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# Don't Panic! Closed String Tachyons in ALE Spacetimes

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We consider closed string tachyons localized at the fixed points of noncompact non-supersymmetric orbifolds. We argue that tachyon condensation drives these orbifolds to flat space or supersymmetric ALE spaces. The decay proceeds via an expanding shell of dilaton gradients and curvature which interpolates between two regions of distinct angular geometry. The string coupling remains weak throughout. For small tachyon VEVs, evidence comes from quiver theories on D-branes probes, in which deformations by twisted couplings smoothly connect non-supersymmetric orbifolds to supersymmetric orbifolds of reduced order. For large tachyon VEVs, evidence comes from worldsheet RG flow and spacetime gravity. For $\mathbb{C}^2/\mathbb{Z}_n$, we exhibit infinite sequences of transitions producing SUSY ALE spaces via twisted closed string condensation from non-supersymmetric ALE spaces. In a $T$-dual description this provides a mechanism for creating NS5-branes via *closed* string tachyon condensation similar to the creation of D-branes via *open* string tachyon condensation. We also apply our results to recent duality conjectures involving fluxbranes and the type 0 string.

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## 1. Motivation and Outline

An understanding of the vacuum structure of String/M theory after supersymmetry breaking is crucial for phenomenology and cosmology. It is also relevant to the question of unification; it is important to understand the extent and nature of connections between different vacua in the theory. A basic issue is the fate of theories that have tachyons in their tree-level spectra. This has long been a source of puzzlement, but for open strings there has been a great deal of progress in recent years [1]. Open string tachyons generally have an interpretation in terms of D-brane annihilation, binding, or decay, and a quantitative description of these processes has been achieved by an assortment of methods from conformal field theory, string field theory, and noncommutative geometry. This has also led to a deeper understanding of the role of K theory, and the reanimation of open string field theory.

For closed string tachyons the understanding is much more rudimentary. These should be connected with the decay of spacetime itself, rather than of branes in embedded in a fixed spacetime. In this paper we study a class of tachyonic closed string theories in which the decay can be followed with reasonable confidence. The key feature of these theories is that the bulk of spacetime is stable, and the tachyons live only on a submanifold. Thus they are similar to the tachyonic open string theories, and we will note certain close parallels, though in the absence of closed string field theory we will not be able to achieve as complete a quantitative control.

The theories we study are noncompact, nonsupersymmetric orbifolds of ten-dimensional superstring theories [2][3]. The simplest case is to identify two dimensions under a rotation by $2\pi/n$, forming a cone with deficit angle $2\pi - 2\pi/n$ ($n$ must be odd, for reasons to be explained in §2). This is the simplest example of an Asymptotically Locally Euclidean (ALE) space, which is defined generally as any space whose geometry at long distance is of the form $\mathbb{R}^k/\Gamma$, with $\Gamma$ some subgroup of the rotation group. The tip of the cone, which is singular, is a seven-dimensional submanifold. The rotation leaves no spinor invariant and so supersymmetry is completely broken, and there are tachyons in the twisted sector of the orbifold theory. Where do these tachyons take us?

There are several plausible guesses, based on experience in other systems:

(I) A hole might appear at the tip, and then expand to consume spacetime. Such a reduction in degrees of freedom is naively suggested by the relevance of the tachyon vertex operator at zero momentum [4], and by the presence of a nonperturbative Kaluza-Klein instability in certain backgrounds [5], and has been argued to be the fate of other tachyonic closed string theories [6][7][8][9].
(II) The tip might begin to elongate, asymptotically approaching the infinite throat geometry that is often found in singular conformal field theories [10].
(III) The tip might smooth out, by analogy to the effect of the marginal twisted sector perturbations in supersymmetric orbifolds. This smoothing might stop at the string scale, or continue indefinitely.

We will argue that it is the last of these that occurs, as was also suggested recently in [3]. At late times, when a general relativistic analysis is valid, an expanding dilaton pulse travels outward with the speed of light. This is depicted in figure 1. The region interior to the pulse is flat, with vanishing deficit angle. The energy contained within the pulse produces the jump to the asymptotic deficit angle of $2\pi - 2\pi/n$ [11]. More generally, by following special directions in the space of tachyons, the decay can take place in a series of steps, where for example $\mathbb{C}/\mathbb{Z}_{2l+1}$ decays via a dilaton pulse to $\mathbb{C}/\mathbb{Z}_{2l-1}$, or to any $\mathbb{C}/\mathbb{Z}_{2l'+1}$ orbifold with $l' < l$.

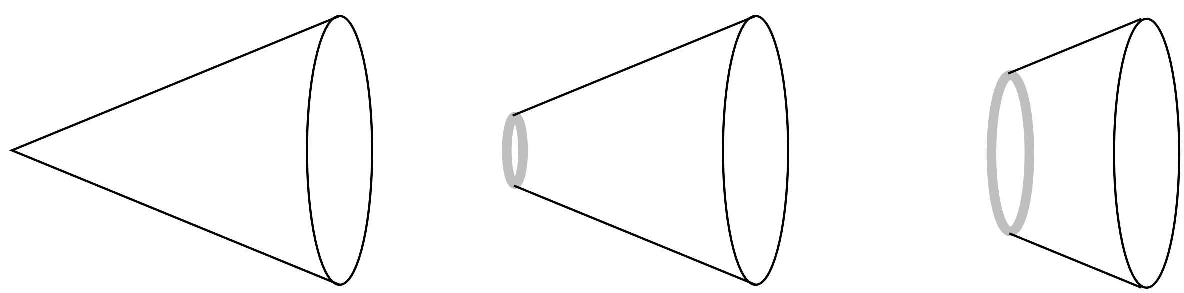

Figure 1: Decay of the conic singularity. The end of the cone is replaced by a flat base. The outward-moving dilaton pulse is shown in gray.

We will analyze this process in two complementary regimes. When the tachyon expectation value is small and so the smoothed region small compared to the string scale, we use D-brane probes [12], whose world-volume theory is a quiver gauge theory [13]. This is the *substring regime*. D-branes on supersymmetric orbifolds have been studied extensively;

we extend these techniques to study non-supersymmetric orbifold compactifications with closed string tachyons. The probes see a smoothed geometry when the tachyon is nonzero. When the smoothing region exceeds the string length scale, we can instead use a general relativistic description, and we argue that the solution has the form in figure 1. This is the *gravity regime*. Because of $\alpha'$ corrections, we do not have a controlled approximation that connects the two regimes, but we argue that together they give a simple and consistent picture of a transition from a conic singularity to flat space via tachyon condensation. These same complementary descriptions have been applied to type I instantons and to supersymmetric ALE spaces [13].

If one or more of these singularities is part of a compact manifold, then the initial stages of tachyon condensation will be the same near each orbifold fixed point, producing a smooth compact geometry. Unlike the noncompact system, which evolves forever, we will argue that the compact space collapses toward a Big Crunch in finite time.

If we put $N$ D3-branes on the fixed plane, and consider the near horizon limit, then the system is expected to be dual to a nonsupersymmetric gauge theory [14]. The fixed plane is partly transverse to the D3-brane, and in the large-radius limit of the $AdS_5 \times S^5/\mathbb{Z}_n$ background we find a dramatic instability that grows toward the boundary of AdS. We argue that at large 't Hooft coupling these nonsupersymmetric field theories are unphysical. This contrasts with the more benign infrared Coleman-Weinberg instabilities evident on the field theory side at weak 't Hooft coupling (small radius), which arise in theories whose gravity duals have tachyons at large radius [2][15].

The analysis can also be applied to orbifolds the form $\mathbb{C}^q/\Gamma$, where $\Gamma$ is a discrete subgroup of the rotation group that fixes a point in $\mathbb{C}^q$. If $\Gamma \not\subset SU(q)$ then the background does not preserve supersymmetry, and there are twisted-sector tachyons localized at the fixed point. For $q = 1$ there is no $\Gamma$ that preserves supersymmetry, but for $q \geq 2$ there will be.

We study several $\mathbb{C}^2/\mathbb{Z}_n$ examples, where analysis of the quiver theories on D-brane probes leads to predictions for transitions between different orbifolds. There is a new effect that can occur in this case: we exhibit infinite sequences of examples with transitions from *nonsupersymmetric* ALE spaces to *supersymmetric* ones. We again expect a gravitational background for large tachyon VEV that involves an expanding shell of dilaton gradient, combined with metric curvature. In this case the total energy of the transition region must vanish, since both the initial and final ALE spaces have vanishing energy as measured by the falloff of the gravitational fields at infinity. (We will explain why this is not inconsistent with positive energy theorems.)

For large $n$ in $\Gamma = \mathbb{Z}_n$, an angular direction is small for a significant range of radii and it is useful to go to a $T$-dual picture involving NS5-branes [16]. These transitions therefore provide a closed string analogue of the open string brane descent relations [17], in that we can realize for example any $A_k$ space (and therefore the equivalent dual system of $k+1$ NS5-branes) via tachyon condensation (and/or marginal deformation) from a non-supersymmetric ALE space. Also, by adding an R-R flux, which has little effect on the geometry, we obtain a system which has a conjectured dual description in terms of fluxbranes [18][19][20][9].

This realization of supersymmetric ALE spaces (and therefore NS branes) by closed-string tachyon condensation is very reminiscent of similar constructions in open string theory [1]. In this regard, we should emphasize that certain puzzles that arise in the open string case arise here as well [3].[1] In particular, in open string tachyon condensation, one finds gauge fields without sufficient perturbative charged matter to Higgs them [21][22], but open string field theory calculations at disk order suggest that they are nonetheless lifted classically [1][23][24]. In the tachyonic closed string systems we study here, there are twisted RR gauge potentials without perturbative charged matter; our evidence suggests that the defect is nonetheless smoothed and the RR potential lifted. In both the open and closed string cases, it would be very interesting to understand the classical stringy effect, evidently going beyond ordinary effective quantum field theory, which allows gauge fields to be so lifted. In the closed string case, it would be interesting to understand an analogue of the quantum confinement effect identified in the open string case in [25]; here as there one has D-branes charged under the gauge group of interest (in our case these are the fractional D-branes, whose condensation might lead to confinement of twisted strings

[1] We thank M. Berkooz, P. Kraus, E. Martinec, and other participants of the Amsterdam Summer workshop for discussions on this.

into untwisted strings).

The organization of this paper is as follows. In §2 we discuss the $\mathbb{C}/\mathbb{Z}_n$ orbifold, including the twisted sector spectrum and the quantum symmetry group of the orbifold theory. We also discuss the difference between orbifolds and ALE spaces that do not have orbifold descriptions. We consider D-brane probes in the orbifold theory, deriving the quiver representation. In §3 we analyze the quiver theory/linear sigma model for the $\mathbb{C}/\mathbb{Z}_n$ orbifolds and their twisted deformations, discussing both generic decays and decays that leave lower-order orbifolds. In §4 we develop the general relativistic description of these same solutions. We discuss renormalization group evolution and time evolution. These are similar, in that both lead to a smoothed region that grows without bound, but there are differences in the details. We discuss the consistency between the renormalization group analysis and the $c$-theorem. We then discuss the fate of compact spaces with nonsupersymmetric orbifold points, and the consequences of our results for AdS/CFT duality. In §5, we analyze transitions in the $\mathbb{C}^2/\Gamma$ case by means of the quiver theories, and exhibit decays from non-SUSY to SUSY ALE spaces. In the general relativistic regime we explain how our results are consistent with positive energy theorems. Finally, in §6 we discuss dual systems, including fluxbranes, and in §7 mention some directions for further research.

## 2. The $\mathbb{C}/\mathbb{Z}_n$ Orbifold

### *2.1. Closed String Spectrum*

These orbifolds are defined by identifying the 8-9 plane under a rotation $R$ through $2\pi/n$. This allows two possible actions on the spinors,

$$R = \exp(2\pi i J_{89}/n) \quad \text{or} \quad \exp(2\pi i J_{89})\exp(2\pi i J_{89}/n)\ , \tag{2.1}$$

where $J_{89}$ is the rotation generator. For either choice, $R^n$ acts trivially on spacetime and so is either 1 or $\exp(2\pi i J_{89}) = (-1)^{\mathbf{F}}$. If $R^n = (-1)^{\mathbf{F}}$, the orbifold group (which is actually $\mathbb{Z}_{2n}$ in this case) includes this operator and so projects out spacetime fermions and introduces tachyons in the bulk. Because we want to have all tachyons localized at the fixed point we must have $R^n = 1$. For the two choices (2.1) one finds

$$R^n = (-1)^{\mathbf{F}} \quad \text{or} \quad (-1)^{(n+1)\mathbf{F}} \ . \tag{2.2}$$

Thus, only the second choice of $R$ is acceptable, and only for $n$ odd:

$$R = \exp\left(2\pi i \frac{n+1}{n} J_{89}\right) \ , \quad n = 2l+1 \ . \tag{2.3}$$

In the sector twisted by $R^k$ ($1 \leq k \leq n-1$), in the light-cone Green-Schwarz description there are six real untwisted scalars, one complex scalar twisted by $k/n$, and four complex fermions twisted by $k/2+k/2n$. The standard calculation of the zero-point energy gives

$$\frac{\alpha'}{4} m^2 = \begin{cases} -k/2n \ , \ k \text{ even} \ , \\ (k-n)/2n \ , \ k \text{ odd} \ . \end{cases} \tag{2.4}$$

Thus the lowest state is tachyonic in every twisted sector. There are also excited state tachyons in many sectors. For example, when $k = 1$ the lowest twisted scalar excitation takes $(1-n)/2n$ to $(3-n)/2n$ and so this state is tachyonic for $n > 3$. Our analysis will be rather coarse, and so we will generally not distinguish the ground state in each sector from excited states with the same symmetries.

We wish to ask, where do these tachyonic perturbations take the system? Since they are in the twisted sectors, their initial effect is in the neighborhood of the fixed point. There are two contexts to consider. First, we could add the tachyonic vertex operators to the Hamiltonian. Tachyonic states correspond to relevant vertex operators, so they change the IR behavior of the world-sheet theory. We are then interested in determining the renormalization group (RG) flow. Second, we could consider a time-dependent string solution that begins as a small but exponentially growing tachyonic perturbation of the orbifold. We are then interested in the subsequent time evolution.

Physically these are distinct questions. The first is an off-shell question from the point of view of string theory, but well posed as question in two-dimensional quantum field theory. The second is an on-shell question in string theory. In fact we will find, as has been seen in other contexts, that the scale and time evolutions are similar. In both cases the question can be posed in the classical string limit, with no string loop effects. If the world-sheet theory were to become singular, for example if the dilaton were to become large, then this framework would break down, but we will find that at least generically this does not happen.

The orbifold preserves an $SO(7,1) \times U(1)$ subgroup of the parent $SO(9,1)$. In addition there is a new "quantum" symmetry that appears in the orbifolded theory [26]: the twist is conserved, mod $n$.[2] The lowest tachyons in general break the quantum symmetry completely but leave the $SO(7,1) \times U(1)$ unbroken (in the RG case) or break it to $SO(7) \times U(1)$ (in the time-dependent case). In some cases we will consider perturbations that leave part of the quantum symmetry unbroken, while in others we will find that a new quantum symmetry, unrelated to the original one, emerges asymptotically.

Actually, the evolution is more restricted than would follow from spacetime symmetry alone. The $X^M$ and $\psi^M$ (of the RNS description) upon which the $SO(7,1)$ or $SO(7)$ acts do not appear in the perturbation. These fields therefore remain free, whereas the symmetry would allow a warp factor depending on the other coordinates.

We will consider processes where a $\mathbb{Z}_{2l+1}$ singularity emits a radiation pulse with just the appropriate energy to leave behind a $\mathbb{Z}_{2l-1}$ singularity. We could also imagine the time-reversed process, sending in a pulse with the appropriate energy.

This raises an interesting issue. We have found that there are no $\mathbb{Z}_{2l}$ orbifolds with supersymmetry broken only locally at the tip of the cone, but what if we consider a solution of type II string theory which describes a pulse sent inward with just the right energy to create such a singularity? When the pulse reaches the origin, the geometry is a cone with deficit angle $2\pi - 2\pi/2l$. The difference between this case and the time reversal of our orbifold decay process is that here there is no simple description of the singularity. Away from the singularity, the lines $\theta = 0$ and $\theta = 2\pi/2l$ are identified under a rotation $\exp(2\pi i J_{89}/2l)$ or $\exp(2\pi i J_{89}) \exp(2\pi i J_{89}/2l)$. This is a sensible configuration, and the dynamical process allows one to reach it. However, on the $2l$-fold covering space, the lines

[2] This is not the same spacetime $\mathbb{Z}_n$ group used to construct the orbifold. All states are invariant by definition under that symmetry, while the quantum $\mathbb{Z}_n$ is carried by the twisted sector states.

$\theta = 0$ and $\theta = 2\pi$ are identified under the action of $(-1)^{\mathbf{F}}$, and so there is a branch cut in the spinor fields. This is the essential difference from the orbifold: in the orbifold the untwisted fields are single-valued on the covering space. We could similarly consider a wedge of any opening angle $\theta_{\rm o}$, where the plane is generically not a covering space. Again, dynamically we could construct a state that has this behavior away from the singularity, but that within a string distance of the singularity has some complicated description, not based on a free CFT, if the singularity is resolved at all. Indeed, we will find many example of orbifolds decaying to such spaces; we will use the terms 'quasi-orbifold' or 'quasi-ALE (QALE)' to refer to these more general spaces that are locally Euclidean but are not obtained as orbifolds of a single-valued theory on Euclidean space.

### *2.2. Open String Spectrum*

We now consider a D$p$-brane probe of the geometry. Here as in many other contexts, D-brane probes [13] and closely related linear sigma model techniques [27] are useful for obtaining a broader view of the space of closed string backgrounds than is available from perturbation theory about a specific world-sheet CFT. In studying a D-brane probe, the low energy quantum field theory on its world-volume is only valid in the substring regime, where the VEVs of world-volume scalars (scaled to have dimensions of length) are sufficiently small compared to the string length $\sqrt{\alpha'}$. We will also study the D-brane probes in the *classical* limit, and in doing so will self-consistently find results consistent with the string coupling remaining bounded throughout the tachyon decay process. It would be an interesting, but distinct, question to relax the $g_s \to 0$ limit we consider here and analyze the quantum dynamics on D-brane probes in these backgrounds, a question that could be considered both before and after the tachyons condense.

The classical world-volume theories of D-branes probing orbifold singularities were worked out in a beautiful paper by Douglas and Moore [13]. The orbifold group $\Gamma$ has both a geometric action $\hat{R}$ and an action $\gamma_R$ on the Chan-Paton indices,

$$|\psi, i, j\rangle \;\to\; \gamma_{R\,ii'}|\hat{R}\psi, i', j'\rangle\gamma^{-1}_{R\,j'j}\ . \tag{2.5}$$

For branes that are free to move away from the orbifold singularity, there must be a distinct image for each element of $\Gamma$ and so the Chan-Paton indices transform in the regular representation. These branes have integer tensions and charges. Irregular representations correspond to fractional branes bound to the fixed locus. We will be interested in the regular case; as we have noted in the introduction, fractional branes are confined once the singularity is resolved, but the full mechanism is not understood.

We will consider a D$p$-brane probe that is extended in the directions $\mu = 0, 1, \ldots, p$ and localized in the directions $m = p+1, \ldots, 7$ and in the orbifolded 8-9 plane. The treatment will be uniform for the IIA or IIB theories, and for all $p$ in the respective theories. We take a single copy of the regular representation, but the discussion readily extends to $N$ copies. For $\Gamma = \mathbb{Z}_n$, $R$ cyclically permutes the D-brane images and so $\gamma_{R\,jk} = \delta_{j+1,k}$. The indices $j, k$ are understood to be defined mod $n$, so in particular $\gamma_{R\,n1} = 1$. It is more convenient to work in a basis in which the spacetime action is not so evident but the spectrum and its quiver representation are simple:

$$\gamma_{R\,jk} = e^{2\pi i j/n} \delta_{jk} \ . \tag{2.6}$$

The low energy theory is itself an "orbifold" of the $\mathcal{N} = 4$ world-volume theory of a D-brane in flat space, obtained by projecting out gauge theory fields that are not invariant under the action (2.5). The massless open string fields are the vector potential $A_{\mu jk}$, the collective coordinates $X^m_{jk}$ and $Z_{jk} = (X^8 + iX^9)_{jk}$, and the spinor $\xi_{jk}$ in the $\mathbf{8}$ of $SO(7,1)$ and with $J_{89} = +\frac{1}{2}$. The real and imaginary parts of $\xi$ form the $\mathbf{16}$ of $SO(9,1)$; we will suppress the $SO(7,1)$ spinor index. The orbifold projection (2.5) on the operation (2.3), (2.6) retains fields with $j - k + (n+1)J_{89} = 0$. The surviving fields are then

$$A_{\mu\, jj} \ , \quad X^m_{jj} \ , \quad Z_{j,j+1} \ , \quad \xi_{j,j-l} \ , \tag{2.7}$$

where $j$ runs from 1 to $n = 2l + 1$ and indices are defined mod $n$. The conjugates are $\overline{Z}_{j+1,j}$ and $\bar{\xi}_{j,j+l}$.

Thus the gauge group is $U(1)^n$, with the collective coordinate $Z_{j,j+1}$ having charge $+1$ under $U(1)_j$ and charge $-1$ under $U(1)_{j+1}$, while $\xi_{j,j-l}$ has charge $+1$ under $U(1)_j$ and charge $-1$ under $U(1)_{j-l}$. The spectrum can be succinctly expressed through "quiver" diagrams [13]. For each factor in the product gauge group, the diagram has a node; for $\Gamma = \mathbb{Z}_n$ these are in one-to-one correspondence with the range of the Chan-Paton indices.

A field with charge $+1$ under $U(1)_j$ and charge $-1$ under $U(1)_k$ is denoted by an arrow from node $k$ to node $j$. For more general representations of $\Gamma$ the gauge group is a product $\prod_j U(N_j)$ and the arrows represent bifundamentals. Arrows beginning and ending on the same node are adjoints, which of course are neutral in the case of $U(1)^n$. For the example $\Gamma = \mathbb{Z}_5$, figure 2 shows the separate quiver diagrams for the various fields.

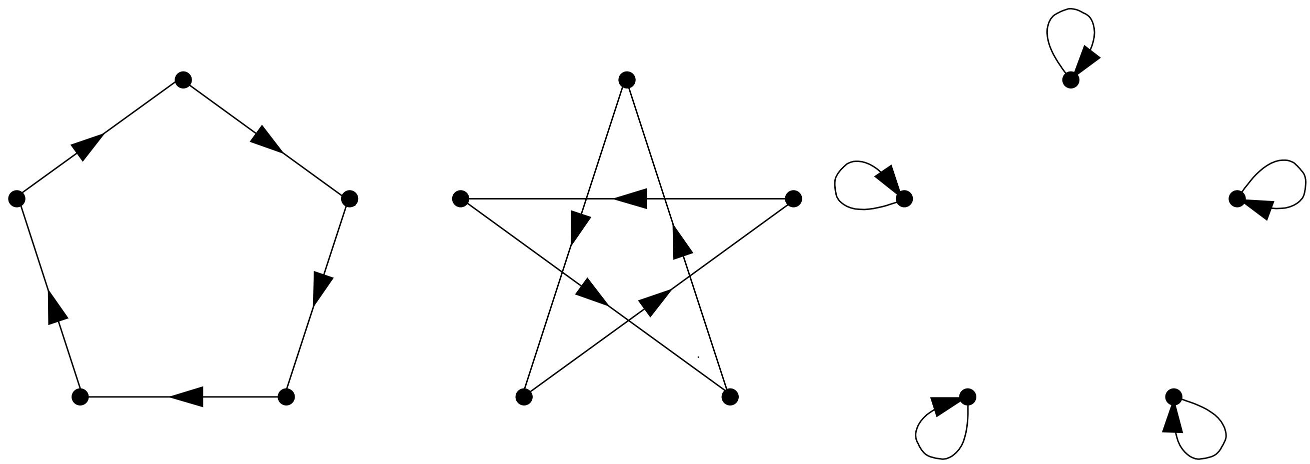

Figure 2: Quiver diagrams for the $\mathbb{C}/\mathbb{Z}_5$ orbifold: for $Z$, for $\xi$, and for $A_\mu$ and $X^m$.

Note that the quiver theory spectrum is invariant under cyclic permutation of the gauge groups; call this symmetry $\Gamma_Q$. All gauge invariant operators inherited from the parent theory, such as $\sum_j F^2_{\mu\nu jj}$, are scalars under $\Gamma_Q$. Gauge invariant operators not descending from gauge invariant operators in the parent theory, such as $F^2_{\mu\nu 11}$, are not $\Gamma_Q$ scalars. Since the lagrangian itself descends from a gauge invariant operator in the parent theory, it is a scalar, and so $\Gamma_Q$ is truly a symmetry of the system. This symmetry is just the realization in the quiver theory of the orbifold quantum symmetry. In particular, bulk twisted modes couple to gauge-twisted operators (*i.e.* not $\Gamma_Q$ scalars) on the brane only in $\Gamma_Q$ invariant combinations — a very useful fact in fleshing out the AdS/CFT dictionary, for example. We will see that quiver diagrams are a very effective tool for following the behavior of the probe theory as the singularity decays.

The potential for the scalars is classically, at the orbifold point,

$$V = \frac{1}{2}\sum_{j,m}(X^m_{jj} - X^m_{j+1,j+1})^2|Z_{j,j+1}|^2 + \frac{1}{2}\sum_j\Big(|Z_{j,j+1}|^2 - |Z_{j-1,j}|^2\Big)^2 , \qquad (2.8)$$

where the overall normalization will not be important. We are interested in the Higgs branch, where $X^m_{jj}$ is independent of $j$ and the $Z_{j,j+1}$ are nonzero. This corresponds to a D-brane probe of the orbifold geometry. On the Coulomb branch the $X^m_{jj}$ depend on $j$ and the $Z_{j,j+1}$ vanish. This branch corresponds to the probe separating into fractional D-branes trapped at the singularity, and it disappears in the deformed geometry. On the Higgs branch there is a Yukawa interaction

$$L_{\rm Y} = \sum_j \xi_{j,j-l}\xi_{j-l,j+1}\overline{Z}_{j+1,j} \ . \tag{2.9}$$

Note that each interaction forms a closed loop on the quiver diagram.

We now consider the geometry of the Higgs branch. The vanishing of the potential (2.8) implies that the magnitude $|Z_{j,j+1}|$ is independent of $j$. Of the $n$ $U(1)$ symmetries, the diagonal decouples. The remaining $n-1$ gauge symmetries can be used to set the phases of the $Z_{j,j+1}$ equal as well, so that the common value $Z_{j,j+1} = Z$ parameterizes the branch. The branch is thus two-dimensional, as it should be for the interpretation of a probe. The gauge choice leaves unfixed a $\mathbb{Z}_n$ gauge symmetry, whose generator is

$$\exp\left(-\frac{2\pi i}{n}\sum_j jQ_j\right) \ . \tag{2.10}$$

This identifies $Z \to e^{2\pi i/n}Z$, so the probe moduli space is indeed the $\mathbb{C}/\mathbb{Z}_n$ spacetime. For each of the fields (2.7) there is one massless mode, where the field is independent of $j$. This is the correct spectrum for a D-brane probe.

The moduli space metric, as measured by the probe kinetic term, is obtained by integrating out the higgsed gauge fields. In a general gauge, the potential requires that on the moduli space $Z_{j,j+1} = re^{i\theta_j}$. The kinetic terms are then

$$\begin{aligned} L_{\rm k} &= \sum_{j=1}^{n}\Big|(\partial_\mu + iA_{\mu\,jj} - iA_{\mu\,j+1,j+1})Z_{j,j+1}\Big|^2 \\ &= \sum_{j=1}^{n}\Big[(\partial r)^2 + r^2(\partial_\mu\theta_j - B_{\mu j})^2\Big] \ . \end{aligned} \tag{2.11}$$

We have defined the relative gauge potentials, $B_{\mu j} = A_{\mu\,jj} - A_{\mu\,j+1,j+1}$, which tautologically satisfy the constraint $\sum B_{\mu j} = 0$. The total $U(1)$ is unbroken and decouples.

Integrating out the broken gauge fields subject to the constraint gives

$$B_{\mu j} = \partial_\mu(\theta_j - \tilde{\theta}) \ , \quad \tilde{\theta} = \frac{1}{n}\sum_{k=1}^{n}\theta_k \ . \tag{2.12}$$

Inserting this into the kinetic term gives the manifestly gauge invariant result

$$L_{\rm k} = n\Big[(\partial r)^2 + r^2(\partial\tilde{\theta})^2\Big] \ . \tag{2.13}$$

As deduced above, the periodicity of $\tilde{\theta}$ is $2\pi/n$. Rescaling to $\theta = n\tilde{\theta}$, with canonical period $2\pi$, the kinetic term becomes

$$L_{\rm k} = n\,(\partial r)^2 + \frac{r^2}{n}(\partial\theta)^2 \ , \tag{2.14}$$

corresponding to the metric of a flat $\mathbb{Z}_n$ cone,

$$ds^2 = n\,dr^2 + \frac{r^2}{n}d\theta^2 \ , \tag{2.15}$$

as expected. For future reference note that we can define $\theta$ as

$$\theta = \arg(Z_{n1}Z_{12}\ldots Z_{n-1,n}) \ ; \tag{2.16}$$

the RHS is gauge invariant, so the period of $\theta$ is manifestly $2\pi$.

The gauge bosons that have been integrated out have masses of order $r/\alpha'$, while excited string states with masses of order $\alpha'^{-1/2}$ have been ignored. The result is therefore valid in the substringy regime [12], $r \ll \alpha'^{1/2}$. We have also ignored quantum corrections in the world-volume theory. This is valid because the world-volume fluctuations are open string fields, and we have taken $g_{\rm s} \to 0$ at the beginning — we have posed the problem in classical string theory.

There is a closely related context in which world-volume quantum corrections would be important. The world-volume theory of the D1-brane provides a linear sigma model construction analogous to those in [27] of the F-string orbifold CFT [28]. In this one must let the quantum world-volume theory flow to the IR fixed point. In the present case we know independently, from the orbifold construction, that the fixed point action is the free action (2.13).

## 3. Decay of $\mathbb{C}/\mathbb{Z}_n$ in the Substring Regime

### *3.1. Generic Tachyon VEVs: Breaking the Quantum Symmetry*

In the initial stage of the instability, the tachyon VEV is small and so the geometry is modified only in the substringy region near the tip of the cone. D-brane probes are therefore the effective tool for investigating the geometry. The closed string background determines the low energy quantum field theory on the probe. This can be obtained directly from a calculation of the disk amplitude with a tachyon vertex operator plus open string vertex operators, as in the appendix of ref. [13]. For our purposes, however, it will suffice to identify the world-volume theory by matching with the quantum numbers of the closed string tachyons.

¿From the discussion in §2.1, the tachyons generically break the quantum symmetry completely, so this will be broken in the world-volume theory. We are in the substringy regime, so we are interested in the leading effects in powers of $Z$. In the potential, this would be a mass term

$$\Delta V = \sum_{j=1}^{n} m_j^2 |Z_{j,j+1}|^2 \ . \tag{3.1}$$

A term of definite quantum charge $k$ would have a coefficient proportional to $e^{2\pi ijk/n}$. Since there are tachyons with all charges except for the untwisted $k = 0$, one obtains arbitrary masses subject to the constraint $\sum_j m_j^2 = 0$. It is then useful to reexpress the mass term as

$$\Delta V = -\sum_{j=1}^{n} \lambda_j \Big( |Z_{j,j+1}|^2 - |Z_{j-1,j}|^2 \Big) \ , \quad \sum_{j=1}^{n} \lambda_j = 0 \ . \tag{3.2}$$

The notation is suggested by the supersymmetric case, where $\lambda_j$ would be the Fayet-Iliopoulos (FI) coefficient for $U(1)_j$.

On the moduli space we now have

$$|Z_{j,j+1}|^2 - |Z_{j-1,j}|^2 = \lambda_j \ . \tag{3.3}$$

For generic $\lambda_j$ the $Z_{j,j+1}$ are therefore distinct, and one of these has magnitude less than the rest, say $Z_{12}$. When this vanishes the remaining $n-1$ $Z_{j,j+1}$ are still nonzero. It follows that $U(1)^n$ is broken to $U(1)$ everywhere on the moduli space, and so there is no orbifold point. The moduli space is smoothed; topologically it is $\mathbb{R}^2$. The gauge-invariant combination $Z_{n1}Z_{12}\ldots Z_{n-1,n}$, which now vanishes linearly when $Z_{12} \to 0$, is a good coordinate.

We can confirm these conclusions by finding the probe metric. Define $\rho_j$ iteratively,

$$\rho_j^2 = \rho_{j-1}^2 + \lambda_j \ , \quad \rho_1 = 0 \ . \tag{3.4}$$

Then with $Z_{j,j+1} = r_j e^{i\theta_j}$, eq. (3.3) implies

$$r_j^2 = r^2 + \rho_j^2 \ , \quad r \equiv r_1 \ . \tag{3.5}$$

The kinetic term is now

$$L_{\rm k} = \sum_{j=1}^{n} \Big[ (\partial r_j)^2 + r_j^2 (\partial_\mu \theta_j - B_{\mu j})^2 \Big] \ . \tag{3.6}$$

Enforcing the constraint $\sum_j B_{\mu j} = 0$ with a Lagrange multiplier $\lambda_\mu$, the equation of motion for $B_{\mu j}$ is $r_j^2(\partial_\mu \theta_j - B_{\mu j}) = \lambda_\mu$. Inserting this into the constraint determines the multiplier,

$$\lambda_\mu \sum_{j=1}^{n} \frac{1}{r_j^2} = \partial_\mu \theta \ , \tag{3.7}$$

where $\theta = \sum_j \theta_j$ is defined as in eq. (2.16) and so has period $2\pi$. The action then takes the simple form

$$L_{\rm k} = n(r)(\partial r)^2 + \frac{r^2}{n(r)} (\partial\theta)^2 \ , \tag{3.8}$$

where

$$n(r) = \sum_{j=1}^{n} \frac{r^2}{r^2 + \rho_j^2} \ . \tag{3.9}$$

The corresponding metric is

$$n(r) dr^2 + \frac{r^2}{n(r)} d\theta^2 \ ; \tag{3.10}$$

for constant $n(r)$ this is the metric (2.15)of a cone cone of deficit angle $2\pi/n$. The function $n(r)$ interpolates smoothly from $n(0) = 1$ (the term $j = 1$) to $n(\infty) = n$. Thus the metric (3.10) is nonsingular at the origin and connects smoothly onto the original $\mathbb{C}/\mathbb{Z}_n$ geometry asymptotically, as in figure 3.

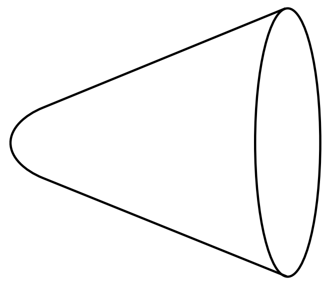

Figure 3: $\mathbb{C}/\mathbb{Z}_n$ singularity with a twisted tachyon VEV, as seen by a D-brane probe.

This smoothed geometry differs somewhat from what will eventually emerge in the gravity regime, depicted in figure 1. The base of the cone is rounded rather than flat. Also, the dilaton is constant: a nontrivial dilaton would lift the moduli space, through the dependence of the DBI term. We will see that in the gravity regime a dilaton must be present, so evidently this is a higher-order effect.

The exact physical meaning of the D-brane probe calculation here is a bit indirect. D-brane probes can observe substringy geometry only on times long compared to the string scale [12][29], while the decay process that we are probing takes place on the string time scale. There are at least two contexts where the calculation above has a precise meaning. First, we could consider a tachyon background which is constant in time and oscillatory in space, where the wavelength is then long compared to the substringy geometry. Second, at large $n$ some tachyon masses-squared are of order $1/n$. Even when neither of these contexts is relevant, we expect that the qualitative conclusion about the geometry is correct, and this is all that we will need.

Again, our analysis of the gauge theory is entirely classical. The non-supersymmetric gauge theories do not look unstable in this approximation at the orbifold point. The tachyon instability is a closed string tree effect and so a one-loop open string effect. In the context of AdS/CFT duality, we would expect to see this instability in the gauge theory; we will return to this point in §4.

Finally, note that the resolved geometry is topologically trivial. Thus, unlike the supersymmetric ALE singularity, there is no interpretation in terms of collapsing cycles at real codimension two. However, in §3.3, and in §5 where we consider the case of real codimension four orbifold singularities, we will see many parallels with the supersymmetric case.

### 3.2. World-sheet Linear Sigma Model

As we noted above, the D1-brane gauge theory provides the starting point for a LSM representation of the F-string world-sheet theory. Let us digress slightly to explain the picture of the tachyon decay process which emerges from this point of view.

The LSM description involves considering a simple gauge theory in the UV which flows to the world-sheet CFT of interest in the IR [27]. In the context of quiver theories on D1-branes at orbifold points, the classical moduli space is the orbifold space (as we reviewed in §2.2), which is the target space for the F-string world-sheet CFT. Based on this and the discrete symmetries of the theory arising for appropriate choice of theta angles, it was argued in [28] that the D1-brane quiver theory provides a linear sigma model formulation of the orbifold CFT, with the caveat that without supersymmetry one must fine-tune away the quantum potential on the moduli space in order to reach the orbifold CFT in the IR (which then enjoys an accidental supersymmetry).

We are interested in the effect on the world-sheet CFT when the tachyon VEVs are turned on in spacetime, which means in terms of a renormalization group analysis that a relevant operator is added to the world-sheet CFT action (taking the tachyons at zero spacetime momentum). We would like to describe this deformation from the UV LSM quiver theory. As we have discussed, the tachyons transform under the quantum symmetry in the IR CFT, and this symmetry exists already in the UV quiver theory. Therefore we can identify twisted operators in the UV theory which will generically mix with the twisted-sector tachyon vertex operators in the IR. The twisted couplings of interest include the $\lambda_j$ in (3.2) above. These are the most relevant twisted deformations in the UV, and we will focus on their effects.

The RG flow diagram of this theory appears as in the following figure.

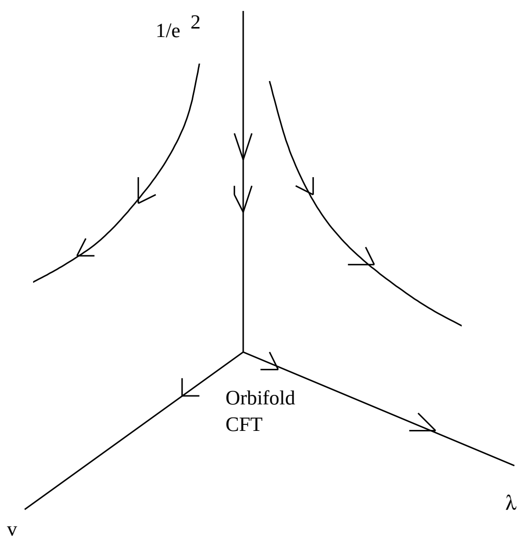


Figure 3.5: Flow diagram for the linear sigma model.

We consider flow toward the IR, keeping track of the indicated couplings $e$ (the gauge coupling) and $\lambda$, and on a third axis the relevant couplings $v$ in the scalar potential of the theory which drive the flow away from the desired IR world-sheet theory; these last we tune away as discussed in [28]. The flow proceeds toward stronger gauge coupling $e$. As we turn on $\lambda$, the vacuum manifold of the LSM smoothes out, as we discussed above. For large $\lambda$, integrating out the massive degrees of freedom in the LSM we obtain a nonlinear sigma model whose RG flow proceeds toward infinite flat space, as we will see in §4. For small $\lambda$, as we flow toward the IR we expect generically for $\lambda$ to mix with the tachyon vertex operators, which are relevant operators so that the flow proceeds away from the orbifold CFT fixed point.

Putting this together, the simplest joining of the two regimes leads again to a picture where the tachyon VEV induces flow from the orbifold CFT to smooth flat space.

### *3.3. Special Tachyon VEVs: Annealing the Quiver*

We have considered a generic tachyon VEV, which in the quiver theory breaks all $U(1)$s and resolves the singularity completely. It is interesting to consider instead partial resolutions of the singularity. Depending on the choice of twisted deformation we turn on, we will find that such resolutions can lead to quasi-orbifolds, which have no free world-sheet CFT description, or to real orbifolds, which do. We will start with an example of the former case and then proceed to the transitions between real orbifolds that are our main interest.

Consider for example the case that $\lambda_1 = -\lambda_2 > 0$, for which eq. (3.3) implies that one bifundamental is greater than the rest,

$$|Z_{12}|^2 = |Z_{j,j+1}|^2 + \lambda_1 \ , \quad j \neq 1 \ . \tag{3.11}$$

The maximum unbroken gauge symmetry is now $U(1)^{n-1}$, where all $Z_{j,j+1}$ other than $Z_{12}$ vanish, so we expect that the symmetry is partially resolved to $\mathbb{Z}_{n-1}$.

The theory near the fixed point can be elegantly described in terms of *annealed* quiver diagrams. As an explicit example, let us analyze the $\mathbb{C}/\mathbb{Z}_5$ orbifold, whose quiver diagrams were given in figure 2. Figure 4 shows the first step in the annealing. In the neighborhood of the fixed point, the bifundamental $Z_{12}$ has a relatively large VEV and breaks $U(1)_1 \times U(1)_2$ to the diagonal $U(1)$. Thus, in the second line of figure 4 we have collapsed nodes 1 and 2.

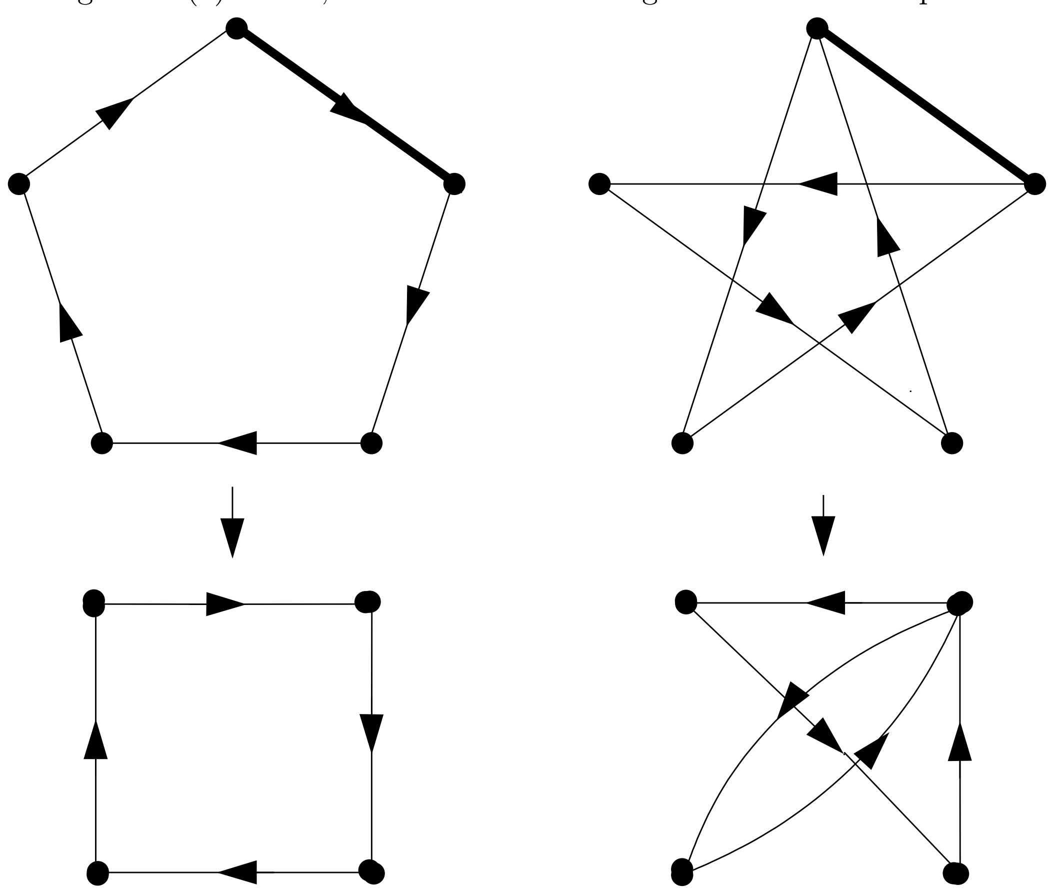

Figure 4: Partially annealed $\mathbb{C}/\mathbb{Z}_5$ scalar and fermion quivers. The scalar $Z_{12}$ is indicated in bold. In the low energy theory the nodes 1 and 2 are identified.

The scalar $Z_{12}$ decouples from the low energy theory, its magnitude fixed by the potential and its phase absorbed by higgsing; thus it is omitted from the annealed diagram. The adjoint scalar $X_{11}^m - X_{22}^m$ accompanying the broken $U(1)$ is also lifted by the potential. Finally, the mass term $\xi_{14}\xi_{42}\overline{Z}_{21}$ removes two fermions, so the final quiver diagram is shown in figure 5.

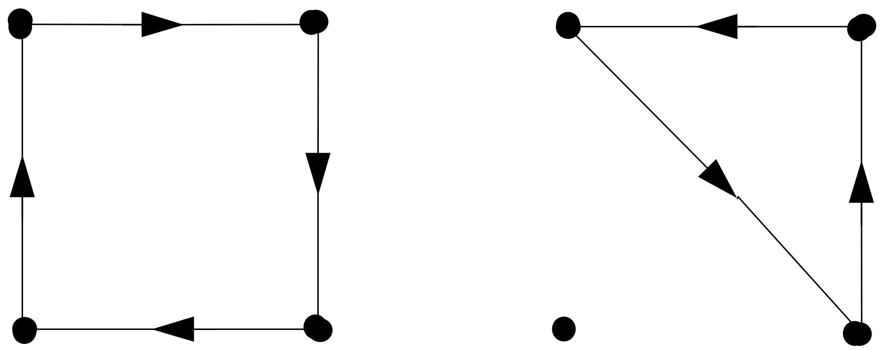

Figure 5: Final annealed $\mathbb{C}/\mathbb{Z}_5$ scalar and fermion quivers.

The scalar spectrum is the same as for a $\mathbb{C}/\mathbb{Z}_4$ orbifold in bosonic string theory, and the metric (3.9) seen by a D-brane probe has a $\mathbb{Z}_4$ singularity. The geometry is as in figure 6, with a $\mathbb{Z}_4$ singularity in a space whose asymptotic geometry is $\mathbb{C}/\mathbb{Z}_5$.

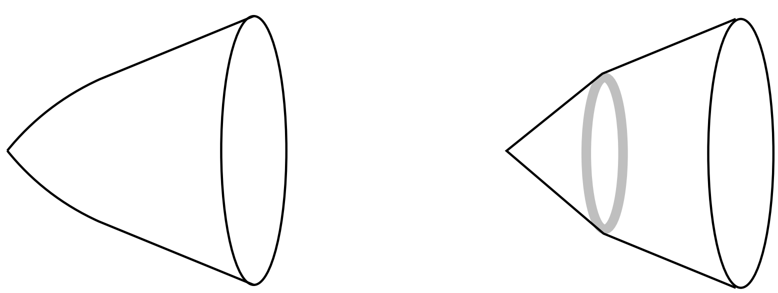

Figure 6: Asymptotic $\mathbb{C}/\mathbb{Z}_n$ geometry with a $\mathbb{C}/\mathbb{Z}_{n'}$ singularity, with $n' < n$, as seen in the substringy and gravity regimes.

The fermion spectrum is not of quiver form. This is not surprising, as we know that there is no orbifold construction of the supersymmetric type II string on the $\mathbb{C}/\mathbb{Z}_4$ singularity. Rather, this must be a quasi-orbifold, not based on a free CFT, as discussed in §2. However, by turning on additional Fayet-Iliopoulos terms, and so a second scalar VEV, we can flow to the $\mathbb{Z}_3$ quiver as shown in figure 7; it is easy to check that the Yukawa terms lift no additional fermions.

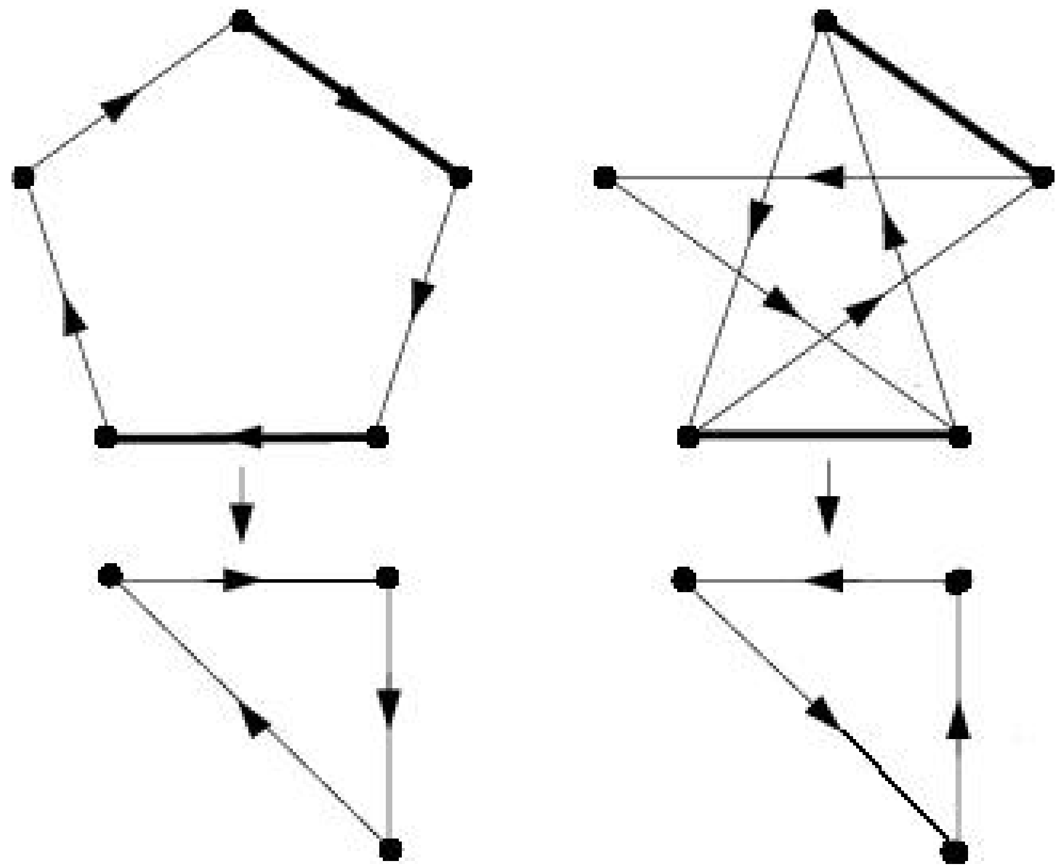

Figure 7: The massless sector of $\mathbb{C}/\mathbb{Z}_5$ with two scalars turned on gives the $\mathbb{C}/\mathbb{Z}_3$ quiver!

More generally, the $\mathbb{C}/\mathbb{Z}_{2l+1}$ singularity can decay to the $\mathbb{C}/\mathbb{Z}_{2l-1}$ singularity, if the FI terms are such that $Z_{12}$ and $Z_{l+1,l+2}$ decouple. (It can also flow to a variety of quasi-orbifold singularities.) For the true orbifold case, the quiver diagram has an obvious $\mathbb{Z}_{2l-1}$ symmetry. This is not a subgroup of $\mathbb{Z}_{2l+1}$, but emerges as an accidental symmetry (in the technical sense) of the low-energy theory. This process can be repeated until we reach the trivial $\mathbb{C}/\mathbb{Z}_1$ orbifold, without tachyons.

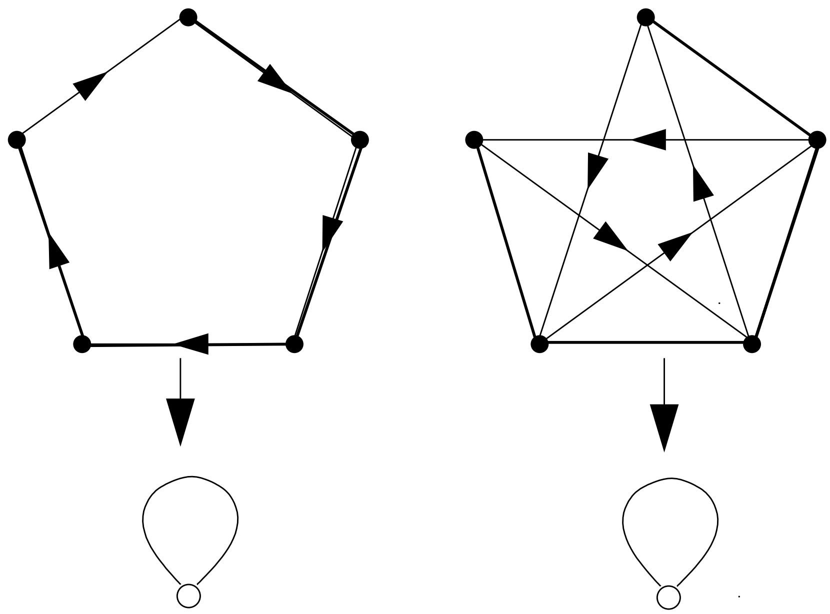

Figure 8: Decay to the $\mathbb{C}/\mathbb{Z}_1$ space.

The spectrum is simply a free chiral supermultiplet, with SUSY reappearing as an accidental symmetry under successive quiver annealings. The order of liftings does not particularly matter. As long as all but one scalar receive generic VEVs the result is inevitably the quiver for SUSY flat space, regardless of the geometries or effective quivers at intermediate scales. Since the bifundamentals couple to relative gauge potentials, this maximally higgses the system; we cannot lift all the scalars without changing the number of degrees of freedom in our theory. In complex codimension two, the story will be much richer, as there is an infinite family of SUSY quivers to which a generic tachyonic quiver can decay.

Note that the decays to lower-order singularities require specific FI terms of no particular quantum symmetry, so they arise from a linear combination of different tachyons. Since the tachyons have different lifetimes, the singular point will not be a static configuration. Presumably it is possible to fine-tune the initial conditions so that the geometry develops the lower order singularity as it enters the gravity regime, where it will remain on top of its tachyon potential.

## 4. Decay of $\mathbb{C}/\mathbb{Z}_n$ in the Gravity Regime

In the preceding section we found that the initial effect of the tachyonic singularity is to smooth the geometry. As with any tachyon, an essential question is the nature of the final state: does the tachyon potential have a minimum, or does the instability continue without end? The D-brane probe analysis in the previous section breaks down when the size of the smoothed region reaches the string scale. We do not have tools to probe this regime, so will study the question by going beyond it to the regime of small curvature. If we were to find that the RG flow in that regime carries us back to higher curvature, this would indicate the presence of a minimum with curvature of order the string scale. In fact, we will find that the flow goes toward ever-smaller curvature.[3] Thus the geometry evolves forever, generating an arbitrarily large region of arbitrarily small curvature, which contains a lower-order singularity if the initial state has been appropriately fine-tuned.

### *4.1. RG Flow*

We now study the RG flow of the world-sheet NLSM corresponding to a background of the massless closed string fields. Owing to discrete symmetries, we need only consider the metric and dilaton. Note that there is no explicit tachyon field in this regime. The instability, whose initial stage is represented by a tachyon in the orbifold description, would now be a property of the solutions to the low energy field equations. The RG equations are

$$\begin{aligned} \dot{G}_{MN} &= -\beta[G_{MN}] + \nabla_M \xi_N + \nabla_N \xi_M \ , \\ \dot{\Phi} &= -\beta[\Phi] + \xi^M \nabla_M \Phi \ , \end{aligned} \tag{4.1}$$

where $G_{MN}$ is the string metric, a dot denotes the logarithmic derivative with respect to world-sheet length scale $\ell\partial_\ell$, and

$$\begin{aligned} \beta[G_{MN}] &= \alpha' R_{MN} + 2\alpha' \nabla_M \nabla_N \Phi \ , \\ \beta[\Phi] &= \alpha' (\nabla\Phi)^2 - \frac{\alpha'}{2} \nabla^2 \Phi \ . \end{aligned} \tag{4.2}$$

The vector field $\xi_M$ is arbitrary and represents the freedom to make a spacetime coordinate change with the change of world-sheet scale. A convenient choice is $\xi_M = \alpha' \nabla_M \Phi$, so that

$$\dot{G}_{MN} = -\alpha' R_{MN} \ , \quad \dot{\Phi} = \frac{\alpha'}{2} \nabla^2 \Phi \ . \tag{4.3}$$

[3] We cannot exclude the possibility of a fixed point with curvature of order the string scale, but the fact that both the substring and gravity geometries evolve toward smaller curvature strongly suggest that this flow continues smoothly through the stringy regime.

In these coordinates the flow of the metric does not depend on the dilaton; this is possible because the dilaton does not appear in the flat world-sheet action.

The perturbation leaves a $(7 + 1)$-dimensional free field theory, so the problem is essentially two dimensional. It is convenient to work in conformal gauge, because the flow (4.3) preserves that gauge. Thus,

$$ds^2 = e^{2\omega}(d\rho^2 + \rho^2 d\theta^2) \ , \tag{4.4}$$

where for generality we consider an arbitrary periodicity $0 \leq \theta \leq 2\pi/\nu$. In this gauge, the metric (2.15) for a cone of opening angle $2\pi/n$ corresponds to

$$\omega = \left(\frac{\nu}{n} - 1\right) \ln \rho + \text{constant} \ . \tag{4.5}$$

In conformal gauge the RG is

$$\dot{\omega} = \frac{\alpha'}{2} e^{-2\omega} \hat{\nabla}^2 \omega \ , \tag{4.6}$$

where $\hat{\nabla}^2$ is the Laplacian for the flat metric $d\rho^2 + \rho^2 d\theta^2$, which is $\partial_\rho^2 + \rho^{-1}\partial_\rho$ for a cylindrically symmetric solution.

Let us analyze this first for the transition $n \to n-2$ at large $n$, where the change in the metric is small. The geometry is as depicted in figure 6, with a $\mathbb{C}/\mathbb{Z}_{n-2}$ cone at the origin, going smoothly to a $\mathbb{C}/\mathbb{Z}_n$ cone at large radius. In coordinates with $\nu = n-2$, the boundary conditions are

$$\omega(\rho \to 0) = \text{finite} \ ; \quad \omega(\rho \to \infty) \to -\frac{2}{n} \ln \rho \ . \tag{4.7}$$

We can then linearize, $\dot{\omega} = \alpha' \hat{\nabla}^2 \omega / 2$. A simple solution, obtained by the Fourier transform on the covering space, is

$$\omega(\rho, \ell) = -\frac{1}{n}\left(\ln \ln(\ell/\ell_0) + \int_0^{u_0} \frac{du}{u}(1 - e^{-u})\right) \to \ -\frac{1}{n} \int_0^{u_0} \frac{du}{u}(1 - e^{-u}) \ , \tag{4.8}$$

where $u_0 = \rho^2 / 2\alpha' \ln(\ell/\ell_0)$; in the second form the $\ln \ln$ term has been conveniently absorbed in an $\ell$-dependent rescaling of $\rho$. The solution depends only on $u_0$, so the radius $\rho_{\rm t}$ of the transition region grows with increasing world-sheet length scale, $\rho_{\rm t} \sim \ln(\ell/\ell_0)^{1/2}$. Pointwise in the IR the system approaches a $\mathbb{C}/\mathbb{Z}_{n-2}$ cone everywhere. Asymptotically, all solutions to the diffusion equation with the given boundary conditions will have the same form. The dilaton satisfies a diffusion equation as well and any initial dilaton gradient will similarly diffuse outward.

For the full nonlinear evolution (4.6) we do not have a simple analytic result, but given the diffusive nature of the equation we expect that in general the smoothed area depicted in figure 3 grows without bound. Hence our conclusion that the flow found in the substring region, toward smaller curvature, continues indefinitely in the gravity region.

There are two reasons that one might doubt this result. The first is the Zamolodchikov $c$-theorem, showing irreversibility of the flow of the central charge [30]. Here we start with an orbifold CFT of canonical central charge (15 in all for the type II string). In the IR, we claim that the theory flows pointwise to flat spacetime, again with canonical central charge. The reason that this is consistent is that the noncompactness of the target space invalidates the $c$-theorem [31]. There are other cases of CFT theorems that are invalid in noncompact target spaces. The classic example is the holomorphicity of conserved currents, which does not hold for the world-sheet currents associated with rotational invariance in noncompact directions [32]. For the $c$-theorem, the basic objects are the vacuum expectation values of operator products. The string world-sheet vacuum fills out the entire target manifold, a familiar IR effect, so the region of curvature makes a contribution of measure zero.

A second reason that one might have expected the opposite result is the example of compact spaces of positive curvature, which flow to greater curvature. We claim that the difference of boundary conditions in the compact and noncompact cases accounts for the differing behaviors. In fact, there is a simple monotonicity result that makes this clear. From the differential equation (4.3) it follows that

$$\ell \partial_\ell \int d^2x \sqrt{G} = -\frac{\alpha'}{2} \int d^2x \sqrt{G} R \ . \tag{4.9}$$

For a manifold of spherical topology, the RHS is $-4\pi\alpha'$ and so the volume is monotonically decreasing. The curvature must at some point become stringy, and the low energy theory break down. For the noncompact manifold the integral is not defined, but one can consider the integral interior to a circle of some given radius (over a region such as depicted in figure 3). The smoothing of the singularity does have the effect of reducing this volume, whereas flow back toward a singular cone would increase the volume in contradiction to the flow (4.9).

Finally, we might also be interested in the case that the original singularity is part of a compact space. Most simply, consider $T^2/\mathbb{Z}_3$, which is a flat space of spherical topology, with three $\mathbb{C}/\mathbb{Z}_3$ singularities each of deficit angle $4\pi/3$. From the $c$-theorem, or from eq. (4.9), one concludes that the space eventually flows to large curvature. The three singularities begin to smooth, until the smoothed regions merge to form a rough sphere, which then evolves toward smaller radius.

*4.2. Dynamical Evolution*

We now consider on-shell evolution,

$$\beta[G_{MN}] = \beta[\Phi] = 0 \ , \tag{4.10}$$

with the same $\beta$-functions (4.2). This is now a three-dimensional problem, since the solution depends on time. It is convenient to work in the Einstein frame, where this system is just a massless scalar canonically coupled to the metric. The initial metric is again assumed to interpolate from $\mathbb{C}/\mathbb{Z}_n$ at infinity to $\mathbb{C}/\mathbb{Z}_{n'}$ at the origin, with $n' < n$. This is true in both the Einstein and the string frames, because we assume that the dilaton is nonsingular at the origin, while it goes to a constant at infinity (where the evolution has not yet reached).

We do not have analytic solutions for this problem, but it is easy to deduce the general form of the solutions. The constraint equations require that the change in deficit angle be accompanied by energy density of matter. Since we can solve the equations with the NS three-form field strength set to zero, so that the only matter involved is the massless dilaton, this energy must be dilaton gradient and kinetic energy. This dilaton field will radiate outward at the speed of light, as in figure 1. The time scale of the initial decay, before the gravity regime, is the string scale, so this sets the initial width of the dilaton pulse and the kink in the geometry, which then gradually broadens due to dispersion. For $n' = n-2$ at large $n$, an analytic treatment is again simple. The dilaton satisfies a massless wave equation in flat spacetime, and the backreaction on the metric is a perturbative effect.

As a check, let us look for static solutions in the gravity regime, which would have corresponded to minima of the tachyon potential that are visible in this regime. We will take the most general form with $SO(7,1) \times SO(2)$ spacetime symmetry:

$$ds^2 = e^{2\sigma(r)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2f(r)} (dr^2 + r^2 d\theta^2) \ . \tag{4.11}$$

This is slightly more general than elsewhere, in that we allow the $(7,1)$ directions to be warped; note also a slight change of notation, $\mu, \nu = 0, \dots, 7$. The dilaton field equation is

$$\frac{\Phi''}{\Phi'} - 2\Phi' = -\frac{1}{r} - 8\sigma' \quad \Rightarrow \quad (e^{-2\Phi})' = \frac{c_1}{r} e^{-8\sigma} \tag{4.12}$$

with integration parameter $c_1$. The $\mu\nu$ curvature equation reads

$$\frac{(r\sigma')'}{r\sigma'} + 8\sigma' = 2\Phi' \quad \Rightarrow \quad e^{2\Phi} = c_2 r \sigma' e^{8\sigma} \tag{4.13}$$

with integration parameter $c_2$. Putting these together gives $\Phi' = -c_1 c_2 \sigma'/2$, and so

$$e^\Phi \propto (\ln r/r_0)^{-c_1 c_2/2(8+c_1c_2)} \ , \quad e^\sigma \propto (\ln r/r_0)^{1/(8+c_1c_2)} \ . \tag{4.14}$$

These are doubly unacceptable: they do not go over to the unperturbed behavior at large $r$, and they have a singularity at finite $r = r_0$. Only the flat cone, the exceptional solution with $\Phi$ and $\sigma$ constant, survives.

It is interesting again to consider the $T^2/\mathbb{Z}_3$ orbifold, with nonsupersymmetric singularities in a compact space. We cannot follow the behavior analytically, but might expect that after the dilaton pulses have begun to cross the compact space, the time-averaged behavior will be that of a positively curved radiation dominated spacetime. Thus, it will reach a Big Crunch in finite time, beyond which we cannot follow the evolution. One supposition would be that the compact dimensions effectively disappear, leaving an eight-dimensional noncritical string theory [4]. However, the simplest background in that theory — the linear dilaton — has the wrong symmetries to be the endstate of our evolution, as the dilaton gradient is spacelike.

### *4.3. Application to AdS/CFT*

In ref. [14] it was argued that orbifolding should commute with AdS/CFT duality, so that the dual of the orbifolded gauge theory is IIB string theory on the orbifolded spacetime.

This expectation is based on the fact that a duality like the AdS/CFT correspondence concerns a single system with two dual descriptions; if orbifolding makes sense on one side of the duality then the procedure can be mapped to the equivalent dual description of the system given a complete duality dictionary translating between them. In the absence of supersymmetry, if the orbifolding procedure produces a consistent physical system, this requires any instabilities that arise to match up between the two equivalent descriptions. The leading instability that arises in a string background is that of interest here, namely the tachyons. In freely-acting orbifolds on the sphere component of the $AdS_p \times S^q$ geometry, the spectrum is classically tachyon-free. Non-freely acting orbifolds on the $S^q$ do have tachyons, and this has been considered at small 't Hooft parameter [2], where the gauge theory is perturbative. Now let us consider the situation at large 't Hooft parameter, where the AdS description is good.

The AdS description starts with $N$ coincident D3-branes extended in the 0123 directions. The orbifold produces a (7+1)-dimensional fixed plane. This plane contains the D3-branes and extends in four transverse directions. The AdS curvature is small on the string scale and so locally on the fixed plane the initial instability is the same as in flat spacetime. In particular, the decay will release a given energy per unit volume of the fixed plane, as measured in a local inertial frame. The invariant volume element is $(r/R_{\rm AdS})^3 d^3x\,(r/R_{\rm AdS})^{-4} r^3 dr$, where $x$ coordinatizes the field theory dimensions and $r$ is a coordinate along the radial direction of $AdS_5 \times S^5$, with metric $(r/R_{\rm AdS})^2 dx^2 + (r/R_{\rm AdS})^{-2}(dr^2 + r^2 d\Omega^2)$. The translation to the global conserved energy brings in an additional factor of $r/R_{\rm AdS}$, so the total energy released per unit gauge theory volume is simply

$$\int_0^\infty dr\, r^3 \sim \Lambda^4 \ . \tag{4.15}$$

That is, it diverges quarticly in the gauge theory.

We can make a simple model of how this divergence might arise in the gauge theory. Consider a state a $U(1)$ gauge theory where we add a $+$ and a $-$ charge in a volume of linear size $\ell$. The kinetic energy is of order $2\ell^{-1}$, but this is reduced somewhat by the gauge theory potential, for a net $\{2 - O(g^2)\}\ell^{-1}$. Extrapolation would suggest a possible instability at large $g^2$ (to be precise, this theory will have a Landau pole in the UV, so we must imagine a cutoff). In a globally supersymmetric theory, positivity of the energy is guaranteed and so this instability is absent; thus, supersymmetric field theories can make sense at large coupling. However, for nonsupersymmetric theories there is no guarantee that they make sense at strong coupling. Indeed the result (4.15) suggests an instability of just this sort: in a conformal theory we can produce pairs on any scale $\ell$, and the integral over all scales produces a quarticly divergent result. Note that this is much more severe than the instabilities normally encountered in field theories (such as symmetry breaking), which are IR effects and release a finite energy per unit volume. It is difficult to see how this instability could have any sensible final state.

Indeed, the AdS picture is similarly pathological. We can quantitatively study the large-$n$ case $\mathbb{Z}_n \to \mathbb{Z}_{n-2}$, because the dilaton essentially satisfies a free wave equation on the $AdS_5 \times S^5$ covering space,

$$\frac{R_{\rm AdS}^2}{r^2}\partial_t^2\Phi = \frac{r^2}{R_{\rm AdS}^2}\partial_\perp^2\Phi \ . \tag{4.16}$$

The orbifold breaks the $SO(6)$ symmetry of $S^5$ so the dilaton is a superposition of different angular states. For angular momentum $L$,

$$\partial_\perp^2 = \partial_r^2 + \frac{5}{r}\partial_r - \frac{L(L+4)}{r^2} \ . \tag{4.17}$$

Imagine that the decay starts everywhere at once at $t = 0$. This condition is conformally invariant so the dilaton is a function only of $rt$. The wave equation (4.16) then becomes an ordinary differential equation for $\Phi_L(rt)$, and $rt = R_{\rm AdS}^2$ is a singular point. From the dominant terms near the singular point one finds that

$$\Phi_L \sim (R_{\rm AdS}^2 - rt)^{-3/2} \tag{4.18}$$

for every partial wave $L$. Thus, the energy density diverges at finite time for any $r$; this occurs when a geodesic from $(r,t) = (\infty, 0)$ reaches a given radius, carrying the information about the divergent energy release at large radius.

Again, this instability is a property of large 't Hooft parameter, and is not inconsistent with the much milder instability found at small 't Hooft parameter in ref. [2]. Note that we have assumed that the 't Hooft parameter does not run, as holds at large $N$. If the full $\beta$ function were in fact asymptotically free, then the theory would be stable in the UV, and the instability that we are discussing would set in only below some scale. In this event it is possible that there would be a stable final state.

## 5. $\mathbb{C}^2/\mathbb{Z}_n$ Orbifolds and Non-SUSY to SUSY Flows

### *5.1. Orbifolds and Quivers*

One of the interesting results of the study of open string tachyons has been the possibility of realizing stable branes, in particular SUSY branes, by open string tachyon condensation [1][17]. In this section, we study closed string tachyon condensation on $\mathbb{C}^2/\mathbb{Z}_n$ orbifolds by generalizing the D-brane probe approach of §3 to this case. We will exhibit various transitions from non-supersymmetric, tachyonic $\mathbb{C}^2/\mathbb{Z}_n$ orbifolds to supersymmetric ALE spaces, and provide an infinite sequence of such flows which allows us to realize any SUSY ALE space via closed-string tachyon condensation (or more generally a combination of marginal deformation and tachyon condensation).

The discussion will parallel the $\mathbb{C}/\mathbb{Z}_n$ case. All the orbifolds that we consider will be based on a twist of the form

$$R = \exp\left\{ \frac{2\pi i}{n}(J_{67} + kJ_{89}) \right\} , \tag{5.1}$$

depending on two integers $n$ and $k$ (mod $2n$). We will denote the group generated by $R$ as $\mathbb{Z}_{n(k)}$. On spinors with $J_{67}$ and $J_{89}$ charge $s_{67} = s_{89} = \pm\frac{1}{2}$, $R$ acts as $e^{\pm 2\pi i(k+1)/2n}$. On spinors with $-s_{67} = s_{89} = \pm\frac{1}{2}$ it acts as $e^{\pm 2\pi i(k-1)/2n}$. The condition that $R^n = 1$ on spinors forces $k$ to be odd. If $k$ is $\pm 1$, then $R$ leaves half of the $D = 10$ spinors invariant and so produces the familiar supersymmetric $A_{n-1}$ orbifold (for reviews see [33][34]). For other values of $k$, at least some of the twisted sector ground states are tachyonic. If $2n$ is divisible by $k+1$ or by $k-1$, then $R^{2n/(k+1)}$ or $R^{2n/(k-1)}$ leaves some spinors invariant. The associated twisted sector ground state is massless, and indeed is the same as the corresponding twisted sector state in the supersymmetric orbifold (but note that the respective cases $k+1 = 2$ and $k-1 = 2$ are trivial).

Let us now consider a D-brane probe in this background. Define

$$Z^1 = X^6 + iX^7 \ , \quad Z^2 = X^8 + iX^9 \ . \tag{5.2}$$

For the world-volume spinor in the **16** of $SO(9,1)$, its component with $(s_{67}, s_{89}) = (-\frac{1}{2}, -\frac{1}{2})$ will be denoted $\chi$ and it component with $(s_{67}, s_{89}) = (-\frac{1}{2}, +\frac{1}{2})$ will be denoted $\eta$ (the remaining two components are the conjugates). The $SO(5,1)$ spinor indices, respectively **4** and **4′**, are suppressed. Using the techniques discussed in [13] and §2, one finds the world-volume theory to be a $U(1)^n$ quiver theory with matter content

$$A_{\mu\, jj} \ , \quad X^m_{jj} \ , \quad Z^1_{j,j+1} \ , \quad Z^2_{j,j+k} \ , \quad \chi_{j,j-q-1} \ , \quad \eta_{j,j+q} \quad (k \equiv 2q+1) \ . \tag{5.3}$$

The classical scalar potential is

$$V = \mathrm{Tr}\Big\{ \frac{1}{2}[Z^1, \overline{Z}^1]^2 + \frac{1}{2}[Z^2, \overline{Z}^2]^2 + \big|[Z^1, Z^2]\big|^2 + \big|[Z^1, \overline{Z}^2]\big|^2 \Big\} \ . \tag{5.4}$$

Using the Jacobi identity this can also be rewritten

$$\begin{aligned} V &= \mathrm{Tr}\Big\{ \frac{1}{2}\big([Z^1, \overline{Z}^1] - [Z^2, \overline{Z}^2]\big)^2 + 2\big|[Z^1, \overline{Z}^2]\big|^2 \Big\} \\ &= \mathrm{Tr}\Big\{ \frac{1}{2}\big([Z^1, \overline{Z}^1] + [Z^2, \overline{Z}^2]\big)^2 + 2\big|[Z^1, Z^2]\big|^2 \Big\} \ . \end{aligned} \tag{5.5}$$

The Yukawa terms are

$$L_{\mathrm{Y}} = \mathrm{Tr}\Big\{ [Z^1, \chi]\, \eta + [Z^2, \chi]\, \overline{\eta} + \mathrm{h.c.} \Big\} \ . \tag{5.6}$$

*5.2. The Example $\mathbb{Z}_{2l(2l-1)}$: Non-SUSY $\mathbb{Z}_{2l}$ to SUSY $\mathbb{Z}_2$*

Now we analyze the case $k = n-1$, where $n = 2l$ must be even because $k$ is odd. Here

$$R = \exp\{2\pi i J_{89}\} \exp\{2\pi i (J_{67} - J_{89})/2l\} \tag{5.7}$$

is the same as in the supersymmetric case except for a factor of $\exp\{2\pi i J_{89}\} = (-1)^{\mathbf{F}}$, which breaks supersymmetry. Note however that the special case $l = 1$, the $\mathbb{C}^2/\mathbb{Z}_{2(1)}$ orbifold, is supersymmetric: $R = \exp\{2\pi i (J_{67} + J_{89})/2\}$ leaves invariant spinors such that $s_{67} = -s_{89}$.

Before exciting tachyons, the geometry is the same as for the supersymmetric orbifold. In particular, on the probe moduli space the condition that $V$ vanish gives

$$Z^1_{j,j+1} = Z^1 \ , \quad Z^2_{j+1,j} = Z^2 \quad \text{(independent of } j\text{)} \tag{5.8}$$

up to gauge transformation. Thus the probe has two complex moduli, as it should. The origin, where the $U(1)^{2l}$ gauge symmetry is restored, is a $\mathbb{Z}_{2l}$ singularity as in §2.2.

Before discussing the generic decay, it is interesting to consider first deformations that preserve a $\mathbb{Z}_2 \subset \mathbb{Z}_{2l}$ quantum symmetry. This $\mathbb{Z}_2$ acts on the $l^{\text{th}}$ twisted sector as $(-1)^l$, so only states twisted by powers of $R^2$ can have VEVs. Since $R^2 = \exp\{2\pi i(J_{67} - J_{89})/l\}$, these sectors are exactly the same as for the *supersymmetric* $\mathbb{C}^2/\mathbb{Z}_{l(-1)}$ orbifold. In particular, there are no tachyons, so we are actually considering marginal deformations. The perturbation of the gauge theory is then a supersymmetric $D$-term

$$\Delta V = -\sum_{j=1}^{2l} \lambda_j D_j \ , \quad D_j = |Z^1_{j,j+1}|^2 - |Z^2_{j+1,j}|^2 - |Z^1_{j-1,j}|^2 + |Z^2_{j,j-1}|^2 \ , \tag{5.9}$$

where the sign of each term is determined by the $U(1)_j$ charge (note that for a $k = -1$ orbifold $Z^1$ and $Z^2$ are in chiral superfields, while for $k = +1$ it would be $Z^1$ and $\overline{Z}^2$). An overall additive constant is ignored. As in §2.2, $\sum_{j=1}^{2l} \lambda_j = 0$, while the $\mathbb{Z}_2$ quantum symmetry requires that $\lambda_j = \lambda_{j+l}$. Now consider the deformed moduli space; focus on the second of forms (5.5) and note that the first term there is just $\sum_{j=1}^{2l} D_j^2$. The vanishing of the second term requires that

$$Z^1_{j,j+1} Z^2_{j+1,j} \equiv \alpha \tag{5.10}$$

be independent of $j$. Minimizing the $D$-terms then sets $D_j = \lambda_j$, which determines all of the magnitudes in terms of $|Z^1_{12}|$ and $\alpha$. Finally, the phases can be gauged away except for $\sum_{j=1}^{2l} \arg Z^1_{j,j+1}$, giving four real moduli in all.

There are still singularities. Consider the subspace $\alpha = 0$. The condition $D_j = \lambda_j$ determines

$$|Z^1_{j,j+1}|^2 - |Z^2_{j+1,j}|^2 = \rho_j + x \ , \tag{5.11}$$

where $\rho_j = \rho_{j-1} + \lambda_j$ and $x$ is undetermined. When $x = -\rho_{j_0}$ for some $j_0$, both $Z^1_{j_0,j_0+1}$ and $Z^2_{j_0+1,j_0}$ vanish. Further, the $\mathbb{Z}_2$ quantum symmetry implies that $Z^1_{j_0+l,j_0+l+1}$ and

$Z^2_{j_0+l+1,j_0+l}$ vanish as well. There are then two unbroken $U(1)$'s, namely $\sum_{j=j_0+1}^{j_0+l} Q_j$ and $\sum_{j=j_0+l+1}^{j_0} Q_j$, where $Q_j$ is the $U(1)_j$ charge and $j$ is defined mod $2l$. Thus, these $l$ points of restored gauge symmetry, which are generically distinct, are $\mathbb{Z}_2$ singularities on the moduli space.

Thus far the discussion is the same as for the resolution of a *supersymmetric* $\mathbb{C}^2/\mathbb{Z}_{2l(-1)}$ singularity while preserving a $\mathbb{Z}_2$ quantum symmetry: the result there would be a $\mathbb{Z}_{2(-1)}$ orbifold of a smooth $\mathbb{Z}_l$ ALE space. The difference for us is that the final orbifold operation here contains an extra factor of $(-1)^{\mathbf{F}}$, so it must be $\mathbb{Z}_{2(1)}$. Naively one might expect this orbifold point to be nonsupersymmetric, but the discussion at the beginning of this subsection shows that it is supersymmetric with the opposite supersymmetry from that respected by $R^2$. One can think of the final picture as follows: we resolve the $\mathbb{C}^2/\mathbb{Z}_{l(-1)}$ orbifold generated by $R^2$ into a smooth ALE space preserving half of the supersymmetry, and then make a $\mathbb{Z}_{2(1)}$ orbifold which locally would preserve the other half. In other words, we have a space of $SU(2)_1 \subset SO(4) = SU(2)_1 \times SU(2)_2$ holonomy, with $l$ orbifold singularities whose holonomy is in $SU(2)_2$. The space as a whole has no supersymmetry, but half of the supersymmetry survives in the smooth region and the other half locally at the orbifold points. In the limit that the marginal deformation is taken to infinity, we simply have a supersymmetric $\mathbb{C}^2/\mathbb{Z}_{2(1)}$ space, without tachyons.

We can verify this by examining the quivers. At the orbifold point, the potential for the vanishing fields $Z^1_{j_0,j_0+1}$, $Z^2_{j_0+1,j_0}$, $Z^1_{j_0+l,j_0+l+1}$ and $Z^2_{j_0+l+1,j_0+l}$ is quartic so they are massless, while all other scalars are massed up. One unbroken $U(1)$ acts on indices $j = j_0,\, j_0+l+1$, and the other on indices $j = j_0+1,\, j_0+l$. Expanding the Yukawa coupling (5.6) in components, one finds that $\eta_{j_0+1,j_0+l}$ and $\eta_{j_0+l+1,j_0}$ do not appear in terms with scalar expectation values, so these remain massless (note that these are neutral under the unbroken $U(1)$'s). There must therefore also be two massless linear combinations of $\chi$'s; these come in the bifundamental representation of the unbroken $U(1)^2$. The correlation between $U(1)$ charges and $SO(5,1)$ quantum numbers is the same as for the $\mathbb{C}/\mathbb{Z}_{2(1)}$ orbifold, namely the spectrum (5.3) at $k = 1$ with $\eta$ in the adjoint and $\chi$ in the bifundamental representation of the gauge group.

We now turn to the generic twisted state background. The full D-brane probe analysis is less useful here, for two reasons. The first is that without any connection to supersymmetry, the quantum symmetry alone does not fix the form of the quadratic mass terms (specifically, the ratio of $Z^1$ and $Z^2$ masses); it requires the calculation of a disk amplitude, as in the appendix to ref. [13]. More critically, for general mass terms allowed by the quantum symmetry, there is no probe moduli space. This is not a problem — from the spacetime point of view it is the same effect that a dilaton background would have — but it makes it difficult to give a geometric interpretation in the substring regime.

Fortunately, we can largely deduce the fate of the instability by expanding around the deformation already considered. Let us first deform $\mathbb{C}^2/\mathbb{Z}_{2l(2l-1)}$ along directions that preserve the $\mathbb{Z}_2$ quantum symmetry as above, so as to have an orbifold of $SU(2)_2$ holonomy in a space of $SU(2)_1$ holonomy. The orbifold locally is supersymmetric and so has marginal deformations in the twisted sector. These correspond to blowing the orbifold points up into smooth $\mathbb{Z}_2$ ALE spaces of $SU(2)_2$ holonomy. Thus we have small patches of $SU(2)_2$ holonomy in a larger region of $SU(2)_1$ holonomy. This is only an approximate solution to the equations of motion, and will in time evolve to a space of generic holonomy and we presume expand indefinitely as in the $\mathbb{C}/\mathbb{Z}_n$ case [35].

Note that the second blowing-up will not be exactly marginal, as the coupling to the $SU(2)$ curvature will break supersymmetry and presumably drive the marginal direction to be tachyonic. If the extent of initial blowing-up is reduced, so as to condense the two steps towards one, the $\mathbb{Z}_{2(1)}$ twisted state will become more tachyonic, so we seem to connect smoothly onto the original string-scale tachyon.

There is a seeming paradox here, whose resolution provides an elegant check on our picture. The initial $\mathbb{C}^2/\mathbb{Z}_{2l(2l-1)}$ orbifold is an exact CFT, and so its tree-level energy (as measured by the $1/r^2$ falloff of the metric) is zero. There is a tree-level tachyon, and so the final state should have negative energy when the kinetic energy of the outgoing pulse is subtracted.[4] Does this not violate a positive energy theorem? In fact, there is no such theorem: negative energy configurations of asymptotic ALE geometry exist [36].[5] There is

[4] This paradox did not arise for $\mathbb{C}/\mathbb{Z}_n$, because in two dimensions a conic deficit angle *is* an ADM energy.

[5] We would like to thank G. Horowitz for informing us about these spaces and explaining their

a negative energy theorem for any geometry that admits spinor fields going to a constant at infinity [37][38]. The geometries of ref. [36] admit spinors, so it must be that any smooth spinor field is *antiperiodic* under the asymptotic ALE identification. This is precisely the geometry of the $\mathbb{Z}_{2l(2l-1)}$ orbifold.

### *5.3. The Example* $\mathbb{C}^2/\mathbb{Z}_{2l(3)}$*: Non-SUSY* $\mathbb{Z}_{2l}$ *to SUSY* $\mathbb{Z}_l$

These results have a resemblance to phenomena that have been observed in open string systems. The existence of a tachyon, which disappears as one goes along a marginal direction, is the same as in a D-brane/anti-D-brane system, where the string-scale tachyon at small separation goes over to a long-range attraction as the branes are separated. The decay of a nonsupersymmetric configuration to a supersymmetric configuration plus outgoing radiation is also familiar.

There are many other similar flow patterns that can be deduced by studying the quiver theories as we have done for the above case. One interesting sequence is for $n = 2l$ and $k = 3$,

$$R = \exp\left\{ \frac{2\pi i}{2l}(J_{67} + 3J_{89}) \right\} , \tag{5.12}$$

whose quiver diagrams are shown in figure 9.

significance.

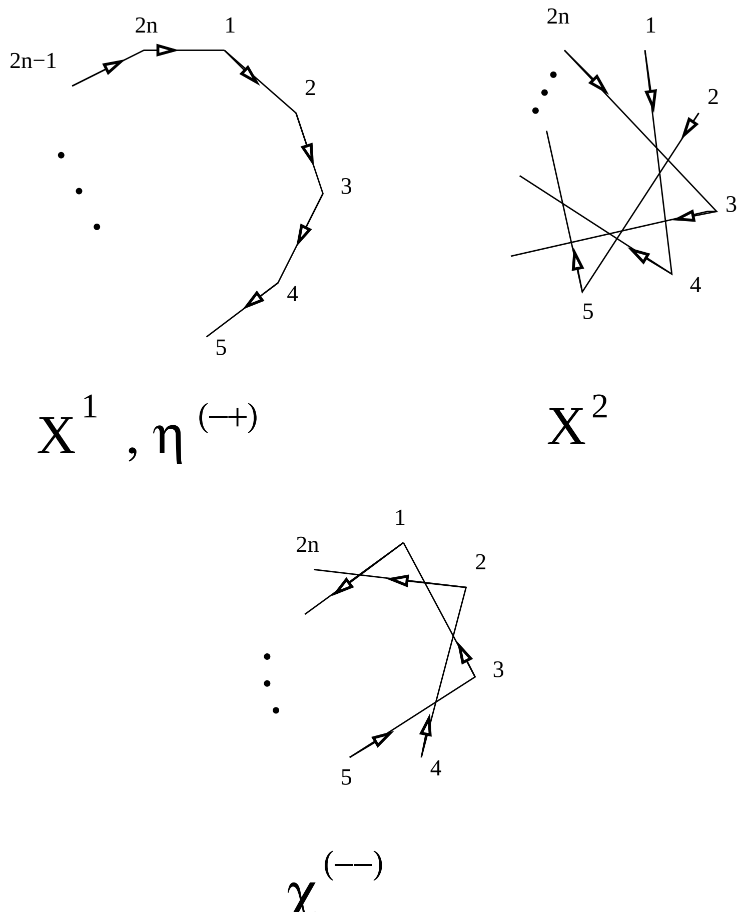


Figure 9: $\mathbb{C}^2/\mathbb{Z}_{2l(3)}$ quivers

In this case $R^l = \exp\{i\pi(J_{67} - J_{89})\}$ is the same as for the supersymmetric $\mathbb{C}^2/\mathbb{Z}_{2(-1)}$ orbifold. In parallel with the previous example, we first excite only marginal states from the sector twisted by $R^l$. This preserves as $\mathbb{Z}_l$ subgroup of the original $\mathbb{Z}_{2l}$ quantum symmetry.

The Fayet-Iliopoulos terms then satisfy

$$\lambda_j = (-1)^{j+1}\lambda \tag{5.13}$$

where we take $\lambda > 0$. The quantum symmetry requires that $Z^1_{2p-1,2p}$, $Z^1_{2p,2p+1}$, $Z^2_{2p-1,2p+2}$, and $Z^2_{2p,2p+3}$ be independent of $p$, and the $D$-terms are minimized when

$$|Z^1_{2p-1,2p}|^2 + |Z^2_{2p-1,2p+2}|^2 = |Z^1_{2p,2p+1}|^2 + |Z^2_{2p,2p+3}|^2 + \lambda \ , \tag{5.14}$$

while $Z^1_{2p-1,2p}Z^2_{2p,2p+3} = Z^1_{2p,2p+1}Z^2_{2p-1,2p+2}$. When $Z^1_{2p-1,2p} = \lambda^{1/2}$ with all other VEVs vanishing, a $U(1)^l$ is restored — namely $Q_{2p-1} + Q_{2p}$ for all $p$ — so this is a $\mathbb{Z}_l$ singularity.

Before taking into account interactions that give mass to some fields, the quiver diagrams thus collapse to those depicted in Figure 10.

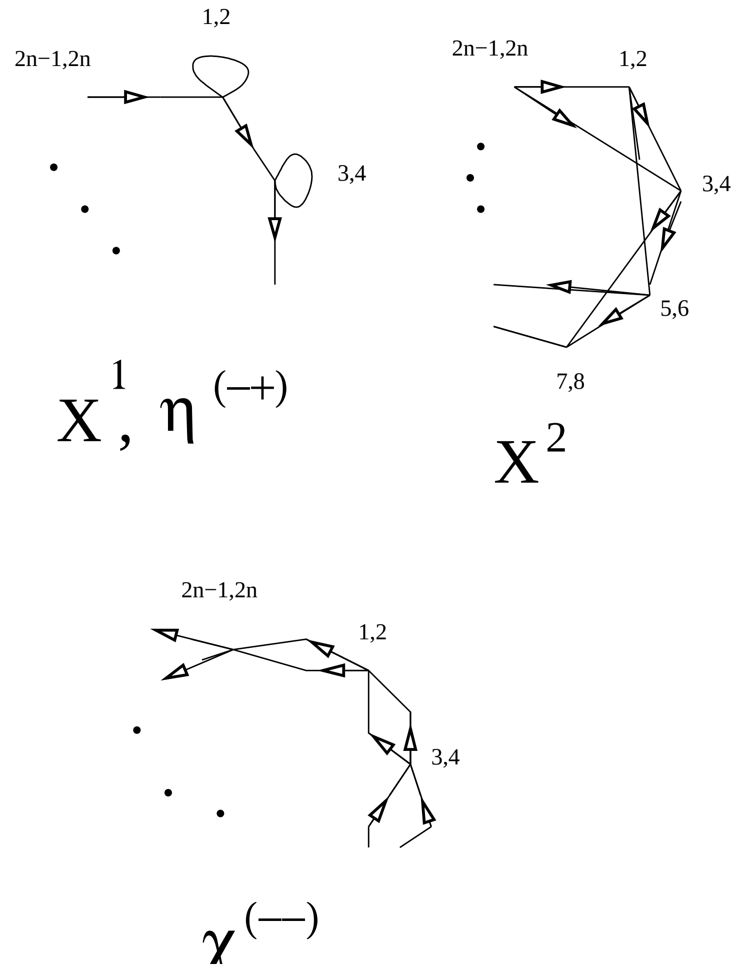


Figure 10: $\mathbb{C}^2/\mathbb{Z}_l$ quivers from collapse of $\mathbb{C}^2/\mathbb{Z}_{2l(3)}$, including massive fields

We next must determine which of the fields in figure 10 mass up in the transition. On the $Z^1$ diagram, the adjoint representations are removed: the potential fixes the magnitudes and the Higgs mechanism removes the phases, leaving the result in Figure 11. On the $Z^2$ diagram, the $|[Z^1, Z^2]|^2$ term gives masses to $Z^2_{2p,2p+3}$, so that the components depicted in figure 11 remain massless. Of the fermions, only half of the components appear in the mass matrix, namely $\eta_{2p,2p}$ and $\chi_{2p+1,2p-1} - \chi_{2p,2p-2}$, leaving the fermions depicted in figure 11. In particular, the $\eta$ are in the adjoint representation and the $\chi$ are in the $(q, q+1)$ bifundamental.

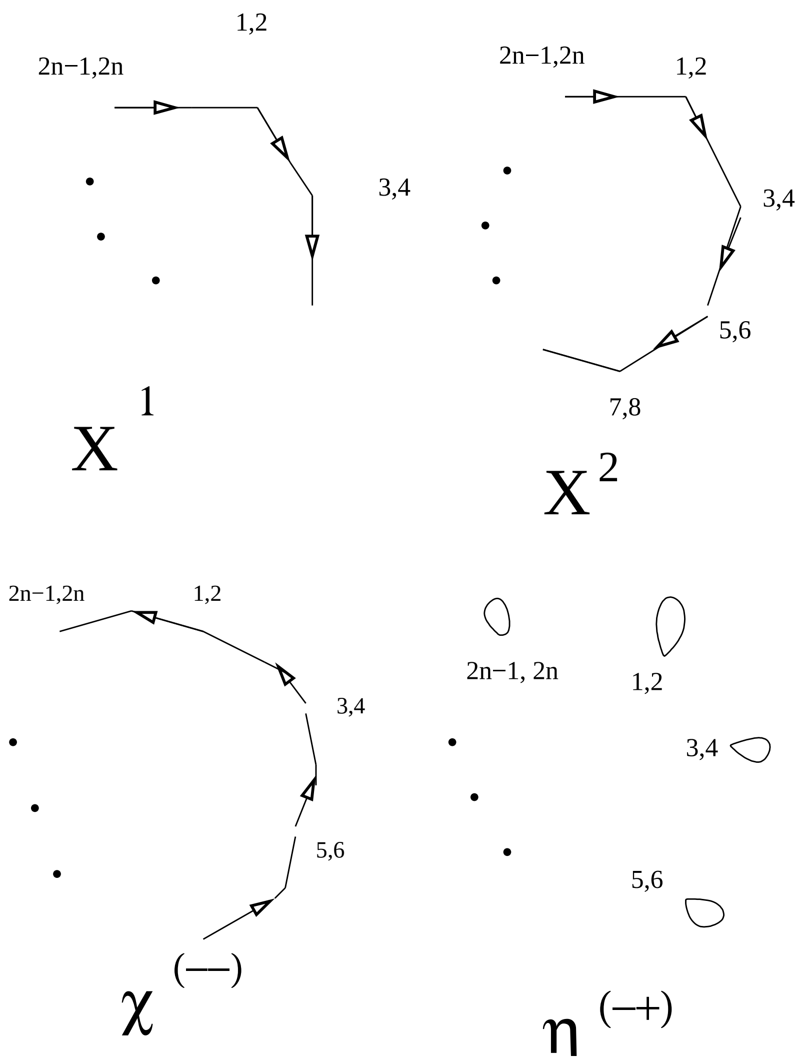


Figure 11: $\mathbb{C}^2/\mathbb{Z}_{2l(3)}$ quivers after twisted-state condensation

Altogether, we are left in Figure 11 with the quiver theory corresponding to D-branes at a $\mathbb{C}^2/\mathbb{Z}_{l(1)}$ orbifold point, which is supersymmetric but with the opposite supersymmetry from the $R^l$ orbifold. Thus the interpretation is parallel to the previous example: the marginal direction blows up the orbifold into a manifolds of smooth $SU(2)_1$ holonomy, which is orbifolded by $\mathbb{Z}_{l(1)} \subset SU(2)_2$.

As a check, consider the low energy theory near the fixed point. We have $R = \exp(2\pi i J/2l)$, where $J = J_{67} + 3J_{89}$. This operator in the original theory becomes

$$J_{67} + 3J_{89} + \frac{1}{2}\sum_{p=1}^{l}(Q_{2p} - Q_{2p-1}) \ , \tag{5.15}$$

in the low energy theory, because this is the linear combination including the broken generators that leaves the background invariant. This acts on the massless fields as

$$Z^1_{2p,2p+1} \to 2Z^1_{2p,2p+1}\ , \quad Z^2_{2p-1,2p+2} \to 2Z^2_{2p-1,2p+2} \tag{5.16}$$

and so it acts as $\tilde{J} = 2(J_{67} + J_{89})$ in the low energy theory. The orbifold operation $\exp(2\pi i\tilde{J}/2l)$ is then $\mathbb{Z}_{l(1)}$ in the low energy theory.

There is another orbifold point, where $Z^2_{2p-1,2p+2} = \lambda^{1/2}$ with all other VEVs vanishing. The analysis of the previous paragraph shows that this is a $\mathbb{Z}_{l(-3)}$, which is nonsupersymmetric for $l > 2$.

In summary, we can obtain all supersymmetric ALE orbifolds by descent from nonsupersymmetric ones. The $\mathbb{C}^2/\mathbb{Z}_{4(3)} \to \mathbb{C}^2/\mathbb{Z}_{2(1)}$ flow is common to both this sequence and the one discussed in §5.2.

### *5.4. The Example $\mathbb{C}^2/\mathbb{Z}_{5(3)} \to \mathbb{C}/\mathbb{Z}_{2(1)}$: Tachyon Condensation*

Since both of the above examples involved marginal as well as tachyonic deformations, it is interesting to ask whether there are in fact examples where such transitions between non-supersymmetric and supersymmetric ALE spaces proceed exclusively by tachyon condensation, without any marginal component. The following simple example exhibits this possibility (which we expect to be generic). We will make the assumption that the twisted deformations we turn on in the quiver world-volume QFT can be accessed by adjusting modes in the tower of twisted states in the closed string sector. It would be interesting to check this generic assumption more explicitly as in [13].

Start with the orbifold $\mathbb{C}^2/\mathbb{Z}_{5(3)}$. We can choose three independent $\lambda_j$ such that the D-terms induce VEVs for $Z^1_{45}$, $Z^1_{51}$, and $Z^1_{23}$. This preserves a $U(1)^2$ subgroup of the $U(1)^5$ gauge symmetry, generated by combinations of charges $Q_4 + Q_5 + Q_1$ and $Q_2 + Q_3$. Plugging these VEVs into the component expansion of the interaction terms (5.5)(5.6) as before, we find that the spectrum reduces to that of the $\mathbb{C}/\mathbb{Z}_{2(1)}$ quiver theory, with gauge group $U(1)^2$, $\eta$ in the adjoint and $\chi$, $Z^1$, and $Z^2$ transforming as bifundamentals. This theory does not have effectively supersymmetric subsectors, in contrast to those in §5.2 and §5.3. So given our assumption about the availability of these deformations in the closed string spectrum (including those that put the Lagrangian in supersymmetric form), this provides an example of a truly tachyonic transition from a non-supersymmetric ALE space to a supersymmetric one.

## 6. Dualities, Fluxbranes, and the Type 0 Tachyon

The results in the preceding sections describe transitions between different ALE spaces (including flat space) by processes in which the string coupling remains bounded. While this is sufficient for our purposes, it is also instructive to consider the predictions that these results imply for processes in dual descriptions of the system. In particular we will consider $T$-dual descriptions, in the angular direction, of the orbifolds that we have considered, as well as the addition of R-R Wilson lines. The duals thus involve NS5-branes, fluxbranes, and the type 0 tachyon.

### *6.1. $\mathbb{C}/\mathbb{Z}_n$ at Large $n$*

An angular direction along which we rotate in performing a $\mathbb{Z}_n$ orbifold projection ends up $n$ times smaller than in the parent theory, so for $n$ large it is of interest to $T$-dualize along this direction in the region near the origin. Near but not at the origin, there is a subspace that looks approximately like a cylinder, with twisted strings playing the role of winding modes around the $S^1$ direction of the cylinder:

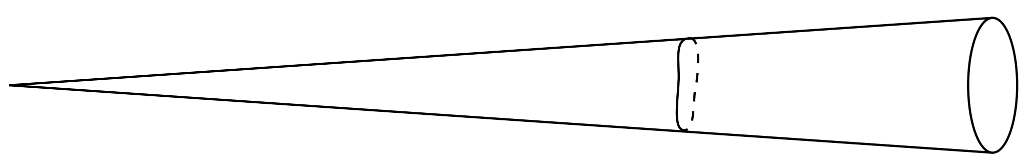

Figure 12: The $\mathbb{C}/\mathbb{Z}_n$ cone at large $n$, with a twisted closed string.

In other words, there are many low-lying twisted states, which become Kaluza-Klein states in the $T$-dual description. The formal $T$-dual of the cone metric (2.15) is

$$ds^2 = n dr^2 + \frac{n\alpha'^2}{r^2} d\tilde{\theta}^2 \ . \tag{6.1}$$

Also, the dilaton is now position-dependent,

$$e^{\Phi} = g_{\rm s} \frac{\sqrt{n\alpha'}}{r} \ . \tag{6.2}$$

In the large-$n$ limit the orbifold operation (2.3) is a small rotation times $(-1)^{\mathbf{F}}$, so on the circle that we are $T$-dualing fields are twisted by $(-1)^{\mathbf{F}}$. Such a twist has three effects on the $T$-dual description. First, the bulk theory is twisted by $(-1)^{\mathbf{F}}$, so it is the type 0 theory (type 0B if we began with IIA, and type 0A if we began with IIB). Second, in going around the $T$-dual circle there is a twist by $(-1)^{\mathbf{Q}}$, where $\mathbf{Q}$ is the quantum symmetry dual to the twist $(-1)^{\mathbf{F}}$. That is, type 0 fields that descend from the type II theory are periodic, while type 0 fields that do not descend (including the type 0 tachyon) are antiperiodic. Third, the periodicity of the dual coordinate is halved, $0 \leq \tilde{\theta} \leq \pi$.

The $T$-dual description is valid out to $r \sim n\sqrt{\alpha'}$, beyond which the $T$-dual circle is small and the original circle is large. It also breaks down for $r < \sqrt{\alpha'}$, where the curvature becomes large. Thus, the apparent divergence of the dilaton (6.2) is irrelevant, as we could have expected since the orbifold description is manifestly weakly coupled. We do not have any good description in this region; it is some sort of effective 'wall' in spacetime, whose properties can be deduced from the exact orbifold description. One property of the wall that is not evident in the metric (6.1) is the breaking of translation invariance in the $\tilde{\theta}$-direction. The twisted modes of the orbifold transform under the finite $\mathbb{Z}_n$ quantum symmetry rather than the infinite group $\mathbb{Z}$ characterizing true winding modes on a cylinder. In the $T$-dual description, this means that the continuous translation symmetry along the dual angular circle is broken to a discrete $\mathbb{Z}_n$ symmetry [39]. This suggests that the wall is actually a line of $n$ branes (defined broadly as defects which break translation invariance) spaced equally along the $T$-dual circle. In the case of $\mathbb{C}^2/\mathbb{Z}_n$, this picture is well understood, as we will review shortly, but for $\mathbb{C}/\mathbb{Z}_n$ we do not know of any suitable candidate branes.

The twisted state tachyon of the original theory is just the bulk type 0 tachyon in the $T$-dual description. The multiplicity of excited tachyons associated with the eight-dimensional fixed plane on the orbifold side maps on the $T$-dual side to the multiplicity of modes of the ten-dimensional type 0 tachyon. Because of the $(-1)^{\mathbf{Q}}$ twist the decay is most rapid at small $r$. The type 0 tachyon in ten dimensions has $\frac{\alpha'}{4}m^2 = -\frac{1}{2}$, and the antiperiodic boundary condition should shift this upward by an amount of order the inverse radius of the dual circle. Indeed, the most tachyonic mode has

$$\frac{\alpha'}{4}m^2 = -\frac{1}{2} + \frac{1}{2n} \tag{6.3}$$

For the partial resolution $n \to n-2$, it seems that the wall relaxes into a lower energy state while a metric and dilaton perturbation (given by the $T$-dual of the picture in §4) propagates outward. For the full decay $n \to 1$, the tip of the cone and the associated low-lying states disappear at the speed of light. In the $T$-dual picture, it seems that the wall, where our control breaks down, is propagating to larger $r$ at the speed of light. At larger $r$, the angular direction gets smaller in this T-dual picture. It would be interesting to try to extract from this a prediction for the type 0 tachyon, but this is not immediate in our system here because the initial wall is present to act as a seed for the decay.

### *6.2.* $\mathbb{C}^2/\mathbb{Z}_n$ *and NS5-Branes*

For the orbifold $\mathbb{C}^2/\mathbb{Z}_n$ at large $n$ and fixed $k$, the angular direction generated by $J_{67} + kJ_{89}$ is again small and a $T$-dual picture is valid. This is best understood in the supersymmetric cases $k = \pm 1$: the $T$-dual description has $n$ evenly spaced NS5-branes [16]. The sequences of transitions between non-supersymmetric and supersymmetric four-dimensional ALE spaces detailed in §5 (and presumably many others like them) allow us to produce any supersymmetric ALE space by closed-string tachyon condensation or marginal deformation. Using the $T$-duality, we can restate this in terms of NS5-branes. Namely, any number of NS5-branes can be obtained by condensation of modes in a non-supersymmetric closed string background.

It is also interesting to look for a brane description of the tachyonic starting point. In particular, in the case $\mathbb{C}^2/\mathbb{Z}_{2l(2l-1)} \to \mathbb{C}^2/\mathbb{Z}_{2(1)}$ one might have expected that since the bosonic action is the same as in a supersymmetric $\mathbb{C}^2/\mathbb{Z}_{2l}$ orbifold, the $T$-duality transformation would produce a similar configuration of $2l$ NS5-branes. However, the factor $(-1)^{\mathbf{F}}$ in the twist (5.7) modifies the $T$-duality as described in §6.1. The $T$-dual circle is only half as large, so there are only $l$ NS5-branes, while the bulk theory is type 0 theory with a $(-1)^{\mathbf{Q}}$ twist around the $T$-dual circle. The marginal deformations that we discussed descend from those of the supersymmetric $\mathbb{Z}_l$ theory, and so correspond to the positions of the $l$ NS5-branes. The tachyons, in the sectors of odd $\mathbb{Z}_2$ quantum symmetry, are modes of the type 0 tachyon.

It would be interesting to pursue this type of dual description of the non-supersymmetric ALE orbifolds further. It is straightforward to apply the general $T$-duality transformation [40], but this results in a smeared 5-brane solution and we do not know the localized form in general.

*6.3. Adding RR Flux*

A simple generalization is to add an RR Wilson line to the $\mathbb{C}/\mathbb{Z}_n$ orbifold,[6] $C_\theta = 1$ in coordinates where the identification is $\theta \sim \theta + 2\pi/n$. The net phase is then $2\pi/n$. In M theory this corresponds to an orbifold by a $2\pi/n$ rotation accompanied by a shift by $1/n$ around the M theory circle. In a dual description where a linear combination of the eleventh direction and the angular direction of the orbifold is taken to be the M direction, this is a fluxbrane [18][19][20][9]. Because of the factor of $(-1)^{\mathbf{F}}$ in the orbifold, it is a fluxbrane in the type 0A theory [41] of strength $BR^2 = 1/n$, or a fluxbrane in the type IIA theory of strength $BR^2 = 1 + 1/n$.

This duality is a strong-weak coupling duality, so that both sides are not simultaneously weakly coupled. However, on the fluxbrane side the coupling varies with radial distance from the origin, becoming weaker toward the origin. If we fix the string coupling to be $g_s < 1$ on the orbifold side, on the fluxbrane side one has a region $r < l_s g_s^{1/3} n = n l_{P,11}$ near the origin which has string coupling $g_s^{(f)} < 1$. For large $n$ and $g_s > 0$, this region can cover many string lengths. We will study the predictions of our results combined with the conjectured orbifold/fluxbrane duality for decay of the Type 0A tachyon in this region. In our analysis in the bulk of this paper, we worked in the classical string limit. As we have just learned, in order to dualize to a fluxbrane side with a significant region of weak coupling near the origin, we must relax this limit somewhat, and consider a nonvanishing orbifold string coupling, though we can keep it weak. For the remainder of this section, we will assume that the decay process we studied proceeds similarly at weak but nonzero coupling.

[6] We thank A. Strominger for discussions on this issue.

RR field strengths couple to extra powers of $g_s$ in the action and so for weak string coupling they have only a small effect on the tachyon decay process we have studied. The decay will proceed as we have described, with the RR flux ultimately dispersing when we reach the flat space endpoint. For the partial decay $n \to n-2$, the outgoing pulse must contain a negative RR flux $2\pi(\frac{1}{n} - \frac{1}{n-2})$. In the dual fluxbrane, the flux near the origin *increases* in the transition, from $\frac{1}{n}$ to $\frac{1}{n-2}$. According to the conjectured duality dictionary in [9], this addition of flux takes the $0A$ theory closer to the flat space IIA theory.

Therefore, by assuming the dualities described in [9], and combining them with our results on classical tachyon decay in orbifolds, we predict that the type 0A tachyon in ten dimensions decays toward the flat ten-dimensional IIA vacuum. This agrees with the conjecture for the fate of the Type 0A tachyon made in [9] based on extrapolating to a regime where *non-perturbative* decays from 0A to IIA occur [5][42]. Our route to this conclusion is somewhat more direct, as we use our classical results on tachyon decays in orbifolds rather than non-perturbative instanton effects. However, these statements are still predicated on the conjectural non-supersymmetric strong-weak coupling duality [41] assumed in [9]. Therefore we regard this as a mild consistency check of the proposal that the type 0A tachyon drives the theory to the type IIA vacuum.

## 7. Conclusions

In this paper we have exhibited strong evidence that tachyonic non-supersymmetric ALE spaces decay to supersymmetric ALE spaces (including flat space). There are several interesting lessons and directions for future work that emerge from our analysis.

On the theoretical side, as we have emphasized at various points, our results are rather similar to ones that emerge in the study of open string tachyon condensation and its relation to unstable brane annihilation. It would be very interesting to understand how far the analogy between twisted strings and open strings goes. Is there a notion of confinement of twisted strings into ordinary untwisted closed strings? Is there a simplification of closed string field theory if one focuses on twisted states and regards untwisted strings as derivative degrees of freedom obtained in internal legs of the diagrams? What does the similarity between closed string and open string processes say about the extent of applicability of K-theoretic techniques as a function of $g_s$?

We should reemphasize, as discussed in the introduction, that there is a similar puzzling issue in the two cases. Namely as in the open string case, our results point to the need for a strictly classical stringy mechanism, different from the Higgs mechanism, for lifting gauge bosons living on decaying defects. It is perhaps a clue that the disappearance of these gauge bosons and the other phenomena we have observed occurs in the closed string as well as open string context: whatever the physics is that gives rise to these processes, it is not tied uniquely to the open string perturbation expansion since it arises for twisted closed strings as well.

Another related lesson is the existence of a large class of non-supersymmetric configurations which, while unstable, do not "decay to nothing", as a class of non-SUSY models without massless fermions are known to do [5][42][7]. Instead, they decay via a relatively well-controlled weakly coupled process to stable supersymmetric configurations. It would be very interesting to understand the fate of *compact* non-supersymmetric orbifolds of the superstring with massless fermions, particularly since as we discussed the time-dependent physics in the compact case is very similar to that of a cosmology heading toward a big crunch singularity.

In terms of model-building, these results, while mostly negative for supersymmetry breaking, at least may help direct attention to more stable possibilities than geometrical orbifolds for breaking SUSY. The fact that the noncompact models decay to SUSY spaces provides a new indication of the intrinsic role of SUSY within the theory. Again, the question of the fate of compact examples which are most relevant for phenomenological model-building is still open.

Finally, it would be interesting to extend these results to other cases. It is presumably straightforward to generalize them to intersecting ALE spaces probed by different combinations of D-branes, and to the type I theory. Noncompact tachyonic orbifolds of the heterotic string may have a similar fate to those we discussed here, but in that case there are no D-brane probes available to study the substring regime. The heterotic case will require an understanding of the dynamics of the vector bundle formed by the gauge bosons as well as the configuration of dilaton and metric. Under RG flow the cases with a standard embedding of the orbifold action into the gauge group will behave as our models here; it would be interesting to study also the time-dependent on-shell spacetime solutions in the heterotic string.

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