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A Canonical Hamiltonian Derivation of Hawking Radiation

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We present a derivation of Hawking radiation based on canonical quantization of a massless scalar field in the background of a Schwarzschild black hole using Lemaître coordinates and show that in these coordinates the Hamiltonian of the massless field is time-dependent. This result exhibits the non-static nature of the problem and shows it is better to talk about the time dependence of physical quantities rather than the existence of a time-independent vacuum state for the massless field. We then demonstrate the existence of Hawking radiation and show that despite the fact that the flux looks thermal to an outside observer, the time evolution of the massless field is unitary.

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Hawking radiation [1] is one of the most interesting phenomena encountered in *pre-quantum* gravity. It is a robust phenomenon in that it has been derived in different ways (see e.g. [2, 3, 4]) and all derivations yield the same conclusion: a black hole of mass M emits (nearly) thermal radiation with a temperature $T_{\rm H} = 1/(8\pi GM)$. To the best of our knowledge however, no derivation of Hawking radiation discusses the problem within the framework of canonical quantization. This is one reason why the issue of whether the time evolution of the semi-classical theory is unitary has been a subject of debate. In this paper we present a simple canonical quantization procedure which leads to the usual results within the framework of a unitary quantum theory. Moreover, it enables us to compute the energy-momentum tensor as a function of position for all times, allowing us, as a matter of principle, to give a self-consistent discussion of the back reaction problem.

The reason why it is not obvious that a canonical quantization scheme is possible in Schwarzschild coordinates is because surfaces of fixed *Schwarzschild time* are not globally space-like. In Lemaître coordinates, however, surfaces of constant time are space-like and extend from $r = 0$ to $r = \infty$. In this coordinate system one can canonically quantize a field on a fixed time surface and then consider its time evolution from that point on. The immediate and most striking result of this approach is that the Hamiltionian of the massless field, computed in the background of the Schwarzschild black hole, is time-dependent. From this point of view, there are no uniquely defined eigenstates (in particular the vacuum-state) of the the quantum mechanical problem, in spite of the fact that we are dealing with a free field theory. It is this time dependence which is ultimately the origin of Hawking radiation.

The metric of the eternal black hole has the following form in Schwarzschild coordinates:

$$\mathrm{d}s^2 = -\left(1-\frac{\alpha}{r}\right)\mathrm{d}t^2 + \left(1-\frac{\alpha}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2, \quad (1)$$

where $\alpha = 2GM$, G is Newton's constant and M is the mass of the black hole. Because the coefficient of dr^2 changes sign at $r = \alpha$ surfaces of constant Schwarzschild time go from being space-like to time-like at this point. This does not occur in Lemaître coordinates, λ, η, which are related to t, r by

$$\lambda = t + 2\sqrt{r\alpha} + \alpha \ln\left|\frac{\sqrt{r}-\sqrt{\alpha}}{\sqrt{r}+\sqrt{\alpha}}\right|, \;\; r = \alpha^{1/3}\left[\frac{3}{2}(\eta-\lambda)\right]^{2/3}, \quad (2)$$

In these coordinates the metric becomes:

$$\mathrm{d}s^2 = -\mathrm{d}\lambda^2 + \frac{\alpha}{r}\mathrm{d}\eta^2 + r^2\mathrm{d}\Omega^2. \quad (3)$$

and from this form we see that surfaces of constant λ are everywhere space-like. Thus, these surfaces can be used to carry out the canonical quantization of the field theory. (Note, since r is a function of λ, the metric is explicitly λ or time-dependent and this translates into the time-dependence of the Hamiltonian.)

The action of a minimally coupled massless scalar field in the background of the Schwarzschild black hole is

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}g^{ij}\partial_i\Phi\partial_j\Phi. \quad (4)$$

Since this action is rotationally invariant we can discuss one angular momentum mode at a time. From now on we will focus on the S-wave component of the field and define $\phi_0 = \Phi/\sqrt{4\pi}$. In Lemaître coordinates standard manipulations lead to the Hamiltonian

$$\mathcal{H} = \frac{1}{2}\int_{\lambda}^{\infty}\mathrm{d}\eta\left\{\frac{2\pi_0^2}{3\alpha(\eta-\lambda)} + \frac{3}{2}r(\eta-\lambda)\left(\frac{\partial\phi_0}{\partial\eta}\right)^2\right\}, \quad (5)$$

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where π_0 is the canonical momentum conjugate to ϕ_0; i.e., $\pi_0 = \alpha(\eta - \lambda)\partial\phi_0/\partial\lambda$. As always, π_0 and ϕ_0 are assumed to satisfy the equal time commutation relation $[\pi_0(\lambda,\eta), \phi_0(\lambda,\eta')] = -i\delta(\eta - \eta')$.

The dynamics of the field ϕ_0 and its canonical momentum π_0 are governed by the Hamilton equations of motion, which are equivalent to the statement

$$\alpha\frac{\partial}{\partial\lambda}\left[(\eta-\lambda)\frac{\partial\phi_0}{\partial\lambda}\right] - \frac{\partial}{\partial\eta}\left[(\eta-\lambda)r\frac{\partial\phi_0}{\partial\eta}\right] = 0. \qquad (6)$$

If we choose the initial space-like surface to correspond to $\lambda = 0$, so that $\eta = 2/3\, r\sqrt{r/\alpha}$ and examine the $\lambda = 0$ Hamiltonian, we see that it is useful to rescale the field and its momentum as follows: $\phi_0 = \phi_1/r$, $\pi_0 = \sqrt{r\alpha}\pi_1$. After this rescaling π_1 and ϕ_1 satisfy the commutation relation: $[\pi_1(r), \phi_1(r')] = -i\delta(r - r')$, and the $\lambda = 0$ Hamiltonian takes the simple form

$$\mathcal{H}_{\lambda=0} = \frac{1}{2}\int_0^\infty \mathrm{d}r\left\{\pi_1^2 + r^2\left(\frac{\partial}{\partial r}\frac{\phi_1}{r}\right)^2\right\}. \qquad (7)$$

Clearly, this is just the Hamiltonian for the radial mode of a free scalar field and it is easily solved. As usual, self-adjointness requires that we impose the boundary condition $\phi_1(r=0) = 0$, which implies that ϕ_1 and π_1 can be written as

$$\phi_1(r) = \int_0^\infty \frac{\mathrm{d}\omega}{\sqrt{\pi\omega}}\sin(\omega r)\left(a_\omega^+ + a_\omega\right);$$

$$\pi_1(r) = i\int_0^\infty \frac{\omega\mathrm{d}\omega}{\sqrt{\pi\omega}}\sin(\omega r)\left(a_\omega^+ - a_\omega\right), \qquad (8)$$

where a_ω^+, a_ω are the usual creation and annihilation operators $[a_\omega, a_{\omega'}^+] = 2\pi\delta(\omega - \omega')$. The $\lambda = 0$ vacuum state is defined by $a_\omega|0\rangle = 0$.

For our purposes it will be sufficient to assume that at $\lambda = 0$ the quantum field is in its vacuum state [5]. To study the time dependence of the theory it is best to work in the Heisenberg representation where the operators are functions of time and the quantum state does not change. The time evolution of the Heisenberg fields are given by the wave equation Eq.(6) supplemented by the condition that ϕ_0 and $\pi_0 \sim \partial\phi_0(\eta)/\partial\lambda$ reduce to their initial values on the $\lambda = 0$ surface. While this is a complicated equation it will be sufficient, for our purposes, to analyze it in the geometric optics approximation. Eventually all formulas for physical quantities will be expressed geometrically, so they will not depend upon the coordinate system. To explain the geometric optics approximation however, it is simplest to work in Painleve coordinates; i.e., λ, r, since the dependence of the solution on λ and r factorizes in these coordinates.

If we look for solutions of the form $\phi_0 = r^{-1}e^{i\omega\lambda}f_\omega(r)$, it is easy to see that, for large ω, $f_\omega(r)$ can be written as

$$\ln f_\omega(r) = i\omega S_{1,2}(r) + \mathcal{O}(\omega^{-1}),$$
$$S_{1,2}(r) = \pm r - 2\sqrt{r\alpha} \pm 2\alpha\ln|\sqrt{r/\alpha} \pm 1|. \qquad (9)$$

Although the assumption of large ω may seem unjustified, these two solutions do provide a very good approximation to the exact solutions both at $r/\alpha \to 1$ and $r \to \infty$, which are the two regions that are of primary interest to us. (Note, however, that these solutions are invalid for $r \to 0$, a point we will have more to say about below.) Examination of these solutions shows that they are characterized by the fact that they are constant along ingoing or outgoing null geodesics. Abstracting this fact we define the geometric optics approximation as the assumption that the time evolution of the field ϕ_0 is given by

$$\phi_0(\lambda, r) = \frac{1}{r}\left[\tilde{\phi}_1(\lambda + S_1(r)) + \tilde{\phi}_2(\lambda + S_2(r))\right], \qquad (10)$$

where $\tilde{\phi}_1$ and $\tilde{\phi}_2$ are two arbitrary functions that can be determined from the requirement that at $\lambda = 0$ they can be written in terms of the field ϕ_0 and its canonical momentum π_0.

Imposing this requirement to determine the two arbitrary functions $\tilde{\phi}_{1,2}(S_{1,2}(r)) = f_{1,2}(r)$ we find

$$\phi_0(\lambda, r) = \frac{1}{r}\left\{f_1(x_1) + f_2(x_2)\right\}, \qquad (11)$$

where

$$f_{1,2}(x) = \frac{1}{2}\int_0^x \mathrm{d}\xi\left[\phi_1'(\xi) \pm \pi_1(\xi) \mp \frac{\alpha^{1/2}\phi_1(\xi)}{\xi^{3/2}}\right], \qquad (12)$$

$\phi_1' = \mathrm{d}\phi_1/\mathrm{d}\xi$ and $S_{1,2}(x_{1,2}) = \lambda + S_{1,2}(r)$. Now, since the field ϕ_1 and the momentum π_1 are expressed through creation and annihilation operators defined at $\lambda = 0$, the above set of equations allows us to compute any Green's function of the field ϕ_0 at any later time.

A quantity which is of special interest is the expectation value of the λ, η component of the energy-momentum tensor, since it appears on the right-hand side of the Einstein equations. If the integral of this quantity over a surface of fixed r approaches a constant as $r \to \infty$, this implies a constant flux of outgoing energy, i.e., Hawking radiation. We wish to emphasize that the technique described below permits us to calculate $T_{\lambda\eta}$ for any finite values of λ and η but for the purposes of this Letter we will concentrate on the energy flux through an infinitely large sphere.

In Lemaître coordinates the total energy flux through an infinitely large sphere is given by

$$\mathcal{J} = \lim_{\eta\to\infty}\int \mathrm{d}\phi\mathrm{d}\theta\sqrt{-g}g^{\lambda\lambda}g^{\eta\eta}\frac{\langle T_{\lambda\eta}\rangle}{4\pi} = -\lim_{\eta\to\infty}\frac{r^{5/2}}{\alpha^{1/2}}\langle T_{\lambda\eta}\rangle. \qquad (13)$$

where $\langle T_{\lambda\eta}\rangle$ is the vacuum expectation value of the off-diagonal component of the stress-energy tensor of the field ϕ_0 (a normalization factor of $(4\pi)^{-1}$ is introduced since ϕ_0 denotes the S-wave component of the massless field). Using the explicit expression for the time evolution of the field ϕ_0, Eq.(10), it is a straightforward matter to compute this flux. In order to do it we need to regularize intermediate expressions and we choose to regulate $T_{\lambda\eta}$ by point-splitting. To this end we introduce $\sigma = \lambda_2 - \lambda_1$ and define

$$\langle T_{\lambda\eta}\rangle = \lim_{\sigma\to 0}\frac{1}{2}\langle 0|\left\{\frac{\partial\phi_0(\lambda_1,\eta)}{\partial\lambda},\frac{\partial\phi_0(\lambda_2,\eta)}{\partial\eta}\right\}|0\rangle, \quad (14)$$

where $\{A,B\}$ denotes the anti-commutator of the fields.

The calculation of this quantity for large values of λ and η is simplest in Painleve coordinates. Since the value of η is fixed, the equation $\lambda_2 - \lambda_1 = \sigma$ implies that r_1 and r_2 are related by

$$r_2 = r_1 - \frac{\sqrt{\alpha}\sigma}{r^{1/2}} - \frac{\alpha\sigma^2}{4r_1^2} - \frac{1}{6}\frac{\alpha^{3/2}\sigma^3}{r_1^{7/2}} + \mathcal{O}(\sigma^4). \quad (15)$$

Taking the derivatives in Eq.(14) and considering the limit $r_{1,2}\to\infty$, we derive the following expression for the flux

$$\begin{aligned}\mathcal{J} = &-\frac{1}{2}\lim_{\sigma\to 0}\left[\langle\{f_1'(x_1),f_1'(y_1)\}\rangle W_1(r_2,r_1,x_1,y_1)\right.\\ &\left.+\langle\{f_2'(x_2),f_2'(y_2)\}\rangle W_2(r_2,r_1,x_2,y_2)\right], \quad (16)\end{aligned}$$

where the functions $f_{1,2}$ are defined in Eq.(12),

$$W_i(r_2,r_1,x,y) = \frac{S_i'(r_2)}{S_i'(x)S_i'(y)}\left(1 - \frac{\sqrt{\alpha}S_i'(r_1)}{\sqrt{r_1}}\right),$$

and $x_{1,2}$, $y_{1,2}$ are defined through the set of equations:

$$\lambda_1 + S_1(r_1) = S_1(x_1), \quad \lambda_2 + S_1(r_2) = S_1(y_1), \quad (17)$$

and the same for x_2, y_2 with the change $S_1 \to S_2$. Given these equations it is easy to derive the relation between $y_{1,2}$ and $x_{1,2}$ as a power series expansion in σ. It is clear from Eq.(17) that the points x_1, x_2 are those points on the $\lambda = 0$ surface from which the null-geodesics which wind up at the point λ,η must start out. From the specific form of the functions $S_{1,2}$ we see that in the limit of large λ and large η, the limiting values for these points are $x_1\to\infty$, $x_2/\alpha\to 1$.

Let us now compute the anticommutator $\langle\{f_1'(x_1),f_1'(y_1)\}\rangle$. Starting from Eq.(12) we derive

$$\begin{aligned}\langle\{f_1'(x_1),f_1'(y_1)\}\rangle = \;& \frac{1}{4}\left[\langle\{\pi_1(x_1),\pi_1(y_1)\}\rangle\right. \quad (18)\\ &\left.+\langle\{\phi_1'(x_1),\phi_1'(y_1)\}\rangle + ...\right],\end{aligned}$$

where the dots represent the terms which do not contribute to the flux in $r_1\to\infty$ limit. The anticommutators in the above formula are computed using Eq.(8) and we find the energy flux

$$\begin{aligned}\langle\{\phi_1'(x_1),\phi_1'(y_1)\}\rangle &= \frac{-2}{\pi}\frac{(x_2^2+x_1^2)}{(x_1^2-x_2^2)^2};\\ \langle\{\pi_1(x_1),\pi_1(y_1)\}\rangle &= -\frac{4x_1y_1}{\pi(x_1^2-x_2^2)^2}, \quad (19)\end{aligned}$$

so that the final result for the anticommutator reads

$$\langle\{f_1'(x_1),f_1'(y_1)\}\rangle = -\frac{1}{2\pi}\frac{1}{(x_1-y_1)^2} + ... \quad (20)$$

Performing a similar calculation for the second term in Eq.(16) and substituting expansions of $y_{1,2}$ and r_2 in terms of $x_{1,2}$ and r_1, we find

$$\mathcal{J} = \frac{1}{192\pi\alpha^2} = \frac{\pi}{12}T_{\rm H}^2, \quad (21)$$

where we have introduced Hawking temperature $T_{\rm H} = 1/(8\pi GM)$.

We should make two comments concerning this result. First, the energy flux at large distances, we have just derived, is finite. This is in accord with the expectations regarding possible divergences in the stress-energy tensor computed in a gravitational background [6]. Quite generally, the result of such a calculation is divergent and the divergences are usually removed by appropriate renormalization. The allowed counterterms have a restricted form and include renormalization of the cosmological and Newtonian constants, as well as some other terms that do not appear in Einstein's action. It turns out that *none* of allowed counterterms can renormalize the *off-diagonal* element of the energy momentum tensor in the gravitational background of a Schwarzschild black hole, both in Schwarzschild and Lemaître coordinates. Therefore the result for the energy flux should come out finite and, as we have seen, it does. The second comment concerns the calculation of the energy flux through the sphere of finite radius. It turns out that the result we obtain in that case is not finite in that $\ln\sigma$ terms remain. We believe this to be due to the fact that we have restricted attention to spherically symmetric configurations and have not considered higher angular momenta. This is supported by the fact that in the case of a two-dimensional black hole, where higher angular momentum modes are absent, our result for the flux is finite for arbitrary values of r. We should emphasize that both the outgoing and the infalling fluxes are separately divergent and only the sum of the two provides a finite, unambiguous answer.

To see that this time-independent flux of energy at large distances corresponds to what one would expect from a body at a Hawking temperature $T_{\rm H}$ it is necessary to weakly couple the massless field to a detector [2, 7] (which acts as a thermometer) located at some fixed Schwarzschild radius r. In order to make the computation more realistic we consider adiabatically switching on the detector at some time $t = t_0$ for a finite amount of time δ. (Note, adiabaticity requires $E_{\rm typ}\delta \gg 1$, where $E_{\rm typ}$ is typical detector level spacing.) To realize this situation we add an interaction term to the free field Lagrangian of the form $V_{\rm int} \sim e^{-(t-t_0)^2/(2\delta^2)}\phi_0(t,r)\hat{M}$, where $\hat{M}$ is an operator which acts in the Hilbert space of detector eigenstates. It follows from second order perturbation theory that the probability of exciting the detector to a state of energy E is proportional to the Fourier transform of the Green's function of the massless field [6]:

$$\mathcal{P}(\Delta E) \sim |\langle E|M(0)|E_0\rangle|^2 \int \mathrm{d}t\mathrm{d}t' e^{-i\Delta E(t-t')} \times e^{-[(t-t_0)^2+(t'-t_0)^2]/(2\delta^2)} \langle \phi_0(t,r)\phi_0(t',r)\rangle, \quad (22)$$

where $\Delta E = E - E_0$ and E_0 is the ground state energy of the detector. As in the calculation of the flux which we already described, the Green's function in Eq.(22) is computed using the evolution equation for the field ϕ_0 which relates it to initial conditions for ϕ_0 and π_0 on the surface $\lambda = 0$. It is convenient to define $x_{1,2}$ and $y_{1,2}$ as in Eq.(17) and identify $r_1 = r_2 = r$, $\lambda_1 = \lambda_1(t,r)$ and $\lambda_2 = \lambda_2(t', r)$. As before, these are the points from which infalling and outgoing geodesics leave the $\lambda = 0$ surface in order to reach the points t, r and t', r. Now consider Eq.(22) for large values of t_0. Using the explicit form of the functions $S_{1,2}$ it is easy to find the following approximate solutions: $x_1 = t + r$, $y_1 = t' + r$, $x_2 = \alpha(1 + 2e^{-(t-r)/(2\alpha)})$, $y_2 = \alpha(1 + 2e^{-(t'-r)/(2\alpha)})$, with $t \sim t' \sim t_0$. It is then clear that, asymptotically, $x_2 \to y_2 \to \alpha$ and $x_1 \to y_1 \to \infty$. In this limit, the Green's function can be written as

$$\langle\phi_0(t,r)\phi_0(t',r)\rangle \approx \frac{-1}{4\pi r^2}\left(\ln|x_1 - y_1| + \ln|x_2 - y_2| + \frac{i\pi}{2}\left[\kappa(x_1, y_1) + \kappa(y_2, x_2)\right] + c\right), \quad (23)$$

where $\kappa(x, y) = \theta(x-y)-\theta(y-x)$ and c is some constant.

It is instructive to consider the terms in Eq.(23) separately. The constant does not contribute to $\mathcal{P}(E)$ since it yields a result proportional to $\exp(-\Delta E^2\delta^2) \ll 1$. The $\ln|x_1 - y_1|$ term and the terms described by the function κ give simple, α independent, contributions that can be written as

$$\mathcal{P}_1(\Delta E) \sim -\frac{\pi\delta}{\Delta E} + \mathcal{O}((\delta\Delta E)^{-1}). \quad (24)$$

The important part of the final result comes from the second term in Eq.(23) which describes the radiation coming from the vicinity of the horizon. Appropriately shifting the integration variables we obtain

$$\mathcal{P}_2(\Delta E) \sim -\int \mathrm{d}t\mathrm{d}t' e^{-i\Delta E(t-t')} e^{-[(t-t_0)^2+(t'-t_0)^2]/(2\delta^2)} \times \ln|e^{-t/(2\alpha)} - e^{-t'/(2\alpha)}|. \quad (25)$$

If we then change the variables to $v = (t+t')$, $u = (t-t')$ and neglect all the suppressed terms we arrive at

$$\mathcal{P}_2(\Delta E) \sim \frac{2\pi\delta}{\Delta E}\left[\frac{1}{e^{\Delta E/T_\mathrm{H}} - 1} + \frac{1}{2}\right]. \quad (26)$$

Since the total probability is given by the sum of $\mathcal{P}_1$ and $\mathcal{P}_2$ we get the final result:

$$\frac{\mathcal{P}(\Delta E)}{\delta} \sim \frac{|\langle E|M(0)|E_0\rangle|^2}{\Delta E} \times \frac{1}{e^{\Delta E/T_\mathrm{H}} - 1}. \quad (27)$$

The interpretation of this formula is straightforward. If, at a large distance from the black hole, an observer switches on a detector which interacts with the massless field for finite amount of time, then the energy levels of the detector get populated as if the detector was in equilibrium with a thermal distribution of particles at a temperature T_H.

In conclusion, in this Letter we have presented a canonical Hamiltonian derivation of Hawking radiation for the case of a Schwarzschild black hole. In this formalism the Hamiltonian of the system is perforce time-dependent and so the discussion of the vacuum state of the massless field is irrelevant to the long time behavior of the system. By the very nature of our construction, the state of the system is always a pure state and for this reason there are correlations of the fields inside and outside the black hole horizon. We also explicitly demonstrated, in agreement with arguments that concern the general structure of the divergences in the energy-momentum tensor in the gravitational background, that the Hawking flux at infinity is a uniquely defined finite quantity. This opens up the possibility of identifying the time-dependent function that should be inserted into the right hand side of the Einstein equations to study the back reaction of the Hawking radiation on the geometry of the space-time in a self-consistent way.

Clearly, this approach allows us to address more issues than we can touch upon in a Letter; in particular, the question of back-reaction. Such issues will be discussed in detail in a long paper which is in preparation. It would be remiss of us, however, to conclude without pointing out that there is a significant difference between the case of the two and four-dimensional black hole. Direct substitution shows that the solutions in Eq.(9) are *exact* for the two-dimensional black hole. This is not true for the four-dimensional black hole where the geometric optics approximation breaks down at $r \to 0$. Thus, the behavior of the quantum theory near $r \to 0$ is different in the two cases. This is important because study of both cases shows that after a finite time some modes of the field no longer appear in the time-dependent Hamiltonian and thus, their time evolution becomes trivial. It requires careful study of the behavior of the system in order to understand whether or not these modes stay decoupled as the hole evaporates and whether they store any information.

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[1] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975); J.B. Hartle and S.W. Hawking, Phys. Rev. **D13**, 2188 (1976).
[2] W.G. Unruh, Phys. Rev. **D14**, 870 (1976).

[3] T. Jacobson and D. Mattingly, Phys. Rev. **D61**, 024017 (2000).

[4] M. Parikh and F. Wilczek, Phys. Rev. Lett. **85**, 5042 (2000).

[5] If the initial state is not the vacuum state but any finite energy state of the instantaneous Hamiltonian, the finite energy will be radiated away in finite amount of time. The structure of radiation at large later times is independent of the initial quantum state of the massless field.

[6] N.D. Birrell and P.C.W. Davies, *Quantum Fields in the curved space*, Cambridge University Press, 1984.

[7] B.S. DeWitt, in *General Relativity*, eds. S.W. Hawking and W. Israel, Cambridge University Press, 1979.