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Solving the strong CP problem with supersymmetry *

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Abstract

We propose a new solution to the strong CP problem based on supersymmetric non-renormalization theorems. CP is broken spontaneously and it's breaking is communicated to the MSSM by radiative corrections. The strong CP phase is protected by a susy non-renormalization theorem and remains exactly zero while loops can generate a large CKM phase from wave function renormalization. We present a concrete model as an example but stress that our framework is general. We also discuss constraints on susy breaking and point out experimental signatures.

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1 Introduction

The strong CP problem [1] has been a puzzle since the 70's when it was understood that the θ parameter of QCD is physical. More recently, the strong CP problem has sharpened because fits to K and B physics data now show that the unitarity triangle has three large angles [2, 3]. Thus superweak models are ruled out, and CP violation in the CKM matrix is large, the phase is order one. The only other CP violating parameter in the Standard Model, the strong CP phase, is experimentally bound to be tiny [4], most recent measurements of the electric dipole moment of the neutron and ^{199}Hg imply $\overline{\theta} \lesssim 10^{-10}$ [5]. This asymmetry is especially puzzling since in the Standard Model the CKM phase ϕ_{CKM} and the strong CP phase $\overline{\theta}$ have a common origin. Both of them come from the Yukawa couplings. The CKM matrix is the mismatch between the basis in which the up and down quark Yukawa matrices are diagonal. In the absence of fine-tuning, a large CKM phase implies large phases in the Yukawa matrices. But if phases in the Yukawa matrices are large then the bound on the strong CP phase

$$\overline{\theta} = \theta - \arg\ \det\ Y_u Y_d \tag{1}$$

implies fine-tuning of one part in 10^{10}! Here and in the following we assume real Higgs vacuum expectation values so that phases in quark masses arise only from phases in Yukawa couplings.

The most popular proposed solutions to this problem are the axion [6], a vanishing up-quark mass [7] and the Nelson-Barr mechanism [8]. For the axion solution $\overline{\theta}$ is promoted to be a field, the axion. QCD dynamics generates a potential for the axion with a minimum at zero as desired. The trouble with this solution is that experimental searches for the axion have found nothing and together with cosmological constraints have reduced the allowed parameter space to a small window [2]. A vanishing up-quark mass would nullify the strong CP problem because it would render $\overline{\theta}$ unphysical. When $m_u = 0$ then the strong CP phase can be removed from the Lagrangian by redefining the phase of the up quark field. However, chiral perturbation theory disfavors $m_u = 0$ [9]. While this possibility is still under debate, the question will eventually be settled by lattice computations [10]. Finally, the Nelson-Barr mechanism stipulates that CP is a good symmetry at high scales. CP is broken spontaneously by a complex vev which is coupled to the quarks in such a way that it induces complex mixing with heavy vector-like fermions. By a clever choice of quark masses and Yukawa couplings, one can arrange for a large CKM phase and $\overline{\theta} = 0$. Loop corrections to $\overline{\theta}$ are dangerous in the Nelson-Barr scheme, however they can be made sufficiently small by taking the coupling to the CP violating vev small.

In this Letter we propose a new solution to the strong CP problem which relies on spontaneous CP violation and uses the non-renormalization theorems of supersymmetry to ensure that $\overline{\theta}$ remains zero. Our basic framework assumes unbroken CP and susy at a high scale, e.g. the Planck scale. Therefore we can choose a basis in which all coupling constants are real and $\theta = 0$. CP breaks spontaneously at the scale M_{CP}, and we assume that the MSSM fields do not couple to complex vevs at tree level. Thus $\phi_{CKM} = \overline{\theta} = 0$ at tree level. However, a sufficiently large CKM phase is generated by loops if the CP violating sector (CPX) is strongly coupled to quarks. Naively one would expect that a strong CP phase of order one is also generated. Happily, $\overline{\theta}$ is protected by a non-renormalization theorem [11, 12]. Thus in susy quantum loops can generate a large CKM phase while $\overline{\theta}$ remains exactly zero. After susy breaking the non-renormalization theorem no longer holds, and a small $\overline{\theta}$ is generated. We show that $\overline{\theta}$ remains sufficiently small if susy breaking occurs at low energies and is CP and flavor preserving. Measurements of the superpartner spectrum, electric dipole moments and CP and flavor physics in the B-system will test the predictions of our framework.

In the following section we discuss the general framework in more detail. We give a specific model of CP violation which exemplifies our mechanism in section 3. In section 4 we discuss susy breaking and in the fifth section we list model independent predictions. We conclude in section 6.

2 CP phases from wave functions

We now discuss our general scenario in more detail. Below the cut-off scale (for example the Planck scale or the string scale) we assume that susy and CP are good symmetries so that we can describe physics at this scale by a local supersymmetric Lagrangian with real couplings and vanishing θ-parameters. CP must be broken spontaneously at a lower scale M_{CP} by the complex vev of one or several scalar fields Σ. In order to prevent a direct large contribution to $\overline{\theta}$ at this scale we assume that there are no tree level superpotential couplings of Σ to the MSSM or other colored fields. This could be enforced by a symmetry, or such couplings may be suppressed for geometric reasons in extra dimensions [13]. In order to communicate CP violation to the MSSM we assume that the CPX sector couples to the quarks through messenger fields which couple to Σ. There may also be arbitrary Kaehler potential couplings of Σ to the MSSM. Such couplings are harmless, because the Kaehler potential is real and cannot contribute to $\overline{\theta}$ [14].

This Lagrangian is renormalized at the loop level. In the following, it is convenient to use the "holomorphic" renormalization scheme in which non-renormalization theorems are manifest. Then both the superpotential and $\overline{\theta}$ are not renormalized. However the Kaehler potential is renormalized, and if the CP messenger sector and it's couplings violate flavor non-canonical complex kinetic terms for the quarks are induced. The most general CP violating kinetic terms are 3×3 hermitian matrices Z for each set of fields with identical gauge quantum numbers. Because the Z's are hermitian and positive definite we can write $Z^{-1} = T^2$ with a hermitian T and change to the canonical basis

$$L \sim \int d^4\theta \; Z^{ij} \hat{Q}_i^\dagger \hat{Q}_j = \int d^4\theta \; \delta^{ij} Q_i^\dagger Q_j \quad \text{where} \quad Q = T^{-1}\hat{Q} \; . \qquad (2)$$

Note that wave-function renormalization by the hermitian matrix leaves θ invariant [1], because θ shifts proportional to $\arg\det T = 0$. In the new basis

[1] This was pointed out to us by Holdom but has been known by others as well [15, 16, 17]

the Yukawa terms are $Q^T Y_u U H_u$, $Q^T Y_d D H_d$, where

$$Y_d = T_q^T \hat{Y}_d T_d \quad \text{and} \quad Y_u = T_q^T \hat{Y}_u T_u \ , \tag{3}$$

and also

$$\overline{\theta} \ = \ \theta - \arg\det Y_d Y_u \ = \ 0 - \arg\det \hat{Y}_d \hat{Y}_u \ = \ 0 \ , \tag{4}$$

reflecting the non-renormalization of $\overline{\theta}$.

However, even though $\overline{\theta}$ remains zero, the new Yukawa matrices are clearly complex and a complex CKM matrix is generated. It is intuitive (but somewhat tedious to show [14]) that a large CKM phase is generated if the wave-function renormalization factors T are not close to the unit matrix. Thus if the CP violating dynamics breaks CP by order one and is strongly coupled to the quarks then we obtain

$$\phi_{CKM} \sim O(1) \ , \quad \overline{\theta} = 0 \tag{5}$$

as desired. It is also easy to show that all values for quark masses, mixing angles and CKM phase can be generated in this way. To see this, pick as an example $\hat{Y}_d = \hat{Y}_u = 1 = T_q$, $T_u \propto \mathrm{diag}(m_u, m_c, m_t)$, $T_d \propto V_{CKM} \mathrm{diag}(m_d, m_s, m_b) V_{CKM}^\dagger$. Before discussing susy breaking and corrections to eq. (5) in section 4 we give an explicit example for the CP violating sector.

3 An explicit model

In this section, we give one of many possible models as an example. This model generates large wave function renormalizations only for the down quark singlet which is sufficient to obtain a large CKM phase. Other models with $SU(5)$ or $SO(10)$ unification and messenger fields in full representations of the GUT group can also be built and may be more attractive/predictive. In addition to the usual MSSM superpotential and canonical kinetic terms we assume the following superpotential at the high scale, M_{Pl}

$$W = r_{ij} D_i F_j T + M_{CP} T \overline{T} + \Sigma_{ij} F_i \overline{F}_j \ . \tag{6}$$

Here T and $\overline{T}$ are a vector-like, fourth-generation down quark singlet with real mass M_{CP}, F and $\overline{F}$ are three vector-like SM singlets which obtain complex masses from their coupling to the complex vev $\|\Sigma\| \sim M_{CP}$ and we have absorbed a coupling constant into the definition of Σ.

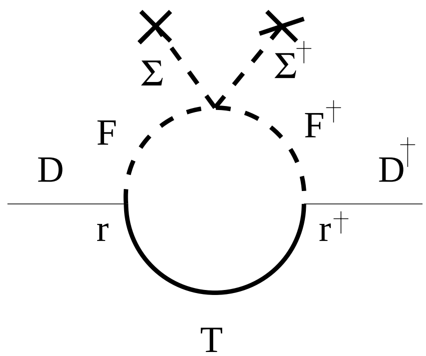

Figure 1: *A diagram contributing to the down quark kinetic term Z_d.*

Integrating out the massive fields at the scale M_{CP} sets the above superpotential to zero, the MSSM superpotential remains unchanged in the "holomorphic" renormalization scheme, and the one-loop diagram of Figure 1 renormalizes the wave function of the right handed down quarks

$$\delta Z_d \sim \frac{r^\dagger \Sigma^\dagger \Sigma r}{16\pi^2 M_{CP}^2} \times \text{Log} . \tag{7}$$

Note that the coupling constant r needs to be strong ($\sim 4\pi$) in order for Z_d and T_d not to be close to the unit matrix. The strong coupling renders this one-loop calculation of Z unreliable, and we simply parameterize the full result by T_d.[2] It is important that there are no true vertex renormalizations of Y_d due to the non-renormalization theorem. This should highlight why susy is crucial for our approach. In a non-supersymmetric theory one-particle-irreducible vertex corrections do arise at some order, and because of the strong coupling diagrams with arbitrarily many loops are dangerous.

[2]Maintaining such a large Yukawa coupling at the scale $M_{CP} < M_{Pl}$ requires new strong gauge interactions for the T's and F's. We have checked that quantum corrections involving these new gauge interactions do not spoil our mechanism, neither perturbatively nor non-perturbatively [14].

At the renormalizable level, the wave-function renormalization factor T_d is the only parameter which remains from the CP violating dynamics at scales below M_{CP}. Higher dimensional operators suppressed by M_{CP} are also generated. In the presence of susy breaking they lead to important corrections to $\overline{\theta}$ as we discuss in Section 4.

Finally, we note that we can relax the condition that Σ not couple at all to MSSM fields in the superpotential to a less stringent constraint by allowing couplings at the non-renormalizable level. Operators such as

$$\mathrm{tr}\,\left(\frac{\Sigma}{M_{Pl}}\right)^n\; W_\alpha W^\alpha \tag{8}$$

result in contributions to $\overline{\theta}\sim(M_{CP}/M_{Pl})^n$ which implies $M_{CP}/M_{Pl}\lesssim 10^{-10/n}$. The existence of such operators is model dependent, and the upper bound on M_{CP} is therefore not mandatory.

4 Supersymmetry breaking

Since susy is broken in nature we need to check that our mechanism for protecting $\overline{\theta}$ from radiative corrections remains stable after susy breaking. In this Letter we only discuss this topic very briefly, a more detailed discussion is in our longer paper [14], and many of the results of this section can also be found in [16, 18, 19, 20].

The non-renormalization theorems are violated in the presence of susy breaking, and $\overline{\theta}$ is renormalized. One contribution to $\overline{\theta}$ that is always there is the well-known heavily GIM suppressed and therefore finite SM contribution which arises at four-loops [15] from the "cheburashka" diagram [21] and gives $\delta\overline{\theta}\simeq 10^{-19}$. The diagram is dominated by loop momenta near the QCD scale and is therefore independent of the mechanism of susy breaking.

However, there are also new contributions which are specific to softly broken susy theories. In the MSSM the expression for $\overline{\theta}$ must be generalized to include the phase of the gluino mass $m_{\tilde{g}}$

$$\overline{\theta}=\theta-\arg\det\,Y_uY_d-3\arg\,m_{\tilde{g}}\;. \tag{9}$$

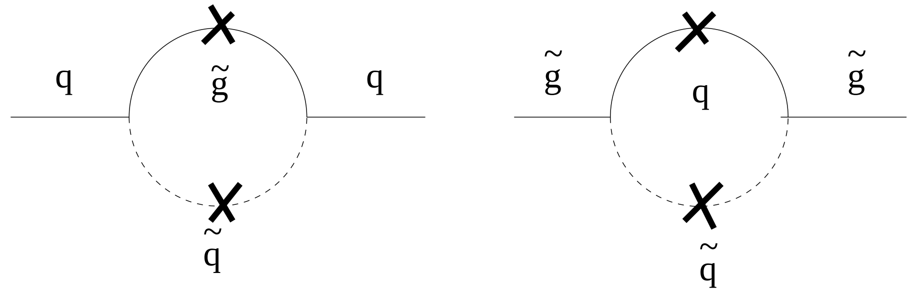

Figure 2: *Lowest order susy diagrams contributing to $\overline{\theta}$. A cross denotes a LR mass insertion.*

Thus the gluino mass must be real to one part in 10^{10}. The same bound applies to the b-term, because a complex b leads to complex Higgs vevs. The phases of the remaining flavor-universal MSSM parameters are also tightly constrained $\sim 10^{-8}$, because they induce contributions to $\overline{\theta}$ from the one-loop diagrams in Figure 2.

But even if all phases in soft terms vanish the diagrams of Figure 2 can still generate large contributions to $\overline{\theta}$ because they involve the complex MSSM Yukawa couplings. The diagrams give expressions such as $\mathrm{Im\,tr}\left[Y^\dagger A\right]$ and renormalize $\overline{\theta}$ unless A is either zero or else proportional to Y. The most natural way to suppress these contributions is to assume that susy breaking is universal and proportional

$$m_{\tilde{u}}^2 \sim m_{\tilde{d}}^2 \sim m_{\tilde{q}}^2 \propto 1 \ , \quad A_{u/d} \propto Y_{u/d} \ . \tag{10}$$

Then $\mathrm{Im\,tr}\left[Y^\dagger A\right] = 0$, and all other similar traces vanish to 12th order in an expansion of Yukawa couplings [14, 16], and therefore contributions from susy breaking to $\overline{\theta}$ are negligibly small.

Very near universality and proportionality as in eq. (10) are required at the weak scale. A susy breaking and communication mechanism which accomplishes this is gauge mediation. This is discussed in more detail in [14], where we find that our mechanism works very naturally with gauge mediation [22]. Other susy breaking scenarios which are compatible are anomaly mediation [23] and gaugino mediation [24]. The scenario cannot, however, be combined with minimal supergravity. The reason being that even if in mSUGRA soft masses are assumed to be universal at the Planck scale they are strongly renormalized by the strong CP and flavor violating dynamics at M_{CP} and become completely non-degenerate. Inserting these non-degenerate soft masses into the diagrams of Figure 2 gives disastrously large contributions to $\overline{\theta}$. This argument implies quite generally (anomaly mediation is an important exception) that susy breaking needs to be communicated to the MSSM at a scale well below M_{CP}.

As the messenger scales of gauge mediation and CP violation get near each other corrections to $\overline{\theta}$ proportional to $(M_{susy}/M_{CP})^2$ arise and give the bound $M_{susy}/M_{CP} \lesssim 10^{-3}$ [14] from requiring $\overline{\theta} \lesssim 10^{-10}$. Note that if this bound is saturated then neutron dipole moment measurements should find non-vanishing results soon. Using the lowest possible messenger scale in gauge mediation $M_{susy} \sim 10^4$ GeV we find $M_{CP} > 10^7$ GeV. Therefore the particles of the CP violating sector cannot be produced at existing or planned accelerators. However there are several indirect predictions from our scenario which allow it to be tested. We discuss them in the next section.

5 Predictions

Even though the CPX dynamics of our solution to the strong CP problem necessarily hides at short distances there are several testable consequences. We predict [14]:

1. Supersymmetry

2. Minimal flavor violation, i.e., no new flavor violation beyond the Yukawa couplings with the well-known implications for B-physics [25, 26].

3. No new CP violation beyond the SM, in particular no new CP violation in the B-system. For example $\sin 2\beta$ is large as in the SM [26].

4. Almost degenerate first and second generation scalars of each gauge quantum number. The splittings are proportional to the square of the

corresponding Yukawa couplings and give $\triangle m < 1$ GeV which should be measurable at a linear collider. We stress that this degeneracy has to hold independent of the susy breaking mechanism.

5. $\overline{\theta}$ is predicted to lie between the current experimental bound of 10^{-10} and 10^{-19} depending on the ratios of scales M_{susy}/M_{CP} and M_{CP}/M_{Pl}. If we are lucky the corresponding hadron electric dipole moments will be measured soon [27]. Lepton dipole moments are expected to be much smaller.

6 Conclusions

The strong CP problem has recently become more urgent because experimental data strongly favor a CKM phase of order one, 10 orders of magnitude larger than the upper bound on the strong CP phase. This represents a puzzle because both appear to arise from Yukawa couplings in the SM. In this Letter we propose a new solution where CP is broken spontaneously and mediated to the SM by radiative corrections. Obtaining a large CKM phase requires the radiative corrections to be large, forcing us to consider strongly coupled models. Whereas such models are very difficult if not impossible to build without susy we have argued that the non-renormalization theorems of susy make such a solution to the strong CP problem very natural. The CKM phase gets $O(1)$ contributions from renormalization whereas the strong CP phase remains exactly zero in the supersymmetric limit. Our picture requires flavor-universal susy breaking and mediation and is compatible with gauge-, anomaly-, and gaugino-mediation but not compatible with minimal supergravity. We presented an explicit model for the CP messenger sector, but we stress that the framework is much more general because it is based on model-independent non-renormalization theorems. It would be interesting to build complete GUT models based on our framework, possibly with flavor and CP violation originating from the same strongly coupled dynamics. A promising avenue to pursue is to combine our framework with the models of Nelson and Strassler [28].

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