Flavor-Change with Ultra-Light Sbottom and Gluinos

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Abstract: Implications of a $2 - 5.5$ GeV sbottom and $12 - 16$ GeV gluino masses for rare $B$ decay phenomenology are discussed. An effective Hamiltonian is constructed in which the gluinos are integrated out and a $\tilde{b}$ squark remains among the light flavor degrees of freedom. Restrictive constraints come from $b \rightarrow s \gamma$ and $b \rightarrow s g$, but they allow a substantially enhanced inclusive $b$ decay rate into charmless hadronic final states, and $O(10\%)$ direct CP asymmetries in $B \rightarrow X_s \gamma$ and $B^{\pm} \rightarrow K^{0} \pi^{\pm}$ decays, which are an order of magnitude larger than in the Standard Model. New contributions to $B_s$ mixing are negligible but significant effects in $B_d$ mixing may be possible.

1. Introduction and Motivation

The measured $b$ quark production cross section at hadron colliders has persistently exceeded NLO QCD predictions by factors of 2. Rather than attributing this discrepancy to additional QCD contributions, e.g., those arising at NNLO, it is interesting to ask whether New Physics could be responsible. In [1] Berger et al. have shown that gluino pair production, followed by decay of each gluino to a bottom-bottom pair can account for the missing rate if the $\tilde{g}$ and light $\tilde{b}$ masses lie in the ranges $m_{\tilde{g}} \cong 12 - 16$ GeV and $m_{\tilde{b}} \cong 2 - 5.5$ GeV, respectively. They have further observed that a light $\tilde{b}$ squark could have evaded direct detection. For example, the additional contribution to $R_{had}$ at large $\sqrt{s}$ would only be $\cong 2\%$, and hence difficult to disentangle. In the resonance region, e.g., $\sqrt{s} \sim 5 - 8$ GeV, a light $\tilde{b}$ squark may resolve a long standing discrepancy in $R_{had}$ between the MARK I and Crystall Ball Collaborations [2].
There are important $Z$-pole constraints on the light $b$, $\tilde{g}$ scenario\cite{ref1}. Most importantly, the light sbottom’s coupling to the $Z$ must be suppressed. The light and heavy sbottom mass eigenstates $\tilde{b}$ and $\tilde{b}_H$, respectively, are admixtures of the left-handed and right handed bottom quark superpartners, $\tilde{b}_L$ and $\tilde{b}_R$, characterized by a mixing angle $\theta_{\tilde{b}}$: $\tilde{b} = \cos \theta_{\tilde{b}} \tilde{b}_R + \sin \theta_{\tilde{b}} \tilde{b}_R$. The coupling of $\tilde{b}$ to the $Z$ is proportional to $-\sin^2 \theta_{\tilde{b}}/2 + \sin^2 \theta_W/3$, and vanishes at tree-level if $\sin \theta_{\tilde{b}} \approx 0.38$. This implies that the light sbottom must be predominantly ‘right-handed’. An overall fit to $Z$ pole observables in $\tilde{b}$, which however only considered the impact of light sbottoms at tree-level, finds a slight improvement over the Standard Model fit for $\sin \theta_{\tilde{b}}$ in the range [0.3, 0.45]. A recent study\cite{ref2} finds very restrictive constraints on $\sin \theta_{\tilde{b}}$ from light $\tilde{b}$-$\tilde{g}$ loop contributions to $R_b$, however it does not include a fit to all $Z$-pole observables and does not take into account other potentially important one-loop supersymmetric contributions.

The presence of a light sbottom and light gluinos alters the running of $\alpha_s$. In the following we will root the evolution of $\alpha_s(\mu)$ at low scales using determinations from $\tau$ decays and deep inelastic scattering at scales $\mu \lesssim 5$ GeV, which would be unaffected by the new particles. We consider $\alpha_s(m_b) \approx 0.19 - 0.22$. The running of $\alpha_s$ at larger scales is slower than in the Standard Model, e.g., $\alpha_s(M_Z) \approx 0.121 - 0.133$, but the lower range of predicted values is compatible with experiment. The comparison is illustrated in Figure 1, where the running of $\alpha_s$ in the two cases is confronted with a compilation of measurements collected in $\tilde{g}$. A new LEP2 average at $\sqrt{s} = 206$ GeV is also included in $\tilde{g}$. A fit to the data appears to be only marginally better in the Standard Model. However, it should be noted that in the supersymmetric scenario the high energy observables used to determine $\alpha_s$ have not been corrected for new contributions due to the light $\tilde{b}$ squark and gluinos.

With regards to potential light $b$ decay modes, a null CLEO search for semileptonic decays\cite{ref3} implies that the branching ratios for the decay modes $\tilde{b} \rightarrow c\ell$, induced by $R$-parity breaking couplings, or $\tilde{b} \rightarrow c\ell\chi^0$, where $\chi^0$ is an ultra-light neutralino, must be highly suppressed. However, the light $\tilde{b}$ squark is allowed to decay promptly via hadronic $R$-parity breaking couplings in the modes $\tilde{b} \rightarrow c\tilde{q}$, $u\tilde{q}$, $q = u, s$. Alternatively, it could be long lived, forming $\tilde{b}$-hadrons.

If we are to take the possibility of a light $\tilde{b}$ squark and light gluinos seriously then the theoretical study of their impact must be extended to $b$ decay phenomenology, which is currently undergoing intensive scrutiny at the $B$ factories. In this talk we report on work in progress in this direction. New sources of flavor violation could arise via supersymmetric $s \rightarrow \tilde{b} - \tilde{g}$ and $d \rightarrow \tilde{b} - \tilde{g}$ ‘Yukawa’ couplings. The overall scale of supersymmetric flavor violating interactions originating from gluino exchange is set by the factor $g^2_s/m_{\tilde{g}}^2$, which is much larger than the

Figure 1: Runnings of $\alpha_s$ in the Standard Model (blue band), and with $m_b = m_b$, $m_q = 15$ GeV (pink band), for $0.19 < \alpha_s(m_b) < 0.22$. The data points are from a compilation of experimental determinations, see text.
corresponding factor $G_F/\sqrt{2} \sim g_W^2/M_W^2$, for weak decays in the Standard Model. Consequently, the new flavor-changing couplings must be much smaller than the corresponding CKM mixing angles. We will find that these couplings must be less than $10^{-4}$, in order to satisfy constraints coming from virtual gluino-sbottom loop contributions to $b \rightarrow s\gamma$, $b \rightarrow sg$, and $B \rightarrow K\pi$. Conversely, even with tiny flavor-changing couplings large deviations from Standard Model predictions are possible. This is of particular interest from a model-building point of view.

The phenomenology of this scenario will depend on whether or not light $\tilde{b}$ squarks can be pair-produced in $b$ decays. If they are too heavy then they only give rise to virtual effects, which we discuss here. However, if $\tilde{b}$ squarks are light enough to be pair produced, new unconventional decay channels would be opened up for $B$ mesons and beauty baryons. Potentially interesting consequences of such decays are briefly mentioned in the Conclusion.

2. The low energy effective Hamiltonian

Flavor-changing processes in the light sbottom-gluino model are most transparently described by means of an effective low-energy Hamiltonian in which the effects of the ‘heavy’ gluino fields are integrated out. The light degrees of freedom are the light quarks $u, d, s, c, b$, the photon and gluons, as well as the light $\tilde{b}$ squark. An expansion of the low-energy Lagrangian in powers of $1/m_{\tilde{b}}$ is justified for $m_{\tilde{b}} \approx 12 - 16$ GeV. For rare $B$ decays we have checked that to good approximation it is sufficient to work to leading order in this expansion.

To parametrize the flavor-violating couplings entering the Hamiltonian, let $\tilde{d}_i$, $i = 1, ..., 6$ denote the down squark mass eigenstates, and $\tilde{d}_L^I, \tilde{d}_R^I$, $I = 1, 2, 3$ denote the interaction eigenstates (superpartners of the left-handed and right-handed down quarks). We write in the usual way \[8\]

\[
\tilde{d}_L^I = (\Gamma_L^I)_{ij} \tilde{d}_j, \quad \tilde{d}_R^I = (\Gamma_R^I)_{ij} \tilde{d}_j,
\]

and identify $\tilde{d}_3$ with the light sbottom. The rest of the squark masses are taken to be of order the generic supersymmetry breaking mass, $M_{SUSY} < 1$ TeV. The new flavor-violating effects arising from light $\tilde{b}$ and $\tilde{g}$ exchange can be parametrized by the dimensionless quantities,

\[
\epsilon_{i3}^{AB} \equiv (\Gamma_A^i)_{i3}^\dagger (\Gamma_B^{33})_{33}, \quad i = 1, 2; \quad A, B = L \text{ or } R.
\]

Note that in general they can be complex, which would lead to new CP violating effects. In terms of the sbottom sector mixing angle introduced earlier, $\Gamma_R^{33} = \cos \theta_b$ and $\Gamma_L^{33} = \sin \theta_b$, implying the following relations,

\[
\epsilon_{23}^{LL} = \epsilon_{23}^{LR} \cot \theta_b, \quad \epsilon_{23}^{RR} = \epsilon_{23}^{RL} \cot \theta_b,
\]

and similarly for the $\epsilon_{13}^{AB}$'s.

In general, new contributions to the $\Delta B = 1$ effective Hamiltonian can be written as

\[
\mathcal{H}_{\Delta B=1} = \frac{4\pi \alpha_s}{m_{\tilde{g}}^2} \sum_i \left( C_i^T (\epsilon_{23}^{AB}) T_i + C_i (\epsilon_{23}^{AB}) O_i \right),
\]
where the dependence of the Wilson coefficients on the flavor-violation parameters $\epsilon^{AB}_{23}$ has been indicated. We briefly describe the operators which arise below. For brevity we explicitly include only the $\Delta S = 1$ operators. The $\Delta S = 0$ operators follow by substituting $s \to d$ everywhere. Their Wilson coefficients follow from the substitutions $\epsilon^{AB}_{23} \to \epsilon^{AB}_{13}$. Color indices are suppressed throughout. The $T_i$ arise from tree-level matching of the full theory onto the effective theory at scales $\mu \sim m_\tilde{g}$:

- Eight $T_i$ operators are present at order $1/m_\tilde{g}$, four of the form $\bar{s}(1 \pm \gamma_5)b\tilde{b}^*\tilde{b}$ with strengths depending linearly on $\epsilon^{LR}_{23}$ or $\epsilon^{RL}_{23}$, and four of the form $\tilde{s}(1 \pm \gamma_5)b\tilde{b}^*\tilde{b}$ with similar dependence on $\epsilon^{LL}_{23}$ or $\epsilon^{RR}_{23}$. The latter could mediate rare $B$ decays to ‘wrong-sign’ kaons.

- Eight $T_i$ operators arise at order $1/m^2_\tilde{g}$, and can therefore be neglected to good approximation. Four are of the form $\bar{s}\gamma_\mu(1 \pm \gamma_5)b\tilde{b}^*D^\mu\tilde{b}$, with strengths depending linearly on $\epsilon^{LR}_{23}$ or $\epsilon^{RL}_{23}$, and four are of the form $\tilde{s}\gamma_\mu(1 \pm \gamma_5)b\tilde{b}^*D^\mu\tilde{b}$, with similar dependence on $\epsilon^{LL}_{23}$ or $\epsilon^{RR}_{23}$.

The Wilson coefficients for the ‘one-loop’ operators $O_i$ are obtained by computing the full theory amplitudes due to light $\tilde{b}-\tilde{g}$ loops and subtracting the corresponding light $\tilde{b}$-loop contributions of the $T_i$.

- Four such operators are present at order $1/m_\tilde{g}$: Two electromagnetic dipole operators (Standard Model and opposite-chirality) which mediate $b \to s\gamma$ decays, of the form $\bar{s}\sigma_\mu(1 \pm \gamma_5)eF^\mu b$, and two chromomagnetic dipole operators (Standard Model and opposite-chirality) which mediate $b \to sg$ decays, of the form $\bar{s}\sigma_\mu(1 \pm \gamma_5)g_5G^\mu b$.

- Eight four-quark QCD penguin operators arise at order $1/m^2_\tilde{g}$: four are of the same form as in the Standard Model and four have the opposite chirality, i.e., $\bar{s}\gamma_\mu(1 - \gamma_5)b\sum_q \tilde{q}\gamma^\mu(1 \pm \gamma_5)q$ and $\tilde{s}\gamma_\mu(1 + \gamma_5)b\sum_q \tilde{q}\gamma^\mu(1 \pm \gamma_5)q$, respectively. Their effects can be neglected compared to those of the chromomagnetic dipole operators.

New contributions to the Standard Model chirality dipole operator Wilson coefficients depend linearly on $\epsilon^{LR}_{23}$ (at leading order in $1/m_\tilde{g}$), whereas the opposite-chirality coefficients depend on $\epsilon^{RL}_{23}$. The Standard Model and opposite-chirality QCD penguin Wilson coefficients depend linearly on $\epsilon^{LL}_{23}$ and $\epsilon^{RR}_{23}$, respectively. Although the latter can be neglected, it is interesting to note that the sbottom mixing angle $\theta_b$ in Eq. (2.3) fixes the ratios of (Standard Model or opposite-chirality) dipole operator to QCD penguin operator Wilson coefficients.

Finally, eight $\Delta B = 2, \Delta S = 2$ operators which can mediate $B_s$ mixing are present at order $1/m_\tilde{g}$ after one-loop matching of the full theory sbottom-gluino box graphs onto the effective theory. They are of the form $\bar{s}\gamma_\mu(1 \pm \gamma_5)b\tilde{s}\gamma^\mu(1 \pm \gamma_5)b$, $\tilde{s}\gamma_\mu(1 \pm \gamma_5)b\tilde{s}\gamma^\mu(1 \mp \gamma_5)b$, and $\bar{s}(1 - \gamma_5)b\tilde{s}(1 - \gamma_5)b$, (as usual color indices have been suppressed). Analogous operators mediating $B_d$ mixing are obtained by substituting $s \to d$ everywhere. We can write the effective $\Delta B = 2$ Hamiltonian in the form $H_{\Delta B=2} = \alpha_s^2/m^2_\tilde{g}\sum D_i Q_i$. The Wilson coefficients $D_i$ for the $\Delta S = 2$ and $\Delta S = 0$ operators depend quadratically on the $\epsilon^{AB}_{23}$ and $\epsilon^{AB}_{13}$, respectively.
3. Rare B decays and B mixing

We begin with a discussion of constraints coming from $B \rightarrow X_s \gamma$ and $B \rightarrow X_{sg}$ decays. Our strategy for describing $B \rightarrow X_s \gamma$ decays is to perform a partial next-to-leading order (NLO) analysis: The Standard Model contributions are included fully at NLO, as in [9, 10, 11], since we know that the NLO corrections, particularly those due to the $\bar{c}L\gamma_\mu bL\gamma^\mu cL$ current-current operator, are substantial. However, the new SUSY contributions are significant, tuning evidently requires a sizable new weak phase (90% c.l.) [14] significantly reduces the allowed region, so that $E_\gamma > 2$ GeV. If the chromomagnetic $b \rightarrow sg$ dipole operators are significantly enhanced, the shape of the photon energy spectrum is modified by new soft contributions from photon bremsstrahlung. Due to this possibility, we compare branching-ratio predictions directly with the CLEO measurement for $E_\gamma > 2$ GeV, using shape function convolutions for the energy spectrum [11]. For simplicity, we limit our discussion to constraints on new contributions to the Standard Model chirality dipole operators, which only depend on $e_{23}^{LR}$ at leading order in $1/m_b$. To very good approximation we can describe these processes using a truncated operator basis, ignoring the QCD penguin operators.

We use the parametrization $e_{23}^{LR} = |e_{23}^{LR}| e^{i\theta_{LR}}$, and exhibit constraints as contours in the $(|e_{23}^{LR}|, \theta_{LR})$ plane. In Figure 2a CLEO ±1σ contours are drawn for $\text{BR}(B \rightarrow X_s \gamma)/E_\gamma > 2$ (CP-averaged). The ratio $z = m_c/m_b$ entering the charm-loop $b \rightarrow s \gamma$ matrix element of the current-current operator has been allowed to vary in the range [0.22, 0.29]. Values near 0.22 are obtained by using the MS mass, $m_c(m_b)$ [13]. Elsewhere, $z = 0.29$ is used. Absence of significant tuning evidently requires $|e_{23}^{LR}| < 1 \times 10^{-4}$. This conclusion is further reinforced by the contours for $\text{BR}(B \rightarrow X_{sg})$ shown in Figure 2b. The CLEO upper-bound\(^1\) of 6.8% (90% c.l.) significantly reduces the allowed region, so that $|e_{23}^{LR}| \lesssim 5 \times 10^{-5}$. However, a sizable new weak phase $\theta_{23}^{LR}$ is allowed. In Figure 2c we draw contours for the direct CP asymmetry $A_{CP}(B \rightarrow X_s \gamma)$ making use of formulae in [13], and have included the CLEO 90% c.l. upper and lower bounds of +10% and -27%, respectively [16]. Finally, in Figure 2d we show the allowed region which survives the three constraints. Comparison with the contours for $A_{CP}$ and $\text{BR}(B \rightarrow X_{sg})$ implies that $A_{CP} \sim 10\%$ is possible, as is a significantly enhanced $b \rightarrow sg$ branching ratio of 5 - 10%. Recall that in the Standard Model $A_{CP} \sim 1\%$ and $\text{BR}(B \rightarrow X_{no\text{ charm}}) \sim 1\%$.

We have not yet taken into account potentially important two-loop $O(1/m_b)$ contributions to the LO anomalous dimension matrix from mixing of the current-current operators $T_i$ into the dipole operators. This work is currently in progress. There may also be important contributions arising at NLO from $b \rightarrow sg$ matrix elements of the operators $T_i$. These corrections will amount to a "k-factor" rescaling of the the $|e_{23}^{LR}|$ axes in Figure 2. It should also be noted that the theoretical predictions for $A_{CP}$ suffer from large renormalization scale dependence. Therefore, at this stage we regard the constraints shown in Figure 2 as illustrative. Nevertheless, our conclusions hold qualitatively.

Rare $B \rightarrow K\pi$ decays are also described to very good approximation at leading order in $1/m_b$, so we need only keep new contributions to the chromomagnetic dipole operators.

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\(^1\)Using more recent inclusive charmonium and charmed baryon yields gives an upper bound of 9%.
Figure 2: (a) ±1σ contours for the CLEO BR(B → Xsγ)E > 2 measurement in the (|εLR|, θLR) plane. (b) Contours for BR(B → Xsg) = 1, 5, 10, 20%. Also shown (blue-dashed) is the CLEO upper bound of 6.8%. (c) Contours for ACP(B → Xsγ) = ±1, ±10, ±30%. Also shown (green-dotted) are the 90% c.l. upper and lower limits from CLEO. (d) Combination of the previous bounds highlighting the allowed region (shaded yellow) in the (|εLR|, θLR) plane. αs(MZ) = 0.125 (αs(mb) = 0.20), μ = mb = 4.8 GeV.

Only Standard Model chirality operators are considered so that constraints can again be simply exhibited in the (|εLR|, θLR) plane. All contributions to the amplitudes have been evaluated using the QCD factorization approach of Beneke et al. (BBNS) [17], taking their default values for the various hadronic and CKM input parameters. Here we report on results for the decays B± → K0π±, which have little sensitivity to the CKM weak phase γ and which therefore are predicted to have a very small direct CP asymmetry in the Standard Model.

In Figure 3a, ±1σ contours have been drawn for the World Average branching ratios quoted in [17]. BR(B± → K0π±)W Avg = (17.2 ± 2.5) × 10⁻⁶. A second set of contours has been added for 1.35 × (+1σ BR) and .65 × (−1σ BR), which is intended to take into account a typical ±35% uncertainty in BBNS branching ratio predictions when varying over all input parameters. By superimposing the allowed region from B → Xsγ, Xsg in Figure 2d, the B → Kπ constraints are seen to be comparable but less restrictive. Contours for ACP(B± → K0π±) obtained using the BBNS approach are shown in Figure 3b. Comparison with the allowed region indicates that large direct CP asymmetries of order
10% are possible, to be compared with $\sim 1\%$ in the Standard Model. A similar result is obtained for $A_{CP}(B^\pm \to \phi K^\pm)$. Although the theoretical uncertainties for $A_{CP}(K\pi)$ are large [17], and two-loop mixing of the $T_i$ into the chromomagnetic dipole operators has not been taken into account, such added effects again will not change our conclusions qualitatively.

![Figure 3:](image)

(a) $\pm 1\sigma$ contours for $\text{BR}(B^\pm \to K^0\pi^\pm)_{\text{world avg}}$ in the $(|\epsilon_{LR}^I|, \theta_{LR})$ plane, using default BBNS inputs (solid-lines), and assuming a $\pm 35\%$ uncertainty (blue-dashed), see text. The allowed region from radiative decays is shaded yellow. (b) Contours for $A_{CP}(K^0\pi^\pm) = \pm 1, \pm 5, \pm 10, \pm 20 \%$. $\alpha_s(M_Z) = 0.125$, $\mu = m_b = 4.2$ GeV.

We conclude this section with a brief discussion of $B$ mixing constraints. Contributions of the $\Delta B = 2$ operators to $\Delta M_{B_d}$ involve several different combinations of the $\epsilon_{13}^{AB}$'s. Requiring that the contribution of each operator by itself should not exceed the measured value of $\Delta M_{B_d}$, and using the vacuum saturation approximation, we find for example that $\sqrt{\text{Re}[\epsilon_{13}^{LR} \epsilon_{13}^{LR}]} < (1 - 4) \times 10^{-4}$. This is not as restrictive as the bounds obtained from radiative $b \to d\gamma$ decays. Given that the CLEO measurement of the inclusive radiative branching ratio actually corresponds to the sum of $b \to s\gamma$ and $b \to d\gamma$, and that $\Gamma(B \to \rho\gamma)/\Gamma(B \to K^{*}\gamma) < .19$ (90\% c.l.) [18], the bound on $|\epsilon_{13}^{LR}|$ from radiative $B$ decays is at least as stringent as the bound on $|\epsilon_{23}^{LR}|$. Therefore, new supersymmetric contributions probably could not account for the bulk of $\Delta M_{B_d}$, but they may significantly modify the CP violating mixing phase. Finally, bounds on $\epsilon_{23}^{AB}$ from radiative $B$ decays imply that new supersymmetric contributions to $B_s$ mixing must be negligible.

4. Conclusion

It has been pointed out that new supersymmetric contributions to $b$ quark production at hadron colliders can account for the long-standing discrepancy between the measured and NLO QCD cross sections if there is a light $b$ squark with mass in the range 2 - 5.5 GeV, and if the gluinos have mass in the range 12 - 16 GeV [1]. In this talk we have explored the phenomenology of rare $B$ decays in such a scenario, and have found very restrictive
constraints on the flavor-violation parameters controlling supersymmetric contributions to $b \to s$ and $b \to d$ transitions, namely $\epsilon_{23}^{AB}, \epsilon_{13}^{AB} \lesssim 10^{-5}$. This implies that certain off-diagonal down squark mass matrix entries must be similarly suppressed compared to the generic squark mass squared. However, interesting New Physics effects are possible. Among these are an enhanced $B \to X_{s\gamma}$ charm rate, which may help explain the low $b$ semileptonic branching ratio and charm multiplicity, and $O(10\%)$ direct CP asymmetries in $B \to X_{s\gamma}$ and $B^\pm \to K^0\pi^\pm$ decays, to be compared with Standard Model CP asymmetries of order 1%. The most restrictive constraints are due to $B \to X_{s\gamma}$ and $B \to X_{s\gamma}$. Less restrictive constraints follow from $B_d$ mixing, and new contributions to $B_s$ mixing must be negligible.

We have not considered the potential consequences of $\tilde{b}$ pair production. The new decay modes $b \to s\tilde{b}\tilde{b}$ and $b \to s\tilde{b}\tilde{b}$ which become accessible when the $\tilde{b}$ is sufficiently light would aect the decay widths of $B$ mesons and $\Lambda_b$ baryons differently, and hence could potentially explain the anomaly of the low $\Lambda_b$ lifetime. A signiﬁcant increase in the $\Gamma_{B_d} - \Gamma_{\Sigma_d}$ lifetime difference may also be possible. We will report on this interesting class of effects elsewhere.

References


