetry-breaking mass parameters have a similar difficulty. They are changed substantially from the enormous energy scale where they are created to the much lower energy scale of accelerator experiments where they can be observed. Fortunately, the changes are predicted by quantum field theory, so it is possible here also to undo their effect by calculation.

The gauginos, the superpartners of the gauge bosons, obey a simple scaling relation. To leading order, they are rescaled by the same factor as the Standard Model gauge couplings. So if, for example, the masses m_1, m_2, and m_3 of the $U(1)$, $SU(2)$, and $SU(3)$ gauginos are equal to a common value m at the energy scale M of grand unification, then at any lower energy scale Q these parameters will obey the relation

$$m_i(Q) = \frac{\alpha_i(Q)}{\alpha_i(M)} m \ . \tag{20}$$

This simple consideration predicts that the three mass values have the ratio

$$m_1 \ : \ m_2 \ : \ m_3 \ = 0.5 \ : \ 1 \ : \ 3.5 \tag{21}$$

for the physical values at accelerator energies. The corresponding relation for the supersymmetry partners of quarks and leptons is more complicated. Quantum field theory predicts an additive contribution resulting from the fluctuation of a squark or slepton into the corresponding quark or lepton plus a massive gaugino. The squarks couple relatively strongly to the gluino, and that particle is also expected to receive a larger mass from (21), so this mechanism typically makes the squarks heavier than the sleptons. In the extreme case in which the squarks and sleptons have zero mass

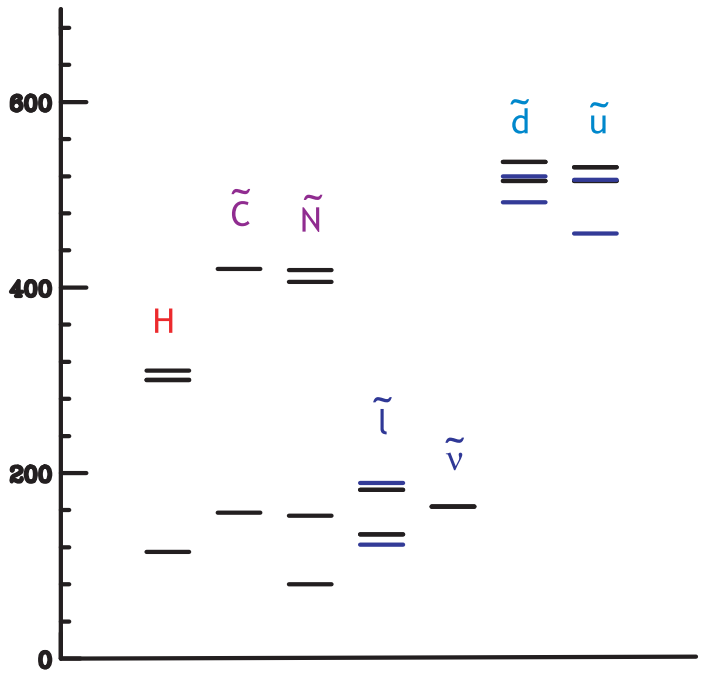

Figure 7: Sample spectrum of supersymmetric partners, based on universal masses for gauginos and sfermions at the energy scale of grand unification.

at the grand unification scale, the physical masses at the TeV scale should be in the ratio

$$\begin{aligned} & m(\widetilde{e}_R) : m(\widetilde{e}_L) : m(\widetilde{d}_R) : m(\widetilde{u}_R) : m(\widetilde{u}_L/\widetilde{d}_L) : m_2 \\ & \quad = 0.5 : 0.9 : 3.09 : 3.10 : 3.24 : 1 \ . \end{aligned} \tag{22}$$

A complete spectrum for the superparticles that illustrates these features is shown in Fig. 7. In this spectrum, I have assumed a common mass for the gauginos and a separate common mass for the squarks and sleptons. The mass splittings between the squarks and sleptons and between the electroweak and strong-interaction gauginos come from quantum field theory corrections. This assumption is the simplest one possible—and, probably, much too simple. In Fig. 8, I illustrate some alternative hypotheses for the underlying supersymmetry-breaking parameters. The figures show the quantum field theory evolution of parameters from the original supersymmetry-breaking parameters on the right to the measurable values of squark and slepton masses on the left. It is a common feature that the squarks are heavier and somewhat degenerate, while the slepton partners of right- and left-handed leptons are lighter and well split in mass. Precision analysis of the spectrum is needed to go beyond this qualitative feature, but the figure indicates that the detailed predictions for the

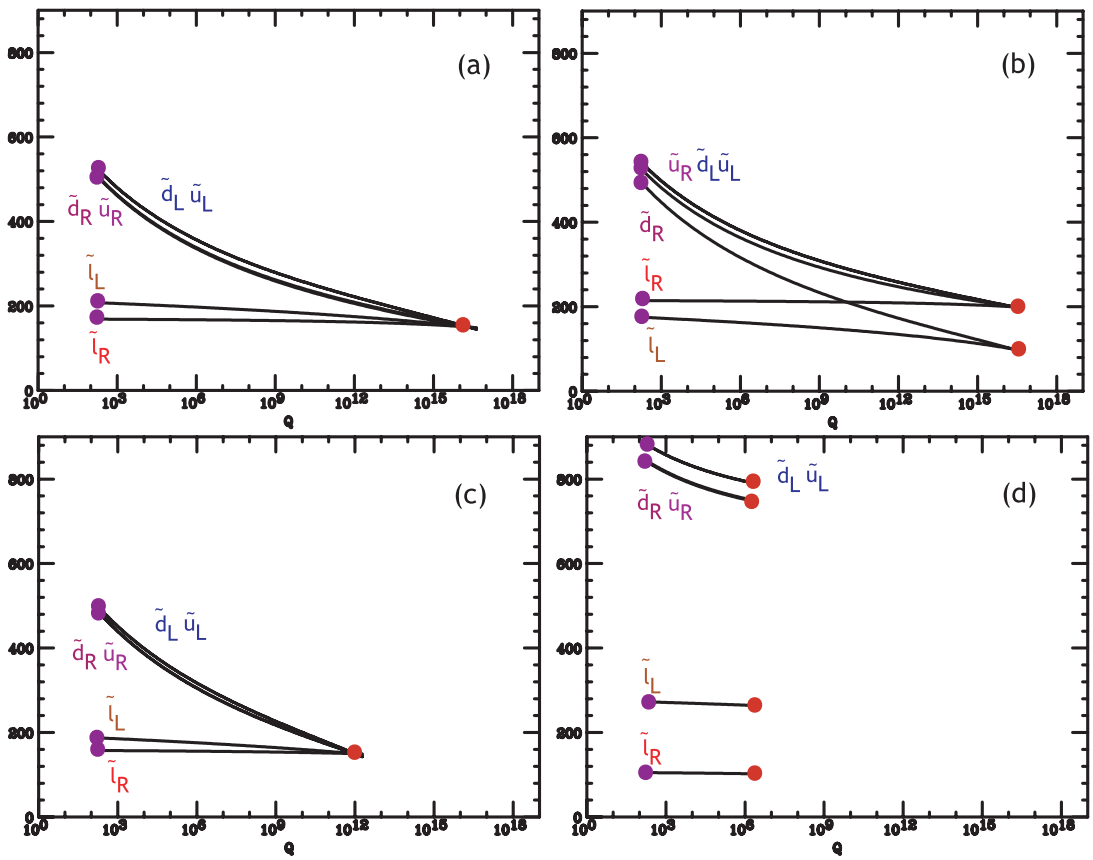

Figure 8: Evolution of squark and slepton masses from the mediation scale M down to the weak interaction scale (100 GeV) in four different scenarios: (a) universal mass at M equal to the grand unification scale; (b) separate masses for each individual $SU(5)$ mutliplet at M; (c) universal mass at M well below the grand unification scale; (d) masses generated at a low mediation scale M by Standard Model gauge and gaugino couplings. The dots on the right are the underlying parameter values; the dots on the left are the masses that would be measured in experiments.

supersymmetry spectrum do vary significantly in a way that can reveal the differences in the original assumptions.

Some other properties of the spectrum should also be noted. The partners of the heaviest quarks and leptons τ, b, and t are split off from the others by two effects. First, there is an additional quantum field theory contribution due to the coupings to the Higgs bosons that are responsible for the larger masses of the quarks and leptons. Second, there are supersymmetry-breaking contributions to the sfermion-sfermion-Higgs couplings that lead to mixing between the partners of the left- and right-handed fermion species.

Mixing of particle states is an issue in many parts of the supersymmetry spectrum, and one that significantly complicates the interpretation of the particle masses. Not only do the two scalar partners of each heavy quark or lepton mix together, but also there can be important mixings among the partners of the gauge bosons and Higgs bosons. In addition to the W^+ partner $\widetilde{w}^+$, there is a fermionic partner of the Higgs boson h^+; after electroweak symmetry breaking, these particles have the same quantum numbers and can mix. The mass eigenstates of this system, which are the observable physical particles, are called 'charginos', $\widetilde{C}_i^+$; they are quantum-mechanical mixtures of the two original states. Typically, one mass eigenvalue is close to m_2 while the other is close to an underlying Higgs mass parameter μ. To determine either parameter with precision, the mixing must be understood. Similarly, the gaugino partners of the photon and the Z^0 combine with two neutral Higgs fermions to form a four-state mixing problem that must be disentangled. The mass eigenstates of this mixing problem are called 'neutralinos', $\widetilde{N}_i^0$.

In addition to their role in the precision analysis of spectra, the mixing parameters just described are of interest in their own right. To check the story I have told in Section 4.1 about the origin of electroweak symmetry breaking, we should use the measured values of the supersymmetry parameters to compute the Higgs boson vacuum expectation value. The parameters of $\widetilde{t}$ mixing turn out to play an important role in this calculation, as does the parameter μ. The mixing parameters also play an important role in the calculation of the abundance of cosmological dark matter left over from the early universe. In Section 4.3, I have identified the dark matter particle with the lightest neutralino, $\widetilde{N}_1^0$. The reaction cross sections of this particle depend on the composition of the lowest mass eigenstate of the four-state mixing problem of neutral fermions. In addition, the pair annihilation of neutralinos often is dominated by the annihilation to tau lepton pairs, which brings in the mixing problem of the tau lepton partners. Both sets of mixing angles need to be measured before we can produce a precise prediction for the dark matter density from supersymmetry that we can compare to the measured cosmological abundance.

7 Measuring the Superspectrum

The complications discussed in the previous section add some difficulty to the interpretation of the supersymmetry spectrum, but these difficulties are no worse than those typically encountered in atomic or nuclear spectroscopy. They are a hint that the experimental determination of the underlying parameters of supersymmetry will be a subtle and fascinating study.

A serious question remains, though, about whether we can actually have the data. The properties of supersymmetric particles cannot be determined on a lab bench. High energies are required, and also a setting in which the properties of the exotic particles that are produced can be well measured. Cosmic rays could potentially provide the required energies, but they do not provide enough rate. To produce massive particles, the quarks or gluons inside colliding protons must come very close together, and this means that the typical cross sections for producing supersymmetric particles in proton-proton collisions are less than 10^{-10} of the proton-proton total cross section. The only known technique for extracting enough of these rare events from very high energy collisions is that of creating controlled reactions at dedicated particle accelerators.

Though it might be possible to glimpse supersymmetry at the currently operating accelerator at Fermilab, a comprehensive study of supersymmetry spectroscopy will require new accelerators with both higher energy and greater capabilities than those that are now operating. The high energy physics community is now planning for these accelerators—the Large Hadron Collider (LHC) at CERN and a next-generation electron-positron collider along the lines of the TESLA project in Germany or the NLC and JLC projects in the US and Japan. In this section, I will review some of the experiments at these facilities that might follow the discovery of supersymmetric particles.

Even given the needed energy and rates of particle production, it is a nontrivial question whether accelerator experiments can be sufficiently incisive to allow us to work out the detailed properties of the supersymmetry spectrum. But, in the next several sections, I will argue that it is so. Despite the fact that experiments at these proposed facilities are far removed from the human scale, they can include many subtle analytic methods. We can have the data to recover and understand the basic parameters of supersymmetry. It will be an adventure to perform these experiments and lay out the spectroscopy of supersymmetric particles—and another adventure to interpret this spectrum in terms of the physics or geometry of deep underlying distance scales.

7.1 Experiments at the LHC

The LHC is a proton-proton collider, with a center-of-mass energy of 14 TeV, now under construction at CERN. At energies so far above the proton mass, proton-proton collisions must be thought of as collisions of the proton's constituents, quarks and gluons. The dominant processes are those from gluon-gluon collisions. Such collisions bring no conserved quantum numbers into the reaction except for the basic 'color' quantum numbers of the strong interactions. Thus, they can produce any species of strongly-interacting particle, together with its antiparticle, up to the maximum mass allowed by energy conservation.

In the sample spectra shown in Fig. 8, the strongly-interacting supersymmetric partners, the squarks and gluinos, are the heaviest particles in the theory. These particles are unstable, decaying to quarks and to the partners of the electroweak gauge bosons. Often, the decays of the heavy particles proceed in several stages, in a cascade. If the quantum number R presented in Section 4.3 is conserved, the lightest supersymmetric partner produced in each cascade decay wil be stable and will exit the detector unobserved, carrying away some energy and momentum from the reaction. These are the particles of cosmological dark matter, and in the laboratory too they appear only as missing mass and energy.

These properties give the LHC events which produce supersymmetric particles a characteristic form. Typical proton-proton collisions at the LHC are glancing collisions between quarks and gluons. These produce a large number of particles, but these particles are mainly set moving along the direction of the proton beams, with relatively small perpendicular (or 'transverse') momentum. When heavy particles are produced, however, the decay products of those particles are given transverse momenta of the size of the particle mass. A quark produced with large transverse momentum materializes in the experiment as a cluster of mesons whose momenta sum to the momentum of the original quark and whose directions are within a few degrees of the original quark direction. Such a cluster, called a 'jet', is the basic object of analysis in experiments at proton colliders. Events with supersymmetric particle production contain multiple jets with large transverse momentum, and also unbalanced or missing transverse momentum carried away by the unobserved stable dark matter particles.

Studies of supersymmetry production carried out by the ATLAS experiment at the LHC make use of a variable that is sensitive to all of these effects. Define

$$M_{eff} = \not{p}_T + \sum_1^4 p_{Ti} \ , \tag{23}$$

the scalar sum of the p_T imbalance and the p_T values of the four observed jets of largest p_T. Events with large M_{eff} come from new physics processes outside the Standard

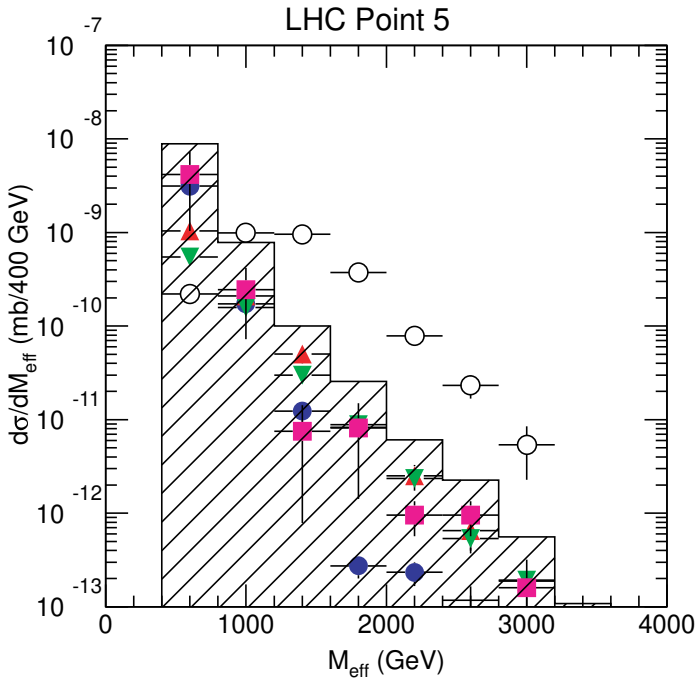

Figure 9: Expected distribution of the quantity M_{eff}, defined by (23), in the ATLAS experiment at the LHC, from Standard Model events and from events with supersymmetric particle production, from [36].

Model. This is shown in Fig. 9, in which the M_{eff} distribution expected from Standard Model events is compared to that expected from supersymmetry production for one specific choice of the spectrum. Not only can one use the variable M_{eff} to select events with supersymmetry, but also the average value of M_{eff} is well correlated with the mass of the strongly-interaction supersymmetric particles. This is shown in Fig. 10, which gives a scatter plot of the average value of M_{eff} versus the lighter or the squark and gluon masses for a number of supersymmetry spectra considered in the ATLAS study.

Once the mass scale of the supersymmetry spectrum is known and a sample of events can be selected, the more detailed properties of these events can give precise measurements of some of the spectral parameters. The observables that are most straightforward to measure are the energy and momenta of jets and leptons produced in the event, and these often do not have an unambiguous interpretation. However, in some cases, these parameters tell a very specific story. Consider, for example, a spectrum in which the mass difference between the second and the lightest neutralino

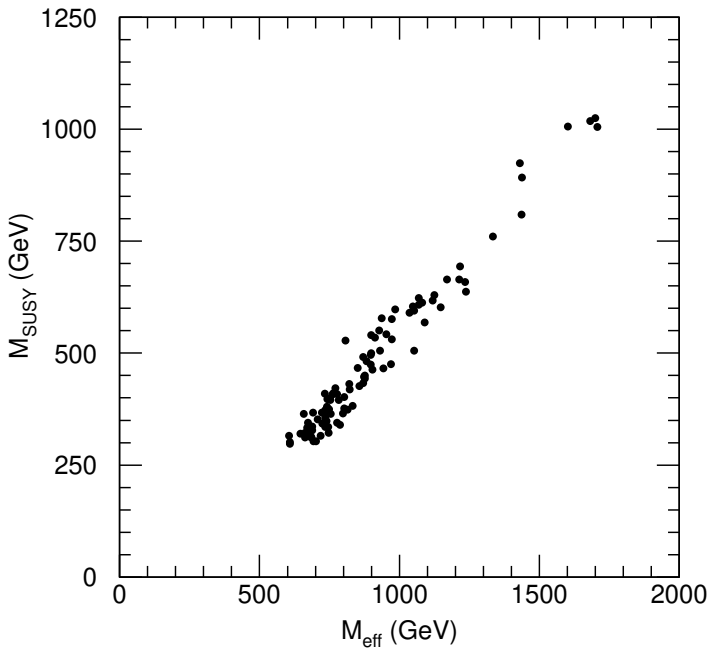

Figure 10: Correlation of M_{eff} with the lighter of the squark and gluino masses, from [36].

is less than the mass of the Z^0 boson. Then the $\widetilde{N}_2^0$ can decay to the light unobserved particle $\widetilde{N}_1^0$ by

$$\widetilde{N}_2^0 \to \widetilde{N}_1^0 + \ell^+\ell^- \ , \tag{24}$$

where ℓ is a muon or an electron. Because there is not enough energy from the mass difference to form a Z^0, the system of two leptons has a broad distribution in mass. However, it cuts off sharply at the kinematic endpoint

$$m(\ell^+\ell^-) = m(\widetilde{N}_2^0) - m(\widetilde{N}_1^0) \ . \tag{25}$$

By identifying this feature, it should be possible, in a scenario of this type, to measure the mass difference of neutralinos to better than 1%. The decay of $\widetilde{N}_2^0$ to $\widetilde{N}_1^0$ is a typical transition at the last stage of the decay cascade of the partners of left-handed quarks.

The $\ell^+\ell^-$ endpoint determination is illustrated in Fig. 11, which gives the lepton pair spectrum at one of the points studied by ATLAS. The background from Standard Model processes is shown explicitly in Fig. 11(a); there is very little. The observed leptons in the selected event then arise dominantly from supersymmetry decays, but

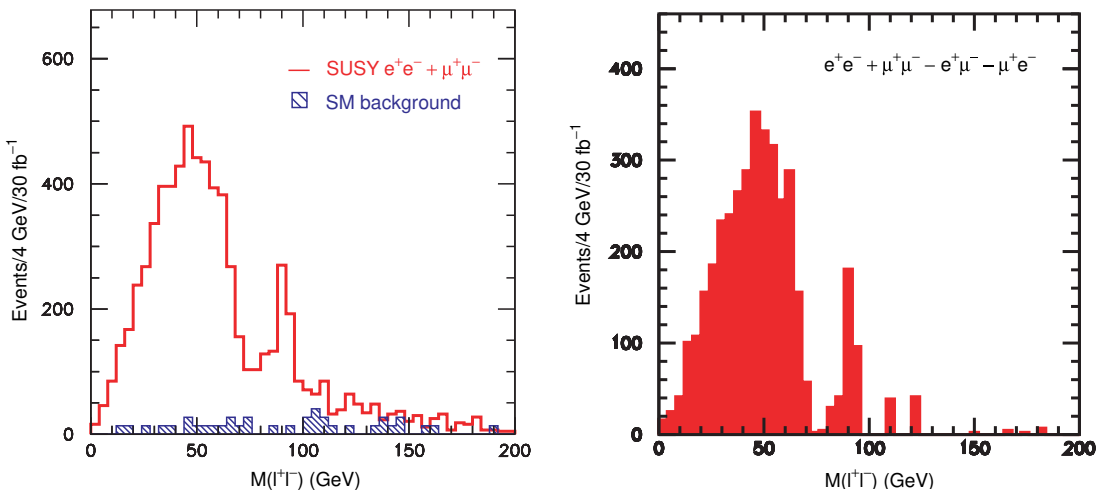

Figure 11: Expected mass spectrum of $\ell^+\ell^-$ pairs in the ATLAS experiment at the LHC, for the supersymmetry point 4 considered in [36].

from a number of different mechanisms. Most of these mechanisms, however, produce charged leptons singly (with neutrinos) and therefore produce one electron and one muon as often as a pair. By subtracting

$$(e^+e^-) + (\mu^+\mu^-) - (e^+\mu^-) - (\mu^-e^+) \tag{26}$$

we can concentrate our attention on the leptons produced in pairs. The subtracted mass spectrum is shown in Fig. 11. The pairs with mass of about 90 GeV arise from decays of the third and fourth neutralinos by emission of a Z^0 boson, which then decays to $\ell^+\ell^-$. The peak at lower mass comes from the $\widetilde{N}_2^0$ decays. The endpoint is very sharp, allowing a precise mass difference to be determined.

In many cases, this step is just the beginning of a deeper investigation. The events near the endpoint in the mass distribution correspond to the special kinematics in which the final $\widetilde{N}_1^0$ is almost at rest in the frame of the $\widetilde{N}_2^0$. This allows the maximum amount of the energy of the $\widetilde{N}_2^0$ to go into the leptons, creating the maximum mass. But this means that, if we can determine the mass of the $\widetilde{N}_1^0$ from another set of measurements, we have the entire momentum vector of the $\widetilde{N}_1^0$, and therefore the momentum vector of the $\widetilde{N}_2^0$. If the $\widetilde{N}_2^0$ was produced in a decay $\tilde{q} \to q\widetilde{N}_2^0$, we can add the momentum of an observed quark jet and attempt to reconstruct the mass of the parent squark. Fig. 12 shows an example of such an analysis. The mass peak at about 270 GeV is the reconstructed squark; its mass is determined in this analysis to percent-level accuracy.

Less straightforward possibilities can also occur. Figure 13 shows the $\ell^+\ell^-$ mass spectrum at another point considered in the ATLAS study in which the $\widetilde{N}_2^0$ decays

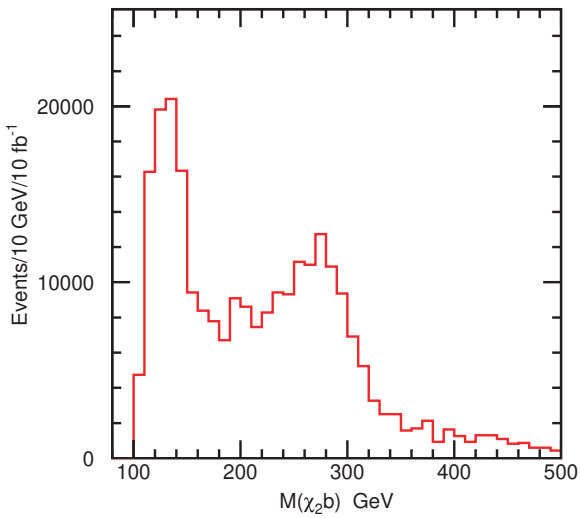

Figure 12: Reconstruction of the $\widetilde{b}$ mass by combining a reconstructed $\widetilde{N}_2^0$ with a b quark jet, from the simulation study of point 4 in [36].

to $\widetilde{\ell}\ell$. It might happen that the $\widetilde{N}_2^0$ has a kinematically allowed decay only to $\widetilde{\ell}_R^{\pm}\ell^{\mp}$. In other scenarios, the $\widetilde{N}_2^0$ could decay to either the $\widetilde{\ell}_L$ or the $\widetilde{\ell}_R$. The latter case is shown as the solid curve in Fig. 13, with two sharp endpoints visible. There is obviously some subtlety in determining the correct decay pattern of the neutralinos from the data. But the clues are there, and, if they are deciphered correctly, many parameters of the supersymmetry spectrum can be obtained. More examples are given in ref. [36].

7.2 Experiments at the Linear Collider

Experiments in electron-positron annihilation should present a quite different view of the supersymmetry spectrum. Electrons and positrons are elementary particles, so they can annihilate to a state of pure energy without leaving over any residue. This state, like that produced by a gluon-gluon collision, is completely neutral in its quantum numbers. So an electron-positron collision can directly produce particle anti-particle pairs of any particle with electromagnetic or weak interaction quantum numbers:

$$e^+e^- \to X\overline{X} \ . \tag{27}$$

The particles are produced back-to-back, each with the original electron energy. It is even possible to control the spin orientations of the particles: In a linear accelerator, the electron can be given a definite longitudinal polarization which is preserved during the acceleration process. Then the $X\overline{X}$ system is produced in annihilation with

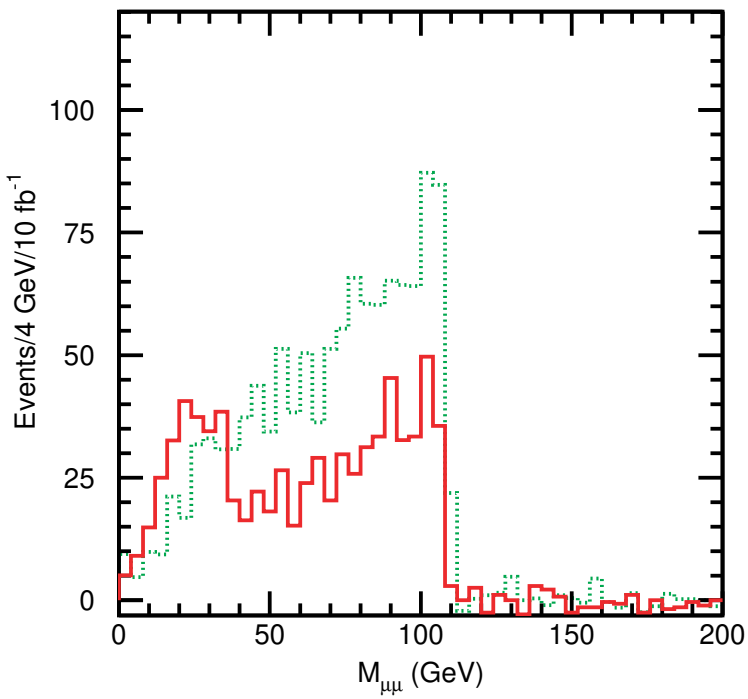

Figure 13: Expected mass spectrum of $\ell^+\ell^-$ pairs in the ATLAS experiment at the LHC, for a supersymmetry parameter set in which $\widetilde{N}_2^0$ can decay to both $\widetilde{\mu}$ states, compared to the mass spectrum (shaded) in which only the decay to the lighter $\widetilde{\mu}$ is allowed, from [36].

angular momentum $J = 1$, oriented parallel to the electron spin direction.

Because electrons and positrons radiate more copiously than protons, it is more difficult to accelerate them to very high energy. So the energies planned for the next-generation electron-positron collider are much lower than that of the LHC, 500 GeV in the first stage, increasing with upgrades to about 1 TeV. This should be enough energy to produce the lightest states of the superspectrum and subject them to a controlled examination.

An example of a simulated supersymmetry event at this facility is shown in Fig. 14. The reaction shown is the production of a pair of charginos, which subsequently decay to the lightest neutralino plus a pair of quarks or leptons:

$$e^+e^- \to \widetilde{C}^+\widetilde{C}^- \to e^+\nu\widetilde{N}_1^0 \quad q\overline{q}\widetilde{N}_1^0 \ . \tag{28}$$

The electron is visible as the isolated stiff track. There are two well-defined jets

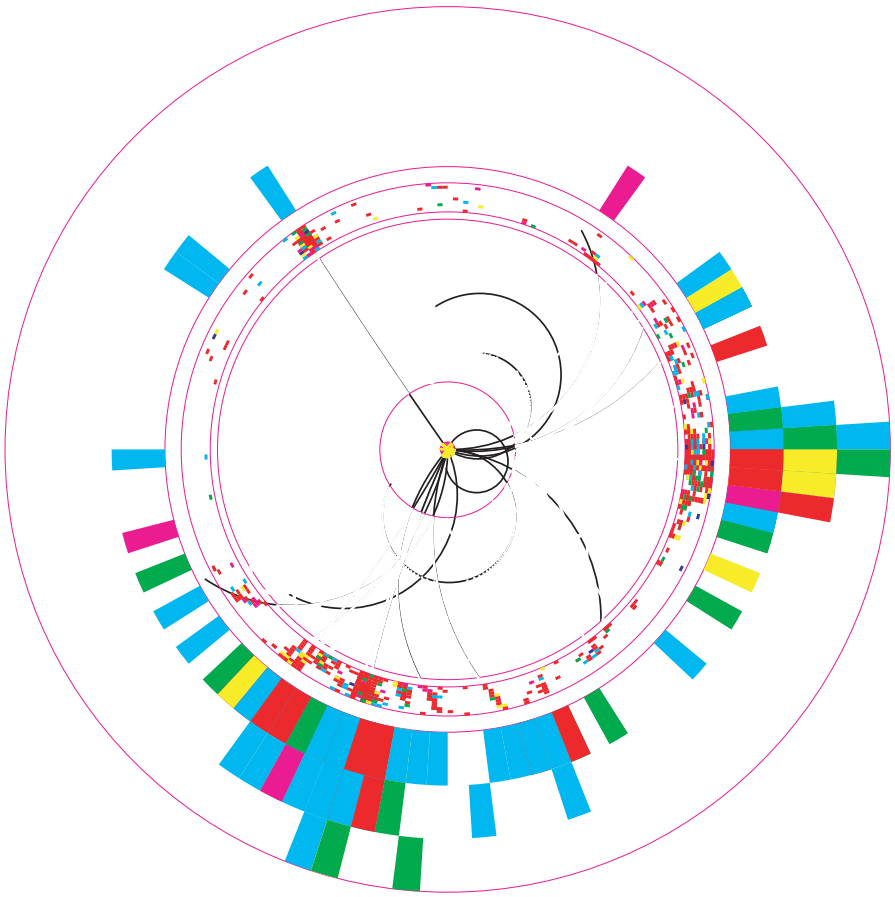

Figure 14: A simulated event of e^+e^- annihilation to a chargino pair, as it would appear in a detector at a linear e^+e^- collider, from [37].

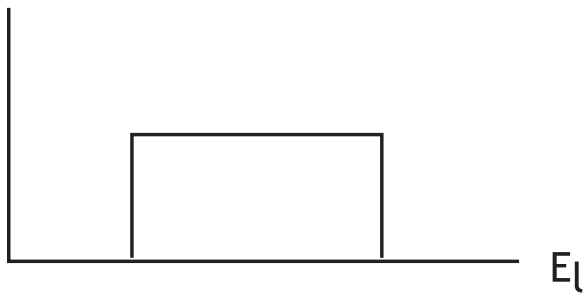

Figure 15: Schematic form of the lepton energy distribution in slepton pair-production events.

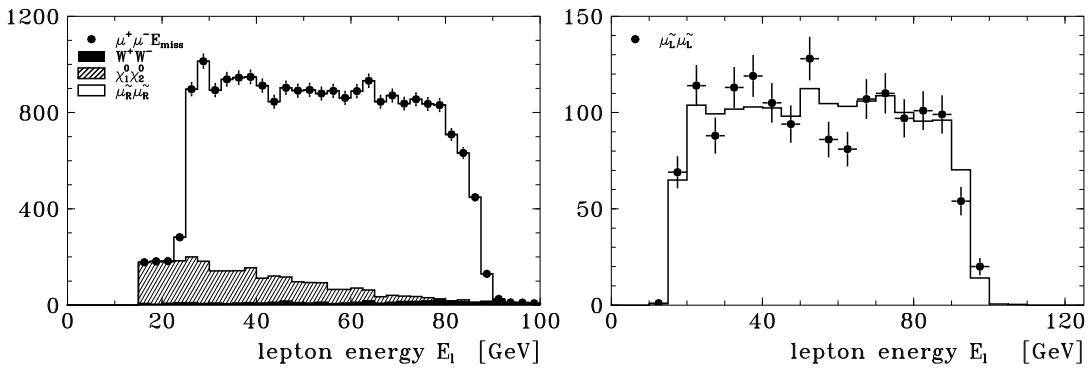

Figure 16: Distributions of $\mu^{\pm}$ energy from simulations of smuon decays in smuon pair-production events, from [39].

which are the signals of the quark and antiquark. The colored cells denote the energy deposition by both charged and neutral particles. The momentum and energy flow from the electron and the jets is simple and readily reconstructed, giving a clear picture of the whole event.

The relation between the momenta of the decay products and the momenta of the parent supersymmetric particles is also very simple. The cleanest correspondence comes in the case of slepton pair production. The slepton decays to the corresponding lepton and a neutralino, for example,

$$\widetilde{\mu} \to \mu \widetilde{N}_1^0 \ . \tag{29}$$

Because the slepton has spin 0, the decay is isotropic in its rest frame. The sleptons are produced in motion, but the boost of an isotropic distribution is a distribution that

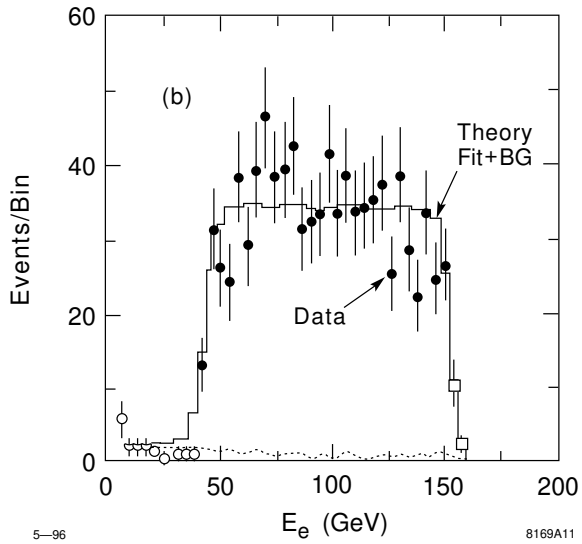

Figure 17: Distribution of e^- energy from $\widetilde{\nu}$ decays in a simulation of sneutrino pair-production events, from [40].

is constant in energy between he kinematic endpoints. So the distribution observed in the lab has the schematic form shown in Fig. 15. From the values of the energy at the two endpoints, one can solve algebraically for the mass of the slepton and the mass of the neutrinalino produced in the decay [38]. The masses can be determined by this technique to better than 1%.

In Fig. 16, I show the energy distributions produced in simulations of smuon pair production for the supersymmetry parameter set considered in [39]. The technique generalizes to other supersymmetric particles. The superpartner of the electron neutrino should often decay by

$$\widetilde{\nu} \to e^- \widetilde{C}_1^+ \ . \tag{30}$$

The chargino decays to a complex final state, but the electron has the same flat distribution that we have just discussed. Figure 17 shows a simulation study of the electron distribution in $\widetilde{\nu}$ pair-production, showing well-defined kinematic endpoints. In chargino pair-production, the energy distribution is more complex, both because the chargino decay is not isotropic and because the chargino decays to a two-quark or two-lepton system of indefinite mass. But the $q\overline{q}$ energy and mass distributions, shown in Fig. 18 still show quite well-defined endpoints and still allow very accurate mass determinations [39].

The simplicity of these reactions can be further exploited along a number of lines to expose more detailed aspects of supersymmetry spectroscopy. Because it is possible

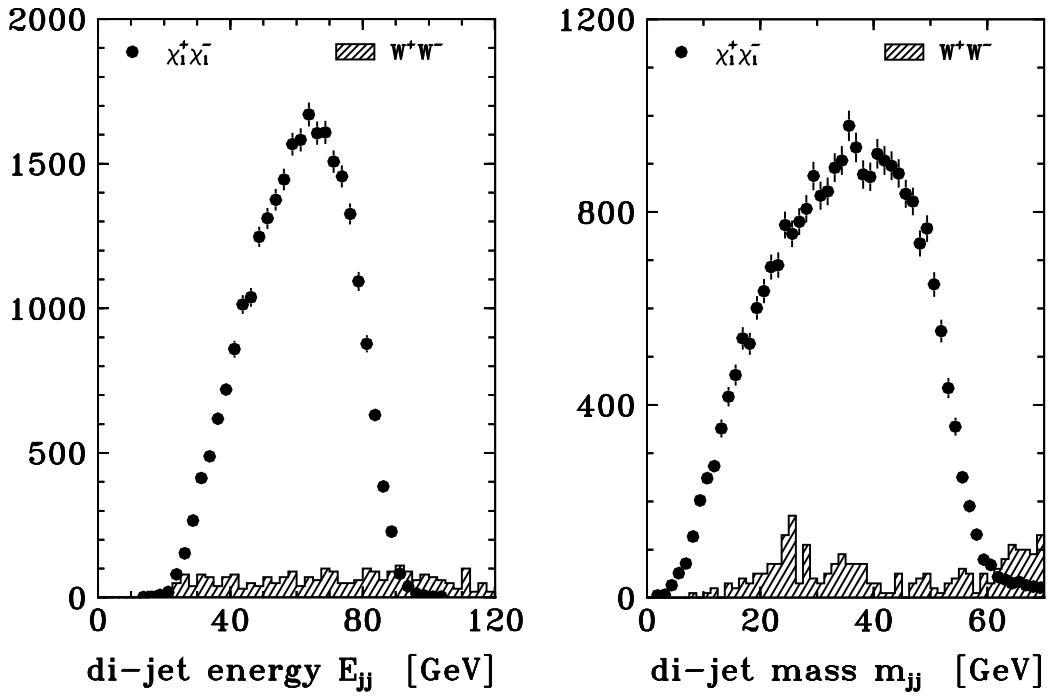

Figure 18: Distributions of $q\overline{q}$ energy and mass distributions in a simulation of chargino pair-production events, from [39].

in e^+e^- annihilation to directly control the e^+e^- center of mass energy, it is possible to precisely locate the threshold energy for a pair production process (27). This technique can produce a mass determination at the 0.1% level. In Fig. 19, I show the dependence of the cross section for the reaction $e^-e^- \to \widetilde{e}^-\widetilde{e}^-$ on center of mass energy in the vicinity of the threshold. A variation of the selectron mass by less than 0.1% is quite visible above the expected statistical errors [41].

A more subtle question is the determination of the mixing angles defining the stop, stau, chargino, and neutralino eigenstates. For this study, the initial electron polarization can be used in a powerful way. For the stop and stau, the pair-production cross section for a given initial-state polarization depends only on the electroweak quantum numbers of the final particles. The mass eigenstate is a mixture of two states with different quantum numbers, and so the cross section is an unambiguous function of the mixing angle. Figure 20(a) shows a determination of the mixing angle in the lighter stop eigenstate by comparing the measured pair-production cross sections from left- and right-handed polarized beams. For the charginos and neutralinos, the pair-production from left- and right-handed beams actually accesses different Feynman diagrams with different intermediate particles. For example, the production from a

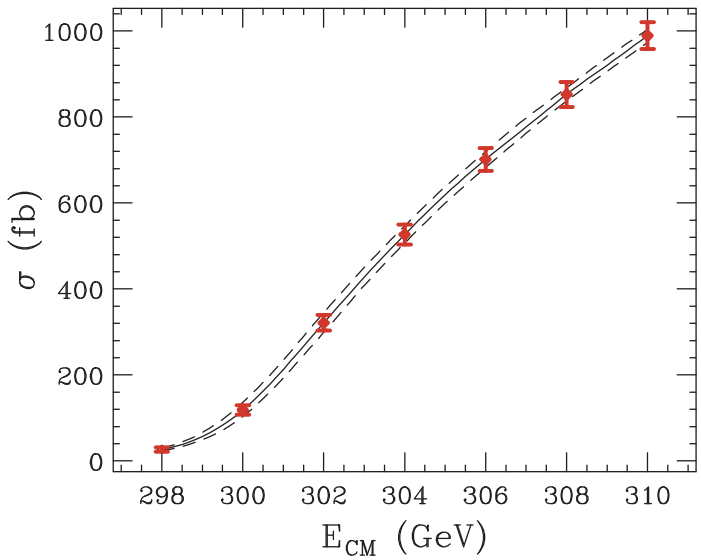

Figure 19: Sensitivity of the threshold cross section in $e^-e^- \to \widetilde{e}^-\widetilde{e}^-$ to the mass of the $\widetilde{e}^-$, from [41]. The three curves correspond to selectron masses differing by 100 MeV. Initial state radiation and other realistic beam effects are included.

right-handed electron beam (at least for center of mass energies much larger than m_Z) produces only the component of the eigenstate that is the partner of the Higgs boson. Figure 20(b) shows the value of this polarized production cross-section as a function of the parameters μ and m_2. The cross section is large is regions where the lightest chargino is mainly a Higgsino and small where it is mainly a gaugino. The measured mass of the chargino picks out a specific point on each contour of constant cross section. With this constraint, the content of the chargino eigenstate can be precisely determined.

8 Conclusions

In this lecture, I have presented a possible picture of the future of high-energy physics based on the existence of supersymmetry, a fundamental symmetry between bosonic and fermionic elementary particles. After reviewing the current status of our understanding of the interactions of elementary particles, I have explained how supersymmetry can address many of the pressing questions that are now unanswered.

But just as supersymmetry provides the solution to our present questions, it will raise a new set of questions that must then be investigated. Chief among these is the question of the mechanism of supersymmetry breaking and the origin of the masses

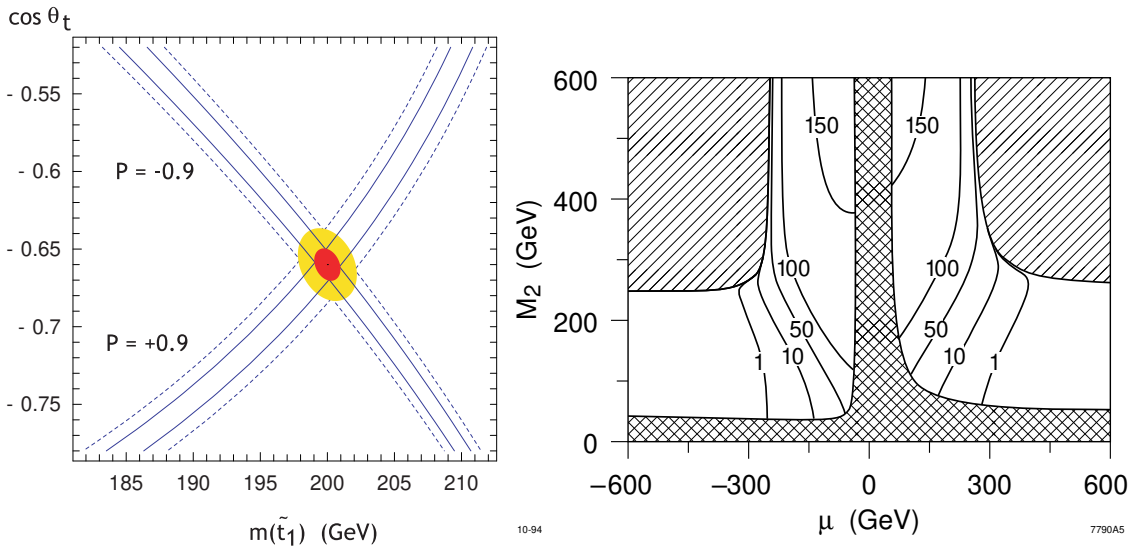

Figure 20: Mixing angle determinations in e^+e^- annihilation to supersymmetric particles: Left: $\widetilde{t}$ mixing angle determination from measurement of the pair-production cross section from left- and right-handed electron beams, from [42]. Right: μ vs. m_2 determination from measurement of the production cross section for chargino pairs from using an e^-_R beam, from [43].

of superpartners. I have argued that these questions might well connect directly to very deep issues of the short-distance geometry of spacetime and to the connection of the observed interactions of particle physics to string theory or another grand theory of unification.

I have argued that these new questions will need to be resolved from experimental data, specifically, the data on the masses and mixing of the new particles predicted by supersymmetry. I have explained how the next generation of particle accelerators will give us the tools to acquire this data. These are huge and expensive technical projects, but they have the capabilities to bring us the information that we need.

This experimental study will bring us into a new regime in fundamental physics, and we must frankly acknowledge that we do not know what its outcome will be. Perhaps the superspectrum measurements will show an anticipated, simple pattern. More likely, as has happened for every other new set of particles and forces, they will present a puzzle that defies straightforward projections.

This is what we hope for whenever we experiment on the laws of physics. We look for a chance to raise puzzles whose resolution will take us deeper into the working of Nature. To solve such puzzles, physicists must organize the facts into newly imagined

patterns and regularities. Today the Standard Model leads us to the need for supersymmetric particles. We look forward to their discovery, and then their painstaking exploration. When the facts about these particles are gathered, we will find ourselves with concrete questions that will challenge us to make another such leap. We will find ourselves then at that moment that we prize, the moment when the next Werner Heisenberg can open our eyes to a yet more unexpected reality.

I am grateful to Professors Gerd Buschhorn and Julius Wess for their invitation to speak at the symposium and to Jonathan Feng, Michael Dine, Keisuke Fujii, Hitoshi Murayama, and many other colleagues at SLAC and elsewhere with whom I have discussed the issues presented in this lecture. I thank Richard Zare for his very useful critique of the manuscript. This work was supported by the US Department of Energy under contract DE–AC03–76SF00515.

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