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We apply a recently proposed mechanism for predicting the weak mixing angle to theories with TeV-size dimensions. “Reconstruction” of the associated moose (or quiver) leads to theories which unify the electroweak forces into a five dimensional SU(3) symmetry. Quarks live at an orbifold fixed point where SU(3) breaks to the electroweak group. A variety of theories – all sharing the same successful prediction of \( \sin^2 \theta_W \) – emerges; they differ primarily by the spatial location of the leptons and the absence or presence of supersymmetry. A particularly interesting theory puts leptons in a Konopinski-Mahmoud triplet and suppresses proton decay by placing quarks and leptons on opposite fixed points.

1. Introduction

The one quantitative success of physics beyond the standard model (SM) is the prediction of the weak mixing angle \( \sin^2 \theta_W \) by supersymmetric grand unified theories (GUTs). The SU(5) prediction is \( \sin^2 \theta_W = 3/8 \) at tree level [1]. Running this value from the GUT scale to the weak scale in supersymmetric theories produces the measured value of 0.231 within theoretical uncertainties [2]. A critical assumption is the existence of a large energy desert above the weak scale. For a theory with a low cutoff, an alternative approach must be taken.

Recently, a new mechanism for predicting the weak mixing angle with TeV-physics was proposed [3]. It leads to the unification of the two electroweak gauge couplings into their SU(3)-symmetric value, giving \( \sin^2 \theta_W = 0.25 \) at tree level. Since this value is close to the experimental value of \( \sin^2 \theta_W = 0.231 \) at \( M_Z \), SU(3)-unification occurs at a few TeV [3]. The proposed mechanism is quite simple: one adds to the SM an SU(3)-gauge group and a scalar \( \Sigma \) whose vacuum expectation value (VEV) breaks the SU(3) and the electroweak gauge sector to the diagonal SU(2) \( \times U(1) \). SM fields are all singlets under the new SU(3). Nevertheless, if the original SU(2) \( \times U(1) \) couplings are somewhat large, the low energy gauge groups reflect the SU(3) symmetry.

Recent studies of “dimensional deconstruction” [4,5] have highlighted the close connection between “moose” or “quiver” theories and those with extra dimensions. With this motivation, we will attempt to “reconstruct” the fundamental moose of this mechanism (figure 1a) into its one-dimensional cousin. The models which result (figure 1b) contain a five-dimensional SU(3) broken to SU(2) \( \times U(1) \) at one orbifold fixed point on which some or all the SM particles are localized. The purpose of this paper is to study the physics of the simplest model and its variations.

In these models quarks must be localized on the SU(2) \( \times U(1) \) orbifold fixed point. If quarks lived in the bulk or the SU(3)-symmetric fixed point, they would have to belong to SU(3) multiplets, and this would conflict with their fractional charges [3]. On the other hand, integrally charged particles – the leptons and the Higgs – can belong to SU(3)-multiplets and therefore can live at either fixed-point or in the bulk. In fact the lepton doublet and singlet together neatly fit into a Konopinski-Mahmoud triplet [5]. Migrating the lepton or the Higgs to different locations gives rise
to different theoretical possibilities with distinct phenomenology.

Another degree of freedom which leads to different theoretical options is the presence or absence of supersymmetry. The unification scale of these theories is in the multi-TeV range. For a Higgs mass is $\sim 100$ GeV, there is typically a moderate fine tuning of order of $\sim 10^{-4}$. One way to avoid it is supersymmetry.

**Orbifolds versus Mooses:**

While the minimal module proposed in reference [3] is quite simple, it is interesting to study five-dimensional realizations of the mechanism. A reason for this is the appeal of geometric intuition. For example, the mechanism of [3] requires that the $SU(3)$ gauge coupling is smaller than that of the original $SU(2) \times U(1)$. Placing $SU(3)$ in a moderately large bulk naturally leads to a Gaussian dilution of its coupling strength. Also, the presence of the bi-fundamental field $\Sigma$ may appear less natural than breaking the symmetry by orbifold boundary conditions. Another tool is locality, which can help suppress proton decay and other dangerous operators [8]. Aesthetic arguments aside, the important feature of the mechanism proposed in [3] is that it is simple and the prediction of $\sin^2 \theta_W$ so robust that it can be embedded in a variety of frameworks, including TeV-dimensions.

In section 2 we discuss the prediction of the weak mixing angle and the associated theoretical uncertainties. This discussion parallels the one in reference [3] and establishes the correspondence between the deconstructed and reconstructed minimal modules. In section 3 we analyze the phenomenology of the minimal reconstructed module and some of its variations, which differ from each other by the location of the lepton and the Higgs. An interesting variation has automatic proton stability due to the separation of quarks and leptons at opposite fixed points [3]. In section 4 we discuss models which address the hierarchy problem via supersymmetry, and we conclude in section 5.

![Figure 1. a) Deconstructed (4D) and b) reconstructed (5D) versions of electroweak $SU(3)$ models.](image)
$Z_2$ “parity” symmetry which exists in the Lagrangian, e.g., all even modes of negative parity states are projected out. This operation can break gauge symmetries, flavor symmetries and supersymmetries and is generically referred to as the Scherk-Schwarz mechanism [13,14]. Within the context of gauge theories, it is also referred to as the Hosotani mechanism [17].

Within the specific example of an SU(3) theory broken by orbifold boundary conditions to SU(2) × U(1), the gauge fields corresponding to the generators of SU(3)/(SU(2) × U(1)) are given odd boundary conditions (parity −1) about a boundary at $y = \pi R$. Because of this, while we can perform arbitrary SU(3) gauge transformations in the bulk, on the brane only SU(2) × U(1) transformations are non-trivial. Due to this less restrictive symmetry we can include incomplete SU(3) multiplets on the boundary, without any inconsistency in the theory. This will allow us to include quarks in our models although they cannot be put into SU(3) multiplets. We will also use orbifolding to break supersymmetry when relevant.

2.2. Theoretical Uncertainties

Because SU(3) is broken at the $y = \pi R$ boundary, we can include contributions to the kinetic terms of the gauge fields on the boundary which are only SU(2) × U(1) invariant.

\[
\mathcal{L} = \int d^4x \, dy \frac{F_3^{ij} F_3^{ij}}{4 g_3^2} + \gamma F_{\mu
u} F^{\mu\nu} \delta(y) + \frac{\delta_Y}{3} F_1^{\mu
u} F_1^{\mu\nu} + \delta_2 F_2^{\mu
u} F_2^{\mu\nu} \delta(y - \pi R). \tag{1}
\]

Here $i, j$ and $\mu, \nu$ are five and four dimensional Lorentz indices, respectively, while 3, 2, 1 index whether it is the field strength tensor for the complete SU(3) multiplet, or just the SU(2) or U(1) subgroup. Note that $g_3^2$ has dimensions here of mass$^{-1}$.

Ignoring quantum effects, we match to the effective four-dimensional gauge theory. The gauge couplings are given by

\[
\frac{1}{g_I^2} = \frac{3\pi R}{g_3^2} + 3\gamma + \delta_Y, \tag{3}
\]

\[
\frac{1}{g_2^2} = \frac{\pi R}{g_3^2} + \gamma + \delta_2,
\]

and thus the degree to which the SU(3) relation $g_2^2/g_3^2 = 3$ depends crucially on the size of $\delta_Y$ and $\delta_2$. The SU(3) universal piece $\gamma$ is irrelevant for this and we will henceforth ignore it. We can gain insight into their size by studying the strength of coupling of the theory at the cutoff scale $\Lambda$. That is, we naturally expect

\[
\frac{1}{\Lambda g_3^2} \sim \delta_i \tag{4}
\]

It is straightforward to make a connection with the uncertainties discussed in [3]. In that case, there were uncertainties due to $g_{1,2}$, which, in the case of strong coupling, perturbed the prediction for $\sin^2 \theta_W$ only slightly. Here, the equivalent uncertainties arise from $\delta_Y, 2$, which are small when the theory is strongly coupled at the cutoff. One can consider the uncertainty plot of [3] to essentially apply to our setup as well. If we require the observed four-dimensional gauge coupling $g_2^2$ is order one, then $g_3^2/\pi R \sim 1$ and hence $\delta_i \sim (\Lambda \pi R)^{-1}$. The result is simply that the larger the volume, the smaller the theoretical uncertainty.

2.3. Running

We can now consider quantum effects. Because the spectrum of the theory, including Kaluza-Klein (KK) modes, is not SU(3) symmetric, we will generate log enhanced contributions to the $\delta_i$ in eq. (4). These are completely calculable within an effective theory. One straightforward approach is to sum the contributions of the KK modes along the lines of [15,16]. We can then calculate the effective four-dimensional couplings as a function of energy

\[
\alpha^{-1}(\mu) = \alpha^{-1}(\mu_0) - \frac{b}{2\pi} \log(\mu/\mu_0) \tag{5}
\]

\[
- \frac{\tilde{b}_{even} r_{even}(\mu, R)}{8\pi} - \frac{\tilde{b}_{odd} r_{odd}(\mu, R)}{8\pi},
\]

We define

\[
\alpha^{-1}(\mu) = \int_{\mu_0}^{\mu} \frac{d\theta_3(it/\pi R^2)}{t} - \frac{1}{t}, \tag{6}
\]

\[
r_{even}(\mu, R) = \int_{\mu_0}^{\pi R^2/4} \frac{d\theta_3(it/\pi R^2)}{t},
\]

\[
r_{odd}(\mu, R) = \int_{\mu_0}^{\pi R^2/4} \frac{d\theta_3(it/\pi R^2) - \theta_3(it/\pi R^2)}{t},
\]

\[\]
where \( \theta_5(t) = \sum_{n=-\infty}^{\infty} \exp(\pi i n^2) \). Here, \( \tilde{b}_{\text{even}} \) and \( \tilde{b}_{\text{odd}} \) are the contributions to the beta functions from the modes at \( n/R \) and \( (2n+1)/2R \), respectively.

While the above expression actually encodes the power law running due to the KK tower, the overwhelming majority will be \( SU(3) \) universal, with only a logarithmic piece distinguishing \( \alpha_Y \) from \( \alpha_Z \). This can be studied by looking at the relative running of the two couplings (suitably normalized)

\[
\delta \alpha^{-1}(\mu) = \alpha_Y^{-1}(\mu) - 3 \alpha_Z^{-1}(\mu) = \frac{b_Y - 3b_Z}{2\pi} \log(\mu/\mu_0)
\]

This quantity encodes the relative running between the two couplings, and by studying where it crosses zero, we can determine the cutoff of the theory.

### 3. Non-supersymmetric models

In this section we present non-supersymmetric theories of \( SU(3) \) electroweak unification in five dimensions. We describe two models explicitly, first a simple extra-dimensional version of the minimal module \( 3 \) and then a version in which quarks and leptons are on different boundaries, with the Higgs in the bulk, thus naturally explaining the absence of proton decay and explaining why leptons appear to come in complete \( SU(3) \) triplets.

In the theories below there are two theoretical inputs, \( \Lambda \) and \( R \), to which the value of \( \sin^2 \theta_W \) is logarithmically sensitive. The range of these parameters is restricted by the hierarchy problem. If the theory is not strongly coupled at \( \Lambda \) we expect one-loop corrections to the squared Higgs mass to be of order a loop factor times \( \Lambda^2 \). To produce a Higgs mass around the electroweak breaking scale without more than 1% fine tuning, we require \( \Lambda < 20 \text{ TeV} \). The value of \( 1/R \) is then restricted to the range \( 1 \text{ TeV} \leq 1/R < \Lambda \). For the cutoff in the range 2–20 \text{ TeV}, the variation of \( \sin^2 \theta_W \) at \( M_Z \) is roughly a few percent.

From the previous section, another contribution to the uncertainty are boundary kinetic terms. A guess at the fractional uncertainty in the squared couplings is \( \sim 1/(\pi R A) \) which can be somewhat large in the scenarios outlined below. However, for large regions in parameter space \( 3 \), the corrections are in fact quite small, again of order a few percent.

#### 3.1. Minimal Reconstruction

This model is simply a continuous version of the two-site moose shown in Figure 1. An \( SU(3) \) gauge theory lives in the full five dimensions but is broken by orbifold boundary conditions at \( y = \pi R \) by requiring the following properties of the gauge fields:

\[
A^\mu(-y) = A^\mu(y)
\]

\[
A^\mu(2\pi R - y) = Z A^\mu(y) Z
\]

and

\[
A^5(-y) = -A^5(y)
\]

\[
A^5(2\pi R - y) = -Z A^5(y) Z
\]

where

\[
Z = \begin{pmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

breaks \( SU(3) \rightarrow SU(2) \times U(1) \). The massive gauge bosons form an \( SU(2) \) doublet with hypercharge \( 3/2 \). The masses of states in the KK towers of the unbroken and broken theories of \( SU(3) \) universal, respectively. The differential running of gauge couplings can be calculated using the techniques outlined in the previous section. For this model the beta function coefficients for the zero modes are \( (b, b_Y) = (-19/6, 41/6) \), and for the KK modes are \( (\tilde{b}_{\text{even}}, \tilde{b}_{\text{even}}; \tilde{b}_{\text{odd}}, \tilde{b}_{\text{odd}}) = (-21/3, 0; -21/6, -63/2) \).

We can now compute the size of the extra dimension as a function of the cutoff and find, for example, for \( \Lambda = (10, 20) \text{ TeV} \) we get \( L^{-1} \equiv (\pi R)^{-1} \sim (1.9, 1.3) \text{ TeV} \). However, a tiny difference in the gauge couplings (percent) results in a
significant difference in the compactification scale (order one). The sensitivity of $1/R$ to the exact gauge couplings translates into an insensitivity of $\sin^2 \theta_W$ to the specific values of $1/R$ chosen above.

Variations of this model involve moving the Higgs and leptons off of the SU(2) × U(1) boundary. This is possible because they carry quantum numbers of components of SU(3) representations while the quarks do not. (Putting leptons in the bulk does, however, require doubling the number of species as the orbifold projections only allow one component of each triplet to survive.) The renormalization group analysis will differ but produces the same generic features of the model above as displayed in Figure 2, namely that the relative running is slowed by the existence of partial multiplets in the KK towers.

3.2. Proton Stability

A particularly intriguing model is motivated by placing the SM fields in their “natural” locations based on their quantum numbers. The quarks must remain on the SU(2) × U(1) while the leptons fall into complete SU(3) multiplets $\bar{3}$ and thus should live on the SU(3) preserving boundary. The Higgs does not fill out a complete multiplet and thus should live in the bulk where the orbifolding splits multiplets. The Higgs could live in a $\bar{3}$ or $\bar{6}$ of SU(3). The former would only allow the highly constrained lepton Yukawa couplings $y^L_i H L^i L^j$ contracted with an epsilon tensor predicting $m_\tau = m_\tau$ and $m_e = 0$. The $\bar{6}$ however is a symmetric tensor and easily allows enough freedom to produce the charged lepton spectrum. The Higgs in the bulk will satisfy

$$H(-y) = H(y)$$
$$H(2\pi R - y) = -ZH(y)Z$$

where $Z$ is defined as in equation 10.

thus projecting out an SU(2) triplet with hypercharge -1 and a singlet with hypercharge +2 and leaving only the zero mode for the charge 1/2 doublet. The zero mode spectrum is just the standard model and the Higgs is expected to get a mass at one loop due predominantly to the top Yukawa coupling, self coupling and gauge couplings.

The other attractive component of this scenario is that it naturally suppresses proton decay as no local counter terms containing both quarks and leptons can be constructed. However, one can imagine exponentially suppressed contributions through the exchange of heavy states. For a cutoff of 20 TeV, an extra dimension of size $1/2$ doublet. The zero mode spectrum is just the standard model and the Higgs is expected to get a mass at one loop due predominantly to the top Yukawa coupling, self coupling and gauge couplings.

This model is particularly interesting as it requires (by gauge invariance) a coupling of the leptons to the exotic singly and doubly charged gauge bosons. If produced, the doubly charged gauge bosons will decay into like-sign leptons with very high $p_T$, something easily seen at RUN II or the LHC, depending on its mass.

4. Supersymmetric Models

While these are elegant models in which $\sin^2 \theta_W$ is predicted, we have only addressed questions of the hierarchy problem in the sense that the cutoff of the theory is low. Moreover, we have no understanding of the origin of electroweak symmetry breaking. To this end, there has been great interest in supersymmetric models in TeV-sized extra dimensions.

Theories with TeV-sized dimensions provide
the simplest framework for explaining electroweak symmetry breaking. This can be realized either with the Scherk-Schwarz mechanism, or with a large, localized supersymmetry breaking term, both of which have the added benefit of solving the supersymmetric flavor problem automatically. These mechanisms naturally relate the scale of EWSB to the size of the extra dimension. In these scenarios, gauginos naturally have mass of $1/R$, while the sfermions are generated at one loop order higher (i.e., $(4\pi R)^{-1}$).

There is still the free parameter $\Lambda$, which is limited by strong coupling to be within a few orders of magnitude of $R^{-1}$. If one makes the additional assumption that $\Lambda$ is the strong coupling scale of the theory, one can then make a more robust prediction of $\sin^2 \theta_W$.

In addition to the location of the Higgs and leptons, one has other possibilities for constructing models. Supersymmetry can be broken by Scherk-Schwarz boundary conditions, such as in \[21,22\]. One can also give a large boundary mass for the gauginos, but this limits the setup to having all matter and Higgs fields on the $SU(2) \times U(1)$ boundary in order to avoid flavor violation and/or very large ($O(R^{-1})$) soft masses for the Higgses.

Such models have a few phenomenologically interesting features: first, with Scherk-Schwarz compactifications and gaugino masses on the $SU(3)$ boundary, it is quite natural to have biinos which are nearly degenerate with the winos. If the leptons live in a triplet on the $SU(3)$ boundary (with a sextet Higgs in the bulk, as described), the right-handed sleptons will receive additional gauge mediated contributions from the broken generators, potentially reflecting the underlying $SU(3)$ symmetry. Interestingly, with quarks and leptons on separated boundaries, we need only ordinary R-parity to forbid dangerous proton decay operators. (Without R-parity, superpotential couplings like $QQQ(H_d + \partial_\mu H_u^\pm)$ and $L(H_u + \partial_\mu H_d^\pm)$ together can lead to unacceptably large proton decay rates.)

Unification is not a generic feature of these models. However, as an example of an interesting model in which unification does occur, consider the following scenario: we will assume both boundaries to be broken to $SU(2) \times U(1)$, with quarks on one boundary, and the leptons on the other. Higgses and gauge fields propagate in the bulk with the charge assignments of figure \[3\]. In this model, all KK modes come in complete $SU(3)$ multiplets, so only the zero modes contribute. The zero mode beta functions are $(b_1 = 25/2, b_2 = -5/6)$. In addition to sfermions which will contribute to the running at a scale $\sim (4\pi R)^{-1}$, there is also the scalar singlet under $SU(2)$ with hypercharge 1, as well as the fifth component of the broken generator gauge bosons which transforms as a doublet under $SU(2)$ with hypercharge $3/2$. These particles should also pick up masses down by a loop factor from $R^{-1}$. With this set up, and a compactification scale of 1 TeV, we find unification at 5.3 TeV.

This model illustrates again the possibility of new, doubly charged particles at the weak scale. The decay modes are model dependent, but generically one would expect again hard charged leptons. The $SU(2)$ singlet field would most likely decay through an off-shell broken gauge boson into a Higgs. The decays of these would then give hard leptons and b quarks (or W bosons if

\begin{figure}[h]
\centering
\begin{tabular}{|c|c|}
\hline
$SU(2)\times U(1)$ & $SU(3)/SU(2)\times U(1)$ \\
\hline
$++ A$ & $- A$ \\
$++ \lambda_1 \lambda_2$ & $- \lambda_1 \lambda_2$ \\
$\phi -$ & $\phi +$ \\
\hline
$H_u, H_d$ & $\sigma^--, \sigma^+$ \\
$\psi^h_h$ & $\psi^h_\sigma$ \\
$++ h h^+$ & $++ \sigma \sigma^{--}$ \\
$\psi^c_h$ & $\psi^c_\sigma$ \\
\hline
\end{tabular}
\caption{Charge assignments for a model with $SU(3)$ broken to $SU(2) \times U(1)$ on both boundaries.}
\end{figure}
the Higgs is sufficiently heavy, which can happen in TeV scale 5D SUSY theories \[24,25\].

Ultimately, there is a wealth of phenomenology to be explored, but aside from these few generic points, it is best studied within the context of a complete model in which EWSB is realized.

**Conclusions:** In this paper we used the mechanism of ref \[3\] to reconstruct TeV-scale five-dimensional theories that successfully predict the weak angle. The best limit to the size of the new dimensions comes from the mass limits of excited Zs and is \(1/R > 1.3\,\text{TeV}\). There is a plethora of experimental predictions, such as the presence of exotic singly and doubly charged gauge bosons. Direct searches place a limit of 700 GeV to their mass. They can decay into pairs of (same sign)leptons. Even the lightest new particles will in general decay through higher dimension operators to particles on the boundaries and may give dramatic signatures at the LHC.

While completing this work, we became aware of refs \[26,27\], which address similar issues.

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