SLAC–PUB–8631
September 2000

Light-Front-Quantized QCD in Light-Cone Gauge*

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Abstract

The light-front (LF) quantization[1] of QCD in light-cone (l.c.) gauge is discussed. The Dirac method is employed to construct the LF Hamiltonian and canonical quantization of QCD. The Dyson-Wick perturbation theory expansion based on LF-time ordering is constructed. The framework automatically incorporates the Lorentz condition as an operator equation. The propagator of the dynamical ψ_+ part of the free fermionic propagator is shown to be causal, while the gauge field propagator is found to be transverse. The interaction Hamiltonian is re-expressed in a form closely resembling the one in conventional theory, except for additional instantaneous interactions. The fact that gluons have only physical degrees of freedom in l.c. gauge may provide an analysis of coupling renormalization similar to that of the pinch technique, which is currently being discussed as a physical and analytic renormalization scheme for QCD.

Presented at
The 30th International Conference on High-Energy Physics
(ICHEP 2000)
Osaka, Japan
July 27–August 2, 2000

*Work supported by Department of Energy contract DE–AC03–76SF00515.

1 Introduction

The quantization of relativistic field theory at fixed light-front time $\tau = (t-z/c)/\sqrt{2}$, was proposed by Dirac[2] half a century ago. It has found important applications[3, 4, 5, 6] in gauge theory and string theory. The light-front (LF) quantization of QCD in its Hamiltonian form provides an alternative approach to lattice gauge theory for the computation of nonperturbative quantities. We discuss here[7] the LF quantization of QCD gauge field theory in l.c. gauge employing the Dyson-Wick S-matrix expansion based on LF-time-ordered products. The case of covariant gauge has been discussed in our earlier work[8].

2 QCD action in light-cone gauge

The LF coordinates are defined as $x^\mu = (x^+ = x_- = (x^0 + x^3)/\sqrt{2},\ x^- = x_+ = (x^0 - x^3)/\sqrt{2},\ x^\perp)$, where $x^\perp = (x^1, x^2) = (-x_1, -x_2)$ are the transverse coordinates and $\mu = -, +, 1, 2$. The coordinate $x^+ \equiv \tau$ will be taken as the LF time, while x^- is the longitudinal spatial coordinate.

The quantum action of QCD in l.c. gauge is described in the standard notation by

$$\begin{aligned} \mathcal{L}_{QCD} &= -\frac{1}{4} F^{a\mu\nu} F^{a}{}_{\mu\nu} + B^a A^a{}_- + \bar{c}^a \mathcal{D}_-^{ab} c^b \\ &\quad + \bar{\psi}^i (i\gamma^\mu D_\mu^{ij} - m\delta^{ij})\psi^j . \end{aligned}$$

Here $\bar{c}^a, c^a$ are anticommuting ghost fields and auxiliary fields $B^a(x)$ are introduced in the *linear* gauge-fixing term. The action is invariant under BRS symmetry transformations. Since B^a carries canonical dimension three, no quadratic terms in them are permitted.

3 Spinor field propagator

The quark field term in LF coordinates reads

$$\begin{aligned} & i\sqrt{2}\bar{\psi}_+^i \gamma^0 D_+^{ij} \psi_+^j + \bar{\psi}_+^i (i\gamma^\perp D_\perp^{ij} - m\delta^{ij})\psi_-^j \\ + \quad & \bar{\psi}_-^i \left[i\sqrt{2}\gamma^0 D_-^{ij} \psi_-^j + (i\gamma^\perp D_\perp^{ij} - m\delta^{ij})\psi_+^j \right] \end{aligned}$$

where[8] $\psi_\pm = \Lambda^\pm \psi$.

This shows that the minus components ψ_-^j are in fact nondynamical fields without kinetic terms. Their equations of motion in l.c. gauge lead to the constraint equation

$$i\sqrt{2}\psi_-^j(x) = -\frac{1}{\partial_-}(i\gamma^0\gamma^\perp D_\perp^{kl} - m\gamma^0\delta^{kl})\psi_+^l(x)$$

The free field propagator of ψ_+ is determined from the quadratic terms (suppressing the color index) $i\sqrt{2}\psi_+{}^\dagger\partial_+\psi_+ + \psi_+{}^\dagger(i\gamma^0\gamma^\perp\partial_\perp - m\gamma^0)\psi_-$ where $2i\partial_-\psi_- = (i\gamma^\perp\partial_\perp + m)\gamma^+\psi_+$. The equation of motion for the independent component ψ_+ is nonlocal in the longitudinal direction. In the quantized theory we find the following nonvanishing *local* anticommutator $\{\psi_+(\tau, x^-, x^\perp), \psi_+{}^\dagger(\tau, y^-, y^\perp)\} = \frac{1}{\sqrt{2}}\Lambda^+\delta(x^- - y^-)\delta^2(x^\perp - y^\perp)$. They may be realized in momentum space through the following Fourier transform

$$\psi(x) = \frac{1}{\sqrt{(2\pi)^3}}\sum_{r=\pm}\int d^2p^\perp dp^+\theta(p^+)\sqrt{\frac{m}{p^+}}$$

$$\left[b^{(r)}(p)u^{(r)}(p)e^{-ip\cdot x} + d^{\dagger(r)}(p)v^{(r)}(p)e^{ip\cdot x}\right]$$

where[8]

$$u^{(r)}(p) = \frac{\left[\sqrt{2}p^+\Lambda^+ + (m + \gamma^\perp p_\perp)\Lambda^-\right]}{(\sqrt{2}p^+ m)^{\frac{1}{2}}}\tilde{u}^{(r)}$$

and the nonvanishing anticommutation relations are given by: $\{b^{(r)}(p), b^{\dagger(s)}(p')\} = \{d^{(r)}(p), d^{\dagger(s)}(p')\} = \delta_{rs}\delta^3(p - p')$. The free propagator $< T\psi_+^i(x)\psi_+^{\dagger j}(0) >_0$ is then shown[8] to be causal

$$\frac{i\delta^{ij}}{(2\pi)^4}\int d^4q\frac{\sqrt{2}q^+\Lambda^+}{(q^2 - m^2 + i\epsilon)}e^{-iq\cdot x}$$

and it contains no instantaneous term.

4 Gauge field propagator in l.c. gauge

In the l.c. gauge the ghost fields decouple. We can obtain the free propagator using the Lagrangian density of abelian gauge theory

$$\frac{1}{2}\left[(F_{+-})^2 - (F_{12})^2 + 2F_{+\perp}F_{-\perp}\right] + BA_-$$

where $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$. Following the Dirac procedure we show that the phase space constraints remove all the canonical momenta from the theory. The surviving variables are $A_\perp$ and A_+. The latter, however, is a dependent variable satisfying $\partial_-(\partial_- A_+ - \partial_\perp A_\perp) = 0$. The construction of the Dirac bracket shows that in the l.c. gauge on the LF we simultaneously obtain the Lorentz condition $\partial \cdot A = 0$ as an operator equation as well. The *reduced* Hamiltonian is found to be ${H_0}^{LF} = \frac{1}{2}\int d^2x^\perp dx^- \; \left[(\partial_- A_+)^2 + \frac{1}{2}F_{\perp\perp'}F^{\perp\perp'}\right]$. The equal-$\tau$ commutators are found to be $[A_\perp(x), A_\perp(y)] = i\delta_{\perp\perp'}K(x,y)$ where $K(x,y) = -(1/4)\epsilon(x^- - y^-)\delta^2(x^\perp - y^\perp)$. They are *nonlocal* in the longitudinal coordinate, like the ones for scalar field, but there is no violation of the *microcausality* principle on the LF. Their momentum space realization is obtained by the Fourier transform[7]

$$
\begin{aligned}
A^\mu(x) \quad &= \frac{1}{\sqrt{(2\pi)^3}} \int d^2k^\perp dk^+ \, \frac{\theta(k^+)}{\sqrt{2k^+}} \sum_{(\perp)} \\
&\quad E^\mu_{(\perp)}(k) \left[b_{(\perp)}(k)e^{-ik\cdot x} + b^\dagger_{(\perp)}(k)e^{ik\cdot x}\right]
\end{aligned}
$$

where k^- is shown[7] to be defined through the dispersion relation, $2k^-k^+ = k^\perp k^\perp$ corresponding to a *massless* photon. Here the nonvanishing commutators are given by $[b_{(\perp)}(k), {b^\dagger}_{(\perp')}(k')] = \delta_{(\perp)(\perp')} \; \delta^3(k-k')$. The free gluon propagator is hence found to be[7]

$$
\frac{i\delta^{ab}}{(2\pi)^4} \int d^4k \; e^{-ik\cdot x} \; \frac{D_{\mu\nu}(k)}{k^2 + i\epsilon}
$$

where

$$
D_{\mu\nu}(k) = -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{(n\cdot k)} - \frac{k^2}{(n\cdot k)^2}\, n_\mu n_\nu
$$

with $n_\mu = \delta^+_\mu$, $E^\mu_{(\perp)}(k) = E^{(\perp)\mu}(k) = -D^\mu_\perp(k)$, $n^\mu D_{\mu\nu}(k) = 0$, and $k^\mu D_{\mu\nu}(k) = 0$.

5 QCD Hamiltonian in l.c. gauge

The interaction Hamiltonian in the l.c. gauge, $A^a_- = 0$, may be rewritten[7] as

$$
\begin{aligned}
&-g\bar{\psi}^i\gamma^\mu A^{ij}_\mu\psi^j + \frac{g}{2}f^{abc}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)A^{b\mu}A^{c\nu} \\
&+\frac{g^2}{4}f^{abc}f^{ade}A_{b\mu}A^{d\mu}A_{c\nu}A^{e\nu} - \frac{g^2}{2}j^+_a\frac{1}{(\partial_-)^2}j^+_a \\
&-\frac{g^2}{2}\bar{\psi}^i\gamma^+\,(\gamma^{\perp'}A_{\perp'})^{ij}\frac{1}{i\partial_-}(\gamma^\perp A_\perp)^{jk}\psi^k
\end{aligned}
$$

where $j^+_a \;=\; \bar{\psi}^i\gamma^+(t_a)^{ij}\psi^j \;+\; f_{abc}(\partial_- A_{b\mu})A^{c\mu}$ and a sum over distinct flavours, not written explicitly, is to be understood.

The Dyson-Wick perturbation expansion based on the time ordering with respect to the LF time τ, is built[7] straightforwardly. There are no ghost interaction terms in l.c. gauge. The instantaneous interactions, which can be treated systematically, are seen to be required, say, from the computation of the classical Thomson scattering limit or of electron-muon scattering. While using LF coordinates together with the dimensional regularization the causal prescription for the $1/k^+$ singularity, as given by Mandelstam and Leibbrandt, is mathematically consistent with the causal form of, say, the fermionic propagator. Computations of the divergent parts of the 1-loop gluon and quark self-energy and a three-gluon vertex corrections have been discussed in reference[7].

The fact that gluons have only physical degrees of freedom in l.c. gauge may provide an analysis of coupling renormalization similar to that of the pinch technique, which is currently being discussed[9] as means to define a physical and analytic renormalization scheme for QCD. In addition, the couplings of gluons in the l.c. gauge provides a simple procedure for the factorization of soft and hard gluonic corrections in high momentum transfer inclusive and exclusive reactions.

Acknowledgments

The financial aids from CNPq and FAPERJ for the participation at ICHEP2000 and the hospitality offered at the Theory Division of SLAC are gratefully acknowledged by PPS.

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[1] Research partially supported by the Department of Energy under contract DE-AC03-76SF00515.
Presented at *ICHEP2000*, Osaka, Japan, Parallel Session PA-15b, July 27-02 August 2000. Scanned *transparencies*, [TALK 15b-03], are available at: http://ichep2000.hep.sci.oaka-u.ac.jp

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