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$\gamma^*\gamma \to \pi\pi$ at large Q^2 *

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Abstract

The QCD analysis of the process $\gamma^*\gamma \to \pi\pi$ at large Q^2 and small center-of-mass energy allows one to access a new hadronic observable describing the exclusive transition from a $q\bar{q}$ or gg state to a pair of mesons. A fruitful study may be envisaged at existing machines.

1 Introduction

Exclusive hadron production in two-photon collisions provides a tool to study a variety of fundamental aspects of QCD and has long been a subject of great interest (cf. [1, 2, 3] and references therein). Recently a new aspect of this has been pointed out, namely the physics of the process $\gamma^*\gamma \to \pi\pi$ in the region where Q^2 is large but W^2 small [4, 5]. This process factorizes [6, 7] into a perturbatively calculable, short-distance dominated scattering $\gamma^*\gamma \to q\bar{q}$ or $\gamma^*\gamma \to gg$, and non-perturbative matrix elements measuring the transitions $q\bar{q} \to \pi\pi$ and $gg \to \pi\pi$. We call these matrix elements generalized distribution amplitudes (GDAs) to emphasize their close connection to the distribution amplitudes introduced long ago in the QCD description of exclusive hard processes [8].

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2 Factorization

We are interested in $e + \gamma \to e + \pi\pi$. To lowest order in QED the interaction between the lepton and the hadron side is mediated by one-photon exchange. The contribution we focus on is the one where the $\gamma^*\gamma \to \pi\pi$ subprocess appears, as depicted in Fig. 1. The two diagrams where both photons attach to the lepton line are referred to as the bremsstrahlung contribution. We notice that in this case the pions are in a C-odd state, in contrast to the former one where they emerge in a C-even state. The phenomenological interest of this observation will be discussed in Sect. 4.

The kinematical regime in which we study the reaction is that of large $Q^2 = -q^2$, where $q = k - k'$ is the momentum tranferred from the electron (see Fig. 1), and small $W^2 = (p + p')^2$. To be specific we shall explore the domain $Q^2 \geq 4$ GeV^2 and $W^2 \leq 1$ GeV^2. The Q^2 at which the leading contribution to be discussed starts to drive the cross section is, however, a matter of experimental determination because present theory can at best estimate the size of power corrections [9].

Let us now discuss the factorization of the process at large Q^2. To do this it is useful to visualize the process in the Breit frame, where the incoming real photon of momentum $-\frac{1}{2}(Q + W^2/Q)\,\hat{z}$ collides with a static virtual photon of momentum $Q\hat{z}$ and the pion pair emerges with momentum $\frac{1}{2}(Q - W^2/Q)\,\hat{z}$. Neglecting W^2 compared with Q^2, a simple spacetime cartoon can be drawn (see Fig. 2), which in addition shows the momentum sharing between the two pions on one hand, and between the two partons on the other hand.

Although a complete proof of factorization relies on a very detailed study of loop corrections, we may motivate it by emphasizing the similarities between the one-pion and the two-pion channels. The result of factorization is that the amplitude of the $\gamma^*\gamma \to \pi\pi$ reaction is a combination of a short distance transition from $\gamma^*\gamma$ to two partons (two quarks or two gluons) and a long

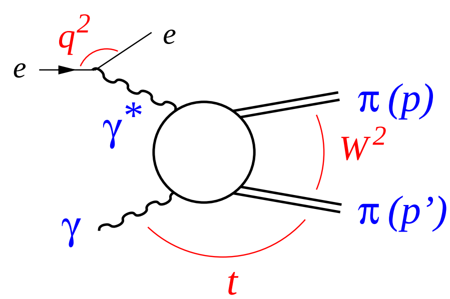

Figure 1: The subprocess $\gamma^*\gamma \to \pi\pi$ in the reaction $e + \gamma \to e + \pi + \pi$.

distance process of hadronization of two partons into two hadrons.

At leading order in α_S, the two photons couple to a $q\bar{q}$ pair. The relevant diagrams are displayed in Fig. 3. The matrix element of the time ordered product of the two electromagnetic current reads

$$T^{\mu\nu} = -g_T^{\mu\nu} \sum_q \frac{e_q^2}{2} \int_0^1 dz \, \frac{2z-1}{z(1-z)} \, \Phi_q^+(z, \zeta, W^2), \tag{1}$$

where Φ_q^+ is the quark component of the two-pion distribution amplitude. The index + indicates that the process selects the C-even component of the quark GDA. We notice that the amplitude (1) is independent of Q^2. This is understood to be true up to logarithmic scaling violation. We also remark that both photons have the same helicity. This property is specific to the $q\bar{q}$ channel. In the gluon channel, i.e., for the $\gamma^*\gamma \to gg$ subprocess, the photons can have opposite helicities [10], but still the virtual photon has to be transversely polarized. The phenomenological consequences of all these facts will be discussed in Sect. 4.

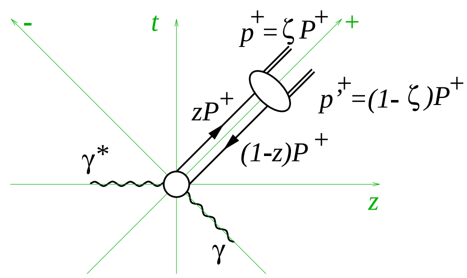

Figure 2: Spacetime cartoon for $\gamma^*\gamma \to \pi\pi$ in the Breit frame.

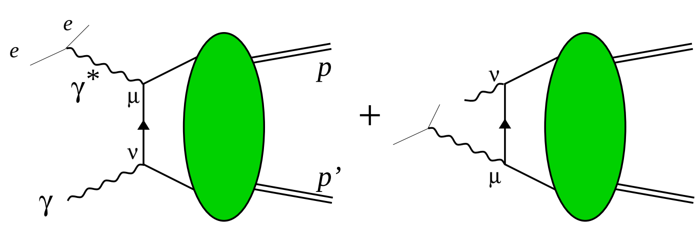

Figure 3: Leading order amplitude.

3 The generalized distribution amplitude

The operator definition of the two-pion distribution amplitude is, in $A^+ = 0$ gauge,

$$\begin{aligned}
\Phi_q(z,\zeta,W^2) &= \int \frac{dx^-}{2\pi} e^{-iz(P^+x^-)} \langle \pi(p)\pi(p')|\bar{q}(x^-)\gamma^+ q(0)|0\rangle, \\
\Phi_g(z,\zeta,W^2) &= \frac{1}{P^+}\int \frac{dx^-}{2\pi} e^{-iz(P^+x^-)} \langle \pi(p)\pi(p')|F^{+\mu}(x^-)F_\mu{}^+(0)|0\rangle.
\end{aligned}$$

The standard analysis of QCD radiative corrections shows that the amplitude (1) is modified by logarithms of Q^2. The leading logarithmic corrections may be factorized into the distribution amplitude, which thus depends on a factorization scale μ^2 through a linear combination of terms $(\log \mu^2)^{-d_n}$ with anomalous dimensions d_n. We refer the reader to Ref. [5] for a review in the present context. We only mention here that in the considered channel quarks and gluons mix under evolution and that all d_n's are positive, except for one which is zero. This implies that the GDA tends to a non vanishing asymptotic form when $\mu^2 \to \infty$. This asymptotic form reads:

$$\begin{aligned}
\sum_{q=1}^{n_f} \Phi_q^+(z,\zeta,W^2) &= 18 n_f z(1-z)(2z-1) \\
&\quad \times \left[B_{10}(W^2) + B_{12}(W^2)\, P_2(2\zeta-1)\right], \\
\Phi_g(z,\zeta,W^2) &= 48 z^2(1-z)^2 \\
&\quad \times \left[B_{10}(W^2) + B_{12}(W^2)\, P_2(2\zeta-1)\right].
\end{aligned}$$

One finds that the first subasymptotic term has the same polynomials dependence in z and ζ, whereas solutions with yet higher d_n involve polynomials of higher degree in both z and ζ.

One interesting phenomenological consequence of the interrelation of z and ζ is that, since the ζ dependence may be rewritten as a partial wave expansion, the $\cos\theta$ dependence of the cross section provides information on the z behavior of Φ. The latter would be otherwise difficult to extract since the amplitude is given as an integral over this variable (see Eq. (1)).

Let us now construct a simple model for the GDA. To perform this task, we take advantage of the energy-momentum sum-rules:

$$\begin{aligned}
\int_0^1 dz\,(2z-1)\,\Phi_q(z,\zeta,W^2) &= \frac{2}{(P^+)^2}\langle \pi^+(p)\pi^-(p')|T_q^{++}(0)|0\rangle, \\
\int_0^1 dz\,\Phi_g(z,\zeta,W^2) &= \frac{1}{(P^+)^2}\langle \pi^+(p)\pi^-(p')|T_g^{++}(0)|0\rangle,
\end{aligned}$$

and of the form factor decomposition of $T^{\mu\nu}$:

$$\begin{aligned}\langle\pi(p)\pi(p')|T^{\mu\nu}_{q,g}(0)|0\rangle &= \frac{1}{2}T^{(1)}_{q,g}(W^2)\,[(p+p')^\mu(p+p')^\nu - W^2 g^{\mu\nu}]\\ &+ \frac{1}{2}T^{(2)}_{q,g}(W^2)\,(p-p')^\mu(p-p')^\nu .\end{aligned}$$

We have to perform an analytic continuation from W^2 to 0, which leads to

$$\frac{9n_f}{10}\,B_{12}(0) = \sum_q T^{(2)}_q(0) = R_\pi$$

where $R_\pi \approx 50\%$ is the total momentum fraction carried by quarks and antiquarks in a pion.

We simplify the discussion on energy dependence by using first that below the inelastic threshold the partial wave phases are related to the phase shifts $\delta_0(W^2)$ and $\delta_2(W^2)$ of $\pi\pi$ elastic scattering as a consequence of Watson's theorem[11]. We then assume $|B_{12}(W^2)|$ to be constant and thus equal to $B_{12}(0)$. Finally, we estimate B_{10} through a soft pion theorem that gives $B_{10}(0) = -B_{12}(0)$ [11].

Our model two-pion distribution amplitude thus reads

$$\Phi_{u/d} = 10z(1-z)(2z-1)R_\pi \left[\frac{\beta^2-3}{2}\,e^{i\delta_0(W^2)} + \beta^2\,e^{i\delta_2(W^2)}\,P_2(\cos\theta)\right]$$

4 Phenomenology

Let us now briefly outline some useful phenomenological features of the $e\gamma \to e\pi\pi$ process (see Ref. [5] for more details).

There are three independent helicity amplitudes A_{++}, A_{0+} and A_{-+}, but the first one dominates at large Q^2 and may be written as

$$A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz\,\frac{2z-1}{z(1-z)}\,\Phi_q^{\pi\pi}(z,\zeta,W^2),$$

at leading order in α_S. The two other amplitudes are non-leading:

$$A_{0+}/A_{++} \propto 1/Q, \qquad A_{-+}/A_{++} \propto \alpha_S(Q^2).$$

In the case of $\pi^+\pi^-$ production, the cross section gets a contribution from the Bremsstrahlung process and can be decomposed as

$$d\sigma = d\sigma_B + d\sigma_I + d\sigma_G.$$

The interference with the bremsstrahlung process allows us to access the $\gamma^*\gamma$ process at the amplitude level, thanks to the different C-conjugation properties of the two processes. This contribution is selected by charge asymmetries such as

$$d\sigma(\pi^+(p)\pi^-(p')) - d\sigma(\pi^-(p)\pi^+(p'))$$

and by angular distributions derived from this. This interference part may be written as

$$d\sigma_I(Q^2, W^2, \cos\theta, \varphi) \propto e_l \Big(C_0 + C_1 \cos\varphi + C_2 \cos 2\varphi + C_3 \cos 3\varphi \Big)$$

with the dominant term at large Q^2 given by

$$\begin{aligned} C_1 &= \mathrm{Re}\Big\{ F_\pi^* A_{++} \Big\} [1 - (1-x)(1+\epsilon)] \sin\theta \\ &- \mathrm{Re}\Big\{ F_\pi^* A_{0+} \Big\} \sqrt{2x(1-x)}\, 2\epsilon \cos\theta \\ &+ \mathrm{Re}\Big\{ F_\pi^* A_{-+} \Big\} (1-x) \sin\theta \end{aligned}$$

With our model distribution presented above we estimate counting rates around 10^4 events for an integrated luminosity of 20–30 fb^{-1} at an e^+e^- c.m. energy of 10 GeV. This would allow one to measure the interference term with some $O(10\%)$ statistical errors.

5 Summary

The study of $\gamma^*\gamma \to \pi^+\pi^-$, $\pi^0\pi^0$ at large photon virtuality and small c.m. energy is a powerful new tool for investigating the confining mechanism which takes place in the transformation of quarks or gluons into two mesons. The simplest model for the two-pion distribution amplitude uses the asymptotic solution to the evolution equations, $\pi\pi$ elastic phase shifts, and R_π.

Encouraging rates are predicted for the kinematics and luminosity of existing B factories, and one may expect data to come soon from the BABAR, BELLE, and CLEO experiments.

The same GDA appears in deep electroproduction processes too[12], where it opens the possibility of extracting skewed parton distributions from 2π electroproduction without careful separation of resonant ρ vs. non-resonant 2π production.

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