Bulk Physics at a Graviton Factory*

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Abstract

A general prediction of the 5-d Randall-Sundrum (RS) hierarchy model is the emergence of spin-2 Kaluza Klein (KK) gravitons with weak scale masses and couplings. The lowest order effective theory of the RS model is given by 5-d Einstein gravity which uniquely fixes the self-interactions of gravitons. We demonstrate that large numbers of light KK resonances could be produced at a future lepton-collider-based “Graviton Factory”. Measuring the self-interactions of these KK gravitons will probe the accuracy of the 5-d Einstein gravity picture and, in addition, yield indirect information on the as yet untested self-coupling of the 4-d graviton. The self-interactions of the gravitons can be studied by measuring the decays of the heavier states to the lighter ones. We show that these decays have sufficient rates to be studied at future colliders and that they are also, in principle, sensitive to higher derivative operators, such as the Gauss-Bonnet term. In a generalized RS model, with non-universal 5-d fermion masses, FCNC’s will be induced. We show that precise measurements of the rare decays of KK graviton/gauge states into flavor non-diagonal final states at a Graviton Factory can be used to map the 5-d fermion mass matrix.

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1 Introduction

The Randall-Sundrum (RS) model offers an interesting geometric view of the hierarchy between the apparent scale of gravity $\overline{M}_P \sim 10^{18}$ GeV and the weak scale $M_w \sim 10^2$ GeV [1]. The RS model background geometry is AdS$_5$ (5-d spacetime with constant negative curvature), truncated by two 4-d Minkowski walls separated by a fixed distance. All the parameters of the model are assumed to be naturally given by various powers of the 5-d fundamental scale $M_5 \sim \overline{M}_P$. In the original RS construct, the Standard Model (SM) fields are confined to one of the 4-d boundaries which we will refer to as the SM wall. The induced metric on the SM wall then generates from $M_5$ a physical scale $\Lambda_\pi \sim M_w$, through a geometric exponential warp factor. Due to this exponentiation, no large hierarchies need to be introduced; for more details on the RS construct please see Ref. [1].

A generic prediction of the RS model is the emergence of spin-2 resonances $G^{(n)}$, $n = 1, 2, 3, \ldots$, which are the Kaluza-Klein (KK) excitations of the 5-d graviton. The masses and couplings of $G^{(n)}$ are set by $\Lambda_\pi$, and hence, these resonances could be relevant to physics at the weak scale. Many recent works have studied the phenomenological effect of $G^{(n)}$ on collider and precision electroweak data [2]. These studies have focused on the leading coupling of $G^{(n)}$ to the SM field content. The RS model is based on an effective 5-d Einstein Gravity (EG$_5$) theory which describes the coupling of the graviton to the SM fields, as well as its self-couplings. The $G^{(n)}$ are the 4-d manifestations of the 5-d graviton. Hence, to test the model at leading order, one must go beyond the $G^{(n)}$–SM coupling and also probe the leading order inter-couplings of $G^{(n)}$ which are fixed, given $\Lambda_\pi$ from collider data. This is similar to testing QCD at tree level, which requires agreement between theory and experiment, not only for the quark-gluon coupling, but also for the gluon self-couplings. The inter-couplings of $G^{(n)}$ can also yield indirect evidence for the as yet unobserved massless 4-d graviton self-coupling, as well as a handle on the coefficients of possible 5-d higher derivative terms, such as the Gauss-Bonnet term.

The original RS model has also been generalized to allow for the SM fields to reside in the 5-d bulk. An important set of parameters that enters this generalized RS model is the 5-d mass matrix of the SM bulk fermions. It has been shown that the value $\nu$ of the 5-d fermion mass in units of $k$, where $k$ is the curvature scale of
AdS$_5$, has a significant role in the low energy phenomenology of the model [3], since it controls the couplings of the zero mode (observed) fermion to $G^{(n)}$ and the KK gauge fields $A^{(n)}$. It has been pointed out that giving various flavors of fermions different 5-d masses induces FCNC’s at tree level [4]. Thus, these masses must be fairly universal in order to avoid unacceptably large FCNC’s. One way of probing the 5-d fermion mass matrix is to search for presumably rare flavor non-diagonal decays of $G^{(n)}$ and $A^{(n)}$.

To probe graviton self-coupling or the 5-d masses of the fermions with precision in the aforementioned manner, a large number of KK excitations are needed. In this paper, we show that if a generic RS-type model is realized in Nature an appropriate weak scale Linear Collider (LC) can be used to produce large numbers, $\sim 10^7$, of light KK gravitons. In this sense, the LC can operate as a “Graviton Factory”. Of course, if the SM fields reside in the 5-d bulk, we expect to produce comparably large numbers of light KK gauge fields as well.

As an example of the $G^{(n)}$ inter-coupling, we study the decay $G^{(3)} \rightarrow G^{(1)}G^{(1)}$, which is the first kinematically allowed decay within the graviton sector. We give the branching fraction for this decay as a function of the ratio of the masses in the initial and final states; it is found that the dependence on the mass ratio is fairly strong. However, at the classical level in the RS model, the mass ratio is fixed by the geometry and the branching fraction for $G^{(3)} \rightarrow G^{(1)}G^{(1)}$ is about 15%. These results are presented in section 2, where brief comments on the effect of 5-d higher derivative terms are also included.

In section 3, we study a model in which all SM fermions except the top quark will be assumed to have the same value of $\nu$. We then show the effects of this mass matrix in terms of flavor non-diagonal decays of $G^{(1)}$ and $A^{(1)}$. Although this model is perhaps too simplistic to be phenomenologically relevant, it demonstrates the utility of studying such decays at future Graviton Factories in probing the bulk fermion mass matrix. Some concluding remarks are presented in section 4.

### 2 Graviton Sector Self-Interactions

As mentioned before, the RS model generically predicts the emergence of spin-2 KK gravitons $G^{(n)}$ in the 4-d effective theory. The equations of motion for $G^{(n)}$ are
obtained through the KK reduction of the EG\(_5\) action \(S_G\), containing the 5-d graviton kinetic term. We have

\[
S_G = 2 M_5^3 \int d^5 x \sqrt{-G} R_5, \tag{1}
\]

where \(G = \text{det}(G_{\mu\nu})\), \(G_{\mu\nu}\) is the RS background metric, and \(R_5\) is the 5-d Ricci scalar.

The 4-d interactions of the \(G^{(n)}\) states with the SM fields are given by the following Lagrangian

\[
\mathcal{L}_{SM} = -\left[ \frac{1}{M_P} h^{(0)}_{\mu\nu} + \frac{1}{\Lambda^\pi} \sum_{n=1}^{\infty} h^{(n)}_{\mu\nu} \right] T^{\mu\nu}, \tag{2}
\]

where the 4-d tensor fields \(h^{(n)}_{\mu\nu}\) represent the \(G^{(n)}\) states, and \(T^{\mu\nu}\) is the energy momentum tensor of SM which is assumed to reside on the SM wall, in Eq. (2). Phenomenological studies of the RS model have so far focused on the interactions coming from \(\mathcal{L}_{SM}\), such as the on-shell decays of \(G^{(n)}\) into SM fields. These interactions are only suppressed by one power of \(\Lambda^\pi\) for \(n = 1, 2, 3, \ldots\).

However, there are also interactions amongst \(G^{(n)}\), resulting from the KK reduction of the self-coupling of the 5-d graviton in \(S_G\). In powers of \(M_5^{-1}\), the leading 5-d self-coupling is the triple graviton vertex. This indicates that in 4-d the \(\{G^{(l)}, G^{(m)}, G^{(n)}\}\) coupling is, in powers of \(\Lambda^\pi^{-1}\), the leading interaction in the KK graviton sector. This can be seen by using the KK graviton wavefunctions [5], and inspecting the triple KK graviton coupling of \(S_G\). Thus, to test the RS model to leading order in \(\Lambda^\pi^{-1}\), one must also study the \(\{G^{(l)}, G^{(m)}, G^{(n)}\}\) couplings. We note that quartic couplings of the \(G^{(n)}\) are higher order in \(\Lambda^\pi^{-1}\), as they are for the 5-d graviton in powers of \(M_5^{-1}\). In principle, there could be higher derivative terms, such as \(R_5^2\), in the 5-d theory. However, their contribution to the \(\{G^{(l)}, G^{(m)}, G^{(n)}\}\) couplings are higher order in \(\Lambda^\pi^{-1}\). We will briefly discuss the \(R_5^2\) terms at the end of this section.

In the following, we study the process \(G^{(3)} \rightarrow G^{(1)} G^{(1)}\) which is the first kinematically allowed two body decay in the KK sector. To do this, we need the triple graviton vertex. This vertex has the same tensor structure regardless of the number of dimensions and the background geometry. Hence, we use the Feynman rules of Ref. [6] for the 3-graviton vertex \(V_{\{\mu_i, \nu_i\}}(K_i), i = 1, 2, 3\), where \(\{\mu_i, \nu_i\}\) are 4-d Lorentz indices and \(K_i\) is the 4-momentum of the particle labeled \(i\), in Ref. [6]. Since \(V_{\{\mu_i, \nu_i\}}(K_i)\) has the same tensor structure in 5-d, we use the same function for the 5-d graviton triple coupling, and treat \(\{\mu_i, \nu_i\}\) and \(K_i\) as 5-dimensional. To get the decay rate for \(G^{(3)} \rightarrow G^{(1)} G^{(1)}\), we must square \(V_{\{\mu_i, \nu_i\}}(K_i)\) and contract it with the
product of three massive graviton polarization sums, to project out the appropriate
degrees of freedom. This sum is given in Ref. [7] for the case of flat extra dimensions.
However, the degrees of freedom of the 4-d KK gravitons are the same for the RS and
the flat cases, independent of the higher dimensional theory.

The amplitude for the on-shell decay is obtained when the vertex is contracted
with the 4-d massive spin-2 polarization vectors of the KK states. In this way, we see
that the only relevant indices are 4-d and we can take \( \{\mu_i, \nu_i\} \) to be 4-d Minkowskian.
However, the scalar products of the 5-momenta, \( K_i \cdot K_j \), in \( V(\mu_i, \nu_i)(K_i) \) must be
treated carefully. By using the 4-d massive graviton polarization sum we have ex-
tracted the appropriate tensor structure, but to complete the reduction from 5-d to
4-d, these scalar products should be expressed in terms of 4-d quantities. The 5-
momenta \( K^M_i, M = 0, 1, \ldots, 4 \), in the Feynman rules correspond to 5-d derivatives
\( \partial^M \), acting on the graviton field \( h_i \). Thus, \( K_i \cdot K_j \) corresponds to terms proportional
to \( G^{MN} \partial_M h_i \partial_N h_j \), in the 5-d action.

The 5-d graviton can be expanded in the KK modes \( h^{(n)}_{\mu\nu} \)

\[
h_{\mu\nu}(x, \phi) = \sum_{n=0}^{\infty} h^{(n)}_{\mu\nu}(x) \frac{e^{-2\sigma \chi^{(n)}(\phi)}}{\sqrt{r_c}}, \tag{3}
\]

where the \( \mathbb{Z}_2 \) even wavefunctions \( \chi^{(n)}(\phi) \) only depend on the 5\(^{th} \) dimension parameterized by \( \phi \in [-\pi, \pi] \); \( x^4 = r_c \phi \), \( \sigma = kr_c |\phi| \), and \( r_c \sim 10k^{-1} \) is the compactification
radius. Note that, in Eq. (3), the graviton is taken to be a 5-d tensor fluctuation
on the AdS\(_5\) background. The 4-d derivatives \( \partial_\mu \) do not act on \( e^{-2\sigma \chi^{(n)}(\phi)} \) and thus,
after integration over the extra dimension, the 4-d part of the dot product \( K_i \cdot K_j \) is
an overall constant times the 4-d Minkowski scalar product, for a given vertex with
fixed KK fields. However, \( \partial_\phi \) acts on \( e^{-2\sigma \chi^{(n)}(\phi)} \) and, hence, the 5-d part of \( K_i \cdot K_j \)
depends on the mode number \( n \) of the KK states whose momenta are \( K_i \) and \( K_j \).
Therefore, for particles \( \{G_i^{(m)}, G_j^{(n)}\} \), we may schematically write

\[
\int d\phi K_i \cdot K_j \rightarrow \alpha k_i \cdot k_j - \beta_{ij}(m, n), \tag{4}
\]

where \( k_i \) are the Minkowski 4-momenta. We again note that \( \alpha \) does not depend on
the mode numbers \( (m, n) \) of the particles whose momenta are in the product, but
\( \beta(m, n) \) does. That is, for the 4-d parts of the vertex, such as the terms proportional
to \( k_i \cdot k_j \), \( \alpha \) is the same and is given by integrating the wavefunctions \( \chi^{(m)}(\phi) \) for
the particles in the vertex, say \( \{ G^{(l)}, G^{(m)}, G^{(n)} \} \), over the extra dimension. However, \( \beta(m, n) \) are given by integrating a product proportional to \( \chi^{(l)} \partial_\phi \chi^{(m)} \partial_\phi \chi^{(n)} \), which depends on the modes \( (m, n) \).

Using the notation of Ref. [5], we have

\[
\chi^{(n)} \simeq \frac{e^{2\sigma}}{N_n} J_2(z_n), \tag{5}
\]

where \( J_l \) is the \( l \)th order Bessel function. The normalization is given by

\[
N_n \simeq \frac{e^{kr_c\pi}}{\sqrt{kr_c}} J_2(x_n), \tag{6}
\]

where \( J_1(x_n) = 0; \; z_n = (m_n/k)e^{\sigma} \), and \( m_n \) is the mass of the \( n^{th} \) KK graviton. Given \( \chi^{(n)} \) from Eq. (5), integration of the \( \{ G^{(l)}, G^{(m)}, G^{(n)} \} \) vertex in \( S_G \) of Eq. (1) over the extra dimension yields

\[
\alpha = \lambda_1 \int_{-\pi}^{\pi} d\phi \; e^{4\sigma} J_2(z_l) J_2(z_m) J_2(z_n), \tag{7}
\]

where

\[
\lambda_1 = \frac{(2M_P)^{-1} k r_c e^{-3kr_c\pi}}{J_2(x_l) J_2(x_m) J_2(x_n)}. \tag{8}
\]

We also obtain

\[
\beta(m, n) = \lambda_2 \int_{-\pi}^{\pi} d\phi \; e^{4\sigma} J_2(z_l) J_1(z_m) J_1(z_n), \tag{9}
\]

where

\[
\lambda_2 = m_m m_n \lambda_1. \tag{10}
\]

Here, we note that the overall coefficient of the vertex \( \{ G^{(l)}, G^{(m)}, G^{(n)} \} \) has been fixed by comparing the KK zero mode contribution to the vertex and matching with the 4-d results of Ref. [6], after integrating over the extra dimension. We may now numerically compute the decay rate for \( G^{(3)} \rightarrow G^{(1)} G^{(1)} \). The branching ratio for the decay as a function of \( m_1/m_3 \) is presented in Fig. (1). Of course, at the classical level in the RS model, \( m_1/m_3 \simeq 3.83/10.17 \), for which the branching fraction is about 15%. However, if the mass ratio is slightly different, perhaps due to quantum effects, the change in the branching ratio is not negligible, as seen in Fig. (1). Higher KK modes may decay into more than two \( G^{(n)} \) resonances, *e.g.*, \( G^{(n+1)} \rightarrow nG^{(1)} \). However, such decays come from Feynman diagrams that are higher order in \( \Lambda_\pi^{-1} \). In addition, perturbative calculations are unreliable at scales much higher than \( \Lambda_\pi \).
Figure 1: Branching fraction for the decay $G^{(3)} \rightarrow G^{(1)}G^{(1)}$ as a function of the mass ratio $m_1/m_3$ assuming $m_3 = 2$ TeV. The fixed value of this ratio in the RS model is shown as the dotted line and yields $B \simeq 15\%$.

Here we note that the decay $G^{(3)} \rightarrow G^{(1)}G^{(1)}$ can also get contributions from higher derivative operators proportional to $R^2_5$, such as the 5-d Gauss-Bonnet term. These contributions will be parametrically non-leading, as the associated vertices are suppressed by $c^{-2/3}\Lambda^{-3}_\pi$ where $c \equiv k/M_P$. One finds their relative contributions to the decay rate for $G^{(3)} \rightarrow G^{(1)}G^{(1)}$ to be $\sim (c^{1/3}x_3)^2$ where $x_3 \simeq 10.1735$. Thus, even if $c \ll 1$ these terms may have a large effect, assuming that their coefficients in the 5-d action are not too small. This allows for the possibility of measuring the 5-d $R^2_5$ coefficients of the theory at the 1\% level from measurements of $G^{(n)}$ decays in future experiments, once the RS resonances are discovered. In the case where the $G^{(n)}$ have decays to three or more final state gravitons the presence of the $R^2_5$-like terms will lead to a distortion of the final state spectrum thus providing a further handle on their relative contribution beyond the total rate.

Probing the RS KK graviton sector and the bulk fermion mass matrix with precision, as discussed in this and the next sections, requires large numbers of events.
Here, we note that a future LC will have ample data to achieve such precise measurements. To see this, consider the cross section for $e^+e^- \rightarrow \mu^+\mu^-$ for the case where the SM is on the wall. The branching fraction for $\mu$ pairs is $\simeq 0.02$ and the light KK resonance cross sections are $10^3 - 10^4 \text{fb}$ [5]. As the integrated luminosities of typical LC’s are in the range of $100 - 500 \text{fb}^{-1}/\text{yr}$, it is clear that one should easily expect $\sim 10^7$ graviton resonances in the TeV range to be produced. Similar arguments apply when the SM lies in the bulk and $\nu > -0.5$ [8]. When $\nu < -0.5$ one can instead use $\gamma\gamma$ collisions to produce graviton resonances at rates of order $10^7/\text{yr}$.

3 Flavor Changing Decays of Bulk Fields

As described above, the large number of KK gravitons, $\sim 10^7$, produced at future lepton colliders will allow for a detailed study of their rare decay modes. We now consider the case where the SM fields reside in the 5-d bulk. Since the parameter $\nu$ controls the strength of the couplings of both the graviton and gauge KK towers to the zero mode fermions, allowing fermions with the same SM quantum numbers to have different values of $\nu$ leads to Flavor-Changing (FC) couplings as noted by several authors [4]. The observation of rare decays induced by FC couplings could then be used to map out the 5-d fermion mass matrix. To be specific we consider a toy model where all of the fermions except the top quark have the same value of $\nu = -0.4$ and we let $\nu_t$ vary. (We choose the value $\nu = -0.4$ since it is middle of region III [8] where all of the lightest graviton and gauge KK excitations are observable at colliders and there are no hierarchically large values of $\Lambda_\pi$.) In this case, the mixing in the $Q = 2/3$ sector will generate FC $u,c,t$ couplings which are somewhat less constrained than those in the $Q = -1/3$ sector [9]. Whether such FC couplings are sufficiently large as to be dangerous for low-energy $D$ meson physics is beyond the scope of the present paper. In either case, the theory presented here is just a toy model for demonstration purposes and is not to be taken too seriously.

Let $c_i^{G,A}$ denote the couplings of a zero mode fermion, having $\nu = \nu_i$, to the lowest graviton or gauge excitation in the weak eigenstate basis, i.e., $c_i^{G,A} = C_{i001}^{G,A}(\nu_i)$ using the notation in Ref. [8]. (Here, $i = 1, 2, 3$ will label the fields $u, c$ and $t$.) Symbolically, we can write this interaction as $\overline{f}_i^{(0)}c_i^{G,A}f_j^{(0)}G^{(1)},A^{(1)}$. Converting to the mass eigenstate basis by a unitary transformation $U$ this becomes $\overline{f}_k^{(0)}U_{ki}c_i^{G,A}U_{ij}f_j^{(0)}G^{(1)},A^{(1)}$. 

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Now consider the case $k = 3$ and $j = 1, 2$ and use the fact that $c_1^{G,A} = c_2^{G,A}$ together with the unitarity condition $\sum_i U_{ki}^\dagger U_{ij} = \delta_{kj}$; the strength of the FC couplings are found to be $(c_3^{G,A} - c_1^{G,A})U_{33}^\dagger U_{31,2}$. The difference in the $c_i$ can be obtained directly by performing some basic integrations over Bessel functions, once the value of $\nu_t$ is specified. To obtain sample numerical results we will assume that $U_{ij} \sim (V_{CKM})_{ij}$ at least qualitatively so that, e.g., $U_{33} = 1$ and $U_{32} \simeq A\lambda^2 \simeq 0.04$. Note that in this toy model the $t\pi$ coupling is thus expected to be an order of magnitude larger than the corresponding $tt\pi$ one.

![Figure 2: The flavor changing branching fractions for a 1 TeV first graviton (solid green) or $\gamma/Z$ (dashed red) excitation as functions of $\Delta \nu$.](image)

It is now straightforward to obtain the desired branching fractions $B$ for $G, A \to t\pi + x\pi$ which are shown in Fig. (2) as functions of $\Delta \nu = \nu_t - \nu$ with $\nu = -0.4$ assuming first KK masses of 1 TeV. (For the gauge boson KK case we assume that we are probing the almost degenerate first $\gamma/Z$ excitation. Comparable results would be obtained for the first KK gluon excitation.) We see immediately that if $\Delta \nu$ is sizeable, e.g. $\Delta \nu \gtrsim 0.1$, branching fractions as large as $10^{-(3-4)}$ are obtainable. Decays at such high rates will be trivial to observe with a sample of $\sim 10^7$ gravitons or gauge KK excitations. For very small $\Delta \nu$ we find the approximate relations $B \simeq 1.4 \times 10^{-6}[\Delta \nu/0.01]^2$ for gravitons and $B \simeq 5.3 \times 10^{-6}[\Delta \nu/0.01]^2$ for neutral gauge bosons.
Using these results it is clear that values of $|\Delta \nu| \sim 0.01$ can be probed through these decays, at LC Graviton Factories.

4 Conclusion

In this paper we have explored some of the physics accessible at LC Graviton Factories which may produce as many as $10^7$ resonant gravitons per year at design luminosities, depending upon their mass. We examined two particular processes: (i) the decay of a heavy graviton tower member into two lighter ones and (ii) the flavor changing decay of a light graviton (or bulk vector field) when the the SM is off the wall.

As described in the text, the tree-level coupling of gravitons to matter occurs at the same order as does the trilinear graviton coupling, in analogy with QCD. In order to explore the underlying theory of gravity in the RS model, it is necessary to probe these gravitational self-couplings. This becomes possible in the RS model at Graviton Factories where we can observe the decay of higher KK states into lighter ones. In particular, we examined the process $G^{(3)} \rightarrow G^{(1)}G^{(1)}$ which due to the RS particle spectrum is the first accessible one at colliders. The branching fraction for this decay was found to be about 15%. This branching fraction yields a precise measurement of the gravitational self-interactions which allows us to probe, e.g., higher derivative contribution in the 5-d action, such as the Gauss-Bonnet term, at the level of 1% or better. The decay rate shows sensitivity to $m_1/m_3$; this ratio could in principle receive quantum corrections.

In the case where the SM fields lie in the bulk, giving fermions of each generation with the same electroweak charges different values of the parameter $\nu$ leads to FCNC-type decays. If the branching fractions for these decays are precisely measured we can map out the values of $\nu$ for all the fermions. In this paper, we demonstrated, within the context of a toy model, that such FCNC’s can be sizeable for both graviton and gauge KK resonances and that the precision measurements at a Graviton Factory can probe differences in $\nu$ values as low as 0.01. Together with ordinary flavor conserving decays, these can be used to map out all of the fermion locations in the extra dimension.

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References


