

Semileptonic Form-factors from $B \rightarrow K^* \gamma$ Decays in the Large Energy Limit [†]

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Abstract

Making use of the measurement of the $B \rightarrow K^* \gamma$ branching ratio together with the relations following from the limit of high recoil energy, we obtain stringent constraints on the values of the form-factors entering in heavy-to-light $B \rightarrow V \ell \ell'$ processes such as $B \rightarrow K^* \ell^+ \ell^-$, $B \rightarrow K^* \nu \bar{\nu}$ and $B \rightarrow \rho \nu$ decays. We show that the symmetry predictions, when combined with the experimental information on radiative decays, specify a severely restricted set of values for the vector and axial-vector form-factors evaluated at zero momentum transfer, $q^2 = 0$. These constraints can be used to test model calculations and to improve our understanding of the q^2 -dependence of semileptonic form-factors. We stress that the constraints remain stringent even when corrections are taken into account.

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Semileptonic decays of B mesons play an important role in our efforts to put together the pieces of the puzzle that the standard model (SM) represents. Through decays such as $B \rightarrow D^{(*)}\ell\nu$ and $B \rightarrow (\pi, \rho)\ell\nu$ some of the fundamental parameters of the SM like the CKM matrix elements V_{cb} and V_{ub} can be measured. Further, in modes mediated by flavor changing neutral currents (FCNC) like e.g. $B \rightarrow K^{(*)}\ell^+\ell^-$ and $B \rightarrow K^{(*)}\nu\bar{\nu}$ decays, the short distance structure of the SM can be tested for contributions from high energy scales, possibly due to new physics. These exclusive modes have distinct experimental signatures in present experiments such as e^+e^- B factories (CLEO, BaBar, Belle), as well as the B -physics programs at high energy colliders (Tevatron Run II, BTeV and LHC-B). However, this great potential is somewhat diminished by the fact the theoretical predictions for exclusive modes carry an uncertainty due to the presence of hadrons in the initial and final states. This comes in the form of hadronic matrix elements parametrized in turn by form-factors, and determined by the non-perturbative, long distance dynamics of QCD.

In the last decade a fair amount of progress has been made. Our understanding of the behavior of hadrons in the heavy quark limit (HQL) has improved since it was discovered that this regime leads to new symmetries [1]. Heavy quark symmetries, and the resulting heavy quark effective theory (HQET) have been of great use in reducing theoretical uncertainty in transitions where a heavy quark is present in both the initial and the final state hadrons. This has translated into very small uncertainties in the extraction of V_{cb} from $b \rightarrow c$ decay modes. On the other hand, the application of HQET to exclusive heavy-to-light transitions has been more limited.

More recently, the large energy limit (LEL), which results in additional symmetries impacting heavy-to-light decays [2], has been resuscitated by the authors of Ref. [3]. In analogy to the HQET, the LEL regime also leads to a controlled expansion in the framework of the so-called large energy effective theory (LEET). In addition to the heavy quark $M \rightarrow \infty$ limit, $E_h \gg \Lambda_{QCD}$ is considered, where E_h denotes the final hadronic energy. It applies to heavy-to-light transitions as the ones we are going to study in the kinematical range not too close to the zero recoil point. In the actual $(M, E_h) \rightarrow \infty$ limit the matrix elements should be fully described by perturbative QCD for exclusive processes in the Brodsky-Lepage formalism [4]. However in practice, m_b is not heavy enough for the perturbative approach to dominate [5] the form-factors, whereas LEET captures the non-perturbative nature of this regime. This was shown in Ref. [3] and will be further discussed below.

In addition to those of HQET, LEET enforces new relations among the relevant form factors. In this paper we show that combining the well understood heavy quark spin symmetry (HQSS) with leading order LEL relations and the measured $B \rightarrow K^*\gamma$ branching ratio, leads to stringent constraints on the semileptonic form-factors. These are particu-

larly important for $B \rightarrow V\ell\ell'$ decays, with $\ell, \ell' = \ell^\pm, \nu$ and $V = K^*, \rho$ denoting a light vector meson and enable the determination of the vector and axial-vector form-factors at zero momentum transfer $q^2 = 0$ in a model independent way. We show that corrections in $1/E_h$ and α_s do not affect our results.

We parametrize the hadronic matrix elements over quark bilinears relevant for semileptonic and radiative B meson decays into a vector meson in terms of form-factors $V, A_{0,1,2}$ and $T_{1,2,3}$. These are functions of q^2 , where q_μ is the momentum transfer into the dilepton pair and/or the photon in the radiative mode. In general, the form factors carry also a flavor index depending on the final quark $q = u, s, (d)$ in the decays under consideration. They are, however, the same in the SU(3) limit. We employ the following decomposition for $B \rightarrow V\ell\ell'$ decays of the ‘‘semileptonic’’ matrix elements over vector and axial vector currents

$$\langle V(k, \epsilon) | \bar{q} \gamma_\mu b | B(p) \rangle = \frac{2V(q^2)}{m_B + m_V} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha k^\beta \quad (1)$$

$$\begin{aligned} \langle V(k, \epsilon) | \bar{q} \gamma_\mu \gamma_5 b | B(p) \rangle &= i2m_V A_0(q^2) \frac{\epsilon^* \cdot q}{q^2} q_\mu + i(m_B + m_V) A_1(q^2) \left(\epsilon_\mu^* - \frac{\epsilon^* \cdot q}{q^2} q_\mu \right) \\ &\quad - iA_2(q^2) \frac{\epsilon^* \cdot q}{m_B + m_V} \left((p+k)_\mu - \frac{m_B^2 - m_V^2}{q^2} q_\mu \right). \end{aligned} \quad (2)$$

and for the FCNC magnetic dipole operator $\sigma_{\mu\nu}$

$$\begin{aligned} \langle V(k, \epsilon) | \bar{q} \sigma_{\mu\nu} (1 + \gamma_5) q^\nu b | B(p) \rangle &= i2T_1(q^2) \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha k^\beta \\ &\quad + T_2(q^2) \left\{ \epsilon_\mu^* (m_B^2 - m_V^2) - (\epsilon^* \cdot p) (p+k)_\mu \right\} \\ &\quad + T_3(q^2) (\epsilon^* \cdot p) \left\{ q_\mu - \frac{q^2}{m_B^2 - m_V^2} (p+k)_\mu \right\}, \end{aligned} \quad (3)$$

where ϵ_μ denotes the polarization four-vector of the vector meson $V = \rho, K^*, \dots$. Notice that $T_1(0) = T_2(0)$ and T_3 does not contribute to the amplitude to the radiative decay into an on-shell photon.

The Heavy Quark Limit: In the HQL $m_b \gg \Lambda_{\text{QCD}}$ the form factors over the vector and axial-vector currents are not independent of the dipole ones. Instead, they obey the following well known relations [6,5]

$$T_1(q^2) = \frac{m_B^2 + q^2 - m_V^2}{2m_B} \frac{V(q^2)}{m_B + m_V} + \frac{m_B + m_V}{2m_B} A_1(q^2), \quad (4)$$

$$\frac{m_B^2 - m_V^2}{q^2} [T_1(q^2) - T_2(q^2)] = \frac{3m_B^2 - q^2 + m_V^2}{2m_B} \frac{V(q^2)}{m_B + m_V} - \frac{m_B + m_V}{2m_B} A_1(q^2), \quad (5)$$

$$\begin{aligned} T_3(q^2) &= \frac{m_B^2 - q^2 + 3m_V^2}{2m_B} \frac{V(q^2)}{m_B + m_V} + \frac{m_B^2 - m_V^2}{m_B q^2} m_V A_0(q^2) \\ &\quad - \frac{m_B^2 + q^2 - m_V^2}{2m_B q^2} [(m_B + m_V) A_1(q^2) - (m_B - m_V) A_2(q^2)]. \end{aligned} \quad (6)$$

In terms of the symmetries of the HQET, eqns. (4-6) are a result of the Heavy Quark *Spin* Symmetry that arises in the heavy quark limit due to the decoupling of the spin of the heavy quark [1].

The Large Energy Limit: We now consider the Large Energy Limit (LEL) for heavy-to-light transitions into a vector meson as the ones we are studying. As a result, one recovers the HQSS form-factor relations (4-6), but now there are additional new relations among the form-factors defined in (1-3). These will receive corrections that roughly go as $(\Lambda_{QCD})/E_h$ and read as [3]

$$V(q^2) = \left(1 + \frac{m_V}{M}\right) \xi_{\perp}(M, E) , \quad (7)$$

$$A_1(q^2) = \frac{2E}{M + m_V} \xi_{\perp}(M, E) , \quad (8)$$

$$A_2(q^2) = \left(1 + \frac{m_V}{M}\right) \left\{ \xi_{\perp}(M, E) - \frac{m_V}{E} \xi_{\parallel}(M, E) \right\} , \quad (9)$$

$$A_0(q^2) = \left(1 - \frac{m_V^2}{ME}\right) \xi_{\parallel}(M, E) + \frac{m_V}{M} \xi_{\perp}(M, E) , \quad (10)$$

and

$$T_1(q^2) = \xi_{\perp}(M, E) , \quad (11)$$

$$T_2(q^2) = \left(1 - \frac{q^2}{M^2 - m_V^2}\right) \xi_{\perp}(M, E) , \quad (12)$$

$$T_3(q^2) = \xi_{\perp}(M, E) - \frac{m_V}{E} \left(1 - \frac{m_V^2}{M^2}\right) \xi_{\parallel}(M, E) . \quad (13)$$

It is apparent from eqns. (7)-(13) that, in the LEL regime, the $B \rightarrow V\ell\ell'$ decays are described by only two form-factors: ξ_{\perp} and ξ_{\parallel} , instead of the seven a priori independent functions in the general Lorentz invariant ansatz of the matrix elements. Here, ξ_{\perp} and ξ_{\parallel} are functions of the heavy mass M and the hadronic energy E , and refer to the transverse and longitudinal polarizations, respectively.

This simplification leads to new relations among the form-factors. For instance, the ratio of the vector form-factor V to the axial-vector form-factor A_1 ,

$$R_V(q^2) \equiv \frac{V(q^2)}{A_1(q^2)} = \frac{(m_B + m_V)^2}{2E_V m_B} , \quad (14)$$

is independent of any of these unknown, non-perturbative functions $\xi_{\perp,\parallel}$ and is determined by purely kinematical factors. Here, $E_V = (m_B^2 + m_V^2 - q^2)/(2m_B)$ denotes the energy of the final light vector meson. A similar relation holds for T_1 and T_2 , since they both are also proportional to the ‘‘transverse’’ form-factor ξ_{\perp} . As we will see below, these predictions have important consequences for observables at large recoil energies (low q^2).

The leading corrections to expressions such as eqn. (14) are expected to be of order $\mathcal{O}(\Lambda_{\text{QCD}}/2E_V)$, so if $q^2 \rightarrow 0$, then $E_V \simeq m_B/2$ and the corrections are expected to be typically below 10%. Additional corrections from perturbative QCD arise through the exchange of hard gluons [7], which are also small and below the 10% level. This confirms the result from Ref. [5] derived for $B \rightarrow \pi\ell\nu$: although pQCD should *formally* dominate the $M \rightarrow \infty$ limit, this is not what actually happens for $M \simeq m_b$, namely hard gluon corrections to the LEL relations are small.

Furthermore, the ratio R_V defined in eqn. (14), does not receive α_s corrections. The reason for this, as well as the physical picture behind expression (14), becomes clear once we look at the transverse helicity amplitudes for a generic $B \rightarrow V\ell\ell'$ transition. Making use of the HQSS relations (4) and (5), these can be written as

$$H_{\pm} = \mathcal{F} \left(V \mp \frac{(m_B + m_V)^2}{2m_B k_V} A_1 \right), \quad (15)$$

where \mathcal{F} is a factor depending on the mode under consideration (e.g. Wilson coefficients, coupling constants, etc...) and k_V is the momentum of the vector meson. Thus, we see from the form of R_V in the large energy limit, that the “+” helicity vanishes $H_+ = 0$ in the LEL regime, up to residual terms of order $m_V^2/2E_V^2$. This is not a surprise: in the limit of an infinitely heavy quark decaying into a light quark, the helicity of the latter is “inherited” by the final vector meson. In the SM, the $(V - A)$ structure in semileptonic decays is reflected in the dominance of the H_- transverse helicity. On the other hand, the amplitude to flip the helicity of the fast outgoing light quark is suppressed by $1/E_h$. This is also the reason why α_s corrections from hard gluon exchange between the spectator quark and the fast light quark do not affect eqn. (14): they are not helicity-changing. By the same reasoning, the same is true for the ratio of T_1 and T_2 .

Finally, we point out that the expression (14) for R_V is expected to hold in most relativistic quark models that compute the form-factors at $q^2 = 0$. This is the case because these model calculations, although rather uncertain in the absolute value of each form-factor *per se*, are likely to respect the helicity conservation property of the fast outgoing light quark. The overall uncertainty in each form-factor comes in as the overlap of meson wave-functions, and largely vanishes in the ratio R_V . This was found in Ref. [8] in the context of predictions for the forward-backward asymmetry in $B \rightarrow K^*\ell^+\ell^-$, where it was shown that the position of the zero of the asymmetry only depends on R_V . Since the zero is located in the low q^2 region, (around $q^2 = 3\text{GeV}^2$ in the SM) and in the region of validity of the LEL, one can use eqn. (14) to predict R_V and the position of the asymmetry zero with very small hadronic uncertainties [15].

Constraints on Semileptonic Form-factors at $q^2 = 0$: We now extract the magnitude of the form factor $T_1(0)$ from the branching ratio of $B \rightarrow K^*\gamma$ decays. It is customary to normalize the exclusive to the inclusive $B \rightarrow X_s\gamma$ branching ratio, thus eliminating the

uncertainties from the CKM factor $V_{tb}V_{ts}^*$ and the SM short distance Wilson coefficient. This results in the ratio¹

$$R_\gamma \equiv \frac{Br(B \rightarrow K^*\gamma)}{Br(B \rightarrow X_s\gamma)} = \frac{m_B^3}{m_b^3} \left(1 - \left(\frac{m_{K^*}}{m_B}\right)^2\right)^3 |T_1(0)|^2, \quad (16)$$

which can be evaluated using data [9,10]:

$$\begin{aligned} Br(B \rightarrow K^*\gamma) &= (4.25 \pm 0.55 \pm 0.29) \times 10^{-5} \\ Br(B \rightarrow X_s\gamma) &= (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4}. \end{aligned}$$

Note that the exclusive branching ratio reflects a weighted average of the charged and neutral modes [11]. We obtain $R_\gamma = 0.135 \pm 0.030$, leading to $|T_1(0)| = 0.333 \pm 0.043$. Here we employed an on-shell b -quark mass in the conservative range of $m_b = (4.8 \pm 0.2)$ GeV to evaluate the phase space factor m_b^3 from the inclusive decay in eqn. (16). However, the dominant uncertainty in the extraction of $T_1(0)$ stems from the experimental input in R_γ .

We recall that HQSS relates form-factors of matrix elements of magnetic dipole operators to those of semileptonic currents. At $q^2 = 0$ eqn. (4) can be written as

$$A_1(0) = \frac{2m_B}{m_B + m_{K^*}} T_1(0) - \frac{m_B - m_{K^*}}{m_B + m_{K^*}} V(0), \quad (17)$$

Using for $T_1(0)$ in (17) the value extracted from the measurement of R_γ translates into a constraint in the $[V(0), A_1(0)]$ plane, which is displayed in Fig. 1 (thicker band). On the other hand the ratio of these form-factors, R_V , which in the large energy limit is given by eqn. (14), constitutes another constraint. The LEL constraint (cone in Fig. 1) is plotted assuming a 10% error in the ratio, which we believe to be conservative. In fact, the typical size of this error is $\mathcal{O}(\Lambda_{\text{QCD}}/2E_h^{\text{max.}}) \simeq 6\%$. By nearly doubling its size we expect to safely account for the fact that this is a non-gaussian error. The intersection of the HQSS plus $B \rightarrow K^*\gamma$ data constraint with the LEL expression for R_V leads to the two ellipses corresponding to the 68% (solid) and 90% (dashed) confidence intervals (i.e. 1.5σ and 2.1σ respectively)². Our fit results in

$$V(0) = 0.39 \pm 0.06, \quad A_1(0) = 0.29 \pm 0.02. \quad (18)$$

We compare our findings for $V(0)$ and $A_1(0)$ with several model predictions in Fig. 1. For illustration, we take the Bauer-Stech-Wirbel (BSW) model from Ref. [12] (cross), the modified version of the Isgur-Scora-Grinstein-Wise (ISGW2) model from Ref. [13] (diamond), a recent relativistic constituent quark model prediction by Melikhov and Stech

¹In fact, even in the presence of physics beyond the SM, the Wilson coefficient cancels in the ratio R_γ as long as there is no sizeable contribution to the “flipped chirality” dipole operator $\bar{s}_R \sigma_{\mu\nu} b_L$.

²To obtain the ellipses from linear fitting, we approximate the cone by a band with thickness given by that of the cone at the intersection with the HQSS constraint.

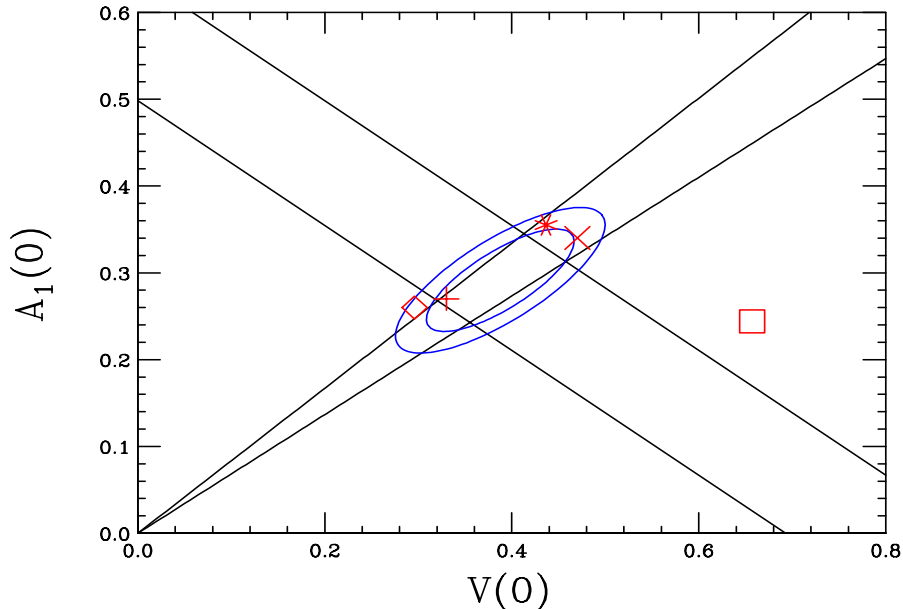


Figure 1: Constraints on the semileptonic form-factors $V(0)$ and $A_1(0)$ from $B \rightarrow K^*\gamma$ data plus HQSS (thicker band) together with the relation from the LEL (cone). The ellipses correspond to 68% and 90% confidence level intervals. Central values of model predictions are also shown and correspond to BSW [12] (vertical cross), ISGW2 [13] (diamond), MS [14] (star), LCSR [15] (diagonal cross) and LW [16] (square), respectively.

(MS) [14] (star), the recent calculation in the Light Cone QCD Sum Rule (LCSR) formalism of Ref. [15] (diagonal cross) and the prediction by Ligeti and Wise (LW) from Ref.[16] (square). We see that relativistic constituent quark models, which directly compute the form-factors at $q^2 = 0$, fall close to the constraint. This is the case with the models of Refs.[12,15,14]. The ISGW2 prediction, although slightly outside the 68% C.L. contour, fares rather well, probably not in small measure due to the relativistic corrections added with respect to the non-relativistic ISGW model [17].

On the other hand, the LW prediction (square in Fig. 1) appears to be excluded. It is based on $D \rightarrow K^*\ell\nu$ data, heavy quark flavor symmetry and assuming monopole q^2 -dependence of the form-factors. This latter assumption is needed in order to extrapolate from the small recoil energies of charm decays ($E_h \leq 1.14$ GeV) to the $q^2 = 0$ region in B decays, which corresponds to $E_{K^*} \simeq m_B/2$. Although the heavy quark flavor symmetry is expected to be affected by large corrections, these are unlikely to produce such a shift with respect to the symmetry predictions. The assumption of monopole behavior for the q^2 -dependence on the other hand, is not well justified far away from the zero recoil point. In fact, it is known that in the deep Euclidean region form-factors should match to the pQCD predictions. For vector form-factors this asymptotic behavior for $q^2 \ll 0$ (but

still $|q^2| < m_B^2 \ln(m_B^2/\Lambda_{\text{QCD}})$ is that of a dipole [5,18]. Thus, it is possible that around $q^2 = 0$ the q^2 -dependence is suppressed with respect to that of a monopole, even if it is not completely a dipole [18]. On the other hand, $A_1(q^2)$ may not be as suppressed due to the additional factor [19] $(1 - q^2/m_B^2)$, which is also present in $T_2(q^2)$. The suppression of $V(q^2)$ could bring the LW prediction into line with the constraint of Fig. 1.

We point out that the LEL relations alone are sufficient to determine $[V(0), A_1(0)]$ from the $b \rightarrow s\gamma$ data and (16), respectively, without employing the HQSS relation (17). Feeding eqn. (11) into eqns. (7) and (8) yields $V(0) = 0.39 \pm 0.05$ and $A_1(0) = 0.29 \pm 0.04$, in agreement with our previous result eqn. (18). Further, LEET predicts a simple relation between T_3 and A_2 , namely $T_3(0)/A_2(0) = (m_B - m_V)/m_B + \mathcal{O}(m_V^2/m_B^2) \sim 0.83$ ³.

With the use of $SU(3)$ flavor symmetry, the constraints obtained above can be directly imposed on the analogous form-factors entering in $B \rightarrow \rho\ell\nu$ decays. Corrections to the $SU(3)$ limit at large values of the hadronic recoil energies are expected to be of the order of [8]

$$\delta \simeq \frac{(m_s - m_q)}{E_h}, \quad (19)$$

with m_q the u, d constituent quark mass. Thus, for typical values of the constituent quark masses, we expect the $SU(3)$ corrections relevant to the constraints in Fig. 1 to be below 10%.

Conclusions: We have derived stringent constraints on the vector and axial-vector form-factors $V(q^2)$ and $A_1(q^2)$ entering in $B \rightarrow K^*\ell^+\ell^-$, $B \rightarrow K^*\nu\bar{\nu}$ and (in the $SU(3)$ limit) $B \rightarrow \rho\ell\nu$ decays. These apply to the highest recoil energy of the vector meson, i.e. $q^2 = 0$. We emphasize that these constraints, which are summarized in Fig. 1, come exclusively from:

- Data on $B \rightarrow K^*\gamma$ and $B \rightarrow X_s\gamma$ branching ratios,
- Heavy Quark *Spin* symmetry (in this case (4)) and
- the Large Energy Limit (in particular eqn. (14), the ratio of (7) and (8)).

Of these three ingredients, the first one is derived from experimental measurements, and the second one is a well established symmetry relation with corrections well below the experimental errors in the branching ratios. As discussed above, the third element is a direct consequence of the helicity conservation property of the strong interactions, which implies that in the LEL, helicity flipping is down by $(\Lambda_{\text{QCD}}/2E_h)$. This leads to a purely kinematical expression for the ratio of the vector-to-axial-vector form factor R_V , eqn. (14), valid to leading order in the $1/E_h$ expansion. We thus conclude that these constraints

³For comparison, using the central values from Ref. [15] we obtain for this ratio the value 0.92, whereas following the procedure of Ref.[16] the obtained value is 1.36.

are fairly solid and model independent. In any event, a rigorous treatment of the leading corrections in LEET is still lacking and should be undertaken. On the other hand, the experimental errors in the measurements of both the exclusive and inclusive radiative decays could be substantially reduced in the B factory era, leading to an even more stringent constraint in Fig. 1.

Lattice gauge theory calculations of the form-factors have made great progress in recent years [20]. However, they are confined to the region of low recoil energy. The constraints derived here allow an extrapolation from this region down to low values of q^2 , without *ad hoc* assumptions about the q^2 -dependence of the form-factors.

In the LEL, $SU(3)$ corrections are at most of order 10%, allowing our constraints to be also imposed on the $B \rightarrow \rho\ell\nu$ form-factors. The use of LEET also results in similar results in $B \rightarrow P$ transitions (with $P = \pi, K$, etc.), as well as in baryon decays such as $\Lambda_b \rightarrow \Lambda\gamma$ and $\Lambda_b \rightarrow \Lambda\ell^+\ell^-$, where only one form-factor is needed to determine the hadronic matrix elements.

The precise knowledge of the form-factors at $q^2 = 0$ gives us a handle to understand their q^2 -dependence, as well as testing model calculations. The reduction of the theoretical uncertainties inherent to the description of exclusive semileptonic heavy-to-light decays such as $B \rightarrow \rho\ell\nu$ facilitates the clean extraction of the SM parameter V_{ub} . At the same time, it allows us to test the short distance structure of the SM for new physics contributions to FCNC mediated decays such as $B \rightarrow K^*\ell^+\ell^-$ and $B \rightarrow K^*\nu\bar{\nu}$ [21].

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