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Standard Model Parameters and the Cosmological Constant*

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Abstract

Simple functional relations amongst standard model couplings, including gravitional, are conjectured. Possible implications for cosmology and future theory are discussed.

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1 Introduction

There are many oft-stated reasons for believing that the standard model of elementary particles is incomplete. One is the large number of "fundamental" parameters, of order 20, for the "old" standard model. And this number increases to about 30 for the "new" standard model, which includes parameters describing neutrino masses and mixings. Either way, one expects the future, better theory to contain fewer parameters, implying that there should exist relationships between the existing standard-model parameters. The search for possible relationships is the topic of this note.

It is most likely that such relationships are very complicated and indirect. Therefore the attempt to find them with the information at hand can and should be viewed with deep suspicion and skepticism. But if there is a nonvanishing chance, however small, that simple, discoverable relationships do exist, then it would seem that there is little to lose by supposing that this is the case and engaging in the pursuit. It is this attitude, with eyes wide open, that we adopt here.

Existence of such relationships will be most likely if the amount of "new physics" between accessible energies, at and below the electroweak scale, and ultrahigh energies, at or beyond the grand-unification (GUT) or Planck scale, is minimized [1]. Therefore an implicit assumption taken here is that new physics at energies between electroweak and GUT scales is absent or of minimal importance. This in turn implies that the "hierarchy problem", *i.e.* why the quadratically divergent Higgs-boson mass is so small, is resolved at a level deeper than supersymmetry at the electroweak scale, perhaps at the same level as for the resolution of the "cosmological constant problem". We shall return to this point in Section 6.

Existence of simple relationships between standard model parameters may also imply that the dynamics of the future theory is relatively simple. Otherwise, why should any such simple relationship exist? This is an additional stimulant for attempting the search.

Our considerations will proceed in three stages. The starting point will be a review of the standard model parameters and of the "gaugeless limit", which expresses in a way the conventional wisdom on how the standard model is constructed. We then discuss an intermediate version which relates parameters in the gauge sector to those in the Higgs sector. Our final, most speculative step is to relate them all to parameters in the gravitational sector.

2 Standard Model Parameters

The standard model parameters include the three gauge coupling constants, which we here assume evolve from a common source at the GUT scale characterized by a GUT fine-structure constant $\alpha_{\rm gut} \approx 1/40$ or so. Many of the remaining parameters lie in the Higgs sector. The two most important are the strength v of the Higgs condensate (the *vev*) and the strength λ of the elastic scattering amplitude of the Higgs boson with itself. In addition there are many Yukawa coupling constants h_i of the Higgs fields to quarks and leptons, responsible for their masses and mixings, including CP violating effects. By far the largest of these couplings is that of the Higgs boson to the top quark, which we simply denote by h. In this note we lack the sophistication to consider the myriad of smaller couplings, and will effectively set them to zero.

We shall keep both parameters from the gravitational sector. The scale of the Planck mass M, which determines Newton's constant, will be considered equivalent to the GUT, grand-unification scale, because again we will lack the sophistication to distinguish them. However we shall not neglect the cosmological-constant scale $\Lambda \sim \mu^4$, with $\mu \sim 30\, cm^{-1} \sim 7 \times 10^{-4}\, eV$.

Finally, we shall have nothing to say about the parameter θ, which controls CP violation in the strong-interaction QCD sector, and will set it to zero.

3 The Gaugeless Limit

At electroweak energy scales and above, all gauge couplings are small, and it is a reasonable approximation, both for phenomenological and conceptual purposes, to set them to zero, most efficiently by letting $\alpha_{\rm gut} \to 0$. The dynamics left behind is that of the Higgs sector, which is brutally exposed in all of its intrinsic ugliness. The intermediate bosons become massless, allowing rapid decay cascades of all quarks to the up quark. All leptons, including the electron, decay to neutrinos via (longitudinal) W emission. The longitudinal W's remain coupled in the gaugeless limit, emerging as the massless Goldstone modes associated with spontaneous symmetry breaking [2].

This gaugeless limit quite accurately expresses (in reverse) the textbook picture of how the standard-model electroweak dynamics works: first the Higgs mechanism in isolation is constructed; then the effect of the gauge interactions is included. But it is possible that this two-step approach is in the long run better viewed as a linked, single step. Something like this is expressed in the "second gaugeless limit", to which we now turn.

4 The Second Gaugeless Limit

If standard-model parameters are linked, it should be the case that if $\alpha_{\rm gut}$ is set to zero, other standard-model parameters are changed. In searching for simple ways this might occur, we shall ask that the limiting theory is not pathological.

An example of one such limit can be obtained by starting with the assumption (true in supersymmetric theories, and in some sense in Coleman-Weinberg scenarios of radiatively induced symmetry breaking) that the quartic Higgs coupling λ is proportional to g^2, or $\alpha_{\rm gut}$:

$$\lambda \sim g^2 \sim \alpha_{\rm gut} \ . \tag{1}$$

Then in order for the Higgs mass $\mu^2 \sim \lambda v^2$ to remain finite, we must have

$$v^2 \sim g^{-2} \sim \alpha_{\rm gut}^{-1} \ . \tag{2}$$

If one demands that fermion masses remain finite and nonvanishing, then the Yukawa couplings h must be proportional to the gauge couplings g:

$$h^2 \sim g^2 \sim \alpha_{\rm gut} \ . \tag{3}$$

Evidently, the gauge boson masses

$$m_{W,Z}^2 \sim g^2 v^2 \tag{4}$$

also remain finite and nonvanishing.

The net result in this "second gaugeless limit" is a noninteracting theory of *massive* quarks, leptons, gauge bosons, and Higgs bosons. Conceptually, it is a radical departure from the conventional picture, if only because the Higgs Yukawa coupling constants are proportional to gauge couplings. How does this occur? Is it through symmetries, or dynamics, or geometry, or some combination? Examples of this kind of behavior do exist in the literature, in terms of attempts to relate the couplings through an assumed cancelation of divergent radiative corrections between gauge and Higgs sectors [3, 4].

It is especially interesting that in this second gaugeless limit the dependence of the standard model Lagrangian density on the coupling constants $g \sim h$ is extremely simple. After appropriate field redefinitions, the residual dependence is an overall multiplicative factor $1/g^2$ in front of the bosonic Lagrangian, with no dependence at all within the fermionic sector (in the limit of only the top quark possessing mass).

To see this, one simply rescales the gauge potentials A in the familiar way, and does the same with the Higgs fields ϕ as well

$$gA \to A \tag{5}$$

$$g\phi \to \phi \ . \tag{6}$$

If one wishes, one may also rescale the fermion fields in the same way,

$$\psi \to g^{-1}\psi \tag{7}$$

in which case the g-dependence of the entire Lagrangian density is simply an overall coefficient g^{-2}.

5 A Third Gaugeless Limit

We now go still further and look for connections between the gauge/Higgs parameters and the gravitational sector. Our starting point is within the gauge sector, and can be motivated by the hypothesis that gauge bosons originate at the GUT scale as composites of other degrees of freedom. In more familiar contexts, this is expressed as a compositeness condition [5],

$$Z_3 = \frac{g}{g_0} \to 0 \ , \tag{8}$$

where the limit $g_0 \to \infty$ implies compositeness: the probability Z_3 of finding a bare boson within the physical boson becomes zero in the limit.

The observed coupling g is typically related to g_0 as follows

$$\frac{1}{g^2} = \frac{1}{g_0^2} + c \log \to c \log \ , \tag{9}$$

where the logarithm is typically an integral over contributions from the boson's constituents. We now adopt the same structure, but using gravitational parameters as arguments of the logarithm:

$$\frac{1}{\alpha_{\rm gut}} = \frac{4\pi}{g^2} \simeq \frac{c_g}{4\pi} \, \ell n \frac{M^2}{\mu^2} \ . \tag{10}$$

If the coefficient c_g of the logarithm is chosen to be three, there is good numerical agreement. But no claim of a "derivation" of that coefficient, however, is implied, nor indeed of the functional form.

Once the gauge couplings are expressed in such a way, it becomes reasonable to assume that the Higgs couplings are also expressed in a similar way

$$
\begin{aligned}
\frac{4\pi}{\lambda} &\sim \frac{c_\lambda}{4\pi}\,\ell n\,\frac{M^2}{\mu^2} \\
\frac{4\pi}{h^2} &\sim \frac{c_h}{4\pi}\,\ell n\,\frac{M^2}{\mu^2}. \qquad (11)
\end{aligned}
$$

Only the Higgs *vev* remains to be estimated. Given the dependence of the other couplings upon the gravitational parameters, a natural choice is the rather well-known relation [6]

$$v^2 \sim M\mu \qquad (12)$$

or, if one wishes

$$v^2 \sim M\mu\ \ell n\,\frac{M^2}{\mu^2}\ . \qquad (13)$$

In this variant, the "second gaugeless limit", is attained in the limit

$$
\begin{aligned}
M &\rightarrow \infty \\
\mu &\rightarrow 0 \qquad (14) \\
0 &< M\mu < \infty\ . \qquad (15)
\end{aligned}
$$

If one chooses to omit the logarithm in Eq. (13), then one obtains in the limit a noninteracting theory of *massless* quarks, leptons, Higgs bosons, and gauge bosons. Clearly both the massless and massive options should be considered.

Numerically, one has for the value of the v^2

$$v^2 \sim 6 \times 10^4\ \mathrm{GeV}^2\ . \qquad (16)$$

If one uses the Planck scale for M in Eq. (12), we obtain

$$M\mu \sim 10^7\ \mathrm{GeV}^2 \qquad (17)$$

which is a little too large. On the other hand, if the GUT scale of, say, 3×10^{15} GeV is used in Eq. (13), then

$$M\mu \sim 10^3 - 10^4\ \mathrm{GeV}^2 \qquad (18)$$

which is a little too small. Inclusion of unknown coefficients and/or the logarithm can in principle provide the needed numerical agreement. More important than that is to find even a hint of such behavior from an underlying theory.

The above speculations can be expressed in differential form, in terms of Gell-Mann-Low equations [7]. Our basic premise is that the GUT scale gauge and Higgs couplings are sensitive to the value of the cosmological constant. This sensitivity is to be expressed in terms of familiar-looking equations

$$\begin{aligned} \mu \frac{dg^2}{d\mu} &= \beta_g \, g^4 + \cdots \\ \mu \frac{dh^2}{d\mu} &= \beta_h \, h^4 + \cdots \\ \mu \frac{d\lambda}{d\mu} &= \beta_\lambda \, \lambda^2 + \cdots \ . \end{aligned} \tag{19}$$

While these look like the usual renormalization-group equations, they are not. They express the dependence of the usual running coupling constants, evaluated at the GUT scale, upon the value of the cosmological constant. While the general form of the dependence has been assumed to be the same, these "cosmological β-functions" differ in detail; in particular the sign is changed for the gauge couplings but not for the Higgs couplings.

We may also write a Gell-Mann-Low equation for the Higgs *vev*. Without the logarithm we have

$$\mu \frac{dv^2}{d\mu} = v^2 + \cdots \tag{20}$$

and with the logarithm

$$\mu \frac{dv^2}{d\mu} = v^2(1 - \beta_g g^2) + \cdots \ . \tag{21}$$

No matter which way this idea is expressed, the main question is whether such a dependence of standard model parameters on the cosmological constant is credible. In its favor are the rough numerical agreements, which we at least regard as unforced. Also perhaps in favor of this scenario is the feature that the dynamics becomes trivial, including the vanishing of the *vev*, in the limit of vanishing of the cosmological constant. This is an avenue for at least reducing the electroweak hierarchy problem to that of understanding the nature of the cosmological constant. And it clearly demands that the role of the cosmological constant in the future theory be a central one.

6 A Possible Connection to Cosmology

Recently there has been a line of argument [8] which utilizes analogies of the standard model vacuum and its excitations with that of quantum liquids, in particular with

^{3}He-A. In this visualization, it is rather natural to expect a nearly vanishing vacuum pressure, characteristic of an infinite liquid in equilibrium at zero temperature. It is not much of a stretch to thereby obtain a vanishing vacuum energy (cosmological constant) as well in that limit. If the liquid has a boundary, to be identified with an event horizon, then there will be corrections, leading to a nonvanishing but small cosmological constant. A rather concrete example of this general idea has been provided by the picture of a black hole recently put forth by Chapline *et al.*[9] They assume that a phase transition occurs on the horizon between the conventional exterior Schwarzschild black-hole spacetime and an unconventional interior black-hole spacetime, taken to be static de Sitter space. This interior space possesses a cosmological constant, which scales as follows:

$$\Lambda \sim \mu^4 \sim R^{-2} \tag{22}$$

where R is the radius of the black hole. With the coupling constant relations obtained in the previous section, this would imply that the standard model parameters within the black hole differ from those outside, in such a way that for infinite radius the gauge couplings and particle masses vanish. In the opposite limit of a Planck-radius black hole, the gauge couplings become strong and the particle masses approach the Planck scale. If our universe contains a similar de Sitter horizon [10], then the standard model parameters will scale in a similar way. In particular, because of the above behavior of the cosmological constant, the electroweak vacuum energy v^4 will scale as

$$v^4 \sim M^2\mu^2 \sim M^3 R^{-1}\ . \tag{23}$$

There will have to be a close connection between cosmology in the large, in particular horizon structure, and the existence and nature of the Higgs condensate [11]. This is reinforcement for the arguments regarding the electroweak hierarchy problem made in the introductory section of this note.

7 Concluding Comments

These ideas are of course extremely speculative. Their value is in proportion to what further, if anything, can be done with them. We do find encouragement in the numerics, and in the simplicity and cogency of the relations which have been presented, especially with respect to the Gell-Mann-Low equations for the couplings. In the case of the renormalization group equations of the standard model, the coefficients are simple and calculable by essentially perturbative techniques. Perhaps there is an analogously simple scheme (but radically different in its physics!!) to be found. And the fact that the logarithmic factors, $\log M^2/\mu^2$, associated with gauge and Higgs couplings only appear as a multiplier of the entire standard-model Lagrangian density might indicate that they are some kind of extra-dimensional phase volume. However, implementation of this idea in more concrete terms is beyond the scope of this note.

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[10] See D. Easson and R. Brandenburger, `hep-th/0103019`, and references therein.

[11] There cannot, however, be too strong a correlation between the horizon radius and the scale size of the universe, as measured by the Hubble expansion, lest the time dependence of standard-model couplings becomes unacceptably large.