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Fleischer–Mannel analysis for direct CP asymmetry

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Abstract

We apply the method of Fleischer and Mannel to extract information on $\sin\alpha$ in the charged B system. Hadronic contributions are fixed through appeal to data allowing one to cleanly interpret the CP asymmetry without assuming dominance of the top quark penguin contribution.

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In the Standard Model (SM) CP violation arises exclusively from a phase in the CKM matrix [1,2]. If the SM is complete this matrix must be unitary giving, for example,

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{1}$$

There are nine such relationships, of which only three are independent. Of these three, Eq. (1), relevant to b quark interaction, has a special status, as the other two contain one term that is very small making them difficult to test experimentally. Eq. (1) can be conveniently represented by a triangle in the complex plane, whose angles are given by

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \quad \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \quad \gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right). \tag{2}$$

The unitarity condition can then be conveniently written in a form independent of the particular convention used for the CKM matrix elements

$$\alpha + \beta + \gamma = \pi\big|_{\mathrm{mod}\ 2\pi}\,. \tag{3}$$

The equality only holds modulo 2π because α, β and γ, in principle, can be either the internal or external angles of a triangle depending on the sign of $\sin\delta_{\mathrm{CKM}}$ [3], though it is usually assumed that $\sin\delta_{\mathrm{CKM}} > 0$ and hence the angles are internal [4] (see however, Ref. [5]). This relation therefore provides a significant test of the Standard Model making a detailed study of the weak decays of B mesons one of the major goals of contemporary physics (for reviews see Ref. [6]).

Usually the neutral B system is considered for this purpose, as the asymmetries from direct CP violation in the charged B sector are often subject to considerable hadronic uncertainty. However the ease of tagging differently charged particles lends motivation to a theoretical study of charged B decay that would eliminate unknown hadronic contributions to the asymmetries and allow for a clean extraction of CKM information. Moreover, neutral B experiments typically yield $\sin 2\alpha$, leading to a discrete four-fold ambiguity in the determination of α [4]. Therefore, our interest lies in deducing complementary information from charged B experiments to lessen this ambiguity.

Recent studies of direct CP violation due to $\rho-\omega$ mixing in decays in both the mesonic [7–9] and baryonic [10] b hadrons suggest the study of systems in which the non-perturbative strong physics is phenomenologically well constrained through vector meson dominance (VMD) (for a review of VMD see, for example, Ref. [11]). Here we shall be concerned with the extraction of CKM information free from hadronic uncertainty.

We shall follow the approach of Fleischer and Mannel (FM) [12] to eliminate the possible hadronic uncertainty in the penguin terms. Let us begin by considering the general definition of the CP asymmetry in the charged system

$$a = \frac{|A^-|^2 - |A^+|^2}{|A^-|^2 + |A^+|^2} = \frac{1 - |A^+/A^-|^2}{1 + |A^+/A^-|^2} \equiv \frac{1 - |\xi|^2}{1 + |\xi|^2} \tag{4}$$

where A^- is the negative particle decay amplitude and A^+ that for the positive, such that $A^+ = CP[A^-]$. We now wish to focus on the CP parameter, ξ, defined in Eq. (4). To do this we need to build up amplitudes in which CP violation is possible, through strong and weak phase differences. As usual, we shall consider tree and penguin contributions to supply the weak phase difference. Following Refs. [7,8] we shall consider $A^- = B^- \to \rho^-(\pi^+\pi^-)$ where the $\pi^+\pi^-$ pair is produced in the $\rho-\omega$ resonance region. As seen in $e^+e^- \to \pi^+\pi^-$ data [13], there is a considerable G-parity violating contribution of the ω to $\pi^+\pi^-$ production. In Ref. [7] it was suggested that this could supply the necessary strong phase difference through $\rho-\omega$ mixing (for a brief review of $\rho-\omega$ mixing see, e.g., Ref. [14]). We thus introduce the hadronic quantity,

$$X \equiv \frac{\widetilde{\Pi}_{\rho\omega}}{s - m_\omega^2 + im_\omega\Gamma_\omega} \equiv \frac{\widetilde{\Pi}_{\rho\omega}}{s_\omega} = \frac{\widetilde{\Pi}_{\rho\omega}}{|s_\omega|^2}(s - m_\omega^2 - im_\omega\Gamma_\omega), \tag{5}$$

introducing the ω propagator term s_ω in an obvious fashion. Though, as an isospin violating quantity, $\widetilde{\Pi}_{\rho\omega} = 3500\text{MeV}^2$ is small compared with, say, m_ρ^2, the narrowness of the ω width ($\Gamma_\omega = 8.4\text{MeV}$) allows X to differ appreciably from unity near $s \sim m_\omega^2$, and to develop the necessary strong phase. All other sources of isospin violation are always small and unimportant.

The problem now reduces to the vector analogue of $B \to \pi\pi$ considered by FM, i.e., $B^- \to \rho^-\rho^0$. For this case, we can factor the weak phase from the tree amplitude, T, through

$$T \equiv e^{i\gamma}t, \tag{6}$$

where the weak phase γ (see Eq. (2)) arises from the tree CKM term $V_{ud}V_{ub}^*$. We therefore have

$$\xi \equiv \frac{A^+}{A^-} = \frac{\overline{T} + \overline{P}X}{T + PX} = \frac{te^{-i\gamma} + \overline{P}X}{te^{i\gamma} + PX} = \frac{e^{-2i\gamma} + e^{-i\gamma}\overline{P}X/t}{1 + e^{-i\gamma}PX/t}. \tag{7}$$

It is now convenient to introduce the unitarity triangle side length

$$R_t \equiv \sqrt{(1-\rho)^2 + \eta^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right| = \frac{\sin\gamma}{\sin\alpha}. \tag{8}$$

The extraction of R_t is possible, for example, through a comparison of the kaon partial decay widths $\Gamma(K^+ \to \pi^+\nu\bar{\nu})/\Gamma(K_L \to \pi^0\nu\bar{\nu})$ (see Eq. (38) of Ref. [15]). With this we have

$$\xi = e^{-2i\gamma}(1 + 2i\lambda R_t|P'|e^{i\delta'}X\sin\alpha/t) \tag{9}$$

and the asymmetry is thus given by

$$a_{2\pi} = 2\lambda R_t \frac{|P'|}{t}\frac{\widetilde{\Pi}_{\rho\omega}}{|s_\omega|^2}\left[(s - m_\omega^2)\sin\delta' - m_\omega\Gamma_\omega\cos\delta'\right]\sin\alpha + \mathcal{O}\left(|P'/t|^2\right), \tag{10}$$

where the 2π represents the decay $B^- \to \rho^-\pi^+\pi^-$ in the $\rho-\omega$ resonance region.

This alone would not be so interesting, but it is possible by examining other decay channels to determine the RHS of Eq. (9) almost totally from experimental input. This is possible because the QCD penguin amplitude for the decay $b \to s$ is given by

$$P^{(s)} = \sum_{q=u,c,t} V_{qs}V_{qb}^*P_q^{(s)} = -|V_{cb}|(1 - \Delta P^{(s)})|P_{tu}^{(s)}|e^{i\delta_{tu}^{(s)}}. \tag{11}$$

In the limit of exact SU(3) symmetry, for which the s and d quarks are equivalent, this coincides exactly with P' of Eq. (7). This approximation for the penguin terms is justified at the b scale where both s and d quarks are effectively massless. From a comparison of masses and lifetimes

$$\begin{array}{lcc} & M(\mathrm{GeV}) & \tau(10^{-12}s) \\ B^{\pm} & 5.28 & 1.62 \pm 0.06 \\ B^0 & 5.28 & 1.56 \pm 0.06 \\ B_s & 5.37 & 1.61 \pm 0.10 \end{array} \tag{12}$$

we see SU(3) symmetry is very good for the B sector. Thus the penguin structure of $b \to s$ decays can can provide experimental input to the general expression for the CP asymmetry. However, to be truly helpful we need to find a decay $B \to f$ with no tree contribution and the same short distance physics as the penguin term in $B^- \to \omega\rho^-$. To analyse this, we shall appeal to the factorisation approximation, where assuming isospin symmetry (ie that ρ and ω are equivalent)

$$P_\omega^{(d)} = \sqrt{2}(a_3 + a_4 + a_5). \tag{13}$$

The decay $B_s \to \phi\phi$ has no tree contribution, and its short distance term is given by

$$\mathcal{A}^{\mathrm{short}}(B_s \to \phi\phi) = P_\phi^{(s)} = 2(a_3 + a_4 + a_5) = \sqrt{2}P_\omega^{(d)}. \tag{14}$$

However, in the factorisation approximation the full amplitude for $B_s \to \phi\phi$ is given by the product of $B_s \to \phi$, for which SU(3) symmetry is very good, and vacuum$\to \phi$, where we might expect SU(2) symmetry to be reasonable, but not SU(3) symmetry. Therefore we need to account for this in Eq. (14) through [16]

$$f \equiv \frac{\langle\phi|V_\mu|0\rangle}{\langle\omega|V_\mu|0\rangle} \sim 1.38. \tag{15}$$

This leads to

$$\left|\frac{P'}{t}\right| = \frac{1}{f}\left[\frac{\Gamma(B_s \to \phi\phi)}{2\Gamma(B^- \to \rho^-\rho^0)}\right]^{1/2}. \tag{16}$$

The asymmetry is therefore given by

$$a_{2\pi} = \frac{2\lambda R_t}{f}\left[\frac{\Gamma(B_s \to \phi\phi)}{2\Gamma(B^- \to \rho^-\rho^0)}\right]^{1/2} \frac{\widetilde{\Pi}_{\rho\omega}[m_\omega\Gamma_\omega \cos\delta' - (s - m_\omega^2)\sin\delta']}{(s - m_\omega^2)^2 + m_\omega^2\Gamma_\omega^2} \sin\alpha. \tag{17}$$

In Eq. (17) $\widetilde{\Pi}_{\rho\omega}$ is accurately known from fitting $e^+e^- \to \pi^+\pi^-$ data [17,18]

$$\widetilde{\Pi}_{\rho\omega} = -3500 \pm 300\ \mathrm{MeV}^2 \tag{18}$$

with no statistically significant imaginary part or momentum dependence in the ω resonance region. Similarly for the isospin conserving ω case, $B^- \to \rho^-\omega$ where $\omega \to \pi^+\pi^0\pi^-$, we have

$$a_{3\pi} = \frac{2\lambda R_t}{f}\left[\frac{\Gamma(B_s \to \phi\phi)}{2\Gamma(B^- \to \rho^-\rho^0)}\right]^{1/2} \sin\delta' \sin\alpha \tag{19}$$

which can be obtained by susbtituting $X = 1$ in Eq. (9).

A comparison of $a_{2\pi}$ and $a_{3\pi}$ therefore allows us to extract $\tan\delta'$ and hence determine the magnitude of $\sin\alpha$, enabling us to halve the four-fold ambiguity. Together with the sign of $\sin\alpha$ obtained through a determination of the short distance strong phase from factorisation calculations [8] this should allow for the elimination of the four-fold ambiguity in $\sin 2\alpha$.

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