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The Light-Cone Fock Representation in QCD*

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Abstract

The light-cone Fock-state representation of QCD encodes the properties of a hadrons in terms of frame-independent wavefunctions, providing a systematic framework for evaluating structure functions, exclusive hadronic matrix elements, including time-like heavy hadron decay amplitudes, form factors, and deeply virtual Compton scattering. A new type of jet production reaction, "self-resolving diffractive interactions" can provide direct information on the light-cone wavefunctions of hadrons in terms of their quark and gluon degrees of freedom as well as the composition of nuclei in terms of their nucleon and mesonic degrees of freedom. The relation of the intrinsic sea to the light-cone wavefunctions is discussed. The decomposition of light-cone spin is illustrated for the quantum fluctuations of an electron.

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1 Introduction

The light-cone wavefunctions $\{\psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)\}$ are the interpolating amplitudes between a hadron and its quark and gluon degrees of freedom. For example, the proton eigenstate of the light-cone Hamiltonian in QCD satisfies: $H_{LC}^{QCD} | \Psi_p\rangle = M_p^2 | \Psi_p\rangle$. The projection of the proton's eigensolution $| \Psi_p\rangle$ on the color-singlet $B = 1$, $Q = 1$ eigenstates $\{| n\rangle\}$ of the free Hamiltonian $H_{LC}^{QCD}(g = 0)$ gives the light-cone Fock expansion: [1]

$$\begin{aligned} \left|\Psi_p; P^+, \vec{P}_\perp, \lambda\right\rangle &= \sum_{n\geq 3,\lambda_i} \int \Pi_{i=1}^{n} \frac{d^2k_{\perp i}dx_i}{\sqrt{x_i}16\pi^3} 16\pi^3 \delta\left(1 - \sum_j^n x_j\right) \delta^{(2)}\left(\sum_\ell^n \vec{k}_{\perp\ell}\right) \\ &\quad \left|n; x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\right\rangle \psi_{n/p}(x_i, \vec{k}_{\perp i}, \lambda_i) \ . \end{aligned} \tag{1}$$

The coordinates of the light-cone Fock wavefunctions $\psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)$ are the light-cone momentum fractions $x_i = k_i^+/P^+$ and the transverse momenta $\vec{k}_{\perp i}$ of its constituents. The λ_i label the light-cone spin S^z projections of the quarks and gluons along the z direction. The physical gluon polarization vectors $\epsilon^\mu(k,\ \lambda = \pm 1)$ are specified in light-cone gauge $A^+ = 0$ by the conditions $k\cdot\epsilon = 0,\ \eta\cdot\epsilon = \epsilon^+ = 0$. Each light-cone Fock wavefunction satisfies conservation of the z projection of angular momentum: $J^z = \sum_{i=1}^n S_i^z + \sum_{j=1}^{n-1} l_j^z$. The sum over S_i^z represents the contribution of the intrinsic spins of the n Fock state constituents. The sum over orbital angular momenta $l_j^z = -\mathrm{i}(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1})$ derives from the $n-1$ relative momenta. This definition automatically excludes the contribution to the orbital angular momentum due to the motion of the center of mass, which is not an intrinsic property of the hadron.[2] A comprehensive review of light-cone quantization and physics can be found in the review.[3]

In the light-cone Hamiltonian (front form) method, a quantum field theory is quantized at a fixed light-cone time $\tau = t + z/c$. [4] The generator of light-cone time translations $P^- = P^0 - P_z = i\frac{\partial}{\partial\tau}$ defines the light cone Hamiltonian of QCD. The light-cone momenta $P^+ = P^0 + P^z$ and $P_\perp$ are kinematical and commute with P^-. It is very useful to define the invariant operator $H_{LC} = P^+P^- - \vec{P}_\perp^2$ since its set of eigenvalues $\mathcal{M}_n^2$ enumerates the bound state and continuum (scattering state) mass spectrum.

Gauge theories are usually quantized in light-cone gauge $A^+ = 0$ in which the propagating gauge fields have transverse polarization. An additional advantage is the absence of ghost fields, even in the case of non-Abelian theories. Srivastava and I [5] have recently used the Dirac procedure for canonically quantizing the theory and shown that light-cone gauge automatically incorporates the Lorentz gauge condition $(\partial \cdot A) = 0$. The interactions in the light-cone Hamiltonian of gauge theories contain additional light-cone-time instantaneous fermion and gauge field interactions, analogous to Coulomb interactions in the instant form. However, one can also use the Dyson-Wick method to derive the Feynman covariant rules for gauge theory in light-cone gauge with causal propagators and no ghost fields. [5] We have also shown that one can also effectively quantize QCD on the light-cone in the covariant Feynman gauge.[5]

A crucial advantage of the light-cone formalism is the Lorentz frame-independence of the light-cone wavefunctions. Knowing the wavefunction for one momentum P^μ is sufficient to determining the wavefunction for any other momentum $P^{\mu\prime}$. Note that the variables x_i and $\vec{k}_{\perp i}$ are relative coordinates, independent of the hadron's momentum $P^+, P_\perp$. The actual momenta of the constituents which appear in the $n-$particle Fock states are $p_i^+ = x_i P^+$ and $\vec{p}_{\perp i} = x_i \vec{P}_\perp + \vec{k}_{\perp i}$. In light-cone quantization all k_i^+ are positive. Since the sum of plus momentum is conserved by the local interactions, vacuum fluctuations cannot occur. Thus the physical vacuum in light-cone quantization coincides with the perturbative vacuum, no contributions to matrix elements from vacuum fluctuations occur.

The frame-independence of the light-cone wavefunctions can be contrasted with the equal-time wavefunctions obtained in the conventional equal-time quantization. In the "instant form", Lorentz boosts are dynamical—boosting the hadron from one frame to another requires a new solution of the non-perturbative Hamiltonian equation. Light-cone quantization thus provides a frame-independent, relativistic quantum-mechanical description of hadrons which encodes multi-quark and gluon momentum, helicity, and flavor correlations in the form of universal process-independent hadron wavefunctions.

Hadronic amplitudes can be generally computed in the light-cone Hamiltonian formalism from the convolution of the light-cone wavefunctions with the underlying irreducible quark-gluon amplitudes. For example, spacelike form factors and other hadronic matrix elements of local operators can be expressed as simple overlaps of the initial and final state light-cone wavefunctions with the same number $n = n'$ of Fock constituents. Particle creation or absorption into the vacuum is not allowed because of the positivity of the k^+. Diagrams in which the current creates or annihilates pairs is forbidden in the special frame $q^+ = 0, q_\perp^2 = Q^2 = -q^2$ [6, 7] for space-like momentum transfer. One can also take matrix elements of "plus" components of currents such as J^+ and T^{++} to avoid instantaneous contributions to the current operator.

The simplicity of the diagonal representation of light cone matrix elements such as $\langle p+q \mid J^+ | p\rangle$ in the front form is in striking contrast with the equal-time formalism. In the instant form the evaluation of current matrix elements not only requires overlaps of states with different particle number, but it also requires the computation of time-ordered amplitudes in which the photon interacts with charged particles arising from vacuum fluctuations, *e.g.*, $| 0\rangle \to | q(\vec{p}_a)\overline{q}(\vec{p}_b)g(\vec{p}_g)\rangle$ with $\vec{p}_g + \vec{p}_a + \vec{p}_b = \vec{0}$, particles which then are absorbed by the final-state equal-time wavefunctions. Thus in the instant form, current matrix elements are not determined by the equal-time Fock wavefunctions alone.

Hwang, Ma, Schmidt and I [2] have recently used the light-cone wavefunction representation of gravitational form factors to prove that the anomalous moment coupling $B(0)$ to a graviton vanishes for any composite system. This remarkable result was first derived from the equivalence principle by Okun and Kobzarev [8]; see also [9, 10, 11, 12]. In our proof we show that – after summing over the couplings of the gravitons to each of the n constituents – the contribution to $B(0)$ vanishes identically for each Fock component ψ_n due to of the Lorentz boost properties of the light-cone Fock representation.[2]

Exclusive semi-leptonic B-decay amplitudes involving timelike currents such as $B \to A\ell\overline{\nu}$ can also be evaluated exactly in the light-cone Fock representation.[13, 14] In this case, the timelike decay matrix elements require the computation of both the diagonal matrix element $n \to n$ where parton number is conserved and the off-diagonal $n+1 \to n-1$ convolution such that the current operator annihilates a $q\overline{q'}$ pair in the initial B wavefunction. This term is a consequence of the fact that the time-like decay $q^2 = (p_\ell + p_{\overline{\nu}})^2 > 0$ requires a positive light-cone momentum fraction $q^+ > 0$. As I shall discuss below, one can also give a light-cone wavefunction representation of the deeply virtual Compton amplitude.[15]

Given the light-cone wavefunctions, one can compute the moments of the helicity and transversity distributions measurable in polarized deep inelastic experiments.[17] For example, the polarized quark distributions at resolution Λ correspond to

$$\begin{aligned} q_{\lambda_q/\Lambda_p}(x,\Lambda) &= \sum_{n,q_a}\int\prod_{j=1}^{n}\frac{dx_j d^2k_{\perp j}}{16\pi^3}\sum_{\lambda_i}|\psi^{(\Lambda)}_{n/H}(x_i,\vec{k}_{\perp i},\lambda_i)|^2 \\ &\quad\times 16\pi^3\delta\left(1-\sum_i^n x_i\right)\delta^{(2)}\left(\sum_i^n \vec{k}_{\perp i}\right) \\ &\quad\times\delta(x-x_q)\delta_{\lambda_a,\lambda_q}\Theta(\Lambda^2-\mathcal{M}_n^2)\ , \end{aligned} \tag{2}$$

where the sum is over all quarks q_a which match the quantum numbers, light-cone momentum fraction x, and helicity of the struck quark. Similarly, the distribution of spectator particles in the final state which could be measured in the proton fragmentation region in deep inelastic scattering at an electron-proton collider are in principle encoded in the light-cone wavefunctions. More generally, all multi-quark and gluon correlations in the bound state are represented by the light-cone wavefunctions. Thus in principle, all of the complexity of a hadron is encoded in the light-cone Fock representation, and the light-cone Fock representation is thus a representation of the underlying quantum field theory.

The key non-perturbative input for exclusive processes at high momentum transfer is the gauge and frame independent hadron distribution amplitude [16, 17] defined as the integral of the valence (lowest particle number) Fock wavefunction; *e.g.* for the pion

$$\phi_\pi(x_i,\Lambda)\equiv\int d^2k_\perp\,\psi^{(\Lambda)}_{q\overline{q}/\pi}(x_i,\vec{k}_{\perp i},\lambda) \tag{3}$$

where the global cutoff Λ in invariant mass is identified with the resolution Q. The distribution amplitude controls leading-twist exclusive amplitudes at high momentum transfer, and it can be related to the gauge-invariant Bethe-Salpeter wavefunction at equal light-cone time. The logarithmic evolution of hadron distribution amplitudes $\phi_H(x_i,Q)$ can be derived from the perturbatively-computable tail of the valence light-cone wavefunction in the high transverse momentum regime.[16, 17] The conformal basis for the evolution of the three-quark distribution amplitudes for the baryons [18] in terms of conformal eigensolutions of the evolution kernel, the Jacobi polynomials, has recently been obtained by V. Braun *et al.*[19] The asymptotic solution for the proton resembles a scalar $I=0$ diquark structure.

2 A perturbative example

Recently Hwang, Ma, Schmidt, and I have shown that the light-cone wavefunctions generated by the radiative corrections to the electron in QED provides an ideal system for understanding the spin and angular momentum decomposition of relativistic systems.[2] The model is patterned after the quantum structure which occurs in the one-loop Schwinger $\alpha/2\pi$ correction to the electron magnetic moment.[20] In effect, we can represent a spin-$\frac{1}{2}$ system as a composite of a spin-$\frac{1}{2}$ fermion and spin-one vector boson with arbitrary masses. A similar model has recently been used to illustrate the matrix elements and evolution of light-cone helicity and orbital angular momentum operators.[21] This representation of a composite system is particularly useful because it is based on two constituents but yet is totally relativistic. We can also explicitly compute the form factors $F_1(q^2)$ and $F_2(q^2)$ of the electromagnetic current, and the various contributions to the form factors $A(q^2)$ and $B(q^2)$ of the energy-momentum tensor.

For example, the two-particle Fock state for an electron with $J^z = +\frac{1}{2}$ has four possible spin combinations: [17, 20]

$$\left\{ \begin{array}{l} \psi^{\uparrow}_{+\frac{1}{2}\,+1}(x,\vec{k}_{\perp}) = -\sqrt{2}\frac{(-k^1+{\rm i}k^2)}{x(1-x)}\,\varphi \ , [\ell^z=-1] \\ \psi^{\uparrow}_{+\frac{1}{2}\,-1}(x,\vec{k}_{\perp}) = -\sqrt{2}\frac{(+k^1+{\rm i}k^2)}{1-x}\,\varphi \ , [\ell^z=+1] \\ \psi^{\uparrow}_{-\frac{1}{2}\,+1}(x,\vec{k}_{\perp}) = -\sqrt{2}(M-\frac{m}{x})\,\varphi \ , [\ell^z=0] \\ \psi^{\uparrow}_{-\frac{1}{2}\,-1}(x,\vec{k}_{\perp}) = 0 \ , \end{array} \right. \tag{4}$$

where

$$\varphi = \varphi(x,\vec{k}_{\perp}) = \frac{e/\sqrt{1-x}}{M^2-(\vec{k}_{\perp}^2+m^2)/x-(\vec{k}_{\perp}^2+\lambda^2)/(1-x)} \ . \tag{5}$$

Each configuration satisfies the spin sum rule: $J^z = S^z_{\rm f} + s^z_{\rm b} + l^z = +\frac{1}{2}$. The sign of the helicity of the electron is retained by the leading photon at $x_\gamma = 1-x \to 1$. Note that in the non-relativistic limit, the transverse motion of the constituents can be neglected, and we have only the $\left|+\frac{1}{2}\right\rangle \to \left|-\frac{1}{2}+1\right\rangle$ configuration which is the non-relativistic quantum state for the spin-half system composed of a fermion and a spin-1 boson constituents. The fermion constituent has spin projection in the opposite direction to the spin J^z of the whole system. However, for ultra-relativistic binding in which the transversal motions of the constituents are large compared to the fermion masses, the $\left|+\frac{1}{2}\right\rangle \to \left|+\frac{1}{2}+1\right\rangle$ and $\left|+\frac{1}{2}\right\rangle \to \left|+\frac{1}{2}-1\right\rangle$ configurations dominate over the $\left|+\frac{1}{2}\right\rangle \to \left|-\frac{1}{2}+1\right\rangle$ configuration. In this case the fermion constituent has spin projection parallel to J^z.

The spin structure of perturbative theory provides a template for the numerator structure of the light-cone wavefunctions even for composite systems since the equations which couple different Fock components mimic the perturbative form. The structure of the electron's Fock state in perturbative QED shows that it is natural to have a negative contribution from relative orbital angular momentum which balances the S_z of its photon constituents. We can thus expect a large orbital contribution to the proton's J_z since gluons carry roughly half of the proton's momentum, thus providing insight into the "spin crisis" in QCD.

3 Light-cone Representation of Deeply Virtual Compton Scattering

The virtual Compton scattering process $\frac{d\sigma}{dt}(\gamma^* p \to \gamma p)$ for large initial photon virtuality $q^2 = -Q^2$ has extraordinary sensitivity to fundamental features of the proton's structure. Even though the final state photon is on-shell, the deeply virtual process probes the elementary quark structure of the proton near the light cone as an effective local current. In contrast to deep inelastic scattering, which measures only the absorptive part of the forward virtual Compton amplitude, deeply virtual Compton scattering allows the measurement of the phase and spin structure of proton matrix elements of the current correlator for general momentum transfer squared t.

To leading order in $1/Q$, the deeply virtual Compton scattering amplitude factorizes as the convolution in x of the amplitude $t^{\mu\nu}$ for hard Compton scattering on a quark line with the generalized Compton form factors $f_i(x, t, \zeta)$ of the target proton.[22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] Here x is the light-cone momentum fraction of the struck quark, and $\zeta = Q^2/2P \cdot q$ plays the role of the Bjorken variable. Integrals of these quantities over x are independent of ζ and are equal to the gravitational form factors $A_q(t)$ and $B_q(t)$ where the graviton couples only to quarks.

Recently, Markus Diehl, Dae Sung Hwang, and I [15] have shown how the deeply virtual Compton amplitude can be evaluated explicitly in the Fock state representation using the matrix elements of the currents and the boost properties of the light-cone wavefunctions. We choose the frame where $P_I = (P^+\ ,\ \vec{P}_\perp\ ,\ P^-) = \left(\ P^+\ ,\ \vec{0}_\perp\ ,\ \frac{M^2}{P^+}\ \right)$, and $P_F = (P'^+\ ,\ \vec{P}'_\perp\ ,\ P'^-) = \left((1-\zeta)P^+\ ,\ -\vec{\Delta}_\perp\ ,\ \frac{(M^2+\vec{\Delta}_\perp^2)}{(1-\zeta)P^+}\right)$. The incident space-like photon carries $q^+ = 0$ and $q^2 = -Q^2 = -\vec{q}_\perp^2$ so that no light-cone time-ordered amplitudes involving the splitting of the incident photon can occur.

The diagonal (parton-number-conserving) contribution to the generalized form factors for deeply virtual Compton amplitude in the domain[30, 31, 33] $\zeta \le x_1 \le 1$ is:

$$\begin{aligned}
&\sqrt{1-\zeta} f_{1\,(n\to n)}(x_1,t,\zeta) \ - \ \frac{\zeta^2}{4\sqrt{1-\zeta}} f_{2\,(n\to n)}(x_1,t,\zeta) \\
= \ &\sum_{n,\ \lambda} \prod_{i=1}^{n} \int_0^1 dx_{i(i\ne 1)} \int \frac{d^2\vec{k}_{\perp i}}{2(2\pi)^3}\ \delta\left(1-\sum_{j=1}^{n} x_j\right)\ \delta^{(2)}\left(\sum_{j=1}^{n}\vec{k}_{\perp j}\right) \\
&\times \psi^{\uparrow\,*}_{(n)}(x'_i,\vec{k}'_{\perp i},\lambda_i)\ \psi^{\uparrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_i)(\sqrt{1-\zeta})^{1-n},
\end{aligned} \tag{6}$$

$$\begin{aligned}
&\sqrt{1-\zeta}\ \left(1+\frac{\zeta}{2(1-\zeta)}\right)\ \frac{(\Delta^1-\mathrm{i}\Delta^2)}{2M} f_{2\,(n\to n)}(x_1,t,\zeta) \\
= \ &\sum_{n,\ \lambda} \prod_{i=1}^{n} \int_0^1 dx_{i(i\ne 1)} \int \frac{d^2\vec{k}_{\perp i}}{2(2\pi)^3}\ \delta\left(1-\sum_{j=1}^{n} x_j\right)\ \delta^{(2)}\left(\sum_{j=1}^{n}\vec{k}_{\perp j}\right) \\
&\qquad\qquad \times \psi^{\uparrow\,*}_{(n)}(x'_i,\vec{k}'_{\perp i},\lambda_i)\ \psi^{\downarrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_i)(\sqrt{1-\zeta})^{1-n},
\end{aligned} \tag{7}$$

where

$$\left\{\begin{array}{lll}
x'_1 = \frac{x_1-\zeta}{1-\zeta}\,, & \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp & \text{for the struck quark,} \\
x'_i = \frac{x_i}{1-\zeta}\,, & \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp & \text{for the } (n-1) \text{ spectators.}
\end{array}\right. \tag{8}$$

One also has contributions from the $n+1 \to n-1$ off-diagonal Fock transitions in the "ERBL" domain $0 \le x_1 \le \zeta$:

$$\begin{aligned}
&\sqrt{1-\zeta} f_{1\,(n+1\to n-1)}(x_1,t,\zeta) \ - \ \frac{\zeta^2}{4\sqrt{1-\zeta}} f_{2\,(n+1\to n-1)}(x_1,t,\zeta) \\
= \ &\sum_{n,\ \lambda} \int_0^1 dx_{n+1} \int \frac{d^2\vec{k}_{\perp 1}}{2(2\pi)^3} \int \frac{d^2\vec{k}_{\perp n+1}}{2(2\pi)^3} \prod_{i=2}^{n} \int_0^1 dx_i \int \frac{d^2\vec{k}_{\perp i}}{2(2\pi)^3} \\
&\times \delta\left(1-\sum_{j=1}^{n+1} x_j\right)\ \delta^{(2)}\left(\sum_{j=1}^{n+1}\vec{k}_{\perp j}\right) [\sqrt{1-\zeta}]^{1-n}
\end{aligned} \tag{9}$$

$$\times\psi^{\uparrow\,*}_{(n-1)}(x'_i,\vec{k}'_{\perp i},\lambda_i)\ \psi^{\uparrow}_{(n+1)}(\{x_1,x_i,x_{n+1}=\zeta-x_1\},$$

$$\{\vec{k}_{\perp 1},\vec{k}_{\perp i},\vec{k}_{\perp n+1}=\vec{\Delta}_\perp-\vec{k}_{\perp 1}\},\{\lambda_1,\lambda_i,\lambda_{n+1}=-\lambda_1\}),$$

$$\begin{aligned}
&\sqrt{1-\zeta}\,\Big(1+\frac{\zeta}{2(1-\zeta)}\Big)\,\frac{(\Delta^1-\mathrm{i}\Delta^2)}{2M}f_{2\,(n+1\to n-1)}(x_1,t,\zeta)\\
=\ &\sum_{n,\ \lambda}\int_0^1 dx_{n+1}\int\frac{d^2\vec{k}_{\perp 1}}{2(2\pi)^3}\int\frac{d^2\vec{k}_{\perp n+1}}{2(2\pi)^3}\prod_{i=2}^{n}\int_0^1 dx_i\int\frac{d^2\vec{k}_{\perp i}}{2(2\pi)^3}\\
&\times\delta\left(1-\sum_{j=1}^{n+1}x_j\right)\ \delta^{(2)}\left(\sum_{j=1}^{n+1}\vec{k}_{\perp j}\right)[\sqrt{1-\zeta}]^{2-n}\\
&\times\psi^{\uparrow\,*}_{(n-1)}(x'_i,\vec{k}'_{\perp i},\lambda_i)\ \psi^{\downarrow}_{(n+1)}(\{x_1,x_i,x_{n+1}=\zeta-x_1\},\\
&\{\vec{k}_{\perp 1},\vec{k}_{\perp i},\vec{k}_{\perp n+1}=\vec{\Delta}_\perp-\vec{k}_{\perp 1}\},\{\lambda_1,\lambda_i,\lambda_{n+1}=-\lambda_1\}),
\end{aligned}\tag{10}$$

where $i=2,3,\cdots,n$ label the $n-1$ spectator partons which appear in the final-state hadron wavefunction with

$$x'_i=\frac{x_i}{1-\zeta}\,,\qquad \vec{k}'_{\perp i}=\vec{k}_{\perp i}+\frac{x_i}{1-\zeta}\vec{\Delta}_\perp\ .\tag{11}$$

The above representation is the general form for the generalized form factors of the deeply virtual Compton amplitude for any composite system. Thus given the light-cone Fock state wavefunctions of the eigensolutions of the light-cone Hamiltonian, we can compute the amplitude for virtual Compton scattering including all spin correlations. The formulae are accurate to leading order in $1/Q^2$. Radiative corrections to the quark Compton amplitude of order $\alpha_s(Q^2)$ from diagrams in which a hard gluon interacts between the two photons have also been neglected.

4 The Physics of Non-Valence Fock states

The light-cone representation allows one to understand the role of higher particle number states in hadron phenomenology. One can identify two contributions to the heavy quark sea, the "extrinsic" contributions which correspond to ordinary gluon splitting, and the "intrinsic" sea which is multi-connected via gluons to the valence quarks.

The leading $1/m_Q^2$ contributions to the intrinsic sea of the proton in the heavy quark expansion are proton matrix elements of the operator [34] $\eta^\mu \eta^\nu G_{\alpha\mu} G_{\beta\nu} G^{\alpha\beta}$ which in light-cone gauge $\eta^\mu A_\mu = A^+ = 0$ corresponds to three or four gluon exchange between the heavy-quark loop and the proton constituents in the forward virtual Compton amplitude. The intrinsic sea is thus sensitive to the hadronic bound-state structure.[35] The maximal contribution of the intrinsic heavy quark occurs at $x_Q \simeq m_{\perp Q}/\sum_i m_\perp$ where $m_\perp = \sqrt{m^2 + k_\perp^2}$; *i.e.* at large x_Q, since this minimizes the invariant mass $\mathcal{M}_n^2$. The measurements of the charm structure function by the EMC experiment are consistent with intrinsic charm at large x in the nucleon with a probability of order $0.6 \pm 0.3\%$.[36] which is consistent with recent estimates based on instanton fluctuations.[34] Similarly, one can distinguish intrinsic gluons which are associated with multi-quark interactions and extrinsic gluon contributions associated with quark substructure.[37] One can also use this framework to isolate the physics of the anomaly contribution to the Ellis-Jaffe sum rule.[38] Neither gluons nor sea quarks are solely generated by DGLAP evolution, and one cannot define a resolution scale Q_0 where the sea or gluon degrees of freedom can be neglected.

The presence of intrinsic sea can have important consequences in B decay, since the sea quarks can enter directly into the weak interaction subprocess, thus effectively evading the hierarchy of the CKM matrix.[39] Similarly, in a semi-exclusive reaction one can have partial annihilation of a quark and anti-quark by the weak interaction leaving the remaining partons to form the final hadrons. [13]

It is usually assumed that a heavy quarkonium state such as the J/ψ always decays to light hadrons via the annihilation of its heavy quark constituents to gluons. However, as Karliner and I [40] have shown, the transition $J/\psi \to \rho\pi$ can also occur by the rearrangement of the $c\bar{c}$ from the J/ψ into the $|\, q\bar{q}c\bar{c}\rangle$ intrinsic charm Fock state of the ρ or π. On the other hand, the overlap rearrangement integral in the decay $\psi' \to \rho\pi$ will be suppressed since the intrinsic charm Fock state radial wavefunction of the light hadrons will evidently not have nodes in its radial wavefunction. This observation provides a natural explanation of the long-standing puzzle [41] why the J/ψ decays prominently to two-body pseudoscalar-vector final states, breaking hadron helicity conservation, [42] whereas the ψ' does not.

5 Applications of Light-Cone Factorization to Hard QCD Processes

The light-cone formalism provides a physical factorization scheme which conveniently separates and factorizes soft non-perturbative physics from hard perturbative dynamics in both exclusive and inclusive reactions.[17, 16] In hard inclusive reactions all intermediate states are divided according to $\mathcal{M}_n^2 < \Lambda^2$ and $\mathcal{M}_n^2 > \Lambda^2$ domains. The lower mass regime is associated with the quark and gluon distributions defined from the absolute squares of the LC wavefunctions in the light cone factorization scheme. In the high invariant mass regime, intrinsic transverse momenta can be ignored, so that the structure of the process at leading power has the form of hard scattering on collinear quark and gluon constituents, as in the parton model. The attachment of gluons from the LC wavefunction to a propagator in a hard subprocess is power-law suppressed in LC gauge, so that the minimal quark-gluon particle-number subprocesses dominate.

There are many applications of this formalism: *Exclusive Processes and Heavy Hadron Decays.* At high transverse momentum an exclusive amplitudes factorize into a convolution of a hard quark-gluon subprocess amplitudes T_H with the hadron distribution amplitudes $\phi(x_i, \Lambda)$. *Color Transparency.* Each Fock state interacts distinctly; *e.g.* Fock states with small particle number and small impact separation have small color dipole moments and can traverse a nucleus with minimal interactions. This is the basis for the predictions for color transparency [43] in hard quasi-exclusive reactions. *Diffractive vector meson photoproduction.* The light-cone Fock wavefunction representation of hadronic amplitudes allows a simple eikonal analysis of diffractive high energy processes, such as $\gamma^*(Q^2)p \to \rho p$, in terms of the virtual photon and the vector meson Fock state light-cone wavefunctions convoluted with the $gp \to gp$ near-forward matrix element.[44] *Regge behavior of structure functions.* The light-cone wavefunctions $\psi_{n/H}$ of a hadron are not independent of each other, but rather are coupled via the QCD equations of motion. The constraint of finite "mechanical" kinetic energy implies "ladder relations" which interrelate the light-cone wavefunctions of states differing by one or two gluons.[45] This in turn implies BFKL Regge behavior of the polarized and unpolarized structure functions at $x \to 0$.[46] *Structure functions at large x_{bj}.* The behavior of structure functions at $x \to 1$ is a highly off-shell light-cone wavefunction configuration leading to quark-counting and helicity-retention rules for the power-law behavior of the polarized and unpolarized quark and gluon distributions in the endpoint domain. The effective starting point for the PQCD evolution of the structure functions increases as $x \to 1$. Thus evolution is quenched at $x \to 1$.[17, 1, 47] *Hidden Color.* The deuteron form factor at high Q^2 is sensitive to wavefunction configurations where all six quarks overlap within an impact separation $b_{\perp i} < \mathcal{O}(1/Q)$. The dominant color configuration at large distances corresponds to the usual proton-neutron bound state. However, at small impact space separation, all five Fock color-singlet components eventually acquire equal weight, *i.e.*, the deuteron wavefunction evolves to 80% "hidden color." The relatively large normalization of the deuteron form factor observed at large Q^2 hints at sizable hidden-color contributions.[48] Hidden color components can also play a predominant role in the reaction $\gamma d \to J/\psi pn$ at threshold if it is dominated by the multi-fusion process $\gamma g g \to J/\psi$.

6 Self-Resolved Diffractive Reactions

Diffractive multi-jet dissociation of high energy hadrons in heavy nuclei provides a novel way to measure the shape of the LC Fock state wavefunctions and test color transparency. For example, consider the reaction [49, 50, 51] $\pi A \to \mathrm{Jet}_1 + \mathrm{Jet}_2 + A'$ at high energy where the nucleus A' is left intact in its ground state. The transverse momenta of the jets balance so that $\vec{k}_{\perp i} + \vec{k}_{\perp 2} = \vec{q}_\perp < {R^{-1}}_A$. The light-cone longitudinal momentum fractions also need to add to $x_1 + x_2 \sim 1$ so that $\Delta p_L < R_A^{-1}$. The process can then occur coherently in the nucleus. Because of color transparency, the valence wavefunction of the pion with small impact separation, will penetrate the nucleus with minimal interactions, diffracting into jet pairs.[49] The $x_1 = x$, $x_2 = 1 - x$ dependence of the di-jet distributions will thus reflect the shape of the pion valence light-cone wavefunction in x; similarly, the $\vec{k}_{\perp 1} - \vec{k}_{\perp 2}$ relative transverse momenta of the jets gives key information on the derivative of the underlying shape of the valence pion wavefunction.[50, 51, 52] The diffractive nuclear amplitude extrapolated to $t = 0$ should be linear in nuclear number A if color transparency is correct. The integrated diffractive rate should then scale as $A^2/R_A^2 \sim A^{4/3}$. Preliminary results on a diffractive dissociation experiment of this type E791 at Fermilab using 500 GeV incident pions on nuclear targets.[53] appear to be consistent with color transparency.[53] The momentum fraction distribution of the jets is consistent with a valence light-cone wavefunction of the pion consistent with the shape of the asymptotic distribution amplitude, $\phi_\pi^{\rm asympt}(x) = \sqrt{3} f_\pi x(1-x)$. Data from CLEO [54] for the $\gamma\gamma^* \to \pi^0$ transition form factor also favor a form for the pion distribution amplitude close to the asymptotic solution [16, 17] to the perturbative QCD evolution equation.

The diffractive dissociation of a hadron or nucleus can also occur via the Coulomb dissociation of a beam particle on an electron beam (*e.g.* at HERA or eRHIC) or on the strong Coulomb field of a heavy nucleus (*e.g.* at RHIC or nuclear collisions at the LHC).[52] The amplitude for Coulomb exchange at small momentum transfer is proportional to the first derivative $\sum_i e_i \frac{\partial}{\vec{k}_{Ti}} \psi$ of the light-cone wavefunction, summed over the charged constituents. The Coulomb exchange reactions fall off less fast at high transverse momentum compared to pomeron exchange reactions since the light-cone wavefunction is effective differentiated twice in two-gluon exchange reactions.

It will also be interesting to study diffractive tri-jet production using proton beams $pA \to {\rm Jet}_1 + {\rm Jet}_2 + {\rm Jet}_3 + A'$ to determine the fundamental shape of the 3-quark structure of the valence light-cone wavefunction of the nucleon at small transverse separation.[50] For example, consider the Coulomb dissociation of a high energy proton at HERA. The proton can dissociate into three jets corresponding to the three-quark structure of the valence light-cone wavefunction. We can demand that the produced hadrons all fall outside an opening angle θ in the proton's fragmentation region. Effectively all of the light-cone momentum $\sum_j x_j \simeq 1$ of the proton's fragments will thus be produced outside an "exclusion cone". This then limits the invariant mass of the contributing Fock state $\mathcal{M}_n^2 > \Lambda^2 = P^{+2} \sin^2\theta/4$ from below, so that perturbative QCD counting rules can predict the fall-off in the jet system invariant mass $\mathcal{M}$. At large invariant mass one expects the three-quark valence Fock state of the proton to dominate. The segmentation of the forward detector in azimuthal angle ϕ can be used to identify structure and correlations associated with the three-quark light-cone wavefunction.[52] An interesting possibility is that the distribution amplitude of the $\Delta(1232)$ for $J_z = 1/2, 3/2$ is close to the asymptotic form $x_1x_2x_3$, but that the proton distribution amplitude is more complex. This ansatz can also be motivated by assuming a quark-diquark structure of the baryon wavefunctions. The differences in shapes of the distribution amplitudes could explain why the $p \to \Delta$ transition form factor appears to fall faster at large Q^2 than the elastic $p \to p$ and the other $p \to N^*$ transition form factors.[55] One can use also measure the dijet structure of real and virtual photons beams $\gamma^* A \to \mathrm{Jet}_1 + \mathrm{Jet}_2 + A'$ to measure the shape of the light-cone wavefunction for transversely-polarized and longitudinally-polarized virtual photons. Such experiments will open up a direct window on the amplitude structure of hadrons at short distances. The light-cone formalism is also applicable to the description of hadrons and nuclei in terms of their nucleonic and mesonic degrees of freedom.[56, 57] Self-resolving diffractive jet reactions in high energy diffractive collisions of hadrons or nuclei on electrons or nuclei at moderate momentum transfers can thus be used to resolve the light-cone wavefunctions of nuclei.

high energy diffractive collisions of hadrons or nuclei on electrons or nuclei

7 Non-Perturbative Solutions of Light-Cone Quantized QCD

In the discretized light-cone quantization method (DLCQ),[58] periodic boundary conditions are introduced in $b_\perp$ and x^- so that the momenta $k_{\perp i} = n_\perp \pi / L_\perp$ and $x_i^+ = n_i / K$ are discrete. A global cutoff in invariant mass of the partons in the Fock expansion is also introduced. Solving the quantum field theory then reduces to the problem of diagonalizing the finite-dimensional hermitian matrix H_{LC} on a finite discrete Fock basis. The DLCQ method has now become a standard tool for solving both the spectrum and light-cone wavefunctions of one-space one-time theories – virtually any $1 + 1$ quantum field theory, including "reduced QCD" (which has both quark and gluonic degrees of freedom) can be completely solved using DLCQ.[59, 60] Hiller, McCartor, and I [61, 62] have recently shown that the use of covariant Pauli-Villars regularization with DLCQ allows one to obtain the spectrum and light-cone wavefunctions of simplified theories in physical space-time dimensions, such as (3+1) Yukawa theory. Dalley *et al.* have also showed how one can use DLCQ with a transverse lattice to solve gluonic $3 + 1$ QCD.[63] The spectrum obtained for gluonium states is in remarkable agreement with lattice gauge theory results, but with a huge reduction of numerical effort. Hiller and I [64] have shown how one can use DLCQ to compute the electron magnetic moment in QED without resort to perturbation theory. One can also formulate DLCQ so that supersymmetry is exactly preserved in the discrete approximation, thus combining the power of DLCQ with the beauty of supersymmetry.[65, 66] The "SDLCQ" method has been applied to several interesting supersymmetric theories, to the analysis of zero modes, vacuum degeneracy, massless states, mass gaps, and theories in higher dimensions, and even tests of the Maldacena conjecture.[67] Broken supersymmetry is interesting in DLCQ, since it may serve as a method for regulating non-Abelian theories.[62] The vacuum state $|0\rangle$ of the full QCD Hamiltonian coincides with the free vacuum. For example, as discussed by Bassetto,[68] the computation of the spectrum of $QCD(1+1)$ in equal time quantization requires constructing the full spectrum of non perturbative contributions (instantons). However, light-cone methods with infrared regularization give the correct result without any need for vacuum-related contributions. The role of instantons and such phenomena in light-cone quantized $QCD(3+1)$ is presumably more complicated and may reside in zero modes;[69] *e.g.*, zero modes are evidently necessary to represent theories with spontaneous symmetry breaking.[70]

Even without full non-perturbative solutions of QCD, one can envision a program to construct the light-cone wavefunctions using measured moments constraints from QCD sum rules, lattice gauge theory, hard exclusive and inclusive processes. One is guided by theoretical constraints from perturbation theory which dictates the asymptotic form of the wavefunctions at large invariant mass, $x \to 1$, and high $k_\perp$.[17, 71] One can also use constraints from ladder relations which connect Fock states of different particle number; perturbatively-motivated numerator spin structures; guidance from toy models such as "reduced" $QCD(1+1)$ [60]; and the correspondence to Abelian theory for $N_C \to 0$ [72] and the many-body Schrödinger theory in the nonrelativistic domain.

8 Conclusions

The universal, process-independent and frame-independent light-cone Fock-state wavefunctions encode the properties of a hadron in terms of its fundamental quark and gluon degrees of freedom. Given the proton's light-cone wavefunctions, one can compute not only the moments of the quark and gluon distributions measured in deep inelastic lepton-proton scattering, but also the multi-parton correlations which control the distribution of particles in the proton fragmentation region and dynamical higher twist effects. Light-cone wavefunctions also provide a systematic framework for evaluating exclusive hadronic matrix elements, including time-like heavy hadron decay amplitudes, form factors, and deeply virtual Compton scattering. The formalism also provides a physical factorization scheme for separating hard and soft contributions in both exclusive and inclusive hard processes. A new type of jet production reaction, "self-resolving diffractive interactions" can provide direct information on the light-cone wavefunctions of hadrons in terms of their QCD degrees of freedom, as well as the composition of nuclei in terms of their nucleon and mesonic degrees of freedom.

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