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Measuring the Photon Polarization in $B \to K\pi\pi\gamma$

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ABSTRACT

We propose a way of measuring the photon polarization in radiative B decays into K resonance states decaying to $K\pi\pi$, which can test the Standard Model and probe new physics. The photon polarization is shown to be measured by the up-down asymmetry of the photon direction relative to the $K\pi\pi$ decay plane in the K resonance rest frame. The integrated asymmetry in $K_1(1400) \to K\pi\pi$, calculated to be 0.25, is measurable at currently operating B factories.

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The Standard Model (SM) predicts that photons emitted in rare $b \to s\gamma$ decays are left-handed [1], up to small corrections of order m_s/m_b, while being right-handed in $\bar{b} \to \bar{s}\gamma$. This feature is common to inclusive and exclusive radiative decays, also when including long-distance effects in the latter case [2]. While measurements of the inclusive rate agree reasonably well with SM calculations, no evidence exists for the helicity of the photons in these decays. In several models beyond the SM the photon in $b \to s\gamma$ acquires an appreciable right-handed component due to the exchange of a heavy fermion in the electroweak loop process. For instance, in $SU(2)_L \times SU(2)_R \times U(1)$ left-right symmetric models [3] this component may be comparable in magnitude to the left-handed component, without affecting the SM prediction for the inclusive radiative decay rate. An independent measurement of the photon helicity is therefore of interest.

Several strategies have been proposed to look for signals of physics beyond the SM through helicity effects in $B \to X_s\gamma$. In one method the photon helicity is probed through mixing-induced CP asymmetries [4]. In two other schemes one studies angular distributions in radiative decays of Λ_b baryons [5, 6] and in $B \to \gamma(\to e^+e^-)K^*(\to K\pi)$ [7, 8]. Whereas in the method using Λ_b decays one measures directly the photon polarization, the other measurements are sensitive to interference between amplitudes involving photons with left and right-handed polarization. All methods can probe deviations from pure left or right-handedness.

In the present Letter we propose to measure the photon polarization in exclusive radiative B decays to kaon resonance states, $B \to K_{\rm res}\gamma$. We will study in particular decays into an axial-vector meson, $K_1(1400)$, and into a tensor meson, $K_2^*(1430)$. An earlier suggestion to look for parity violation in $B \to K_1(1400)\gamma$ was made in [9].

Radiative decays into $K_2^*(1430)$ were observed both by the CLEO [10] and Belle [11] collaborations with branching ratios

$$\begin{aligned} \mathcal{B}(B \to K_2^*(1430)\gamma) &= (1.66^{+0.59}_{-0.53} \pm 0.13) \times 10^{-5} \qquad \text{(CLEO)} \\ &= (1.26 \pm 0.66 \pm 0.10) \times 10^{-5} \qquad \text{(Belle)} \end{aligned} \tag{1}$$

In these experiments K_2^* states were identified through the $K\pi$ decay mode. K_1 states, which do not decay in this mode, are expected to be observed in the $K\pi\pi$ channel. As we will argue below, in order to probe the photon helicity, one must study excited kaon decays into final states involving at least three particles.

Let us explain first the necessary conditions for a theoretically clean measurement of the photon helicity in radiative B decays. Since the photon helicity is odd under parity, and since one only measures the momenta of final decay products, spin information cannot be obtained from two body decays of the excited kaon. It requires at least a three body decay in which one can form a parity-odd triple product $\vec{p}_\gamma \cdot (\vec{p}_1 \times \vec{p}_2)$. Here $\vec{p}_\gamma$ is the photon momentum, and $\vec{p}_1$, $\vec{p}_2$ are two of the final hadron momenta, all measured in the K-resonance rest frame. The average value of the triple product has one sign for a left-handed photon and an opposite sign for a right-handed photon.

The above correlation is, however, also T-odd. In order not to violate time-reversal in the excited kaon decay, the decay amplitude must involve nontrivial final state interactions. Usually this poses the difficulty of introducing an unknown final state phase. In order to have a measurement which can be cleanly interpreted in terms of the photon helicity, this phase difference must be calculable. This is the case in $K_{\rm res} \to K^*\pi \to K\pi\pi$, where two isospin-related $K^*(892)$ resonance amplitudes interfere. Parametrizing resonance amplitudes in terms of Breit-Wigner forms, known to be a very good approximation for the narrow K^*, yields a calculable strong phase. In this respect, this method is similar to measuring the τ neutrino helicity in $\tau \to a_1\nu_\tau$, where the corresponding phase-difference is calculable in terms of the two interfering $a_1 \to \rho\pi$ amplitudes [12, 13].

Considering cascade decays of $\bar B(b\bar q)$ $(q = u, d)$, $\bar B \to \bar K_{\rm res}\gamma \to \bar K\pi\pi\gamma$, we denote weak $\bar B \to \bar K_{\rm res}\gamma$ amplitudes involving left and right-handed photons by c_L and c_R, and corresponding strong $\bar K_{\rm res}$ decay amplitudes by $\mathcal{M}_L$ and $\mathcal{M}_R$, respectively. Amplitudes involving left and right-handed photons do not interfere since *in principle* the photon polarization is measurable. Therefore,

$$|A(\bar B \to \bar K_{\rm res}\gamma,\ \bar K_{\rm res} \to \bar K\pi\pi)|^2 = |c_L|^2|\mathcal{M}_L|^2 + |c_R|^2|\mathcal{M}_R|^2 \ . \qquad (2)$$

In the SM the photon in $\bar B$ decays is dominantly left-handed, $|c_R|^2 \ll |c_L|^2$. The corresponding B decay amplitudes obey a reversed hierarchy implying a right-handed photon. We denote the photon polarization by λ_γ,

$$\lambda_\gamma \equiv \frac{|c_R|^2 - |c_L|^2}{|c_R|^2 + |c_L|^2} \ , \qquad (3)$$

such that in the SM $\lambda_\gamma \approx 1$ holds for radiative B decays, while $\lambda_\gamma \approx -1$ applies to $\bar B$ decays.

The weak amplitudes $c_{R,L}$ are given by $c_{R,L} = g_+^{K_{\rm res}}(0)C_{7R,L}$, where $g_+^{K_{\rm res}}(0)$ are hadronic form factors at $q^2 = 0$, which have already been computed using several models [14]. (For most part, we will not rely on these calculations). $C_{7R,L}$ are Wilson coefficients appearing in the effective weak radiative Hamiltonian

$$\mathcal{H}_{\rm rad} = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\left(C_{7R}\mathcal{O}_{7R} + C_{7L}\mathcal{O}_{7L}\right) \ , \qquad \mathcal{O}_{7L,R} = \frac{e^2}{16\pi^2}m_b\bar s\sigma_{\mu\nu}\frac{1 \pm \gamma_5}{2}bF^{\mu\nu} \ . \ (4)$$

Since the form factors $g_+^{K_{\rm res}}$ are common to c_L and c_R, a measurement of the ratio c_R/c_L can be translated into information about the underlying new physics entering the Wilson coefficients.

We now describe details of the method based on the decays $B \to K_1\gamma$, beginning with formalism and ending with an estimate demonstrating the high sensitivity of the measurement to the photon polarization. We also discuss an alternative scheme, based on $K\pi\pi$ decays of K_2^*.

The decay processes $K_1(1400) \to K\pi\pi$ are dominated by $K^*(892)\pi$, for which the decay branching ratio is $94 \pm 6\%$ [15]. A smaller branching ratio into ρK, $3 \pm 3\%$ [15], will be neglected at this point. We will study the modes

$$K_1^+ \to \left\{ \begin{array}{c} K^{*+}\pi^0 \\ K^{*0}\pi^+ \end{array} \right\} \to K^0\pi^+\pi^0 \ , \qquad K_1^0 \to \left\{ \begin{array}{c} K^{*+}\pi^- \\ K^{*0}\pi^0 \end{array} \right\} \to K^+\pi^-\pi^0 \ . \qquad (5)$$

The process $K_1(1400) \to K^*(892)\pi$ occurs both in S and D waves. The measured D/S ratio of rates is 0.04 ± 0.01 [15, 16]. At this point we will neglect the D-wave amplitude. The S-wave amplitudes for (5), describing a K_1 of momentum p and polarization ε^μ, decaying into two pions $\pi^\pm$, π^0 of momenta p_1, p_2, and a final K meson of momentum p_3, are given by

$$\mathcal{M} = C_1\,\varepsilon^\mu\,J_\mu\ , \qquad J_\mu = p_{1\mu}\left[\left(1 - \frac{m_K^2 - m_\pi^2}{m_{K^*}^2}\right) B_{K^*}(s_{23}) - 2B_{K^*}(s_{13})\right] - (p_1 \leftrightarrow p_2)\ , \tag{6}$$

where $B_{K^*}(s)$ is a Breit-Wigner form,

$$B_{K^*}(s) = \left(s - m_{K^*}^2 - im_{K^*}\Gamma_{K^*}\right)^{-1}\ , \qquad s_{ij} = (p_i + p_j)^2\ . \tag{7}$$

Isospin symmetry implies that the two K^* contributions are antisymmetric under the exchange of the two pion momenta. The constant C_1 describes the product of strong couplings of K^* and K_1 resonances.

Let us express the amplitudes $\mathcal{M}_{L,R}$ in the rest frame of the K_1. The polarization vectors corresponding to right and left-handed K_1 of helicity ±1, $\varepsilon^\mu_{\pm1}$, are defined in this frame by $\varepsilon^0_\pm = 0$, and $\vec{\varepsilon}_{\pm1} = \mp\frac{1}{\sqrt{2}}(\hat{e}_x \pm i\hat{e}_y)$. The two unit vectors $\hat{e}_x$ and $\hat{e}_y$ are perpendicular to $\hat{e}_z = -\hat{p}_\gamma$, which points along a direction opposite to the photon (or B) momentum. Thus

$$\mathcal{M}_{R,L} \propto \vec{\varepsilon}_{\pm1} \cdot \vec{J}\ . \tag{8}$$

Denoting by θ the angle between the normal to the decay plane, $\hat{n} \equiv (\vec{p}_1 \times \vec{p}_2)/|(\vec{p}_1 \times \vec{p}_2)|$, and the direction opposite to the photon, $\cos\theta = \hat{n}\cdot\hat{e}_z$, one finds

$$\mathcal{M}_{R,L} \propto \frac{1}{\sqrt{2}}\left(\mp J_x - i\cos\theta\, J_{y'}\right)\ , \tag{9}$$

where x, y' and $\hat{n}$ form a set of orthogonal axes. (We choose these axes such that the plane perpendicular to the photon direction and the decay plane intersect on the x axis.)

Squaring the amplitudes and integrating over a common rotation angle ϕ of $\vec{p}_1$ and $\vec{p}_2$ in the decay plane, one obtains

$$\frac{1}{2\pi}\int_0^{2\pi} d\phi |\mathcal{M}_{R,L}|^2 \propto |\vec{J}|^2(1 + \cos^2\theta) \pm 2\mathrm{Im}\left(\hat{n}\cdot(\vec{J}\times\vec{J}^*)\right)\cos\theta\ . \tag{10}$$

Using Eqs. (2) and (3), one obtains the $B \to (K\pi\pi)_{K_1}\gamma$ decay distribution

$$\frac{d\Gamma}{ds_{13}ds_{23}d\cos\theta} \propto |\vec{J}|^2(1 + \cos^2\theta) + \lambda_\gamma 2\mathrm{Im}\left(\hat{n}\cdot(\vec{J}\times\vec{J}^*)\right)\cos\theta \tag{11}$$

The asymmetry between decay distributions corresponding to right and left-handed photons, from which the photon polarization can be determined, is contained in the second term in Eq. (11). It describes an up-down asymmetry of the photon momentum with respect to the K_1 decay plane. In order to measure λ_γ one would fit the B

and $\bar{B}$ decay distributions to (11), which has a well-defined dependence on θ and on the energy variables s_{13}, s_{23} occurring in the Breit-Wigner forms (Eqs. (7) and (6)).

While the highest sensitivity to the photon polarization will eventually be achieved by fitting data to the above energy and angular dependence, it is also useful to consider integrated observables. When averaging over some variables care must be taken in order not to wash out the dependence on the photon polarization. Note that the angular variable $\cos\theta$ changes sign under the exchange of s_{13} and s_{23}, corresponding to interchanging the two pion momenta. We thus define a new angle $\tilde{\theta}$ which is independent of s_{13} and s_{23}, $\cos\theta \equiv \mathrm{sgn}(s_{13}-s_{23})\cos\tilde{\theta}$. An equivalent definition of $\tilde{\theta}$ is the angle between $-\vec{p}_\gamma$ and the normal to the decay plane defined by $\vec{p}_{\rm slow}\times\vec{p}_{\rm fast}$, where $\vec{p}_{\rm slow}$ and $\vec{p}_{\rm fast}$ are the momenta of the slower and faster pions in the K_2^* rest frame.

In order to obtain a conservative estimate for the sensitivity of the decay distribution to the photon polarization, let us examine a specific observable which is proportional to λ_γ, namely the integrated up-down asymmetry,

$$\mathcal{A}_r = \frac{\int_0^{\pi/2} d\Gamma - \int_{\pi/2}^{\pi} d\Gamma}{\int_0^{\pi} d\Gamma} . \tag{12}$$

Here integration is performed over specified ranges of $\tilde{\theta}$, and over any region r in the Dalitz plot. Using Eq. (11) we find

$$\mathcal{A}_r = -N\frac{\langle |\vec{p}_1\times\vec{p}_2| \mathrm{Im}\,(B_{K^*}(s_{13})B^*_{K^*}(s_{23}))\ \mathrm{sgn}(s_{13}-s_{23})\rangle_r}{\langle |\vec{J}\,|^2\rangle_r}\,\lambda_\gamma\ , \tag{13}$$

where

$$N \equiv \frac{3}{2}\left[4-\left(1-\frac{m_K^2-m_\pi^2}{m_{K^*}^2}\right)^2\right] = 5.24 \tag{14}$$

Note that while the variable in the denominator of (13) is positive, the one in the numerator, containing a factor $(s_{23}-s_{13})\mathrm{sgn}(s_{13}-s_{23})$, is negative.

Integrating the numerator and denominator in (13) over the entire Dalitz plot, one obtains

$$\mathcal{A} = 0.25\lambda_\gamma\ . \tag{15}$$

In the SM this asymmetry is positive for B decays, where $\lambda_\gamma \approx +1$, and negative for $\bar{B}$ decays, where $\lambda_\gamma \approx -1$. Namely, *in B^- and $\bar{B}^0$ decays, the photon prefers to move in the hemisphere of $\vec{p}_{\rm slow}\times\vec{p}_{\rm fast}$, while in B^+ and B^0 decays it prefers to move in the opposite direction.* We also calculate an asymmetry $\mathcal{A}_s$, integrated over a square region (s), defined by $0.71\mathrm{GeV}^2 \le s_{13}, s_{23} \le 0.89\ \mathrm{GeV}^2$, where the two K^* bands of widths $2\Gamma_{K^*}$ overlap. We expect this asymmetry to be larger than the total up-down asymmetry $\mathcal{A}$, and we find $\mathcal{A}_s = 0.32\lambda_\gamma$. The region (s) contains 23% of all events. The about four times larger number of events in the entire Dalitz plot clearly overcomes the slight loss in sensitivity. For a three standard deviation measurement of a total up-down asymmetry, $\mathcal{A} \simeq 0.25\ (-0.25)$, expected in the SM for B^+ (B^-) and B^0 $(\bar{B}^0)$ decays, one needs to observe a total of about 150 charged and neutral

B and $\bar{B}$ decays to $(K\pi\pi)_{K_1}\gamma$. Since a detailed fit of data to the distribution (11) is more sensitive to λ_γ than the integrated asymmetry, such a fit may require fewer events.

In order to estimate the number of $B\bar{B}$ pairs needed for this measurement, we will assume that the branching ratio of $B \to K_1(1400)\gamma$ is 0.7×10^{-5}, as calculated in some models [14]. We use $\mathcal{B}(K_1(1400) \to K^*\pi) = 0.94$ [15], and note that 4/9 of all $K^*\pi$ events in K_1^+ and K_1^0 decays occur in the two channels specified in Eq. (5). Including a factor 1/3 for observing a K_S (from K^0) through its $\pi^+\pi^-$ decay, we estimate a branching ratio of $\mathcal{B} = 0.7 \times 10^{-5} \times (4/9)0.94 \simeq 0.3 \times 10^{-5}$ into $(K^+\pi^-\pi^0)_{K_1(1400)}$ and $\mathcal{B} \simeq 0.1 \times 10^{-5}$ into $(K_S\pi^+\pi^0)_{K_1(1400)}$. Ignoring experimental efficiencies and background, 150 $(K\pi\pi)_{K_1}\gamma$ events can be obtained from a total of 4×10^7 $B\bar{B}$ pairs, including charged and neutrals. This number of B mesons has already been produced at e^+e^- colliders [17, 18, 19].

The calculated asymmetry (15) involves theoretical uncertainties from two sources. We neglected small contributions from a ρK amplitude and a small D-wave amplitude in $K_1 \to K^*\pi$. Each of these amplitudes could be at most about 20% of the dominant S-wave $K^*\pi$ amplitude. These amplitudes interfere with the dominant one, thus contributing to the numerator of the up-down asymmetry terms which depend on the two corresponding final state phase differences. (The denominator of the asymmetry obtains terms which are quadratic in these small amplitudes). One phase, $\delta(\rho/K^*)$, is the relative intrinsic phase between the ρK and $K^*\pi$ amplitudes, while the other, $\delta(D/S)$, is the relative phase between D and S-wave amplitudes. K_1 production experiments [16] measure a negligible value for $\delta(D/S)$ and find $\delta(\rho/K^*) \simeq 30^\circ$ for the other phase. We calculate the effect of a possible nonzero ρK contribution (for which there only exists an upper limit), and find that it would tend to decrease the asymmetry by about 10% with an uncertainty of 10%. (See our discussion below of the corresponding effect in K_2^*.) Interference of two $K^*\pi$ amplitudes, in S and D-waves respectively, leads to an uncertainty of only about 4%. (The interference does not involve the enhancement factor N occurring in the pure S-wave asymmetry.) Details of these calculations will be reported elsewhere [20].

Studies similar to the above can be carried out for other kaon resonance states in radiative B decays. K_J^* states with parity $P = (-1)^J$ decay to $K^*\pi$ and $K\rho$ in a single partial wave, while K_J states with parity $P = (-1)^{J+1}$ decay to these modes in two partial waves. The analyses based on other resonance states are similar to the above, however θ-dependence and the form of the vector $\vec{J}$ occurring in the corresponding decay distributions depend on the spin and parity of the resonance. We give two examples, for 2^+ and 1^- K^* resonances.

In the case of $K_2^*(1430)$ one finds, when both $K^*\pi$ and ρK contributions are included,

$$
\begin{aligned}
\frac{d\Gamma}{ds_{13}ds_{23}d\cos\theta} \quad \propto \quad & |\vec{p}_1 \times \vec{p}_2|^2 \left[|\vec{J}|^2(\cos^2\theta + \cos^2 2\theta) \right. \\
+ \quad & \left. \lambda_\gamma 2\mathrm{Im}\left(\hat{n}\cdot(\vec{J}\times\vec{J}^*)\right)\cos\theta\cos 2\theta \right] , \qquad (16)
\end{aligned}
$$

where

$$\vec{J} = \vec{p}_1[B_{K^*}(s_{23}) + \kappa_\rho B_\rho(s_{12})] + \vec{p}_2[B_{K^*}(s_{13}) + \kappa_\rho B_\rho(s_{12})] \ , \tag{17}$$

$$B_\rho(s) = \left(s - m_\rho^2 - i m_\rho \Gamma_\rho\right)^{-1} \ . \tag{18}$$

The complex parameter κ_ρ, parametrizing the relative strength and final state phase difference of the $K^*\pi$ and ρK contributions, is given by

$$\kappa_\rho = |\kappa_\rho| e^{i\delta} = \sqrt{\frac{3}{2}} \frac{g_{K_2^*\rho K}}{g_{K_2^* K^*\pi}} \cdot \frac{g_{\rho\pi\pi}}{g_{K^*K\pi}} \ . \tag{19}$$

The two ratios of couplings are obtained from the corresponding measured partial widths [15]

$$\frac{|g_{K_2^*\rho K}|^2}{|g_{K_2^* K^*\pi}|^2} = \frac{\mathcal{B}(K_2^* \to \rho K)}{\mathcal{B}(K_2^* \to K^*\pi)} \cdot \frac{|\vec{p}_{K^*\pi}|^5}{|\vec{p}_{\rho K}|^5} = 1.20 \ , \tag{20}$$

$$\frac{|g_{\rho\pi\pi}|^2}{|g_{K^*K\pi}|^2} = \frac{2\Gamma_\rho}{\Gamma_{K^*}} \cdot \frac{|\vec{p}_{K\pi}|^3}{|\vec{p}_{\pi\pi}|^3} = 3.16 \, , \tag{21}$$

where central values are used, and p^5 and p^3 are phase space factors occurring in D and P-waves, respectively. This gives $|\kappa_\rho| = 2.38$. The phase δ, which vanishes in the SU(3) limit can be argued to be small. The phase of $g_{\rho\pi\pi}/g_{K^*K\pi}$ is likely to be small, since the magnitude of this ratio is only 8% away from its SU(3) value of $\sqrt{8/3}$. The other phase, of $g_{K_2^*\rho K}/g_{K_2^*K^*\pi}$, was obtained in a K_2^* resonance production experiment [16], where a value smaller than 30° was measured.

The integrated up-down asymmetry in Eq. (16) vanishes. A useful observable which is proportional to λ_γ is the expectation value $\langle \cos\tilde{\theta}\rangle$. One finds for the above defined overlap region (s) of the two K^* bands

$$\langle \cos\tilde{\theta}\rangle_s = R_s \lambda_\gamma \ , \tag{22}$$

where, neglecting the ρ contribution,

$$R_s \approx -\frac{1}{3} \frac{\langle |\vec{p}_1 \times \vec{p}_2|^3 \mathrm{Im}\ (B_{K^*}(s_{13}) B^*_{K^*}(s_{23}))\,\mathrm{sgn}(s_{13} - s_{23})\rangle_A}{\langle |\vec{p}_1 \times \vec{p}_2|^2 |\vec{p}_1 B_{K^*}(s_{23}) + \vec{p}_2 B_{K^*}(s_{13})|^2 \rangle_A} = 0.091 \ . \tag{23}$$

Including the ρ contribution modifies the above value to become $R_s = 0.071 \pm 0.002$, where we use $|\kappa_\rho| = 2.38$ and we let the strong phase δ vary in the range $(0° \pm 30°)$.

The value of R obtained when integrating over the entire Dalitz plot is considerably smaller. For the above value of κ_ρ, we find a variation between a very low value $R = 0.01$ and a somewhat larger value $R = 0.05$. The uncertainty is due to the strong phase of κ_ρ. The increase in statistics relative to region (s) is not sufficient to overcome the loss of sensitivity.

We see clearly that a photon polarization measurement through K_2^* is much less efficient than through K_1. For comparison with the $K_1(1400)$ case, a three standard deviation measurement of the up-down asymmetry at the K_2^*, $\langle\cos\tilde{\theta}\rangle_s \sim 0.07$, requires about two thousand events. We calculate that only 9% of all $K^0\pi^0\pi^+$ or $K^+\pi^-\pi^0$ events are contained in region (s). Using (1) we compute for this region a radiative branching ratio [15] $\mathcal{B} = 1.5 \times 10^{-5} \times 0.17 \times 0.09 \simeq 2.3 \times 10^{-7}$ into $K^+\pi^-\pi^0$ and $\mathcal{B} \simeq 0.8 \times 10^{-7}$ into $K_S\pi^0\pi^+$. Namely, about 6×10^9 $B\bar{B}$ pairs, including both charged and neutral B's, would be required to observe an asymmetry if the asymmetry came only from K_2^*.

For an excited K_1^*, one finds a decay distribution

$$\frac{d\Gamma}{ds_{13}ds_{23}d\cos\theta} \propto |\vec{p}_1 \times \vec{p}_2|^2 \sin^2\theta |B_{K^*}(s_{13}) + B_{K^*}(s_{23}) + \kappa_\rho B_\rho(s_{12})|^2 \,, \qquad (24)$$

which is insensitive to the photon polarization. This can be simply understood by noting that the only parity invariant decay amplitude, which can be constructed from the K_1^* polarization vector $\vec{\varepsilon}$ and the final mesons momenta, is proportional to $\vec{\varepsilon} \cdot (\vec{p}_1 \times \vec{p}_2)$. Its square is invariant under $\vec{\varepsilon}_{+1} \leftrightarrow \vec{\varepsilon}_{-1}$ and therefore cannot be used to measure the photon polarization.

We conclude with a few practical comments. Since charged and neutral $B \to (K\pi\pi)_{K_1(1400)}\gamma$ decay distributions provide a sensitive probe for the photon polarization, experiments measuring radiative B decays should look at $K\pi\pi$ invariant mass around 1400 MeV. This region includes contributions from other resonances, $K_1^*(1410)$ which leads to no asymmetry, and $K_2^*(1430)$ which adds a relatively small asymmetry. The two asymmetries from K_1 and K_2^* have equal signs. The overall up-down asymmetry is diluted relative to the asymmetry from K_1 alone. Using the different energy and angular dependence of the three resonances, Eqs. (11)(16) and (24), should help isolate the K_1 contribution which has the largest asymmetry. This contribution is distinguished by its second term which is up-down antisymmetric, while the distributions for K_1^* and K_2^* are symmetric under $\theta \to \pi - \theta$. Isolating of the K_1 contribution reduces the theoretical uncertainty in interpreting the measured asymmetry in terms of the photon polarization. Some uncertainty may occur when integrating over a $K\pi\pi$ invariant mass range including the two overlapping K_1 resonances at 1400 and 1270 MeV [15], since the two resonances decay with different branching ratios and corresponding different strong phases into $K^*\pi$ and $K\rho$. In order to minimize this uncertainty, one may carry out this measurement in the high invariant mass range, e.g. above $m(K\pi\pi) = 1400$ MeV, thereby suppressing the $K_1(1270)$ contribution. As we have shown when investigating the case of $K_1(1400)$, this study is feasible at currently operating B-factories.

Finally, our study focused on decay modes of higher K resonances which involve one neutral pion. This was necessary in order to have two interfering $K^*\pi$ amplitudes, in which case the up-down asymmetry was calculated assuming only isospin symmetry. An asymmetry is also expected in $K^\pm\pi^\mp\pi^\pm$ channels, involving only charged particles, which were measured very recently by the Belle collaboration [11]. In this case the asymmetry originates in the interference between $K^*\pi$ and ρK (or f_0K) amplitudes. The latter amplitude is significant in $K_1(1270)$ and $K_2^*(1430)$ decays. (In $K_1(1270) \to K^*\pi$ one must also consider the effect of a possibly significant D-wave amplitude, for which the upper limit is rather loose [15].) Here the asymmetry is calculable in the SU(3) limit. SU(3) breaking can be taken from resonance production experiments. These asymmetries, and their implication in determining the photon polarization in radiative B decays, will be studied elsewhere [20].

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References

[1] For a recent review, see T. Hurth, hep-ph/0106050.

[2] B. Grinstein and D. Pirjol, Phys. Rev. D **62**, 093002 (2000).

[3] K. Fujikawa and A. Yamada, Phys. Rev. D **49**, 5890 (1994); K. S. Babu, K. Fujikawa and A. Yamada, Phys. Lett. B **333**, 196 (1994); P. Cho and M. Misiak, Phys. Rev. D **49**, 5894 (1994).

[4] D. Atwood, M. Gronau and Soni, Phys. Rev. Lett. **79**, 185 (1997).

[5] T. Mannel and S. Recksiegel, Acta Phys. Polonica **B28**, 2489 (1997).

[6] G. Hiller and A. Kagan, SLAC-PUB-8752 (in preparation). We thank Gudrun Hiller for bringing this work to our attention.

[7] C. S. Kim, Y. G. Kim, C. D. Lü and T. Morozumi, Phys. Rev. D **62**, 034013 (2000).

[8] Y. Grossman and D. Pirjol, JHEP **0006**, 029 (2000).

[9] T. Islam and A. Weinstein, CLEO Internal Note, July 1999 (unpublished).

[10] CLEO Collaboration, T. E. Coan *et al.*, Phys. Rev. Lett. **84**, 5283 (2000).

[11] Belle Collaboration, K. Abe *et al.*, paper submitted to LP01, XX International Symposium on Lepton and Photon Interactions at High Energies, Rome, Italy, July 23-28, 2001, BELLE-CONF-0109, hep-ex/0107065.

[12] J. H. Kühn and F. Wagner, Nucl. Phys. B **236**, 16 (1984); J. H. Kühn and A. Santamaria, Zeit. Phys. C **48**, 445 (1990).

[13] ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. B **250**, 164 (1990); OPAL Collaboration, K. Ackerstaff *et al.*, Zeit. Phys. C **75**, 593 (1997); DELPHI Collaboration, P. Abreau *et al.*, Phys. Lett. B **426**, 411 (1998); CLEO Collaboration, D. M. Asner *et al.*, Phys. Rev. D **61**, 012002 (2000).

[14] T. Altomari, Phys. Rev. D **37**, 677 (1988); A. Ali, T. Ohl and T. Mannel, Phys. Lett. B **298**, 195 (1993); D. Atwood and A. Soni, Zeit. Phys. C **64**, 241 (1994); S. Veseli and M. G. Olsson, Phys. Lett. B **367**, 309 (1996); D. Ebert, R. N. Faustov, V. O. Galkin and H. Toki, Phys. Lett. B **495**, 309 (2000); Phys. Rev. D **64**, 054001 (2001).

[15] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[16] C. Daum *et al.*, Nucl. Phys. B **187**, 1 (1981).

[17] J. Dorfan, Rapporteur talk at LP01, Ref. [11].

[18] S. Olsen, Rapporteur talk at LP01, Ref. [11].

[19] D. Cassel, Rapporteur talk at LP01, Ref. [11].

[20] M. Gronau, Y. Grossman and D. Pirjol, work in progress.