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Isospin Symmetry Breaking within the HLS Model: A Full (ρ, ω, ϕ) Mixing Scheme

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Abstract

We study the way isospin symmetry violation can be generated within the Hidden Local Symmetry (HLS) Model. We show that isospin symmetry breaking effects on pseudoscalar mesons naturally induces correspondingly effects within the physics of vector mesons, through kaon loops. In this way, one recovers all features traditionally expected from $\rho-\omega$ mixing and one finds support for the Orsay phase modelling of the $e^+e^- \to \pi^+\pi^-$ amplitude. We then examine an effective procedure which generates mixing in the whole ρ, ω, ϕ sector of the HLS Model. The corresponding model allows us to account for all two body decays of light mesons accessible to the HLS model in modulus and phase, leaving aside the $\rho \to \pi\pi$ and $K^* \to K\pi$ modes only, which raise a specific problem. Comparison with experimental data is performed and covers modulus and phase information ; this represents 26 physics quantities successfully described with very good fit quality within a constrained model which accounts for SU(3) breaking, nonet symmetry breaking in the pseudoscalar sector and, now, isospin symmetry breaking.

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I. INTRODUCTION

Despite the large difference in the u and d current quark masses, isospin violation in the strong interaction is typically at the order of a few percent, such as the $\pi^\pm - \pi^0$ mass difference. This is because the scale is set not by $(m_u - m_d)/(m_u + m_d)$ but $(m_u - m_d)/m_s$ [1]. Interest in the contribution of isospin violation is therefore usually confined to systems where both theoretical (or at least phenomenological) and experimental precision are high; for example a_μ [2], CP violation in $B \to 2P$ (where $P \equiv$ pseudoscalars) [3–5] and various aspects of charge symmetry violation in the NN system [6].

However, in $e^+e^- \to \pi^+\pi^-$ the isospin violating process of ρ–ω mixing produces a large effect on the interaction. This is due to both the isospin independence of the initial vertex (the coupling of the ω to the photon is only a third of the coupling of the ρ^0 to the photon) and narrow width of the ω (in the region of the ω resonance the cross-section is approximately 40% larger than it would be without $\rho - \omega$ mixing). Therefore any strongly interacting system where the ρ^0 and ω have significant (if not necessarily large) production amplitudes can expect a similar enchancement in $\pi^+\pi^-$ pair production in the ρ–ω interference region. Lipkin realised this would apply to various decays in the B system [7]. Building on this, Enomoto and Tanabashi discovered a decay channel that would show a sizeable direct CP asymmetry, $B^- \to \rho^-(\rho^0/\omega) \to \rho^-\pi^+\pi^-$. Here the penguin term exists only for $B^- \to \rho^-\omega$ not the $B^- \to \rho^-\rho^0$ and the necessary penguin/tree interference arises through ρ–ω mixing with the strong phase courtesy of the ω propagator [8] (for further details see Ref. [9]). This gives a renewed interest to the description of isospin symmetry breaking.

Having just said that $\rho - \omega$ mixing can lead to large effects, it is important to explain the quoted figure ($\sim 2\%$) for the $\omega \to 2\pi$ branching fraction. The pion form factor can be defined (and a definition is a useful thing) through [10]

$$F_\pi(s) = F_\rho(s)\left(1 + \frac{f_{\omega\gamma}}{f_{\rho\gamma}} \frac{\tilde{\Pi}_{\rho\omega}}{s - m_\omega^2 + i m_\omega \Gamma_\omega}\right). \tag{1}$$

Though the mixing amplitude $\tilde{\Pi}_{\rho\omega} \simeq -4300$ MeV^2 is small compared with the scale of m_ω^2 the extremely narrow $\Gamma_\omega = 8.4$ MeV allows the isospin violating contribution to be sizeable. Correspondingly the $\omega \to 2\pi$ decay must pass through the ρ^0 and thus the attenuation factor is $\tilde{\Pi}_{\rho\omega}/m_\rho\Gamma_\rho$ and so is down by an order $\Gamma_\omega/\Gamma_\rho \simeq 0.05$.

This question of scales and the meaning of the vector meson resonance states themselves must be firmly kept in mind when considering the effects of $\rho - \omega$ mixing (or indeed any isospin violation). We shall see that this has a recent application.

With this respect, it is useful to introduce isospin symmetry breaking within a context especially designed in order to account for physics of light vector and pseudoscalar mesons simultaneously and fully. The framework of vector meson dominance (VMD) models is certainly the most appropriate and, among them, the Hidden Local Symmetry (HLS) Lagrangian, with its non–anomalous [11] and anomalous sectors [12] is a good candidate, taking into account its phenomenological success.

In this paper we extend our previous work on symmetry breaking within this context by readdressing first the $\rho - \omega$ mixing [13,14] and then the full $\rho - \omega - \phi$ mixing. From our previous studies, we already know how SU(3) symmetry breaking has to be introduced

[15] close to lines first proposed in Ref. [16] ; in order to yield an appropriate description of physics information for decay processes involving η and η' mesons, it has been shown that nonet symmetry in the pseudoscalar (PS) sector should also be broken ; a way was proposed in [17] which provides a good understanding of all radiative decays of light mesons. Slightly later [18], we showed that this way of breaking nonet symmetry in the PS sector can be derived from Chiral Perturbation Theory.

However, tree level amplitudes are not sufficient in order to account for the physics of vector mesons. As clear from the observed shape of the pion form factor [19,20], pion loop effects can hardly be neglected when describing the ρ meson [21–25]. Without a $\omega - \phi$ mixing mechanism, any description of their decay modes becomes definitely poor. This is traditionally introduced by means of a mixing angle. It has been shown that kaon loop effects are the simplest mechanism within HLS for generating $\omega - \phi$ mixing [26]. These loop effects can be accomodated within the HLS model in an effective way, by introducing vector meson self–masses and (loop) transition amplitudes like $\omega \leftrightarrow \phi$ within the Lagrangian. In this way one generates the appropriate pion form factor shape and all corrections to ω/ϕ decays [26].

However, effects of isospin symmetry breaking (like $\omega/\phi \rightarrow \pi^+\pi^-$, for instance) still remain outside the HLS framework. We shall show that the loop mechanism which generates the $\omega - \phi$ mixing gives a handle to introduce isospin symmetry breaking also by considering loop effects, mainly kaon loops. This mechanism, together with the U(3)/SU(3) symmetry breaking procedure recalled above, will be shown to provide a clear understanding of (almost) all decay modes accessible to a VMD approach.

The outline of the paper is as follows. In Sections II, III and IV we define a mechanism for isospin symmetry breaking and examine its consequences on the $\rho - \omega$ sector in isolation. In Section V, we illustrate with the pion form factor the consistency of this approach ; more precisely, we show that the HLS model, broken as we propose, leads to an expression of the pion form factor which is nothing but the usual formulation of this in terms of the so–called Orsay phase. Besides, it shows that the $\rho - \omega$ mixing is the mechanism which figures out the origin of the decay process $\omega \rightarrow \pi\pi$ within the HLS model at one–loop order.

Having shown that this approach is consistent, we extend it in Section VI to a scheme involving the full $\rho - \omega - \phi$ sector of the HLS Model. We describe in this Section how the coupling constants for all two body decays (VPP, $VP\gamma$, VVP, $P\gamma\gamma$, Ve^+e^-) can be derived from elementary information provided. In this approach, the $\rho - \omega - \phi$ mixing appears to be the at the origin of the $\omega/\phi \rightarrow \pi\pi$ decay processes, which are described both in modulus and phase.

In Section VII we apply this model to fit all data related to VMD, except for $\rho \rightarrow \pi\pi$ and $K^* \rightarrow K\pi$ which settle a specific problem, not examined here. The picture obtained is impressively successful. Finally, we conclude in Section VIII. We give in the Appendix most formulae and in three Tables most of our results which cover 26 physics quantities simultaneously fitted within a unified framework.

II. PHYSICAL FIELDS OR IDEAL FIELDS

In this Section and in the following one, we concentrate on the $\rho - \omega$ mixing in isolation. This allows to outline the method we use in order to construct a full mixing scheme for vector mesons.

Within a context of an effective field model where we definitely stand, one can assume without any loss of generality that the $\rho - \omega$ mixing is produced by an effective Lagrangian term of the form

$$\mathcal{L}_{mixing} = \Pi_{\omega\rho}(s)\rho_I\omega_I \tag{2}$$

The possible origin of such a term within the HLS model will be discussed later on. Its relevance as origin for the $\rho-\omega$ mixing is a usual assumption (for a review see [21,22], where a throughout discussion of the origin, properties and values of $\Pi_{\omega\rho}(s)$ can also be found).

We have denoted above ρ_I and ω_I the ideal field combinations (*i.e.*, non–strange pure isospin states) ; the corresponding physical fields will be denoted ρ and ω. For the present purpose, and following general ideas [21,22], we only need to assume that $\Pi_{\omega\rho}(s)$ is a real analytic function of s (*i.e.* $\Pi^*_{\omega\rho}(s) = \Pi_{\omega\rho}(s^*)$ where the symbol * denotes the usual complex conjugation).

Whatever its origin, the term in Eq. (2) plays by modifying the $\rho - \omega$ mass term by adding a non–diagonal piece, as for the kaon loop effects responsible for the $\omega-\phi$ mixing [26]. Among the possible origins of the term in Eq. (2), pion loop effects have been considered. Although this specific contribution is ruled out within the HLS model [27] – which rather previleges kaon loop effects – it has been studied in bi-local field models [28,29]. Following the Renard argument [13], the expected sizeable contribution to the imaginary part of $\Pi_{\rho\omega}$ from the pion loop is cancelled by a direct $\omega \to 2\pi$ term.

A. Loop Effects and Mixing

There is a consistent way to account for leading loop effects within the HLS context ; this turns out to modify the mass term in the Lagrangian by including all vector meson self–energies and transition amplitudes like $\phi_I \to \omega_I$, as discussed in [26], but also[1] $\rho_I \to \omega_I$.

Limiting oneself to the $\rho - \omega$ issue, one has simply to perform a transform from ideal to new (canonical) fields, in order to remove the $\omega\rho$ term in the new field basis. In the approach of Ref. [26], loop effects are considered by their effects at tree level only through modified vector meson masses.

In this case, the relevant piece of the effective Lagrangian, quadratic in the fields, is given by :

$$\mathcal{L} = \frac{1}{2}\{ \ [m^2 + \Pi_\rho(s)] \ \rho_I^2 + [m^2 + \Pi_\omega(s)] \ \omega_I^2 + 2\Pi_{\omega\rho}(s) \ \rho_I\omega_I \ \} \tag{3}$$

Following the approach developed for the $\omega - \phi$ mixing [26], we have introduced the ρ and ω self–energies as given in Ref. [26] ; these are real analytic function of s. We have also assumed the ρ_I and ω_I (Higgs–Kibble, HK) masses occuring in the Lagrangian to be equal to the same m (we neglect possible isospin breaking effects on HK masses, as this is not an

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[1] At the same order, one might have to include also the parent $\rho_I \to \phi_I$ transition amplitude, as will be seen in Section IV.

essential feature for the problem under study). Within the HLS model, this common mass is [11] $m^2 = a f_\pi^2 g^2$ in terms of the HLS parameter a, of the universal vector coupling g and the pion decay constant f_π.

The diagonalization procedure of the HLS Lagrangian with one loop corrections is presented in detail in Ref. [26]. We simply recall that the desired diagonalization is obtained by performing the following tranformation :

$$\begin{pmatrix} \rho \\ \\ \omega \end{pmatrix} = \begin{pmatrix} \cos\delta(s) & \sin\delta(s) \\ \\ -\sin\delta(s) & \cos\delta(s) \end{pmatrix} \begin{pmatrix} \rho^I \\ \\ \omega^I \end{pmatrix} \tag{4}$$

which connects the physical fields to their ideal combinations. It leads to physical fields which behave like analytic functions of s ; this can be interpreted as a non–local effect which could be expected as we are not dealing with fundamental (quark and gluon) degrees of freedom. Additionally, it implies at tree level (in this approach) analytical shapes which fit well with physics observations ; for instance, the broad shape of the ρ meson propagator which shows up in the pion form factor is generated here mostly by the pion loop.

The angle $\delta(s)$ – possibly complex – should be chosen in such a way that the mixed $\rho\omega$ term, appearing still in Eq. (3) after the change of fields, identically vanishes. This provides[2] :

$$\tan 2\delta(s) = \frac{2\Pi_{\omega\rho}(s)}{\Pi_\rho(s) - \Pi_\omega(s)} \tag{5}$$

and does not depend on the difference of ρ_I and ω_I HK masses, as this vanishes identically in the approximation where we stand. Moreover, the s dependence of the $\rho - \omega$ mixing exhibited by Eq.(5) is a property already considered [21,22,32]. This s dependence should be expected, as the mixing function $\Pi_{\omega\rho}(s)$ should vanish at $s = 0$ [14] in any model where the vector mesons couple to conserved currents.

B. Analytic Properties of the Angle $\delta(s)$

As noted in [26] for the purpose of $\omega - \phi$ mixing in isolation, angles like $\delta(s)$ above are not real for any real s. In fact, as is clear from Eq. (5), $\sin\delta(s)$, $\cos\delta(s)$ are real analytic functions[3] of s in an analyticity domain with the same branch point singularities as the various self–energies or transition amplitudes ; additional algebraic branch points may occur at odd order zeros or poles of the expression in Eq. (5). For our purpose, one only needs to make weak assumptions which ensure that Eq. (4) can be inverted as an analytic

[2] It should be noted that the denominator in Eq. (5) is nothing but the difference of the ρ_I and ω_I *effective* running masses as they occur in the one–loop corrected mass term of the Lagrangian Eq. (3).

[3] Actually, they might only be meromorphic in the physical sheet, as $\Pi_\rho(s) - \Pi_\omega(s)$ might have zeros in the physical sheet, at $s = 0$ for instance.

matrix function : we assume that the analyticity domain contains the upper and lower lips of the physical region $\{s > 4m_\pi^2\}$ and that both lips can be connected with each other by a continuous path while staying inside this domain. This implies that a segment of $\{s < 4m_\pi^2\}$ should also belong to this analyticity domain.

The function $\delta(s)$, itself, can have logarithmic singularities ; however, it never appears as such in the expressions we have to handle.

C. Interaction Terms and Self-Energies

For completeness, and in order to fix notations for coupling constants, let us recall the relevant interaction piece of the HLS Lagrangian as given in [26] in terms of renormalized fields :

$$\begin{array}{ll} \mathcal{T}_1 = & -\dfrac{iag}{4} Z \, [\omega_I + \sqrt{2}\ell_V \phi_I] \, \left[K^+ \overleftrightarrow{\partial} K^- + K^0 \overleftrightarrow{\partial} \overline{K}^0 \right] \\ & -\dfrac{iag}{4} \rho_I \left[Z \left(K^+ \overleftrightarrow{\partial} K^- - K^0 \overleftrightarrow{\partial} \overline{K}^0 \right) + 2\pi^+ \overleftrightarrow{\partial} \pi^- \right] \end{array} \tag{6}$$

We do not introduce here loop corrections to vertices, as there is no compelling evidence in favor of observable effects of these with the present data accuracy, even for the $e^+e^- \to \pi^+\pi^-$ cross section [23] known nowadays with very good accuracy over a wide range of invariant mass [19,20].

This Lagrangian piece depends on two SU(3) breaking parameters, generated by the BKY breaking mechanism [16,15] $Z = [f_\pi/f_K]^2 \simeq 2/3$ and ℓ_V fit as $\simeq 1.4$ (see Refs. [17,26]).

Up to anomalous contributions we neglect (they were estimated negligible in [26]), the ρ_I, ω_I and ϕ_I self-energies are :

$$\begin{array}{l} \Pi_\rho(s) = 2g^2_{\rho_I K\overline{K}} \Pi(s) + g^2_{\rho_I \pi\pi} \Pi'(s) \\ \Pi_\omega(s) = 2g^2_{\omega_I K\overline{K}} \Pi(s) \\ \Pi_\phi(s) = 2g^2_{\phi_I K\overline{K}} \Pi(s) \end{array} \tag{7}$$

(see Eqs. (D1) and (D4) in [26]) in terms of $\Pi(s)$ and $\Pi'(s)$, respectively the generic kaon and pion loops [26], *i.e.* the loops amputated from their coupling constants to vector mesons. The coupling constants can be read off Eq. (6) and obviously fulfill $|g_{\rho_I K\overline{K}}| = |g_{\omega_I K\overline{K}}|$.

Let us also recall that the Lagrangian derived after the change of fields fulfills the condition of hermitian analyticity, as in the case of the $\omega - \phi$ mixing in isolation [26].

III. THE $\rho - \omega$ MIXING "ANGLE"

Using Eqs. (7), Eq. (5) becomes

$$\tan 2\delta(s) = \frac{2\Pi_{\omega\rho}(s)}{g^2_{\rho_I\pi\pi}\Pi'(s)} \tag{8}$$

It is quite an interesting feature of the HLS model that the difference between the ω_I and ρ_I self-energies is the pion loop to which only ρ_I couples.

In order to estimate this denominator in the $\rho - \omega$ peak mass region, one should also keep in mind that this is practically the difference of the (complex and s–dependent) ρ and ω square masses as they occur in the one–loop corrected Lagrangian. As the width of the ω meson is negligible compared to those of the ρ meson, this gives an input for the real part of the pion loop in Eq. (8) valid in the neighborhood of the $\rho - \omega$ peak. Indeed, in this region, the real part can be identified with the difference of the (Breit–Wigner) masses squared as given in the RPP [30]. Indeed, these are defined as the energy point where the real part of the corresponding propagators goes to zero, or equivalently where the phase goes through $\pi/2$. Therefore, writing :

$$g^2_{\rho_I\pi\pi}\Pi'(s) = R(s) - iI(s), \tag{9}$$

(see Eqs. (A8) in [26]), we have *locally*, using the RPP masses[4] :

$$R(s \simeq m^2_\rho) \simeq m^2_\rho - m^2_\omega \simeq -(1.1 \div 1.9)\ 10^{-2}\text{GeV}^2 \tag{10}$$

depending on the definition used for the *observed* ρ mass [30] ; this has to be compared with the imaginary part

$$I(s \simeq m^2_\rho) \simeq m_\rho \Gamma_\rho = 0.12\ \text{GeV}^2 \tag{11}$$

Therefore, in the mass region of the ρ and ω mesons, the real part of $\Pi'(s)$ is negligible and then the denominator in Eq. (5) locally reduces to its imaginary part with a good approximation ; additionally, $I(s)$ is positive there, as can be inferred from its explicit expression [26].

Within the HLS model, $\Pi_{\omega\rho}(s)$ arises naturally as the *difference* of both (neutral and charged) kaon loops [27]. These are perfectly defined analytic functions [26] of s and each contains a subtraction polynomial which should be different for neutral and charged kaon loops, at least in order to account for isospin symmetry breaking for pseudoscalar mesons. This issue will be discussed in some more detail in the next Section, but for the present purpose it is enough to remark that, even neglecting isospin breaking effects on masses (then, some logarithm functions cancel out identically), the HLS expression for $\Pi_{\omega\rho}(s)$ is essentially *a real valued subtraction polynomial* which has to be determined through renormalization conditions [26]. Standard renormalization conditions [26] imply that this polynomial is minimally of the form $c \cdot s$, with real c, in agreement with general considerations [21,22,14].

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[4] Actually, $R(s)$ is a function of s which contains logarithms and a subtraction polynomial, minimally of the form λs, with λ real to be fixed by means of appropriate renormalization conditions. The Breit–Wigner formulation turns out to approximate locally this real part by a constant which corresponds to the mass given in the RPP [30]. We call this mass definition *observed* mass and keep in mind that it may have little to do with the masses as they occur in Lagrangians.

Thus, within the HLS model, the numerator is essentially real, and the denominator is largely dominated by the imaginary part of the pion loop in the mass region of interest. Using the coupling constants which can be read off from Eq. (6) above, and writing cs the amplitude for $\Pi_{\omega\rho}(s)$ amputated from the coupling constants to ω_I and ρ_I, we have :

$$\tan 2\delta(s) \simeq i \frac{Z^2}{2} \frac{cs}{I(s)} \tag{12}$$

Therefore, in contrast with the customary mixing case of $\omega - \phi$, the mixing angle is close to being purely imaginary in the mass region of interest (the $\rho - \omega$ peak value).

The mixing scheme presented here is not in contradiction with more standard formulations in terms of a perturbation parameter (see [21] for instance, or more recently [34]). However, writing it as a complex angle makes the connection with the $\omega - \phi$ mixing more transparent. Indeed, the nature – real or complex – of these mixing angles follows from peculiarities which could look like kinematical accidents, essentially the relative values of meson masses.

Indeed, if the $\omega - \phi$ mixing angle is *observed* real within the HLS model at one loop [26], it is essentially because it depends only on the kaon loop which is real below the 2–kaon theshold and because the ϕ meson mass is well approximated by $m_\phi \simeq 2m_K$. Therefore, as the excursion above this threshold is tiny, the imaginary part to the mixing angle is tiny ; anomalous contributions are also numerically small [26].

If, instead, the $\omega - \rho$ mixing angle is observed complex, this is simply because it depends on both the pion and kaon loops, and because the pion loop carries a large imaginary part at the ρ peak and above. If it happens that the $\omega - \rho$ mixing angle is additionally close to purely imaginary, the very reason is simply that the *observed* masses of the ω and ρ mesons are so close together that the real part of the mixing angle is very small (but detectable).

Therefore, even if their physics origin is quite different, there is no mathematical difference between the $\omega - \rho$ mixing and the $\omega - \phi$ mixing. Both are analytic functions of s and, even, there exists certainly a region where both are real ($0 \leq s \leq 4m_\pi^2$). Moreover, as the sine and cosine functions can be continued analytically into the complex s–plane in a trivial manner and from there anywhere along the physical region, there is no inconsistency in using an angular formulation which allows to consider the mixing phenomena in a unified framework.

In order to check physically the consistency of the picture just developed, the most appropriate place is certainly the pion form factor. We postpone to the Section V examination of this issue in some detail.

Using the RPP world average mass and width values for the ρ and ω mesons, a better local approximation than Eq. (12) for the mixing "angle" can be written

$$\tan 2\delta(s \simeq m_\rho^2) = d\ m_\rho^2\ \exp\left\{ i \left[\pi - \arctan \frac{m_\rho \Gamma_\rho - m_\omega \Gamma_\omega}{m_\omega^2 - m_\rho^2} \right] \right\} \tag{13}$$

where d is a real constant to be fit. The explicit phase (referred to below as φ) becomes [5] :

$$\varphi = \mathrm{Arg}[\tan 2\delta(s \simeq m_\rho^2)] = 100.7 \pm 0.7 \text{ degrees} \tag{14}$$

and does not account for a possible negative sign in the fit value for d. We will see shortly that this value has to be compared with the so–called Orsay phase frequently fit within the pion form factor in the timelike region ; among recent fit values, let us quote the value obtained within the HLS framework [23] $104.7^\circ \pm 4.1^\circ$.

IV. SU(2) BREAKING WITHIN THE HLS MODEL

A priori, a straightforward way to introduce isospin symmetry breaking within the HLS model could be through the BKY mechanism [15,16] proved successfull when analyzing SU(3) symmetry breaking effects for radiative decays of light mesons [17] or the properties of the $\eta - \eta'$ system [18]. Actually, such an attempt has been already considered in the context of radiative decays of light mesons [31].

For the present purpose, the non–anomalous HLS Lagrangian piece which generates the vector mass term would be written [15] :

$$f_\pi^2 g^2 \mathrm{Tr}[V X'_V V X'_V] \tag{15}$$

where V is the matrix of vector mesons [15,17,26] also recalled below. In the case of SU(3) breaking the matrix X'_V is named traditionally X_A [16,15] and is $X_A = \mathrm{Diag}(1, 1, 1 + c_A)$, with $1 + c_A = [f_K/f_\pi]^2$. Quite naturally, in the general case where SU(2) is additionally broken, one might choose $X'_V = \mathrm{Diag}(1, 1+\epsilon, 1+c_A)$ with ϵ small compared to 1. Focussing provisionally on isospin symmetry breaking and thus discarding issues related with the ϕ meson, one can examine $X'_V = \mathrm{Diag}(1, 1 + \epsilon, 1)$.

Unexpectedly, this does not generate a mass breaking between ρ_I and ω_I, in contrast with SU(3) breaking which indeed generates a mass difference between ϕ_I on the one hand and both ρ_I and ω_I on the other hand. The reason can be traced back to the very structure of the diagonal in the vector meson field matrix which contains $\rho_I \pm \omega_I$ in the first two diagonal entries while the third one is simply $-\phi_I$. In the more general case where $X'_V = \mathrm{Diag}(1, 1+\epsilon, 1+c_A)$, a $\rho - \omega$ mass difference is nevertheless generated, but essentially because of the $\omega - \phi$ mixing [26] ; this is not discussed here any further as we do not plan to focus on mass problems.

It is an easy matter to verify that, consequently, the mixed $\rho - \omega$ contribution to the mass term in Eq (15) cannot be cancelled out through a rotation corresponding to a small ϵ. Therefore, extending the BKY mechanism to SU(2) breaking, does not provide (alone) a satisfactory picture for isospin symmetry breaking within the HLS context.

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[5] If we use for ρ parameters the values given in the entry "$\tau\ e^+e^-$" of the RPP [30], instead of the world average which is somewhat less secure, this angle value becomes 95.5 ± 0.8 degrees. Therefore a non–negligible systematic error (5°, about $7\sigma_{stat}$) can affect this number.

Another solution is naturally proposed by the HLS model in close correspondence with the $\omega - \phi$ mixing. Let us name provisionally $\ell(K^+K^-)$ and $\ell(K^0\overline{K}^0)$ the kaon loops amputated from coupling constants to external vector meson lines. These loops are given by dispersion relations [26] which should be subtracted minimally twice in order that the dispersion integrals converge.

This gives rise in both cases to a first degree polynomial in s with real coefficients in order to satisfy usual analyticity properties [26] ; let us denote by $P_\pm(s)$ and $P_0(s)$ resp., the subtraction polynomials associated with the kaon loops just referred to above. Their coefficients are a priori arbitrary and should be fixed by means of appropriate renormalization conditions. The constant term is always chosen to vanish in theories where vector mesons couple to conserved currents [14] as in the HLS model ; the same effect ensures the masslessness of the photon. If isospin is conserved, the first degree terms of both polynomials should clearly be equal. However, if SU(2) is broken, there is no longer any reason for this requirement to be made.

Therefore, breaking of SU(2) symmetry can be implemented by having different renormalization conditions for $P_\pm(s) = c_\pm\ s$ and $P_0(s) = c_0\ s$. Allowing $c_\pm \neq c_0$ appears to be a consistent effective way to break isospin symmetry within the HLS model at one–loop order.

For clarity, let us denote by $\ell(K^+K^-) + P_\pm(s)$ and $\ell(K^0\overline{K}^0) + P_0(s)$, the full kaon loops, exhibiting this way the (free) subtraction pieces. Up to inessential coefficients related with vector coupling constants and SU(3) breaking effects, we have [27] :

$$\left\{ \begin{array}{l} \Pi_{\phi_I\omega_I}(s) \simeq\ \ell(K^+K^-) + \ell(K^0\overline{K}^0) + P_\pm(s) + P_0(s) \\[2ex] \Pi_{\rho_I\omega_I}(s) \simeq\ \ell(K^+K^-) - \ell(K^0\overline{K}^0) + P_\pm(s) - P_0(s) \\[2ex] \Pi_{\rho_I\phi_I}(s) \simeq\ \ell(K^+K^-) - \ell(K^0\overline{K}^0) + P_\pm(s) - P_0(s) \end{array} \right. \tag{16}$$

Then, quite generally, the HLS model at one loop allows for transition among the ideal combinations of all neutral vector mesons. It should be remarked that these transitions are associated with kaon loops rather than with the pion loop[6]. It is also interesting to note that, even if one neglects the $K^\pm - K^0$ mass difference, the transition amplitudes $\Pi_{\rho_I\omega_I}(s)$ and $\Pi_{\rho_I\phi_I}(s)$ do not drop out, even if their imaginary parts identically vanish. Moreover, as loops are analytic functions of s, real for real s smaller than the loop threshold, the transition amplitude $\Pi_{\rho_I\omega_I}(s)$ is certainly real in the region of the $\rho - \omega$ peak (up to anomalous loop effects). Additionally, it certainly fulfills $\Pi_{\rho_I\omega_I}(s = 0) = 0$.

Therefore, the HLS model allows to have naturally a real $\Pi_{\rho_I\omega_I}(s)$ in the $\omega - \rho$ peak region, as obtained from fits [21,10,24]. This illustrates that loop effects can be used as the main mechanism in order to break isospin symmetry by allowing different renormalization conditions to different kaon loops. Stated otherwise, isospin symmetry breaking in the pseudoscalar sector already induces corresponding effects in the vector sector.

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[6]In order to be complete, we recall that anomalous terms produce loop effects like $P\gamma$ or VP loops which contribute to the transition amplitudes ; these have been estimated to be numerically small [26].

Moreover, one observes that the HLS model at one loop, predicts that the full mixing pattern concerns all three neutral vector mesons and establishes the $\rho_I - \phi_I$ mixing as the physics mechanism for the $\phi \to 2\pi$ decay. This mixing pattern happens to produce 3 complicated equations involving 3 "angles". In contrast with the $\rho - \omega$ mixing angle (close to purely imaginary) and with the $\omega - \phi$ mixing angle (close to purely real), the $\rho - \phi$ mixing angle is certainly complex. We shall rediscuss this issue at the appropriate place.

Let us now comment on what is going on with the ρ and ω (running) masses. Performing the change of fields defined by Eq. (4) in Eq. (3) and using Eq. (5), the $\rho - \omega$ mass term can be written[7] :

$$\mathcal{T}_{\rm M} = \frac{1}{2}\left[\lambda_\rho(s)[\rho(s)]^2 + \lambda_\omega(s)[\omega(s)]^2\right] \tag{17}$$

The effective running masses squared of the $\rho(s)$ and $\omega(s)$ fields become :

$$\begin{cases} \lambda_\rho(s) = m^2 + \dfrac{\Pi_\rho(s)+\Pi_\omega(s)}{2} + \dfrac{\Pi_\rho(s)-\Pi_\omega(s)}{2\cos 2\delta(s)} \\ \lambda_\omega(s) = m^2 + \dfrac{\Pi_\rho(s)+\Pi_\omega(s)}{2} - \dfrac{\Pi_\rho(s)-\Pi_\omega(s)}{2\cos 2\delta(s)} \end{cases} \tag{18}$$

If $\cos 2\delta(s)$ is small enough, the correction to the corresponding quantities associated with ideal field combinations is of order $[\delta(s)]^2$. This is in clear correspondence with the $\omega - \phi$ mixing case treated in [26]. Therefore, the breaking of isospin symmetry we propose also generates a mass difference in the $\rho - \omega$ system ; this should be considered together with the additional mass difference provoked by the $\phi - \omega$ mixing [26], which is also an effect of kaon loops within the HLS model, without reflecting symmetry breaking effects.

V. THE PION FORM FACTOR

In order to compute the pion form factor in the timelike region, the relevant piece of the interaction Lagrangian, before changing to physical vector fields[8], is :

$$\mathcal{L} = \cdots \; -i\left[\frac{ag}{2}\rho_I + e(1-\frac{a}{2})A\right]\cdot[\pi^+ \overleftrightarrow{\partial} \pi^-] - aef_\pi^2 g\left[\rho_I + \frac{1}{3}\omega_I\right]\cdot A + \cdots \tag{19}$$

where A is the electromagnetic field, e the unit electric charge, g the universal vector coupling constant and a the intrinsic HLS parameter fit to $2.35 \div 2.45$ [23,17,26]. This Lagrangian piece is not affected by SU(3)/U(3) symmetry breakdown.

After the change to physical fields given by Eq. (4), it is obvious that SU(2) symmetry breaking generates a direct coupling of ω to $\pi^+\pi^-$. Denoting by $g^0_{\rho\pi\pi} = \frac{ag}{2}$ the unbroken coupling of ρ_I to a pion pair, the coupling constants for physical ρ and ω are :

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[7] This can be written in a more symmetric way by using the analyticity properties of the field transform which implies that $\rho(s) = \rho^\dagger(s^*)$ and $\omega(s) = \omega^\dagger(s^*)$. This property, named hermitian analyticity, is then fulfilled by the full one–loop corrected Lagrangian.

[8] We still skip in this Section mixing with the ϕ meson.

$$g_{\rho\pi\pi} = g^0_{\rho\pi\pi} \cos\delta(s) \ \ , \ g_{\omega\pi\pi} = -g^0_{\rho\pi\pi} \sin\delta(s) \tag{20}$$

As $\delta(s)$ is close to purely imaginary, this leaves the broken ρ coupling close to real and the generated coupling of ω close to purely imaginary[9].

For sake of conciseness, let us also define :

$$f^0_{\rho\gamma} = a f^2_\pi g \ \ , \ f^0_{\omega\gamma} = \frac{a f^2_\pi g}{3} \quad , \tag{21}$$

the ρ_I and ω_I couplings to a photon, as they come out of the standard HLS Lagrangian.

Using the Lagrangian piece in Eq. (19), it is an easy matter to compute the pion form factor :

$$F_\pi(s) = 1 - \frac{a}{2} - \frac{f^0_{\rho\gamma} g^0_{\rho\pi\pi}}{D_\rho(s)} \ \cos\delta[\cos\delta + \frac{1}{3}\sin\delta] + \frac{f^0_{\omega\gamma} g^0_{\rho\pi\pi}}{D_\omega(s)} \ \sin\delta[\cos\delta - 3\sin\delta] \tag{22}$$

where the inverse propagators are of the form $D_V(s) = s - \lambda^2_V(s)$ (see Eqs. (18)) and include an imaginary part; these are also written $D_V(s) = s - m^2_V - i m_V \Gamma_V(s)$ in most phenomenological studies[10] (varying width Breit–Wigner formulae).

If one limits oneself to leading terms in $\delta(s)$, the expression above simplifies to :

$$F_\pi(s) = 1 - \frac{a}{2} - \frac{f^0_{\rho\gamma} g^0_{\rho\pi\pi}}{D_\rho(s)} \ \cos^2\delta + \frac{f^0_{\omega\gamma} g^0_{\rho\pi\pi}}{D_\omega(s)} \ \sin\delta\cos\delta \tag{23}$$

In order to make the correspondence with Eqs. (13) and (14), and with usual formulae for the pion form factor [23], let us state $d/2 = -A$ and use φ, the phase in Eq. (14). Assuming d is small enough, we also have $\tan 2\delta \simeq \sin 2\delta$ and we can approximate the above expression in the neighborhood of the $\rho - \omega$ peak by :

$$F_\pi(s) = 1 - \frac{a}{2} - \frac{f^0_{\rho\gamma} g^0_{\rho\pi\pi}}{D_\rho(s)} - A e^{i\varphi} \frac{f^0_{\omega\gamma} g^0_{\rho\pi\pi}}{D_\omega(s)} \tag{24}$$

This is nothing but the HLS expression of the pion form factor [23] expressed in terms of the so–called Orsay phase, named here φ.

So, isospin breaking expressed in terms of loop effects gives a consistent picture for the pion form factor and reaches the correct Orsay phase value (see Eq. (13)). Therefore, an "imaginary angle" occuring when breaking isospin symmetry is what permits to recover a quite standard and traditional formulation for the pion form factor.

It is an interesting feature that the $\omega - \rho$ mixing, which expresses isospin symmetry violation, appears in correspondence with the $\omega - \phi$ mixing, produced by the same sort of loop effects. The specific character of the $\rho - \omega$ mixing is the dominance of the subtraction

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[9] We recall that $\sin i\alpha = i \sinh\alpha$ and $\cos i\alpha = \cosh\alpha$.

[10] Then, one accepts to release locally the assumption of analyticity.

term, which carries most of the SU(2) symmetry breaking information in the approach we propose.

In order to be complete, one can also give an estimate of the parameter $[c_\pm - c_0]$ which carries the modulus information of δ. Locally, *i.e.* in the vicinity of the ω meson mass, one has:

$$|\tan 2\delta|^2 \simeq \frac{\Gamma(\omega \to \pi^+\pi^-)}{\Gamma(\rho \to \pi^+\pi^-)} = (1.24 \pm 0.17)\ 10^{-3} \tag{25}$$

which corresponds to a negligible "angle" of about 1 degree times i. This is indeed very small but comparable in magnitude to the (real) $\omega - \phi$ mixing angle (about 3 degrees), except that it is purely imaginary.

On another hand, the isospin breaking parameter is mostly given by $[c_+ - c_0]s$; by propagating back the result in Eq. (25) to this parameter, one gets $|c_+ - c_0| \simeq (1.73 \pm 0.04)\ 10^{-2}$, which indeed gives a reasonable order of magnitude for SU(2) breaking.

From a practical point of view, this computation illustrates that :

i/ An imaginary mixing "angle" is consistent with the most prominent case of SU(2) breaking, namely the pion form factor,

ii/ if the $\rho - \omega$ mixing "angle" is very small, it is nevertheless comparable in absolute magnitude to the $\omega - \phi$ mixing.

VI. THE FULL MIXING PATTERN

It follows from the Sections above that, basically, the mixing pattern exhibited by the HLS model at one loop involves the full triplet (ρ_I, ω_I, ϕ_I) as soon as isospin symmetry is broken, as it is in real life. In most physics studies of light meson decays it is usual to neglect isospin breaking effects ; in this case only the $\omega_I - \phi_I$ mixing survives.

A noticeable exception[11] is, of course, the pion form factor where a reasonable account of isospin breaking effects cannot be avoided because of the important $\rho - \omega$ interference structure. In this case, one generally focuses on the $\rho - \omega$ region and skips the $\rho - \phi$ interference which shows up through the decay mode $\phi \to \pi^+\pi^-$ [30,38,39]. Among approaches to the full mixing pattern, let us quote Ref. [56]. An interesting account of the $\rho - \phi$ mixing can also be found in the recent [34] in connection with the $\phi\omega\pi$ coupling.

However, from the final remarks in the Section above, one could ask oneself whether accounting only partly for vector meson mixing effects is legitimate. Indeed, we have just seen that the $\omega_I - \phi_I$ mixing (measured by its –real– angle) and the $\rho_I - \omega_I$ mixing (measured by its –imaginary– angle) are quite comparable in magnitude. Therefore, accounting simultaneously for the full mixing pattern could well influence the picture in radiative decays of light mesons [17,26], or the interpretation of the mixing pattern in some scattering processes [33].

We now aim at examining, using the largest possible data set, the consequences of the full mixing pattern within the framework of the HLS model at one–loop order. To be more

[11]See, however, Ref. [31] for an attempt to describe radiative decays of light mesons.

precise, the aim of the present paper is to describe within a unified framework most two body decays of light mesons. As commented in Ref. [17], the major decay modes $K^* \to K\pi$ and $\rho \to \pi\pi$ have large difficulties to fit within the HLS scheme, while all radiative decay modes of all light mesons, leptonic decay modes of vector mesons and all two body decay modes of the ϕ meson fit quite well [17,26]. Preliminary studies tend to indicate that parametrization and modelling of objects as broad as the ρ meson should be the reason. This will be the subject of a separate study, especially devoted to this specific issue.

A. Diagonalization Procedure

When the ϕ field is "switched on", the effective Lagrangian piece quadratic in the fields changes from Eq. (3) to :

$$\begin{aligned}\mathcal{L} = & \ \tfrac{1}{2}\{ \ [m^2 + \Pi_\rho(s)] \ \rho_I^2 + [m^2 + \Pi_\omega(s)] \ \omega_I^2 + [\ell_V m^2 + \Pi_\phi(s)] \ \phi_I^2 \\ & +2\Pi_{\omega_I\rho_I}(s) \ \rho_I\omega_I + 2\Pi_{\omega_I\phi_I}(s) \ \omega_I\phi_I + 2\Pi_{\rho_I\phi_I}(s)\rho_I\phi_I\} \ \ .\end{aligned} \tag{26}$$

Self-energies and transition amplitudes have been defined in Eqs. (7) and (16) respectively. The amputated loop expressions have been given in [26] and are all real analytic functions of s in well defined domains of the complex s plane.

In order to compute amplitudes involving the *physical* ρ, ω and ϕ mesons, Eq. (26) should be diagonalized. This gives the physical fields as algebraic expressions in terms of the ideal field combinations ρ_I, ω_I and ϕ_I. In these expressions the coefficients of the (linear) relations are analytic functions of s, which basically depend on three "angles" through relations much more complicated than Eq. (4).

One can obviously define three such "angles" corresponding each to the case where one among ρ_I, ω_I and ϕ_I is "switched off". As already stated above, these "angles" are actually (analytic) functions of s and can be real, imaginary or complex depending on the specific s value along the physical region.

The $\omega - \phi$ mixing has been studied in isolation in [26] and the corresponding mixing angle has been found real as long as s is smaller than the two–kaon threshold ; practically, this remains true up the ϕ meson mass region. The $\omega - \rho$ mixing angle has been considered in the previous Sections and has been found close to purely imaginary. The third mixing angle, if considered in isolation, is certainly complex ; indeed it is given by :

$$\tan 2\gamma(s) = \frac{2\Pi_{\rho_I\phi_I}(s)}{\Pi_\rho(s) - \Pi_\phi(s)} \tag{27}$$

in terms of already defined quantities. The denominator here can also be written :

$$\Pi_\rho(s) - \Pi_\phi(s) = \frac{a^2g^2}{4}\left[\Pi'(s) + \frac{Z^2}{4}(1-2\ell_V^2)\Pi(s)\right] \tag{28}$$

in terms of the pion and kaon loops amputated from their coupling to external vector meson lines[12]. In the mass region of interest ($4m_\pi^2 \leq s \leq 4m_K^2$), the pion loop $\Pi'(s)$ is complex while the kaon loop $\Pi(s)$ is real. A rough estimate of this difference shows that the imaginary part is non negligible. The real part of Eq. (28) is certainly smaller than the imaginary part in absolute magnitude, however a precise estimate of its value necessitates assumptions on the subtraction polynomials far beyond the scope of the present paper (see for instance [25]).

B. Transformation from Ideal to Physical Fields

Therefore the general transformation we are interested in is certainly linear and depends on three angles, each a function of s.

Moreover, relying on the angle functions obtained by switching off one among ρ^0, ω, ϕ, it is likely that these angles vary little along the mass range we are interested in[13] (from the ρ mass to the ϕ mass) and the accuracy of the data which can allow their study as functions of s is too poor and limited to be conclusive.

This leads us to approximate these three analytic functions by three constants, over the mass range covered by the light vector and pseudoscalar mesons. This is a somewhat violent assumption and the ability of the model supplied with this constraint to describe experimental data will teach us about its validity.

This being stated, the transformation which allows to define the physical ρ, ω and ϕ fields in terms of ρ_I, ω_I and ϕ_I is formally a rotation and the angles are defined by requiring the vanishing of all mixed terms, $\rho\omega$, $\rho\phi$, $\omega\phi$ in Eq. (26) after the change of fields. The transformation is a real rotation – with real angles – when analytically continued below the two–pion threshold. In this case, the rotation matrix is certainly the most general one and its analytic continuation up to the physical region keeps the most general form, provided one considers the (trivial) analytic continuations of the sine and cosine functions.

This rotation matrix can be chosen as the following CKM–like matrix [30] which was also used in order to study a possible glue component coupled to the $\eta - \eta'$ system [17,18] :

$$M = \begin{bmatrix} \cos\delta\cos\beta & -\sin\delta\cos\beta & \sin\beta \\ \sin\delta\cos\gamma + \cos\delta\sin\beta\sin\gamma & \cos\delta\cos\gamma - \sin\delta\sin\beta\sin\gamma & -\cos\beta\sin\gamma \\ \sin\delta\sin\gamma - \cos\delta\sin\beta\cos\gamma & \cos\delta\sin\gamma + \sin\delta\sin\beta\cos\gamma & \cos\beta\cos\gamma \end{bmatrix} \tag{29}$$

and we define the requested field transformation by :

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[12]Accounting for anomalous loops would add tiny contributions carrying small imaginary parts in the mass region of light vector meson resonances. This does not influence the picture we present now.

[13]See Ref. [26] for the $\omega - \phi$ mixing case in isolation.

$$\begin{bmatrix} \omega \\ \\ \rho \\ \\ \phi \end{bmatrix} = M \begin{bmatrix} \omega_I \\ \\ \rho_I \\ \\ \phi_I \end{bmatrix} \text{ and } \begin{bmatrix} \omega_I \\ \\ \rho_I \\ \\ \phi_I \end{bmatrix} = \widetilde{M} \begin{bmatrix} \omega \\ \\ \rho \\ \\ \phi \end{bmatrix} \tag{30}$$

One can indeed check that $M^{-1} = \widetilde{M}$ whether β, δ and γ are real or complex. As stated above, the sine and cosine functions here are defined through their underlying exponential expressions and coincide with the standard ones for real values of their arguments. We recall that trigonometric functions satisfy all their known properties, even for complex values of their arguments.

If the "angle" functions may become complex, one may wonder that we may be violating hermiticity, as one could have rather expected $M^{-1} = M^{\dagger} = \widetilde{M}^{*}$. This is not true, as can be seen by going a step prior to the approximation by constants. In this case[14], the relation fulfilled by $M(s)$ along the physical region can be written, using obvious notation :

$$M(s+i\varepsilon)M^{\dagger}((s+i\varepsilon)^{*}) = M(s+i\varepsilon)M^{\dagger}(s-i\varepsilon) = 1 \tag{31}$$

assuming that the two lips of the physical region can be connected by a path fully contained in the physical sheet and which does not cross any cut ; additionally, $M(s)$ is real below the 2–pion threshold.

The sine and cosine functions defining $M(s)$ are certainly algebraic functions of the transition amplitudes and self–energies ; therefore, they have essentially the same branch point singularities (plus possible additional ones we will not discuss). Therefore, $M(s)$ is certainly a real analytic function of s in a domain sketched several times above. Then, we should have along the physical region :

$$M^{*}(s-i\varepsilon) = M(s+i\varepsilon) \tag{32}$$

which leads to

$$M(s+i\varepsilon)\widetilde{M}(s+i\varepsilon) = 1 \tag{33}$$

as has been inferred from Eq. (30). The precise analyticity domain where this is valid is not easy to study in the present case ; this is, furthermore, of no consequence for the present study[15].

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[14] We denote by * the simple complex conjugation of matrices (with no transposition) and variables.

[15] In order that the framework of what follows holds, one has only to *assume* that this analyticity domain contains a band along $\mathrm{Re}(s) > 4m_\pi^2$ on both sides of the real axis and, connectedly, a part of the semi–axis $\mathrm{Re}(s) < 4m_\pi^2$. This working assumption does not look severe.

C. A Few Qualitative Remarks

Let us make some comments about the kind of values expected for the angle functions :

j/ By dropping out each among ρ_I, ω_I and ϕ_I successively, one can relate each of these three "angles" with each pair of vector fields. In this way, it is easy to find that δ is mostly related with the pair $\omega - \rho$, β with $\omega - \phi$ (it coincides with the angle named δ_V in [17,26]), and γ with $\rho - \phi$. This might allow to guess which kind of values are expected for these in a global fit : δ mostly real, β mostly imaginary ; the imaginary part of γ can be expected important, but nothing can be seriously inferred for the real part.

jj/A priori, the angle functions occuring in Eq. (29) do not exactly coincide with the angles obtained by "switching off" each among the ideal vector fields (as Eqs (5) or (27)). However, if these angles are small, the correspondence is certainly true for each angle, up to higher order terms in the other angles. This can be checked by comparing the couplings predominently influenced by one angle (when one field is switched off) to its expression in the general case.

jjj/ It remains, nevertheless that the mixing scheme exhibited by Eq. (29) is somewhat unusual. For instance, even if the function γ were chosen to vanish identically, it would remain a piece of ρ_I inside the ϕ field with a second order weight ($\sin\delta \sin\beta$). This is a basic consequence of 3–dimensional rotation matrix properties which certainly underlie diagonalization of Eq. (26) and cannot be avoided in general.

jv/ We have shown above that the $\omega-\phi$ mixing and the $\rho-\omega$ mixing were of comparable magnitudes ; therefore, Eq. (26) shows that the diagonalization has to be done in the general form from start. It is indeed clear that the origin of the transition amplitudes matters much less than their orders of magnitude.

D. Radiative and Vector Decays of Light Mesons

The first important data set we shall analyze are the radiative decays of light mesons. The Lagrangian which allows us to derive their coupling constants can be written :

$$\mathcal{L}_{WZW} = K\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left[\partial_\mu(eQA_\nu + gV_\nu)\partial_\rho(eQA_\sigma + gV_\sigma)P\right] \tag{34}$$

where $Q = \mathrm{Diag}(2/3, -1/3, -1/3)$ is the quark charge matrix and A is the electromagetic field. P is the pseudoscalar field matrix and can be found in [17] with the conventions used here. The vector field matrix is repeated here :

$$V = \frac{1}{\sqrt{2}}\begin{pmatrix} (\rho_I + \omega_I)/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & (-\rho_I + \omega_I)/\sqrt{2} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\phi_I \end{pmatrix}. \tag{35}$$

in order to exhibit that the traditional field ρ^0 is actually the ideal isospin 1 field combination, while the physical field associated with the ρ^0 meson is ρ in Eqs. (30). The coefficient K in Eq. (34) is [26] $K = -3/(4\pi^2 f_\pi)$.

The various $VP\gamma$ coupling constants can be derived from Eq. (34) in a straightforward way ; before rotating to physical fields, they are given in the Appendix.

It should be remarked that Eq. (34) is an expression for the VMD assumption which connects the usual anomalous Wess–Zumino Lagrangian for $P\gamma\gamma$ to its VMD partner $VP\gamma$ through a common normalization factor (K). Therefore, the treatment of $VP\gamma$ and $P\gamma\gamma$ couplings differ only by specific symmetry breaking effects.

The physical $\rho P\gamma$, $\omega P\gamma$ and $\phi P\gamma$ couplings are easily derived using these ideal couplings and the second Eq. (30), by collecting all contributions to the same field combination coupling. Fully developped, they are algebraically rather complicated, even if conceptually simple. They can, however, be easily dealt with within a minimization program.

Let us illustrate one case and, for this purpose, write down symbolically a piece of Eq. (34) :

$$\cdots G_{\rho_I \gamma P}[\rho_I A P] + G_{\omega_I \gamma P}[\omega_I A P] + G_{\phi_I \gamma P}[\phi_I A P] \tag{36}$$

As symbolically, one can derive from Eqs. (29) and (30) three relations :

$$\rho_I = v_{\rho_I}(\omega)\omega + v_{\rho_I}(\rho)\rho + v_{\rho_I}(\phi)\phi \tag{37}$$

and the corresponding ones for ω_I and ϕ_I with, correspondingly, v_{ω_I} and v_{ϕ_I}. The three vectors just defined are simply the columns in Eq. (29). Rewriting Eq. (36) using Eq. (37) and the two other parent ones, it remains only to collect all terms contributing to $[\rho A P]$, $[\omega A P]$ and $[\phi A P]$ in order to get the coupling constants associated with physical vector mesons.

This allows to include all radiative decay modes in our data sample, *i.e.* the possible 15 decay modes presently all measured. Actually, the $\pi^0\gamma\gamma$ partial width is not used but replaced by the pion decay constant world average value [30] $f_\pi = 92.42$ MeV.

Beside the radiative $VP\gamma$ coupling constants, the Lagrangian Eq. (34) defines also the VVP couplings. From a practical point of view, the interesting piece derived from Eq. (34) can be written :

$$\mathcal{L}_1 = -\frac{g^2}{8\pi^2 f_\pi}\left\{[\omega_I \rho_I \pi^0] + [\omega_I \rho^+ \pi^-] + [\omega_I \rho^- \pi^+]\right\} \tag{38}$$

using obvious notations. From this, we can derive the $\phi\omega\pi^0$ coupling which allows to include the corresponding decay mode in our data sample. The coupling constants for $\phi\rho\pi$ [33] is derived from fits to the $e^+e^- \to \pi^+\pi^-\pi^0$ cross section ; the most recent fit value [33] $g_{\phi\rho\pi} = 0.815 \pm 0.021$ GeV^{-1} is seemingly well established. Its parent $\omega\rho\pi$ is subject to more controversy [34–37] and the reported values range between 11.7 ± 0.5 GeV^{-1} [34] for the smallest to 16.1 ± 0.4 GeV^{-1} for the largest, with a prefered value [35] around 14.3 GeV^{-1} ; until clarification, it seems more secure to leave this information outside fits and simply compare with our predictions.

Finally the relative phase of the coupling constants $\phi\omega\pi^0$ and $\omega\rho\pi$ comes from a fit to $e^+e^- \to \omega\pi^0$; the most recent estimate [37] is $-49° \pm 7° \pm 1°$ and will be included into the data sample we shall fit.

E. Information from e^+e^-, $\pi^+\pi^-$ and $K\overline{K}$ Decays

Concerning the decay of vector mesons to e^+e^-, the relevant Lagrangian piece is [17] :

$$\mathcal{L}_{em} = -aef_\pi^2 g \left[\rho_I + \frac{1}{3}\omega_I + \ell_V \frac{\sqrt{2}}{3}\phi_I\right] \cdot A \tag{39}$$

which depends on the breaking parameter [16,15,17] ℓ_V. It allows to derive the corresponding couplings for the physical fields ρ, ω and ϕ using Eqs. (30) above.

The other HLS Lagrangian piece given in Eq. (6) provides the coupling constants of the physical ϕ meson to both K^+K^- and $K_L K_S$ final state.

Finally, the $\pi^+\pi^-$ term in Eq. (6) gives the $\omega\pi^+\pi^-$ and $\phi\pi^+\pi^-$ couplings which allows us to include these partial widths inside our data sample.

Therefore, in addition to the 14 modes, the $\phi \rightarrow \omega\pi^0$ decay width and its phase relative to $\omega\rho\pi$, and the $\phi\rho\pi$ coupling as stated in the section above, we can add 7 more decay modes to our working data sample ($\rho/\omega/\phi \rightarrow e^+e^-$, $\phi \rightarrow K^+K^-/K_L K_S$, $\omega/\phi \rightarrow \pi^+\pi^-$).

As clear from the above Sections dealing with the $\rho - \omega$ mixing, the phase of the ω term (denoted above φ) relative to the ρ term carries as much physics information as the $\omega \rightarrow \pi^+\pi^-$ partial width (one gives the phase of the breaking term, the other its modulus). Referring to the Review of Particle Properties [30], there is no reported average value and the latest fit which could have produced such information did not include this measurement [20], therefore, we shall use the latest published fit value [23] $104.7^\circ \pm 4.1^\circ$ as reference data.

There are however, former fit values for the Orsay phase which give information on its model dependence. By varying the model for the ρ lineshape in the pion form factor, [40,41] yielded values ranging from $94.3^\circ \pm 2.9^\circ$ to $111.4^\circ \pm 4.8^\circ$; Ref. [42] approximating the ρ lineshape by a truncated Laurent series yielded $116.7^\circ \pm 5.8^\circ$. The reference value we choose is somewhat median and has the virtue to reproduce the threshold behavior predicted by Chiral Perturbation Theory with a good accuracy [23] ; however, systematic errors of about 10° might not be considered unlikely.

On the other hand, the fit of the $\phi \rightarrow \pi^+\pi^-$ rate has been done several times [30,38,39], but only one value for the phase is currently reported in the literature [38] and provides valuable physics information. The reported value is $\psi = -34^\circ \pm 4^\circ$; this corresponds to a definition where the ϕ inverse propagator is written $m_\phi^2 - s - im_\phi\Gamma_\phi$, opposite in sign to the definition currently used (see Section V). In order to recover consistency with the rest of the information we use, a minus sign should be absorbed in this phase which thus becomes $\psi = 146^\circ \pm 4^\circ$.

These phases are the phases of the following quantities :

$$F_V = f_{V\gamma} G_{V\pi^+\pi^-} \quad , \quad V = \rho,\ \omega,\ \phi \tag{40}$$

which are allowed to be complex for all vector mesons. It is a condition to check that for ρ the absolute phase remains consistent with zero.

Therefore, taking into account these 2 additional phases, our data sample contains 26 physics quantities; indeed, all modes reported above, except for $\pi^0 \rightarrow \gamma\gamma$ replaced by the world average value for f_π, the controversial coupling $g_{\omega\rho\pi}$, to which we shall nevertheless compare, as well as the phase of $G_{\rho\pi\pi}$ (unavoidably generated by isospin breaking) which should be (and is found) very small.

VII. FITTING THE DATA SAMPLE

We have fit the set of data listed above within the model presented in the above Section concerning the full mixing pattern and in the Appendix concerning the rest of the parameter set.

Concerning the data, all have been taken from the last issue of the Review of Particle Properties (RPP) [30][16]. For data which have no existing entry in the RPP, we have chosen the latest reference. Therefore, among all physics quantities which could be accessed by the model we present, only the major modes $K^* \to K\pi$ and $\rho \to \pi\pi$ are left aside, as already stated.

A. Analysis of the Fit Conditions

The unbroken HLS model [11] basically depends on a very few parameters which are not predicted and should be extracted from data. These are the unit electric charge e, the universal vector meson coupling g, the pion decay constant $f_\pi = 92.42$ MeV and a, a dimensionless parameter specific of the HLS model. In standard VMD models, one has $a = 2$, however within the HLS model this condition can be relaxed ; in this case, fits to experimental data [52,17,18,20] indicate that $a = 2.3 \div 2.5$ should preferred. This merely means that a small coupling $\gamma\pi^+\pi^-$ survives beside vector mesons exchanges.

Concerning symmetry breaking parameters, previous studies [17,18,26] have already reduced the fit freedom by relating, and/or fixing the breaking parameters specific to the pseudoscalar sector : we have already $Z = [f_\pi/f_K]^2 = 2/3$, as a consequence of $F_K(0) = 1$ [16,15] ; it has been checked that the set of radiative and leptonic decays favors this value unambiguously [17].

Finally, it is not a specific feature of the HLS model to require mixing in the η/η' sector, however, the two parameters involved there (the mixing angle θ_P and the nonet symmetry breaking parameter x) are algebraically related by the HLS phenomenology [18] (see Eq. (A1)) with a high accuracy.

In the vector sector of the HLS model, two breaking parameters, denoted here ℓ_V and ℓ_T, seem unavoidable and free, even if ℓ_V might be fixed sometime, when vector meson masses would be clearly understood[17] [15,17,26].

This being stated, there remain 3 complex "angles" (6 parameters) which are the body of the present study. Within some approximations (mostly, neglecting anomalous loops),

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[16] An update of the Particle Data Table can be found at http://pdg.lbl.gov ; some minor modifications have been made to the decay rates considered in the present paper. They do not affect our analysis.

[17] We mean by this, that the relation between theoretical masses as they occur in Lagrangians and the corresponding measured quantities is unclear for broad objects like ρ or K^*. This problem is certainly related with the apparent difficulty to accomodate the major decay modes of K^* and ρ and all other decay modes *simultaneously* within the HLS framework.

one has already noted some clear guesses : one should be mostly real (it corresponds to the standard $\omega - \phi$ mixing angle [26]), another close to purely imaginary (it corresponds to the $\omega - \rho$ mixing angle examined in Sections III–V). Of course, when going to numerical analysis, the validity of these guesses can be controlled. Moreover, the expectations just referred to have been established above or elsewhere [26] by considering mixing patterns in isolation ; therefore, slight departures from these expectations are not unlikely.

i/ As first attempt, we have left these 6 parameters free in the fit. We reached the good fit quality of $\chi^2/\mathrm{dof} = 13.41/15$ (26 data, 11 parameters) which corresponds to a 57% probability. The fit correlation matrix was observed to exhibit large correlations between fit values for β and γ and between Re(γ) and Im(γ) ; as this could well influence the fit procedure, we have looked for equivalent parametrizations. The most appropriate we found was to use Re(β), Im(β), Im(γ) and a parameter k defined by Re(γ)=k Im(γ). In this case we improved the fit quality to $\chi^2/\mathrm{dof} = 12.59/15$, corresponding to a fit probability of 63%. The fit then returned $k = -0.23^{+0.40}_{-0.60}$, which indicates that the angle γ can be chosen imaginary ($k = 0$) ; the global fit assuming this constraint is given in the first data column of Table I.

ii/ We have explored several strategies in order to reduce the freedom in fits by fixing several subsets of parameters. The most interesting results, with reasonable probabilities (above the percent level), are given in the second and third data columns in Table I. The former relies on the observation that the $\omega - \phi$ mixing angle is well fitted real [26] ; this leads to try requesting Im[β]$= 0$ in order to lessen the fit freedom. The latter relies on the observation that both Re[γ] and Re[δ], basically related with the mixing of ω and ϕ to ρ, are quite generally yielded small compared to the corresponding real parts ; this is true in the general framework under examination and also in studies were these mixing phenomena were considered in isolation. Even if somewhat brutal, these approximations lead both to quite reasonable fit quality.

iii/ We have also considered that there can be a functional relation between some "angles". From Eqs.(16), (5) and (27), one might guess that the "angles" γ and δ could be functionally related. As we are dealing with slowly varying functions over the range of interest, we have tried requesting :

$$\gamma = (\mu_1 + i\mu_2)\ \delta \quad . \tag{41}$$

It happens that this relation is well accepted by the data. The fit returned $\mu_1 = (0.030^{+0.007}_{-0.006})$ and $\mu_2 = (0.011^{+0.131}_{-0.114})\ 10^{-1}$ with $\chi^2/\mathrm{dof} = 12.58/15$ (probability 63%). Therefore, requiring the condition in Eq. (41) is certainly justified and additionally, one gets phenomenological motivation to require $\mu_2 = 0$ from start. The corresponding fit results are displayed in the fourth data column of Table I. The fit quality reached can hardly be better.

Finally, an additional fit (not shown) assuming Im[β] $= 0$ and leaving free μ_1 and μ_2 has been performed in order to test the stability of other fit parameters, by requiring a condition expected if one interprets β as strictly equivalent to the $\omega - \phi$ mixing angle in isolation. The result practically coincides with the second data column in Table I, including its fit quality, and returns $\mu_1 = (0.354 \pm 0.044)\ 10^{-1}$ and $\mu_2 = (-0.270 \pm 0.040)\ 10^{-1}$. The various contributions to the χ^2 implies that this fit and the second data column in Table I give the same description of the data with the same probability.

iv/ From the results given in Table I, it is clear that most parameter values do not depend sensitively on the fit strategy considered. As all fit qualities are especially favorable, no strategy can be privileged. The single parameter which seems floating is Re[γ] which cannot be better constrained before improving the accuracy of existing information for $\phi \rightarrow \pi\pi$ in modulus and phase, and/or improving the phase of the $\phi\omega\pi$ coupling constant. Whether γ could be removed as a whole has been considered with a negative answer. Indeed, performing a fit with $\gamma = 0$ leads to a quality which becomes really poor (χ^2/dof = 36.34/17, probability 0.4%).

v/ We have 11 parameters in order to describe the set of 26 experimental data. Out of these, six parameters (Re[β], the $\omega - \phi$ mixing angle named δ_V in [17,26], a, g, θ_P, ℓ_V, ℓ_T) have little to do with isospin symmetry breaking and are fixed mostly by radiative and leptonic decays.

On the other hand, the information prominently affected by isospin symmetry breaking represents 6 measurements ($\omega/\phi \rightarrow \pi\pi$, $\phi \rightarrow \omega\pi$ in modulus and phase), which requires in our approach 4 parameters (δ, μ_1 and μ_2/Im[β]). Therefore, even in this sector, the set of parameters is reasonably constrained and only waits for more accurate data.

B. Analysis of Fit Parameter Values

As first remark, it is clear that all fit parameters not connected with vector meson mixing, (the five first lines in Table I), are quite stable and their values compare well with previous attempts along the present lines [17,26,18]. We note, however, the correlation between ℓ_V and a which reaches -90% ; this correlation is purely numerical and reflects that the dependence upon ℓ_V within the set of coupling constants is actually a dependence upon the product $a\ell_V$.

The value obtained for the pseudoscalar mixing angle has been discussed in [18] and agrees quite well with recent estimates from lattice QCD [43]. It has been shown in Ref. [18] this this angle is (algebraically) related with the mixing angle in favor within the ChPT community by a factor which can be predicted close to 2.

As stated above, the mixing "angle" β can be considered as intimately associated with $\omega - \phi$ mixing. It should be noted that the value of Re(β) varies little when constraints are put on other parameters. This real part is a 30σ effect and corresponds to an $\omega - \phi$ mixing angle of $-(3.2 \div 3.5)^\circ \pm 0.11^\circ$, that is smaller than the ideal mixing angle, as found in [17,26]. These remarks allow to conclude that introducing isospin symmetry breaking, as we propose does not affect sensitively the sector of radiative and leptonic decays. For most parameters not intimately related with isospin symmetry breaking, this follows expectations (see the first five lines in Table I) ; however, because of the "rotation" matrix structure, it was not obvious that Re[β] could not shift by a few degrees, pushing the $\omega - \phi$ mixing angle slightly above its ideal value. This is not observed, whatever the fit strategy.

All uncertainties in the fits are connected mainly with the values for Im(β), Re(γ) and Re(δ). This reflects that, even though valuable, most isospin breaking data are still of rather poor accuracy.

C. Reconstruction of Physics Quantities

The fit parameter values allow to reconstruct branching fractions, coupling constants and phase factors as predicted by our model. Dealing with errors is done by Monte Carlo methods using the full covariance matrix of each fit in order to account properly for correlations. Let us denote V_{ij} the covariance matrix element for parameters x_i and x_j, by λ_α its eigenvalues and by a_i^α the i^{th} component of the α^{th} normalized eigenvector ; then any measured parameter x_i can be considered as a random variable given by :

$$x_i = x_i^0 + \sum_{\alpha=1}^{n} \varepsilon^\alpha \, \sqrt{\lambda_\alpha} \, a_i^\alpha \tag{42}$$

where x_i^0 is the central value returned by the fit and $\{\varepsilon^\alpha, \ \alpha = 1, \cdots n\}$ is a set of independent gaussian random variables of zero mean and unit standard deviation ($< \varepsilon^\alpha \varepsilon^\beta >= \delta_{\alpha\beta}$).

The fit quantities were the coupling constants for each process. These have been derived from the accepted branching fractions [30] –taking into account their accuracy– and assuming that the full widths and masses of mesons are random variables.

In order to reconstruct the physics (measured) quantities, in addition to considering the fit parameters as correlated random variables, we have assumed the mass and width of each vector meson as independent random variables with standard deviation given by the accepted errors [30] ; instead, all masses of pseudoscalar mesons were considered as fixed, except for the η' meson. Finally, for the ρ meson (charged and neutral) we have considered the value given in the τ/e^+e^- entry of the RPP [30] for its mass and width. We thus follow the conclusion of the ALEPH Collaboration who saw no difference for these parameters – within errors – between the charged and neutral modes [45].

1. Radiative and Leptonic Decay Modes

We give in Table II the reconstructed branching fractions for radiative and leptonic decays together with the recommended values [30].

It is interesting to compare these reconstructed values with previous fits done using the model we present, without introducing isospin symmetry breaking (see Table III and IV in [17] and Table 2 in [18], where nonet symmetry breaking and the pseudoscalar mixing angle have been algebraically related). All changes are actually tiny, confirming that breaking of isospin symmetry contributes little in this realm. However, two small changes can be noticed.

The first is that BR($\rho^0 \to \pi^0\gamma$) becomes larger than BR($\rho^\pm \to \pi^\pm\gamma$) by 4.7 % and the predicted branching fraction BR($\phi \to \pi^0\gamma$) increases by 8%. These are clearly consequences of breaking isospin symmetry. Otherwise, whatever the additional conditions stated, the general agreement of the reconstructed physics quantities with the data collected and averaged in the RPP can hardly be better.

Among the recent changes in the RPP, one should notice the branching fraction for $\phi \to \eta'\gamma$ which has now a central value in much better agreement with our model prediction. On the other hand, some new measurements have been recently reported which have not influenced the RPP recommended values (neither our fits) and might be commented.

First, the new measurement $\mathrm{Br}(\rho^0 \to e^+e^-) = (4.67 \pm 0.15)\ 10^{-5}$ reported by CMD-2 [46] remains in good agreement with our fit values. The second new measurement [47] $\mathrm{Br}(\rho \to \eta\gamma) = (3.28 \pm 0.36 \pm 0.24)\ 10^{-4}$ has a higher central value in better agreement with our reconstructed value, as for $\mathrm{Br}(\phi \to \eta\gamma) = (1.287 \pm 0.013 \pm 0.063)\ 10^{-2}$. The third new measurement [47] $\mathrm{Br}(\omega \to \eta\gamma) = (5.10 \pm 0.72 \pm 0.34)\ 10^{-4}$ is in relatively poorer agreement with our predictions than the RPP mean value [30].

The SND Collaboration has also published new results on $\eta\gamma$ decays of vector mesons [48–50]; the branching fractions reported are in good correspondence with our predictions. However, as for the CMD-2 result reported above, the new SND data for $\mathrm{Br}(\omega \to \eta\gamma) = (4.62 \pm 0.71 \pm 0.18)\ 10^{-4}$ might indicate that our prediction for this mode is slightly too large.

As the predictions for $\mathrm{Br}(\omega \to \eta\gamma)$ are alike whatever the conditions on the model, this (possible) 2σ disagreement could point towards a mass dependence of the mixing "angles".

Before closing this Section, it is of relevance to comment on a recent claim [51] that isospin symmetry breaking might be much larger in $\rho^0 \to \pi^0\gamma$ than anywhere else. From what has just been commented, it is clear that a $\simeq 5\%$ effect of isospin symmetry breaking is well accepted by all data, the former [30,52] and the recent SND datum as well ($\mathrm{Br}(\rho^0 \to \pi^0\gamma) = (4.3 \pm 2.2 \pm 0.04)\ 10^{-2}$) [53]. We have checked that the central value claimed by [51] (about a factor of 2 in rates) cannot be reproduced in consistency with the rest of radiative decays.

2. $\pi\pi$ Decay Modes

For the ϕ and ω decays to $\pi\pi$, we have used the recommended branching fractions [30] and the phases fit resp. by [38] and [23]. Table II shows that they are well reproduced by any of our fits. Taking into account the uncertainties already quoted for the Orsay phase, we even cannot rule out the solution given by the third data column.

The Collaboration CMD–2 has recently provided [20] $\mathrm{Br}(\omega \to \pi\pi) = (1.32 \pm 0.23)\%$ significantly smaller than the recommended value $(2.21 \pm 0.30)\%$ we have used, and $\mathrm{Br}(\phi \to \pi\pi) = (1.60 \pm 0.49)\ 10^{-4}$ about 2σ larger than the RPP value $(0.75 \pm 0.14)\ 10^{-4}$. No phase measurement has been correspondingly reported.

It is worth commenting on the possible effects of these new measurements. These have been examined within the framework of our preferred fit strategy (the one reported in the fourth data column of Table I).

We have first changed $\mathrm{Br}(\omega \to \pi\pi)$ to the new CMD–2 datum. The best fit obtained provides $\chi^2/\mathrm{dof} = 13.68/16$ (probability 62 %). The parameter values and errors are the same as in the fit reported in Table I, except that $\mathrm{Im}\delta$ yields a reduced magnitude (-0.029 ± 0.002 becomes -0.023 ± 0.002) ; on the other, hand μ_1 changes from 0.031 ± 0.005 to 0.020 ± 0.007. Finally, the contribution of the Orsay phase to the global χ^2 is about 0.12 and does not change, showing that the datum used remains consistent with the rest.

Having restored $\mathrm{Br}(\omega \to \pi\pi)$ to the RPP recommended value, we have changed the datum for $\mathrm{Br}(\phi \to \pi\pi)$ to the new result of CMD–2. The single significant change with respect to Table I is the value of μ_1 (0.031 ± 0.005 becomes 0.017 ± 0.001) and the fit returned $\chi^2/\mathrm{dof} = 13.44/16$ (probability 64 %).

Finally performing both changes simultaneously provides a fit with $\chi^2/\text{dof} = 14.83/16$ (probability 54%) with results merging the changes mentioned just above.

Therefore, even if some uncertainty remains for the values of the γ and δ angles, the model exhibits enough flexibility in order to accomodate significant changes in some crucial data. Actually, the two modes just commented determine almost solely the magnitude of isospin symmetry breaking.

It should also be noted that the changes just mentioned in the branching fractions do not give rise to inconsistencies with the phases of the corresponding coupling constants we have used, which thus look more firmly established.

3. VVP Couplings and 3π *Decays*

In all fit strategies and even by changing to new data as reported just above, the information concerning the VVP processes is remarkably stable.

One should thus note the nice agreement with the data reported by the SND Collaboration on the $\phi \to \omega\pi^0$ process [34,37] both in branching fraction and phase.

The SND datum [33] for $|g_{\phi\rho\pi}|$ is also reproduced with good accuracy. As the phase of this coupling constant is unfortunately not reported we have no reference datum to which our prediction could be compared. Such information is in principle accessible from fit to $e^+e^- \to \pi^+\pi^-\pi^0$ data [33], but the existence of a (complex) non resonant term[18] in the amplitude renders this extraction hasardous. It could also be accessed from $e^+e^- \to \omega\pi^0$ data but nothing is reported in this respect [36]. Such information, if reliable, could have been valuable as it could dismiss at least one of the fit strategies (see Table III).

Finally, the coupling constant $g_{\rho\omega\pi}$ is found consistent with real and its value falls indeed in the expected range [36]. It is found slightly but significantly smaller than the value prefered by [35] (14.3 GeV^{-1}). Its value is however extremely stable in all fits we attempted and looks accurate ; it should be noted that this parameter is only marginally influenced by isospin symmetry breaking and follows essentially from the set of radiative and leptonic decays.

The decay rates for $\omega/\phi \to \pi^+\pi^-\pi^0$ are, of course, determined by $g_{(\phi/\omega)\rho\pi}$ coupling constants *and* a model for the ρ propagator and the $\rho \to \pi\pi$ decay amplitude . Therefore, our model can be considered as giving a good description of these, up to effects related with modelling the ρ meson propagator for phenomenological purposes.

4. $\phi \to K\overline{K}$ *Decay Modes*

In all attempts we have performed, a non–negligible contribution to the χ^2 comes from both $\phi \to K\overline{K}$ decay modes. Whatever the strategy, the charged mode contributes to the χ^2 by 2.2 and the neutral mode by 1.9. However, when taking into account all sources of errors, Table I, clearly shows that the disagreement with reported data is not really dramatic.

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[18] This term might account for the box anomaly, but also for high mass resonances and this last effect seems hard to model in both modulus and phase.

On the other hand, it is admitted that model predictions for $\phi \to K^+K^-$ have to be corrected for Coulomb interactions [54,33], which was not done above. It has been recently shown [55] that there is a slight discrepancy between the branching fractions for charged and neutral decay modes (about 2σ) and that, accounting for Coulomb interactions among the (very) slow charged kaons, increases this discrepancy to 3σ.

Aware of this question, we have redone our fits by removing $\phi \to K^+K^-$ from the fit data set ; in this case we reached a fit quality of $\chi^2/\text{dof} = 9.04/15$ (88% probability). For symmetry, we have tried removing instead $\phi \to K^0\overline{K^0}$; we reached a fit quality of $\chi^2/\text{dof} = 9.94/15$ (82% probability). Trying to correct the model coupling constant as indicated in [54] only degraded the nominal fit quality. Therefore, we confirm in an independent way the problem raised by A. Bramon *et al.* [55].

In order to identify a (possible) faulty measurement, we have redone our fits by removing both ϕ modes from our fits. Focusing still on the model as given in the fourth data column in Table I, we reach a fit quality of $\chi^2/\text{dof} = 8.42/14$ (probability 87%). What is interesting here is to consider the χ^2 distance of the measurements to what is predicted by our model by relying only on the rest of the data (24 measurements). We got $\chi^2 = 3.76$ for the charged mode (a 2σ effect as pointed out by [55]), while the neutral mode yields $\chi^2 = 0.90$. When correcting the prediction for the charged mode by the Coulomb factor its χ^2 distance increased to $\chi^2 = 16.6$, a 4σ deviation.

Therefore, we confirm the issue raised by Bramon *et al.* [55], with an additional information : if one among $\phi \to K^+K^-$ and $\phi \to K^0\overline{K^0}$ is faulty, it should be the former, which seems overestimated[19]. Indeed, if we correct the model coupling constants in order to account for Coulomb correction (1.042 for the rate), the global fit quality sharply degrades ($\chi^2/\text{dof} = 21.40/16$, probability 16 %).

5. Comment on the Fit Quality

One may wonder about the fit quality reached in most of the likely fits we presented here. This comment applies also to all fits we performed prior to this paper when dealing with radiative and leptonic decays only [17,26,18]. Without being too upset about why "la mariée est trop belle", it is indeed an issue to understand why we quite easily get a mean χ^2 per degree of freedom of about $0.6 \div 0.8$, while one might rather expect fluctuating around 1.

Without really knowing the answer, we guess that the very reason is that the (fit) data recommended by the PDG [30] do not behave exactly like the mean value of a gaussian random variable with standard deviation given by the corresponding recommended error.

The motivation for this guess is the S–factor technique [30] widely used throughout the Review of Particle Properties in order to get reasonable average values for physics quantities. This technique allows to make consistent a collection of data affected by various systematic

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[19] It is interesting to note that systematics on $\phi \to K^+K^-$ are harder to estimate than those on $\phi \to K_L K_S$, because the modelling of nuclear interactions of low energy charged kaons is not still fully satisfactory. Instead, the signature of $K_S \to \pi^+\pi^-$ is much cleaner.

errors which would be inconsistent if one really trusts the reported errors. Therefore, one can guess that the final given errors, even if quite reasonable, are somewhat overestimated. When using a large amount of such reference values, this "S–factor effect" might indeed allow an appropriate model to reach easily quite favorable fit probabilities.

VIII. CONCLUSION

In previous work done with other coauthors, we focused on introducing SU(3) symmetry breaking and nonet symmetry breaking within the framework of the Hidden Local Symmetry Model [17]. We introduced also the $\omega - \phi$ mixing, generated by kaon loops effects, which does not correspond to any symmetry breakdown [17,26]. This framework, supplemented with these symmetry breaking mechanisms has been shown to provide quite a successful picture of all radiative and leptonic decays of light vector and pseudoscalar mesons accessible from inside the VMD framework. We have also shown that this framework was able to explain the main features of the $\eta - \eta'$ mixing phenomenon [18] in perfect agreement with all expectations of Chiral Perturbation Theory (ChPT) ; this led us to get a relation between the pseudoscalar (wave function) mixing angle, basically at work in VMD modelling ($\simeq -10^\circ$), and the ChPT mixing angle recently renamed θ_8 ($\simeq -20^\circ$).

In the present work, we have shown that isospin symmetry breaking can be accounted for within an effective HLS model by means of – essentially – kaon loop effects. In contrast with the case of $\omega_I - \phi_I$ mixing where both kaon loops (charged and neutral) come additively, in the case of $\rho_I - \omega_I$ and $\rho_I - \phi_I$ mixings, it is their difference which occurs. Relying on the properties of Dispersion Relations, this difference should be essentially a polynomial in s with real coefficients, which is additionally constrained to vanish at $s = 0$. We argued that this polynomial should not be identically zero, at least to account for isospin symmetry breaking in the pseudoscalar sector. Indeed, when isospin symmetry is not broken, it is quite legitimate to choose the same renormalization conditions for both the K^+K^- and $K^0\overline{K}^0$ loops ; instead, when isospin symmetry is broken, this requirement has certainly to be relaxed.

Other mechanisms than kaon loops could also be imagined. If they play by generating $\rho_I - \omega_I$ and $\rho_I - \phi_I$ transition amplitudes, the angle formalism we presented here still applies without any change. However, we have shown on the pion form factor, that all properties expected from isospin symmetry breaking are strikingly reproduced by the kaon loop mechanism we advocate. We then naturally recover all properties traditionally expected from the $\rho - \omega$ mixing amplitude : $\Pi_{\rho_I\omega_I}(s)$ is practically real in the $\rho - \omega$ peak invariant mass region, it is s–dependent and vanishes at the chiral point.

Moreover, we were able to derive the pion form factor in the Orsay phase formulation from our (effective) broken Lagrangian ; the Orsay phase was shown to be strictly equivalent to a "rotation" by a complex angle, additionally close to purely imaginary.

Using this framework, it has been possible to extend our breaking scheme in order to include isospin symmetry breaking. Actually, taking into account the various orders of magnitude of the breaking parameters and of the $\omega - \phi$ mixing, it is mathematically safer to define a full mixing scheme involving the triplet ρ, ω, ϕ as a whole. This leads us to define a priori a s–dependent rotation matrix, depending on three angles which can be real or complex.

We have thus formulated an effective Lagrangian model which is able to account quite successfully for practically all physics quantities related to VMD : radiative decays ($VP\gamma$, $P\gamma\gamma$) , leptonic decays (Ve^+e^-), VVP couplings, and all decays related with isospin symmetry breaking ($\omega/\phi \to \pi\pi$, $\phi \to \omega\pi$) in modulus and in phase. This represents 26 physics quantites all well reconstructed.

It should be noted that all results we previously obtained without introducing isospin symmetry breaking are confirmed, including the $\eta - \eta'$ and $\omega - \phi$ mixing angles.

Using all data from the Review of Particle Properties, or – for physics information not given there – results available from experimental literature, we have shown that isospin symmetry violations are always numerically small. This is especially true for corrections to the radiative decay mode $\rho \to \pi^0\gamma$ despite a recent claim. The most prominent effects of isospin violations remain the $\omega \to 2\pi$ and $\phi \to 2\pi$ decays which fix essentially their order of magnitude, while $\phi \to \omega\pi$ plays as a strong constraint.

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TABLES

Fixing Angle Properties	$\rho-\phi$ Imaginary only	$\omega-\phi$ Real only	$\omega-\rho$ Imaginary and $\rho-\phi$ Imaginary	$\omega-\rho$ and $\rho-\phi$ proportional
g	5.651 ± 0.017	5.652 ± 0.017	5.641 ± 0.017	5.652 ± 0.017
θ_P[deg.]	-10.33 ± 0.20	-10.32 ± 0.20	-10.33 ± 0.20	-10.34 ± 0.20
a [HLS]	2.517 ± 0.035	2.523 ± 0.034	2.485 ± 0.033	2.513 ± 0.035
ℓ_V	1.343 ± 0.021	1.337 ± 0.021	1.366 ± 0.021	1.346 ± 0.021
ℓ_T	1.231 ± 0.052	1.230 ± 0.052	1.232 ± 0.052	1.230 ± 0.052
Re[β]	-0.058 ± 0.003	-0.061 ± 0.002	-0.054 ± 0.003	-0.056 ± 0.003
Im[β]	-0.020 ± 0.005	**0.**	-0.028 ± 0.003	-0.029 ± 0.002
Re[δ]	$(0.52 \pm 0.18)\ 10^{-2}$	$(0.54 \pm 0.19)\ 10^{-2}$	**0.**	$(0.55 \pm 0.19)\ 10^{-2}$
Im[δ]	$(-0.29 \pm 0.02)\ 10^{-1}$	$(-0.29 \pm 0.02)\ 10^{-1}$	$(-0.31 \pm 0.02)\ 10^{-1}$	$(-0.29 \pm 0.02)\ 10^{-1}$
Re[γ]	**0.**	$(-0.57 \pm 0.15)\ 10^{-3}$	**0.**	$(.031 \pm .005)$ Re[δ]
Im[γ]	$(-0.96 \pm 0.18)\ 10^{-3}$	$(-1.16 \pm 0.16)\ 10^{-3}$	$(-1.06 \pm 0.18)\ 10^{-3}$	$(.031 \pm .005)$ Im[δ]
χ^2/dof Probability	12.88/16 63 %	17.06/16 38 %	20.94/17 23%	12.59/16 70%

TABLE I. Fit results under various strategies. Parameter values written boldface means that they are not allowed to vary ; this translates mathematically the fit condition given on the top of the Table.

Fixing Angle Properties	$\rho-\phi$ Imaginary only	$\omega-\phi$ Real only	$\omega-\rho$ $\rho-\phi$ Imaginary	$\omega-\rho$ $\rho-\phi$ proportional	PDG
$\rho \to \pi^0\gamma$ $[\times 10^4]$	5.36 ± 0.12	5.37 ± 0.13	5.18 ± 0.10	5.37 ± 0.13	6.8 ± 1.7
$\rho \to \pi^\pm\gamma$ $[\times 10^4]$	5.13 ± 0.10	5.13 ± 0.10	5.10 ± 0.10	5.13 ± 0.10	4.5 ± 0.5
$\rho \to \eta\gamma$ $[\times 10^4]$	3.18 ± 0.08	3.18 ± 0.08	3.15 ± 0.08	3.18 ± 0.08	$2.4^{+0.8}_{-0.9}$
$\eta' \to \rho\gamma$ $[\times 10^2]$	33.91 ± 3.16	33.92 ± 3.13	33.52 ± 3.04	33.93 ± 3.16	30.2 ± 1.3
$K^{*\pm} \to K^\pm\gamma [\times 10^4]$	9.89 ± 1.01	9.78 ± 1.01	9.85 ± 1.03	9.89 ± 1.01	9.9 ± 0.9
$K^{*0} \to K^0\gamma [\times 10^3]$	2.31 ± 0.33	2.31 ± 0.32	2.30 ± 0.32	2.31 ± 0.33	2.3 ± 0.2
$\omega \to \pi^0\gamma$ $[\times 10^2]$	8.49 ± 0.10	8.49 ± 0.10	8.48 ± 0.11	8.49 ± 0.10	8.5 ± 0.5
$\omega \to \eta\gamma$ $[\times 10^4]$	7.72 ± 0.15	7.74 ± 0.16	7.86 ± 0.14	7.69 ± 0.15	6.5 ± 1.0
$\eta' \to \omega\gamma$ $[\times 10^2]$	2.79 ± 0.26	2.77 ± 0.26	2.89 ± 0.26	2.79 ± 0.26	3.03 ± 0.31
$\phi \to \pi^0\gamma$ $[\times 10^3]$	1.37 ± 0.09	1.36 ± 0.09	1.38 ± 0.09	1.38 ± 0.09	1.26 ± 0.10
$\phi \to \eta\gamma$ $[\times 10^2]$	1.29 ± 0.02	1.28 ± 0.02	1.29 ± 0.02	1.29 ± 0.02	1.297 ± 0.033
$\phi \to \eta'\gamma$ $[\times 10^4]$	0.58 ± 0.02	0.59 ± 0.02	0.58 ± 0.02	0.58 ± 0.02	$0.67^{+0.35}_{-0.31}$
$\eta \to \gamma\gamma$ $[\times 10^2]$	39.45 ± 3.74	39.43 ± 3.74	39.32 ± 4.02	39.45 ± 3.74	39.33 ± 0.25
$\eta' \to \gamma\gamma$ $[\times 10^2]$	2.13 ± 0.20	2.13 ± 0.20	2.18 ± 0.19	2.13 ± 0.20	2.12 ± 0.14
$\rho \to e^+e^-$ $[\times 10^5]$	4.70 ± 0.16	4.73 ± 0.16	4.54 ± 0.15	4.69 ± 0.16	4.49 ± 0.22
$\omega \to e^+e^-$ $[\times 10^5]$	6.96 ± 0.21	6.94 ± 0.21	7.06 ± 0.22	6.97 ± 0.21	7.07 ± 0.19
$\phi \to e^+e^- [\times 10^4]$	2.96 ± 0.04	2.96 ± 0.04	2.96 ± 0.04	2.96 ± 0.04	2.91 ± 0.07
χ^2/dof Probability	12.88/16 63 %	17.06/16 38 %	20.94/17 23%	12.59/16 70%	

TABLE II. Reconstructed Branching fractions for radiative and leptonic decays using the various fit strategies. The last column displays the recommended values from the Review of Particle Properties [30]. The last line gives a reminder of the fit quality given in Table I.

Fixing Angle Properties	$\rho-\phi$ Imaginary only	$\omega-\phi$ Real only	$\omega-\rho$ $\rho-\phi$ Imaginary	$\omega-\rho$ $\rho-\phi$ proportional	PDG / Reference
$\phi \to K^+K^- [\times 10^2]$	50.25 ± 0.72	50.26 ± 0.71	50.24 ± 0.72	50.22 ± 0.73	49.2 ± 0.7
$\phi \to K^0_S K^0_L [\times 10^2]$	32.95 ± 0.47	32.95 ± 0.47	32.94 ± 0.48	32.92 ± 0.48	33.8 ± 0.6
$\omega \to \pi^+\pi^- [\times 10^2]$	2.23 ± 0.30	2.19 ± 0.29	2.32 ± 0.31	2.26 ± 0.30	2.21 ± 0.30
phase of $g_{\omega\pi^+\pi^-}$ [degr]	103.50 ± 4.02	106.3 ± 3.81	92.34 ± 0.69	103.40 ± 3.88	[52] 104.7 ± 4.1
$\phi \to \pi^+\pi^- [\times 10^5]$	7.93 ± 1.40	8.15 ± 1.45	7.60 ± 1.24	7.70 ± 1.43	7.5 ± 1.4
phase of $g_{\phi\pi^+\pi^-}$ [degr]	146.30 ± 3.95	147.5 ± 4.09	146.0 ± 3.75	145.95 ± 3.93	[38] 146.0 ± 4.0(*)
$\phi \to \omega\pi^0 [\times 10^5]$	4.10 ± 0.48	3.62 ± 0.41	4.24 ± 0.49	4.18 ± 0.49	4.8 ± 2.0
phase of $g_{\phi\omega\pi^0}/g_{\omega\rho\pi^0}$ [degr]	-50.90 ± 3.63	-61.91 ± 3.05	-52.38 ± 3.27	-47.92 ± 3.52	[37] -49 ± 7.07
coupling $g_{\phi\rho\pi^0}$ GeV^{-1}	0.802 ± 0.026	0.799 ± 0.026	0.803 ± 0.026	0.803 ± 0.026	[33] 0.815 ± 0.021
phase of $g_{\phi\rho\pi^0}/g_{\omega\rho\pi^0}$ [degr]	19.28 ± 4.71	0.22 ± 0.11	27.98 ± 3.53	23.98 ± 3.47	–
coupling $g_{\omega\rho\pi^0}$ GeV^{-1}	13.14 ± 0.09	13.14 ± 0.09	13.09 ± 0.081	13.14 ± 0.09	(see text) $11.7 \div 16.1$
χ^2/dof Probability	12.88/16 63 %	17.06/16 38 %	20.94/17 23%	12.59/16 70%	

TABLE III. Reconstructed Branching fractions from various fit strategies, cont'd. The last column displays the recommended values from the Review of Particle Properties [30]. The last line reminds the global fit quality given in Table I. The datum indicated by (*) has been corrected in order to absorb a minus sign (see text).

APPENDICES

In order to be self–contained, we collect in this Appendix formulae for coupling constants and partial widths ; we do not insist much on how U(3)/SU(3) breaking is performed in the present paper, as it is the matter of already published work [15,26,17,18] to which the interested reader can refer.

APPENDIX A: DETAILS OF THE BREAKING MODEL

Our framework is the HLS model and the SU(3) breaking procedure we follow has been defined first in [16,15]. Focusing on the anomalous sector [12], all details can be found in [17,26,18]. Here, we mainly recall breaking parameter properties or values of concern for the present study.

Breaking the non–anomalous sector of the HLS model [11,16,15] introduces a breaking parameter Z strongly associated with the pseudoscalar (PS) sector ; it is not a free parameter but fulfills $Z = [f_\pi/f_K]^2 = 2/3$.

Concerning the PS sector, we have a priori 2 additional parameters. The first is named x and its departure from 1 measures breaking of nonet symmetry in the PS sector. Another parameter affecting the PS sector is the PS mixing angle θ_P (which describes the η/η' sector in terms of mixtures of singlet and octet components) *or* δ_P (when one prefers referring to departures from ideally mixed states). Both angles are used and are trivially related to each other [17].

When studying the connection between VMD, the Wess–Zumino–Witten Lagrangian and Chiral Perturbation Theory, it has been found [18] that the PS mixing angle θ_P and the nonet symmetry breaking parameter x fulfill :

$$\tan\theta_P = \sqrt{2}\frac{Z-1}{2Z+1}x \tag{A1}$$

with high accuracy (in [18], this relation is given in terms of $z = 1/Z$). Preliminary fits have shown that this relation is still satisfied in the present framework without any degradation ; thus it is assumed. We remind that the mixing angle θ_P relates to the (now) more usual ChPT mixing angle θ_8 by [18] $\theta_8 \simeq 2\theta_P$. Therefore, concerning the PS sector, our model depends only on one free parameter which can be either of θ_P or x. We choose the former.

Associated with the vector sector and, more precisely with vector meson masses, another breaking parameter occurs named here ℓ_V ; it relates with another breaking parameter (c_V) [16,15] by $\ell_V = (1+c_V)^2$. It is a priori subject to fit, and thus free, as the connection between reported vector meson masses [30] and the corresponding masses occuring in the HLS Lagrangian is unclear [26].

Concerning vector mesons, another breaking parameter is necessary in order to account for the anomalous K^* sector ; it is named [17,26] ℓ_T. It was first considered as somewhat ad hoc [17] ; however, it has been shown [26] that it strictly corresponds within VMD to a breaking parameter defined independently by G. Morpurgo [44] within the non–relativistic quark model and found in agreement with low energy QCD. If the partial width value for $K^{*\pm} \to K^\pm\gamma$ is confirmed, this parameter looks unavoidable within VMD ; its precise meaning is still to be understood [26].

A possible break up of nonet symmetry in the vector sector has been found previously undetectable (the parameter y defined and studied in [26]). Preliminary fits in the present study confirmed this conclusion and, therefore, the parameter y was set to 1 definitely.

Thus, concerning SU(3) symmetry breaking, the HLS vector sector depends already on 2 free parameters ℓ_V and ℓ_T, independently of mixing among the ideal field combinations ω_I, ϕ_I and ρ_I associated with neutral vector mesons. This last point is the actual subject of the present paper.

APPENDIX B: BASIC COUPLING CONSTANTS AND PATIAL WIDTHS

We give in this Section all coupling constants which cannot be trivially read off the Lagrangian pieces given in the main text.

1. Radiative Decays

Starting from the Lagrangian in Eq. (34), and using the breaking procedure as defined in Refs [26,17,18], one can compute the coupling constants for all radiative and leptonic decays of relevance. Let us define :

$$G = -\frac{3eg}{8\pi^2 f_\pi} \quad , \quad G' = -\frac{3eg}{8\pi^2 f_K} \quad , \quad Z = [f_\pi / f_K]^2 \quad . \tag{B1}$$

Some $VP\gamma$ coupling constants are not affected by the parameters of isospin symmetry breaking. These are :

$$\left\{ \begin{array}{ll} G_{\rho^\pm \pi^\pm \gamma} = & \frac{1}{3} G \\ G_{K^{*0} K^0 \gamma} = & -\frac{G'}{3} \sqrt{\ell_T} (1 + \frac{1}{\ell_T}) \\ G_{K^{*\pm} K^\pm \gamma} = & \frac{G'}{3} \sqrt{\ell_T} (2 - \frac{1}{\ell_T}) \quad . \end{array} \right. \tag{B2}$$

The $\rho_I P\gamma$ coupling constants are :

$$\left\{ \begin{array}{ll} G_{\rho_I \pi^0 \gamma} = & \frac{1}{3} G \\ G_{\rho_I \eta \gamma} = & \frac{1}{3} G \left[\sqrt{2}(1-x) \cos\delta_P - (2x+1) \sin\delta_P \right] \\ G_{\rho_I \eta' \gamma} = & \frac{1}{3} G \left[\sqrt{2}(1-x) \sin\delta_P + (2x+1) \cos\delta_P \right] \quad . \end{array} \right. \tag{B3}$$

The other single photon radiative modes provide the following coupling constants :

$$
\left\{
\begin{array}{ll}
G_{\omega_I \pi^0 \gamma} = & G \\
G_{\phi_I \pi^0 \gamma} = & 0 \\
G_{\omega_I \eta \gamma} = & \frac{1}{9} G \left[\sqrt{2}(1-x) \cos\delta_P - (1+2x) \sin\delta_P \right] \\
G_{\omega_I \eta' \gamma} = & \frac{1}{9} G \left[(1+2x) \cos\delta_P + \sqrt{2}(1-x) \sin\delta_P \right] \\
G_{\phi_I \eta \gamma} = & -\frac{2}{9} G \left[Z(2+x) \cos\delta_P - \sqrt{2} Z(1-x) \sin\delta_P \right] \\
G_{\phi_I \eta' \gamma} = & -\frac{2}{9} G \left[\sqrt{2} Z(1-x) \cos\delta_P + Z(2+x) \sin\delta_P \right] \quad .
\end{array}
\right. \tag{B4}
$$

2. $P\gamma\gamma$ and $V-\gamma$ Modes

The 2–photon decay modes are not affected by isospin symmetry breaking in the vector sector and keep their usual form within the HLS model [17,26,18] :

$$
\left\{
\begin{array}{ll}
G_{\eta\gamma\gamma} = & -\frac{\alpha_{em}}{\pi\sqrt{3} f_\pi} \left[\frac{5-2Z}{3} \cos\theta_P - \sqrt{2} \frac{5+Z}{3} x \sin\theta_P \right] \\
G_{\eta'\gamma\gamma} = & -\frac{\alpha_{em}}{\pi\sqrt{3} f_\pi} \left[\frac{5-2Z}{3} \sin\theta_P + \sqrt{2} \frac{5+Z}{3} x \cos\theta_P \right] \\
G_{\pi^0\gamma\gamma} = & -\frac{\alpha_{em}}{\pi f_\pi} \quad .
\end{array}
\right. \tag{B5}
$$

As stated in the text, we actually replace this last coupling by the world average value for f_π as given in the RPP [30].

Finally, the leptonic decay widths of vector mesons depend on the HLS $V-\gamma$ couplings. For the ideal combinations, we have :

$$
\left\{
\begin{array}{ll}
f_{\rho_I \gamma} = & a f_\pi^2 g \\
f_{\omega_I \gamma} = & \frac{f_{\rho_I \gamma}}{3} \\
f_{\phi_I \gamma} = & \frac{f_{\rho_I \gamma}}{3} \sqrt{2} \ell_V \quad .
\end{array}
\right. \tag{B6}
$$

3. Partial widths

We list for completeness in this Section the expressions for the partial widths in terms of the coupling constants for the various cases.

The two–photon partial widths are :

$$
\Gamma(P \to \gamma\gamma) = \frac{m_P^3}{64\pi} |G_{P\gamma\gamma}|^2 \quad , \quad P = \pi^0, \ \eta, \ \eta' \quad . \tag{B7}
$$

The leptonic partial widths are :

$$\Gamma(V \rightarrow e^+e^-) = \frac{4\pi\alpha^2}{3m_V^3}|f_{V\gamma}|^2 \quad . \tag{B8}$$

The radiative widths are :

$$\Gamma(V \rightarrow P\gamma) = \frac{1}{96\pi}\left[\frac{m_V^2 - m_P^2}{m_V}\right]^3 |G_{VP\gamma}|^2 \quad , \tag{B9}$$

where V is either of ρ^0, ω, ϕ and P is either of π^0, η, η', and :

$$\Gamma(P \rightarrow V\gamma) = \frac{1}{32\pi}\left[\frac{m_P^2 - m_V^2}{m_P}\right]^3 |G_{VP\gamma}|^2 \quad . \tag{B10}$$

The decay width for a vector meson decaying to $V + P$ is :

$$\Gamma(V' \rightarrow VP) = \frac{1}{96\pi}\left[\frac{\sqrt{[m_{V'}^2 - (m_V + m_P)^2][m_{V'}^2 - (m_V - m_P)^2]}}{m_{V'}}\right]^3 |G_{V'VP}|^2 \quad . \tag{B11}$$

Finally, the partial width for a vector meson decaying into two pseudoscalar mesons of equal masses is :

$$\Gamma(V \rightarrow PP) = \frac{1}{48\pi}\frac{[m_V^2 - 4m_P^2]^{3/2}}{m_V^2}|G_{VPP}|^2 \quad . \tag{B12}$$

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