## The *CP* Puzzle in the Strong Interactions<sup>\*</sup>

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## Abstract

This lecture, directed to an broad audience including non-specialists, presents a short review of the problem of strong CP symmetry maintenance. The problem is defined and the possible solutions briefly reviewed. I discuss the way in which Roberto Peccei and I came up with one solution, generally known as Peccei-Quinn symmetry.

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I would like to begin by thanking the ICTP for this honor, which I greatly appreciate receiving. To Miguel Virasoro and to the rest of the selection committee I want to express my sincere gratitude. I am greatly honored to be invited to join the company of the distinguished physicists who have received this award in the past. I also want to thank my collaborators on the work cited in this award, Steve Weinberg and my co-recipient Howard Georgi on the unification of the couplings [1] and Roberto Peccei, with whom I did the work I will talk about today [2]. He was not included in this award as it was principally focused on the other work, on Grand Unified Theories. I know that Jogesh Pati, the third co-winner this year, has talked on that topic, not only in his Dirac lecture [3] but also in his lectures for the Particle Physics school [4]. So I have chosen to devote my talk today to the topic of strong CP Violation and how to avoid it. This lecture thus also is an extension of the course on CP violation that I am giving in the school here this week [5]. This lecture is intended for a broader audience than usual for such a topic, so I begin by an attempt to explain the topic to the less technically-expert part of the audience.

The term CP is the technical name of a symmetry that appears to be almost but not quite an exact symmetry of nature, namely the symmetry between the laws of physics for matter and those for antimatter [6]. We know from experiment that this symmetry applies to very high precision in the strong interactions. These are the interactions responsible for binding quarks together to make the observed particles such as protons and neutrons, and also for the forces between such particles, for example those that cause the protons and neutrons to bind together in atomic nuclei. We also find that this symmetry applies for electromagnetic interactions (those due to electric and magnetic charges and fields). The surprise of the 1960's was that it is not quite true for the weak interactions. Weak interactions are those responsible for many types of particle decays, in particular all those where one type of quark changes to a different type. The surprise was the observation of weak decay processes that would be forbidden if the matter-antimatter symmetry were exact, rare decays of the long-lived neutral kaon, which should be a CP odd state, to the CP-even state consisting of two pions [7].

In physics exact symmetries are easy to include in a theory. Equally easily we can write theories that do not have the symmetry at all. The hardest situation to explain is one where we seem to have an almost exact symmetry. Typically once a symmetry is broken there is no reason for it to appear to be close to true, rather it just disappears altogether. So when we find an almost true symmetry we need to find a mechanism that can explain that property. In most cases it is no problem to have different symmetry structure for the different interactions. Indeed that is really what distinguishes one type of interaction from another in our theory. So one might think that all we had to do was explain why there is a small CP violation in weak decays. That can be readily accommodated in the three-generation Standard Model.

It turns out however that for the particular case of matter-antimatter symmetry, or CP this separation in the symmetry properties of the different interactions is not



Figure 1: Potential well with two degenerate minima.

so readily achieved. In our current Standard Model theory (and in any extension of it which maintains the well-established QCD theory for the strong interactions) any breaking of CP symmetry in the weak interactions can, and generally will, induce a breaking of that symmetry in the strong interactions. So the challenge is to find a class of theories where this is not the case, or at least where the magnitude of the effect can be tightly controlled.

In order to describe how this strong CP-breaking comes about, and then how it can be avoided, I will begin by describing some similar quantum mechanical properties for a much simpler system. The features I will stress in this more familiar problem all have parallels in the strong CP problem. Consider a particle in a (one-dimensional) potential well such as that shown in Fig. 1. We want to examine the ground states of a particle in this well. The potential is symmetric about the line x=0, but rather than having a single lowest-energy point at that location it has two equally lowenergy locations at  $x = \pm x_0$ . If this were a classical physics problem, say a ball rolling on a hilly surface, and you were asked to find the lowest energy state for the ball, you could readily see that there would be two equally low energy states, one centered at each of the two minima. Indeed, even in quantum mechanics, by making an approximation that each well is independent, one can find two such states, which are described by probability distributions centered on either minimum. Let us call these states  $\psi_{\text{Left}}$  and  $\psi_{\text{Right}}$ . But the quantum phenomenon of tunneling allows a particle located at some time at one minimum to have a finite probability to appear at the other minimum some time later. The two wells are not truly separate. The states  $\psi_{\text{Left}}$  and  $\psi_{\text{Right}}$  cannot be the true stable states (eigenstates) of the system.

To find the correct stable lowest-energy states we apply a general rule of quantum physics. Any symmetry of the energy function, the Hamiltonian, must also be manifest in the stable states of the system. all such states can be labeled by the way in which they transform under the symmetry. The potential energy has a symmetry under the transformation of x to -x. This is called a Parity transformation, it transforms left into right and vice-versa. (In a three-dimensional space parity reverses all directions). The rest of the energy, the kinetic energy, is clearly also invariant under this transformation, since it proportional to is the square of velocity. This means that the entire Hamiltonian for our problem has an invariance under the Parity transformation. Quantum physics then tells us that all stable physical states must also be transformed into themselves under Parity. Since two reflections is equivalent to no reflection at all, the only consistent choices for the constant coefficients, or quantum numbers, of the states under the Parity operation are  $\pm 1$ . The superpositions  $[\psi_{\text{Left}} \pm \psi_{\text{Right}}]/\sqrt{2}$  are the stable ground states with these properties.

As previously stated, the states  $\psi_{\text{Left}}$  and  $\psi_{\text{Right}}$  are broad probability distributions rather than the simple point locations of the classical solutions. Each distribution has a small tail that reflects the fact that there is some small probability to find the particle in the other minimum. Thus the two distributions overlap slightly. Hence they interfere with one another. In one of the definite-Parity states the interference adds a little to the energy, and in the other it reduces it slightly. There is a unique lowest energy state, which turns out to be the combination  $[\psi_{\text{Left}} + \psi_{\text{Right}}]/\sqrt{2}$ . This state transforms into exactly the same state under parity, with coefficient +1. The lowest state which goes into itself times -1 under the Parity transformation,  $[\psi_{\text{Left}} - \psi_{\text{Right}}]/\sqrt{2}$ , has a slightly higher energy. We call this an odd-Parity state. Not only do the left and right states get mixed up because of tunneling, but also the two different Parity admixtures have slightly different energy.

All the higher-energy states of the system can likewise be divided into those that are even and those that are odd under the Parity transformation. If we add electromagnetic interactions in our theory and investigate the possibility of radiative transitions between states of different energies, we find that all such transitions occur only between states that have the same Parity. The Hilbert space, the set of all states of the system, is split into two disjoint parts, which act as if they are two separate worlds, knowing nothing of one another. All this is quite familiar to anyone who has taken a quantum mechanics course. I review it here because the story about strong CP violation concerns a very similar phenomenon, but in a less familiar context.

QCD, the theory of the strong interactions, is a gauge theory. This name labels a class of theories where there are a set of locally-defined redefinitions of all fields, known as gauge transformations, that change the form of the fields everywhere but do not change the energy associated with these fields. This invariance, like the Parity invariance in the example above, leads to a potential that does not have a unique minimum. In the case of QCD, because of the non-Abelian nature of the algebra of gauge transformations it turns out that there are not just two minima to the potential energy, but an infinite number. Any state, that is any static configuration of fields that can be defined by making a time-independent gauge-transformation of the configuration with all QCD fields equal to zero has the same energy, namely zero, as does the state with no fields at all. We call these pure-gauge field configurations. We also require that the fields vanish at spatial infinity. (I here use the language of states, which I describe by static gauge field configurations, to make this story a little more understandable, I hope . I may gloss over some of the finer distinctions between Hamiltonian quantum mechanics and Euclidean field theory in this tale. I do not attempt to define what I really mean by states in a continuum field theory, rather I am trying to give you an intuitive picture how the strong CP-violating term in the theory arises. One usually sees this problem discussed in the Eucliean Field theory, I find the Hamiltonian discussion more intuitive, so that is why I present that language. If you want to see this language are discussed in some detail for a similar situation read my paper with Marvin Weinstein on the two dimensional Abelian Higgs theory [8].)

Among the possible gauge transformations there are some for which the fields so generated cannot be deformed back to the zero-field case by changing them smoothly. For any static field configuration one can compute the the quantity

$$n = (1/32\pi^2) \int d^4x \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} .$$
(1)

This is a topological quantity which must be an integer for any pure gauge field, it is called the winding number [9]. The quantity  $(1/2)\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$  can also be seen to be a total derivative of the quantity

$$K^{\mu} = \epsilon^{\mu\nu\rho\sigma} [A^a_{\nu} F^a_{\rho\sigma} + f^{abc} A^a_{\nu} A^b_{\rho} A^c_{\sigma}].$$
<sup>(2)</sup>

(For more details of this story see for example the TASI lectures on the strong CP problem given by Michael Dine [10].) The winding number can thus be written as a surface integral over  $(1/16\pi^2)K_{\mu}$ . For fields which vanish at spatial infinity this integer must in fact be the difference between two integers, integers that label the state at time minus infinity and time plus infinity. So our static gauge field configurations can be labelled by integers n, which are the spatial integral of  $(1/16\pi^2)K_0$ . Any gauge field where this quantity is 1 cannot be continuously transformed to one where it is zero, since that is a discontinuous jump. To get smoothly such a configuration with label 1 to one with label 0 we must pass through some field configurations that are not pure-gauge fields and that therefore have higher energy than a pure gauge field.

So now we can make a cartoon of this situation, shown as Fig. 2. We plot the energy of the gauge-field configurations as the vertical axis and the horizontal axis labeled A schematically represents all possible field configurations over all of space. The only meaningful part of this picture is that there are now a series of minima to the potential. We can label these minima by the integers n, (the spatial integral of  $(1/16\pi^2)K_0$  for the pure gauge field configuration corresponding to such a minimum). We denote these pure-gauge states by  $|n\rangle$ . There is a well-defined gauge transformation, which I will call G, which transforms the state with field configuration  $|n\rangle$  into  $||n + 1\rangle$  for any n. In the two-well case we saw a tunneling probability between

the different minima of the potential, likewise here there is a tunneling possibility between states of different n. Such a tunneling event is called an instanton. (More precisely an instanton is a classical solution of the Euclidean field theory that has winding number 1.)



Figure 2: Cartoon of the potential for a non-Abelian Gauge Theory.

Just as quantum mechanics told us we must find states of definite Parity for the double well potential, the rules of quantum physics here say that the physical ground states must be those that map into themselves, times a constant, under the non-trivial gauge transformation G. Such states can easily found, they are the superpositions

$$|\theta\rangle = \Sigma_n e^{-in\theta} |n\rangle \quad . \tag{3}$$

These are an infinite set of such states, one for each choice of  $\theta$ . Such a state is not changed by G, except that it is multiplied by a constant or quantum number:

$$G \mid \theta \rangle = e^{i\theta} \mid \theta \rangle \quad . \tag{4}$$

In the problem where the potential had two minima there were two possible ground states, here with infinitely many degenerate minima of the potential there are infinitely many possible ground states. States of different theta are the physically distinct vacua for the theory, each with a distinct world of physics built upon it, just as the Parity odd and Parity even worlds of the two-well potential are distinct. (Translated back into the Euclidean field theory this discussion tells us there are an infinite set of theories, those with a term of the form of equation 1 added to the Lagrangian with coefficient  $\theta$  [9].)

All this may seem to be just formal manipulation, but a little examination shows that CP conservation in the strong interactions is only exactly true in the world built on the state  $\theta = 0$ . The integrand in Equation 1 is a CP-violating quantity. Thus the  $\theta$ -dependent term in the action induces CP-violating effects. Conversely, any CPviolating interaction, even in the weak interaction sector of the theory, will, in general, induce a non-zero value of  $\theta$  via loop-effects. When we investigate the consequences of living in a world with non-zero theta we immediately find this gives a problem. The electric dipole moment of the neutron is constrained by experiments to be extremely small. But the theory predicts a value proportional to  $\theta$ . This constrains our world to be one in which theta is of order  $10^{-12}$  or smaller [11]. This is then the strong CP puzzle: why is this parameter  $\theta$  so tiny? A priori it could have any value, from  $-\pi$  to  $\pi$ . What properties of the theory can ensure such a small value?

There are, to the best of our current knowledge, three possible answers to this question. One of them is the result of work that was included in my citation for this award, the work that I did with Roberto Peccei. I will briefly summarize all three, and then give a more detailed story of the Peccei-Quinn mechanism and the way in which we found it. We do not yet know which, if any, of these answers are correct, just as we do not yet know whether Grand Unified Theories describe nature. One bold thing that the Dirac Medal committee has chosen to do is to honor work which may in the end turn out not to be the correct theory. However it has stood the test of 25 years of examination and experimental probing, it is still a possible answer to the problem.

Before we can discuss the three solutions there is one more feature of the theory that we must discuss, and that is the role of quark fields in the  $\theta$  determination. If one transforms a quark field by a factor  $e^{i\gamma_5\alpha}$  it turns out that this transformation induces a shift in the value of theta by an amount  $-\alpha$ . But such a transformation also changes the phase of the mass term for that quark by an amount  $\alpha$ . The quantity that is unchanged by such transformations is the difference  $\theta_{\text{effective}} = \theta - tr \, \ell n \, \text{det}M$ where M is the matrix of all quark masses. This is the actual (physically meaningful) CP-violating quantity in the theory. In my lecture yesterday I stressed that one must examine all possible phase redefinitions in order to know what differences of phases are the physically meaningful CP-violating phases; this is another example of that rule.

The result immediately suggests the first and perhaps the simplest solution to the problem. In the Standard Model, if there is any one quark with zero bare mass then a chiral transformation of this quark field can be used to set  $\theta_{\text{effective}}$  to zero with no other consequences. Thus, in the presence of a massless quark  $\theta$  is not a physically meaningful parameter. The question is then whether nature chose this solution. The lightest quark is the u quark. Its mass could simply be a renormalization effect, that is to say the bare up-quark mass could be zero and the measured mass is that induced via weak interaction loop diagrams due to the down-type quark masses. This does not seem to give a large enough up quark mass to fit the observed values [12], but it is just possible that the effect could be big enough in some extensions of the Standard Model. Higher loop effects in any such theory will also induce an effective theta parameter, which may be sufficiently small. I think it is very unlikely that the final solution of the problem will involve a massless quark, but it is not completely ruled out with current understanding of quark masses.

The second choice is to impose CP as a symmetry of the full Lagrangian, thereby setting theta to zero at the renormalization scale where the theory is first defined. However since there is an observed CP violation in nature, albeit in the weak interaction sector, such a theory must be constructed so CP breaking occurs spontaneously, via soft operators which acquire CP-violating vacuum expectation values. There will then be a non-zero theta parameter induced by loop effects. If the theory is constructed to suppress these effects at the one and two loop level then the resulting theta parameter can be small enough. Indeed in the Standard Model the theta parameter only receives renormalization corrections at the three loop-level. A number of examples of theories of this type have been suggested in the literature [13]. This answer remains a viable one.

I want to talk about the third approach in a little more detail, it is the one devised by me together with Roberto Peccei. Remember that quark masses arise in the Standard Model because the Higgs field has a non-zero vacuum value. Roberto and I saw that one could add an additional symmetry to the theory in such a way that it is automatic that the vacuum energy is minimized for  $\theta_{\text{effective}} = 0$ . Technically this new global U(1) symmetry is not quite an exact symmetry. Like the strong CP symmetry itself, it is a pseudo-symmetry, broken only by non-perturbative or instanton (tunneling) effects. This is exactly why it works as desired. The trick is to make the Higgs field energy depend on the  $\theta$  value in such a way that, for any initial value of  $\theta$ , the Higgs field will choose a vacuum value such that the resulting physical parameter  $\theta_{\text{effective}}$  is zero. The Higgs vacuum expectation values aquire phases such that the phase of the quark mass matrix cancels against the initial  $\theta$ .

The way this idea occurred to us was very much a consequence of the first solution, the fact that the theta-parameter is irrelevant for zero quark mass. In the Standard Model quark masses are indeed zero in the early Universe, before the phase transition in which the Higgs field obtains its vacuum expectation value. This greatly puzzled me. How could the theta parameter be irrelevant in one phase but become relevant in another? The answer is that this statement is not quite true; in a general theory with Higgs fields a chiral redefinition of the quark fields such as that described above also changes the phases of certain Yukawa couplings. Now the question of whether the theta parameter is physical or not looks a lot like the usual rephasing-invariance question. This became clear to me during a conversation in which Steve Weinberg explained the usual issues of rephasing invariance and CP violation to me, a conversation which took place while Roberto and I were still completely mystified by the QCD  $\theta$ -dependence.

We know that the quark-Higgs Yukawa couplings are the source of quark masses once the Higgs field gets a non-zero vacuum value. This suggested to me the notion that it should be possible to design a Higgs potential, and choose Yukawa couplings, such that, no matter what the initial theta value, one would get  $\theta_{\text{effective}} = 0$  as the Universe cooled, once the quark masses were induced by the Higgs vacuum expectation value. Roberto and I soon found models where this was so. When I described this trick to Sidney Coleman he pointed out that what we had done was to add to the theory a U(1) symmetry broken only by instanton effects. I agreed that was indeed what we had done, and that is the way we presented it in our paper.

Let me describe a simple extension of the Standard Model to illustrate how this idea works. This simple theory is already ruled out by experiment, but the generic idea survives. In the simplest version of the Standard Model there is a single Higgs weak-doublet field that gives mass to both the up-type and the down-type quarks. If we introduce an additional U(1) symmetry (now called PQ symmetry) under which the right-handed up and down type quarks transform differently then, to maintain this symmetry, we must add a second Higgs doublet. The two Higgs doublets also transform differently under the PQ symmetry. One of them has the Yukawa couplings which give mass for the up-type quarks and the other for the down-type. The symmetry also forbids terms of the type  $\phi_1\phi_2^*$  (and higher powers of this quantity) in the Higgs potential energy. However the effects of QCD-instantons provide an additional contribution to the Higgs potential that violates this rule, inducing just such a term with a ( $\theta$ )-dependent coefficient. The minimum of the potential then correlates the phases of the quark masses real are exactly those that cancel the initial  $\theta$  value.

The additional pseudo-symmetry has a consequence, as was pointed out by Weinberg [14] and Wilczek [15], namely that there is an additional pseudo-Goldstone boson, now known as the axion, associated with it. The fact that Roberto and I did not notice this obvious phenomenological consequence of our model shows that we were focused on the general solution to the strong CP problem. I, at least, was so happy to find a general solution to that that I did not stop to examine other phenomenological implications of the model we built to demonstrate the idea before we published it. But the axion implication is common to all such models, for it arises from the symmetry itself. Steve Weinberg called me when he noticed that the theory had an almost zero-mass particle. He wanted to ascertain whether we knew it was there. Our conversation, as I remember it, went something like this: Steve asked whether I had noticed that the U(1) symmetry implies a pseudo-Goldstone boson. I saw immediately that he was right, but told him indeed we had not noticed it. Then Steve told me that he too had not at first noticed the obvious symmetry argument but had found the zero mass eigenstate the hard way, by calculating the Higgs spectrum in the theory. He wanted to check whether we already knew about it before he wrote his paper. Frank Wilczek noticed the same effect independently. By the time the papers were written Weinberg and Wilczek both used the name "axion" for this particle. I rather liked the alternate name "higglet" which was floating around for a while.

Constraints on the existence of such particles rule out the simple model described above. It was quickly eliminated by existing data and further direct laboratory searches for the predicted axion. Models which so far elude all constraints have been suggested, the so-called invisible axion models [16]. The constraints are of three types, from direct laboratory searches, from astrophysics, and from cosmology. The initial laboratory searches looked for the interactions of a penetrating particle in a detector placed some distance behind the beam dump of an accelerator [17]. More recent searches assume that the axion is a principle component of the dark matter in our galactic halo and try to detect the conversion of such a particle to a photon in an intense electromagnetic field set up in a carefully-tuned resonant cavity [18].

Astrophysical constraints on light weakly interacting particles such as an axion arise chiefly from the fact that such a particle would provide an additional mechanism for energy transport from the interior of a star to its surface, and hence additional cooling of the star's core. Constraints of this type can be made from observation of various astrophysical objects, for example from the life-time of red giant stars [19]. Astrophysical constraints also come from the observations of neutrinos produced by the Supernova event known as SN1987A [20]. Although only a few neutrino events were observed they provide quite stringent restrictions on changes to the model of such supernova explosions. The number of neutrinos seen, the duration of the signal, and its timing relative to the optical observation of the event were in good agreement with models. Any additional particle type that could carry off large amounts of energy in the early stages of this explosion could strongly alter the predictions. This puts a bound on the axion parameters.

Cosmological constraints come from the fact that axions are produced from random Higgs-field fluctuations in the early Universe and survive as dark matter. The constraint that axionic dark matter must not overclose the Universe limits the allowed range of axion parameters. This constraint is interesting because it acts as an upper bound on the axion mass, while all other constraints provide only lower bounds. We are left with only a relatively small window in parameter space for the axion. Recent results suggest that the dark matter density is probably only about 1/3 of closure density [21] which will further narrow the available parameter-space for axionic dark matter.

I find it fascinating that an idea to solve a particle physics problem, that of the small value of the strong interaction CP-violating  $\theta$  parameter, should predict a particle of possible cosmological and astrophysical relevance. This is a beautiful example of the unity and universality of physics. In particle physics we try to understand the physics of the smallest things, seeking for the basic constituents of matter and their interactions. But once we postulate anything at this level it has consequences. We must examine whether our theory survives all possible types of constraints. Since astrophysical objects and the early Universe provide more extreme environments than even our highest energy accelerators can produce, we must often look to these for evidence of effects that we cannot directly observe. Eventually we may even find an axion by a laboratory search based on its role as a constituent of the dark matter clustered in the halo of our galaxy, a search that combines cosmology, astrophysics and earth-bound laboratory physics in a most beautiful way. A positive result would certainly be exciting!

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