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Ponderomotive Laser Acceleration and Focusing in Vacuum for Generation of Attosecond Electron Bunches*

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Abstract

A novel approach for the generation of ultrabright attosecond electron bunches is proposed, based on acceleration in vacuum by a short laser pulse. The laser pulse profiles is tailored such that the electrons are both focused and accelerated by the ponderomotive force of the light. Using time-averaged equations of motion analytical criteria for optimal regime of acceleration are found. It is shown that for realistic laser parameters a beam with up to 10^6 particles and normalized transverse and longitudinal emittances $< 10^{-8}$ m can be produced.

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Ponderomotive Laser Acceleration and Focusing in Vacuum: Application for Attosecond Electron Bunches

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When a powerful laser beam is focused on a free electron in vacuum, the ponderomotive force of the laser light pushes electrons in the direction opposite to the gradient of the light intesity and can accelerate them to relativistic energies. Significant progress has been made on laser-based particle acceleration in vacuum in recent years both theoretically [1–3] and experimentally [4,5]. Despite these advances, many challenges remain.

Because the ponderomotive force is proportional to the energy flux of the light, laser acceleration requires a tightly focused high-power beam. A small radial size of the beam, however, results in the large transverse gradient which makes the electron motion unstable in the radial direction. Also, the amplitude of electron oscillations along the polarization direction (so called *quiver* motion) increases with the magnitude of the electric field, and for a small focal spot can easily exceed the beam size, causing the electrons to be *scattered* in the transverse direction. As a result, the electron leaves the acceleration zone prematurely and the phase volume of the accelerated electron beam turns out to be relatively large. This regime of acceleration has been intensively studied theoretically [1,3] and was also observed in the experiment [4,5].

In this paper we propose a novel approach to the laser acceleration, that avoids the transverse scattering during acceleration and allows to achieve extremely small phase volume for the electron beam. We also determine the maximum electron energy that can be obtained in such acceleration and show how it scales with the laser parameters.

Since a laser beam in the lowest approximation can be considered as a plane electromagnetic wave, we first review the main results of the electron motion in a plane electromagnetic wave [6]. Let $A_x(z-ct)$ be the x component of the vector potential corresponding to a plane wave propagating in the z-direction. The electric and magnetic fields in the wave are $E_x = B_y = \partial A_x/\partial z$. Solving equations of motion and assuming that initially, when $A_x = 0$, the accelerating electron was at rest one finds that $p_y = 0$, and

$$p_x = mc\mathcal{A}, \qquad p_z = \frac{mc}{2}\mathcal{A}^2, \qquad \gamma = 1 + \frac{1}{2}\mathcal{A}^2, \tag{1}$$

where $\mathcal{A}$ is the dimensionless vector potential, $\mathcal{A} = eA_x/mc^2$. Although this solution is valid for arbitrary function A_x, in the laser pulse the vector potential can be represented as an oscillation with the frequency ω and a slowly varying envelope such that $A_x \to 0$ when $t \to \pm\infty$. In the limit $\mathcal{A} \gg 1$, the electron is accelerated to relativistic energies. The velocity in the z direction inside the wave is close to the speed of light, $v_z/c \approx 1 - 2/\mathcal{A}^2$. We can estimate the *interaction* (overtake) time during which the particle remains within the wave packet of length l as $T_{\rm int} = l/(c-v_z) = l\mathcal{A}^2/2c$ and the corresponding interaction length $L_{\rm int}$

$$L_{\rm int} = cT_{\rm int} \sim l\frac{\mathcal{A}^2}{2}. \tag{2}$$

It follows from Eq. (1) that after the wave overtakes the electron, the electron comes to rest. Hence, there is no net energy gain in the plain wave, $\Delta\gamma mc^2 = 0$. This result remains also valid in a more general case when the initial velocity of the electron is not equal to zero.

Another important characteristic of the particle motion in the wave is the amplitude of the high-frequency quiver oscillations perpendicular to the direction of the propagation of the wave. Their amplitude Δx is [6]

$$\Delta x \approx \mathcal{A}\lambda\!\!\!^{-}, \tag{3}$$

where $\lambda\!\!\!^{-} = c/\omega$.

In order to explain an energy gain one has to take into account the three dimensional geometry of the laser beam. To simplify description of electron motion in this case, we will use equations of motion averaged over fast oscillations in the wave [7–10]:

$$\frac{d\bar{x}^i}{d\tau} = \frac{\bar{p}^i}{m}, \qquad \frac{d\bar{p}^i}{d\tau} = \frac{mc^2}{2}\frac{\partial U}{\partial \bar{x}_i}, \tag{4}$$

where τ is the proper time, $\bar{x}^i$ and $\bar{p}^i$ are the 4-dimensional coordinate and momentum vectors averaged over the fast oscillations, and the *ponderomotive potential* U is equal to the averaged over the proper time square of the 4-dimensional vector potential of the laser light, $U = -\langle\mathcal{A}_i\mathcal{A}^i\rangle$ with $\mathcal{A}_i = eA_i/mc^2$. Eqs. (4) are valid if the number of oscillations that the particle executes in the laser beam is large. In addition, the gradient scale of the laser beam in the transverse direction should be much larger than the amplitude of the quiver oscillations.

An important advantage of using the averaged description is that in paraxial approximation U only depends on the absolute value of the transverse component of the vector-potential.

In the lowest order of paraxial approximation the electric field in the laser beam is given by a linear combination of eigenmodes $\boldsymbol{E}^{l,m}$ [12]

$$\boldsymbol{E}^{l,m} = \mathrm{Re} E_0^{l,m} \boldsymbol{e}^{l,m} e^{-i\omega t + i\psi_{l,m}(r,z)} H_l \left(\frac{x}{\sqrt{2}\sigma_\perp} \right) \times H_m \left(\frac{y}{\sqrt{2}\sigma_\perp} \right) \frac{\sigma_{\perp 0}}{\sigma_\perp} \exp \left[-\frac{r^2}{4\sigma_\perp^2} - \frac{(z-ct)^2}{4\sigma_z^2} \right], \quad (5)$$

where $E_0^{l,m}$ is the amplitude value of the electric field, $\boldsymbol{e}^{l,m}$ is a unit polarization vector perpendicular to the z-axis, $\psi_{l,m}(r,z)$ is the phase, $\sigma_{\perp 0}$ is the rms beam size in the transverse direction, $\sigma_\perp = \sigma_{\perp 0}\sqrt{1 + z^2/Z_R^2}$, Z_R is the Rayleigh length, σ_z is the beam pulse length, and H_l is the Hermite polynomial.

Consider first a Gaussian pulse corresponding to the lowest mode of the laser beam with $l = m = 0$ and the amplitude electric field $E_0^{0,0} = E_0$. Introducing the amplitude $\mathcal{A}_0$,

$$\mathcal{A}_0 = \frac{e\lambda\!\!\!^{-}}{\sqrt{2}mc^2} E_0, \quad (6)$$

we find for the ponderomotive potential

$$U_0(r,z,t) = \mathcal{A}_0^2 \frac{\sigma_{\perp 0}^2}{\sigma_\perp^2} \exp \left[-\frac{r^2}{2\sigma_\perp^2} - \frac{(z-ct)^2}{2\sigma_z^2} \right]. \quad (7)$$

One can also find the energy $\mathcal{E}$ of the laser pulse in terms of $\mathcal{A}_0$,

$$\mathcal{E} = \frac{\sqrt{2\pi}}{4} \mathcal{A}_0^2 \frac{Z_R \sigma_z}{\lambda\!\!\!^{-} r_e} mc^2. \quad (8)$$

Consider now the acceleration of a particle located initially at rest on the axis, $r = 0$, $z = z_0$. Due to the axisymmetry the particle will remain on the axis all the time, and the averaged equations of motion reduce to

$$\frac{dz}{dt} = \frac{p_z}{m\gamma}, \qquad \frac{dp_z}{dt} = -\frac{mc^2}{2\gamma} \frac{\partial U_0}{\partial z}, \quad (9)$$

where $\gamma = \sqrt{1 + p_z^2/m^2c^2 + U_0}$ and U_0 is now evaluated at $r = 0$.

For given parameters of the laser pulse there is an optimal regime of acceleration which is determined by a special relation between the Rayleigh length and the interaction length. If the Rayleigh length is too large, then in the region of length $L_{\rm int}$ near the focal point the laser pulse can be considered in a good approximation as a plane wave. As was pointed out above, in this case there is no energy transfer between the wave and the particle. On the other hand, if the Rayleigh length is very short, then due to the radial expansion of the laser beam, its amplitude decays before the interaction occurs on the time scale $T_{\rm int}$. Hence the optimal relation between Z_R and $L_{\rm int}$ corresponds to the case when they are of the same order (for $\mathcal{A}_0 \gg 1$),

$$Z_R \sim L_{\rm int} \sim \sigma_z \mathcal{A}_0^2/2. \quad (10)$$

To find the optimal regime of acceleration, we solved numerically Eqs. (9) for several values of $\mathcal{A}_0$ varying z_0 and Z_R. The final value of the factor γ, γ_f, as a function of the ratio σ_z/Z_R is shown in Fig. 1. As follows from this figure the maximum γ_f is achieved when $Z_R \approx \sigma_z \mathcal{A}_0^2/2$, in agreement with Eq. (10), and is approximately equal

$$\gamma_f \approx \frac{\mathcal{A}_0^2}{2}. \quad (11)$$

In the optimal regime the particle's initial position is close to the laser focus, $z_0 \approx 0$.

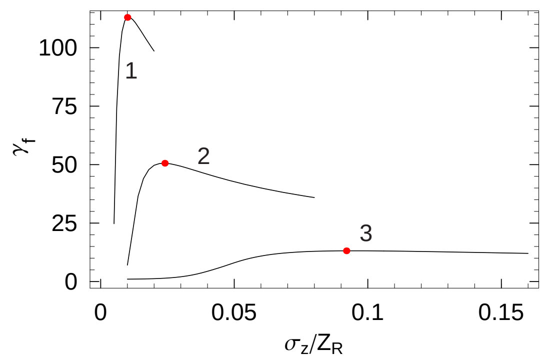

FIG. 1. The value of γ_f after acceleration in a laser pulse given by Eq. (7) as a function of the ratio σ_z/Z_R for three different values of $\mathcal{A}_0$: *1* – $\mathcal{A}_0 = 15$, *2* – $\mathcal{A}_0 = 10$, *3* – $\mathcal{A}_0 = 5$. Each point on these plots was optimized with respect to the initial position z_0. The dots on the curves indicate the maximum values of γ_f for a given $\mathcal{A}_0$.

The optimization for a constant value of $\mathcal{A}_0$ and varying Z_R means varying laser energy. We also carried out optimization for a constant laser energy $\mathcal{E}$. In this case the optimal regime is approximately given by the following relation

$$Z_R \approx \sigma_z \frac{\mathcal{A}_0^2}{4}, \quad (12)$$

and the value of γ_f in optimum is still given by Eq. (11), if the particle is initially located near the focus, $z_0 = 0$. The value of γ_f can be somewhat increased by putting the particle at $z_0 < 0$, and making $\mathcal{A}_0$ larger, but this regime is less preferable because it also increases the amplitude of the quiver oscillations.

From Eqs. (12) and (11) one can find an analytical formula for γ_f in terms of the laser parameters,

$$\gamma_f = \frac{2^{3/4}}{\pi^{1/4}} \left(\frac{\mathcal{E}}{mc^2} \right)^{1/2} \frac{(\lambda\!\!\!^{-} r_e)^{1/2}}{\sigma_z}. \quad (13)$$

Note that γ_f scales as $\mathcal{E}^{1/2}$ and inversely proportional to the bunch length. This emphasizes the effect of shortening of the laser pulse for effective acceleration.

As a numerical example, we assume a Gaussian laser beam with $\mathcal{E} = 1\,\mathrm{J}$, $\lambda = 0.8\,\mu\mathrm{m}$, and the FWHM pulse length 50 fs. Then the optimal parameters are

$$\mathcal{A}_0 = 5.2, \qquad Z_R = 42\ \mu\mathrm{m}, \qquad \sigma_{\perp 0} = 1.4\ \mu\mathrm{m}, \tag{14}$$

and Eq. (13) gives $\gamma_f \approx 13$.

To verify the accuracy of the averaged equations, we also solved the exact three dimensional equations of motion of electron moving in a Gaussian laser beam. The comparison of γ as a function of current position z for the averaged and exact equations of motion for the laser pulse parameters given by Eqs. (14) is shown in Fig. 2. We see that the solution of averaged equations well approximates the smoothened behaviour of the exact equations, with the final value of γ_f very close to each other. In addition to the oscillations of $\gamma(z)$, the exact solution also demonstrates the quiver motion of the electron during acceleration. The amplitude of the quiver oscillations relative to the beam waist for the above example can be estimated using Eqs. (3) and (12),

$$\frac{\Delta x}{\sigma_{\perp 0}} \sim \frac{\mathcal{A}_0 \lambda\!\!\!^{-}}{\sigma_{\perp 0}} \sim \frac{2\lambda\!\!\!^{-}(Z_R/\sigma_z)^{1/2}}{(Z_R\lambda\!\!\!^{-}/2)^{1/2}} \sim \sqrt{8\frac{\lambda\!\!\!^{-}}{\sigma_z}}. \tag{15}$$

For our parameters, we find $\Delta x/\sigma_{\perp 0} \approx 0.4$, in good agreement with simulations, which is small enough to avoid the scattering phenomenon discussed above. Note that acceleration gradient corresponding to Fig. 2 is in the range of ~ 50 GeV/m.

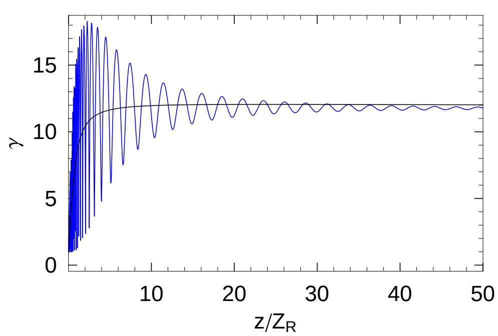

FIG. 2. Comparison of the exact (oscillating curve) and averaged (smooth curve) solutions for the example of Eqs. (14).

Eq. (13) for the final value of γ refers to the case when the electron is initially at rest. One might think that it is possible to achieve much larger final energy by repeating the acceleration process sequentially several times. Unfortunately, it turns out that this is not true. For an electron moving in the direction of the laser beam with the initial energy $mc^2\gamma_0$, inside the laser beam it is accelerated to $\gamma_f \sim \gamma_0 \mathcal{A}$ (we assume $\gamma_0 \gg 1$) [6]. Although this value is $2\gamma_0$ times larger than for the case of $\gamma_0 = 1$ (see Eq. (11)), it turns out that the interaction length in this case also increases and is of the order of $L_{\mathrm{int}} \sim 2l\gamma_0\mathcal{A}^2$. Requiring again $Z_R \sim L_{\mathrm{int}}$, for a given laser energy, we find that $\mathcal{A}$ decreases with the growth of γ_0 resulting in the final energy of acceleration, by order of magnitude, given by the same Eq. (13). At the same time, the amplitude of the quiver oscillations in this regime increases with γ_0 as $\Delta x \sim 2\gamma_0\mathcal{A}\lambda\!\!\!^{-}$ making it more prone for the scattering. For this reasons, we limit our analysis below by the case $\gamma_0 = 1$.

The above results refer to the special case of particles initially located on the axis. Off axis particles in a Gaussian laser beam are accelerated by the radial ponderomotive force and are quickly expelled from the beam in the radial direction. To avoid this instability of motion it is necessary to create a potential well in the radial direction, that would focus the accelerating particles toward the axis [11]. Such focusing can be achieved by superposition of a Gaussian mode given by Eq. (5) with $l = m = 0$ and higher-order modes of the laser light with $l, m \geq 0$. For example, a linear combination of $l = 1$, $m = 0$ and $l = 0$, $m = 1$ with the same polarization vectors $\boldsymbol{e}^{1,0} = \boldsymbol{e}^{0,1}$, equal amplitudes and relative $\pi/2$ phase shift, so that $E^{1,0} = iE^{0,1}$ has the following ponderomotive potential U_1,

$$U_1(r,z,t) = \frac{\mathcal{A}_1^2 r^2 \sigma_{\perp 0}^2}{\sigma_\perp^4} \times \exp\left[-\frac{r^2\sigma_{\perp 0}^2}{2\sigma_\perp^4} - \frac{(z-ct)^2}{2\sigma_z^2}\right] \tag{16}$$

where $\mathcal{A}_1$ is the dimensionless vector potential. This ponderomotive potential is axisymmetric and has a minimum on the axis, at $r = 0$. If the Gaussian mode does not interfere with the higher-order mode, the ponderomotive potentials add, $U = U_0 + U_1$, and for the energy of U_1 mode large enough the total potential exhibits a minimum in the radial direction, as shown in Fig. 3.

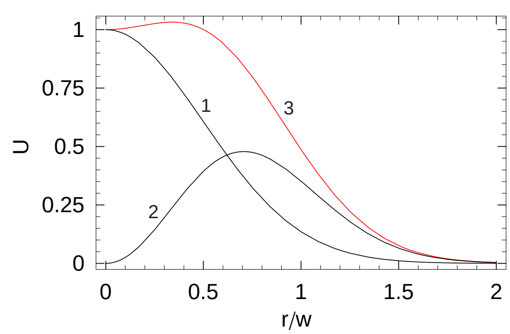

FIG. 3. Radial profile of the ponderomotive potential (in arbitrary units) for the Gaussian mode (*1*), the second mode (*2*) and their sum (*3*) when the modes have the same frequency and the energy of the second mode is 1.3 times larger than the energy of the first one.

To avoid interference of the modes (7) and (16) one can either use different frequencies, or the same frequency with orthogonal polarization of the modes, $\boldsymbol{e}^{0,0} \cdot \boldsymbol{e}^{0,1} = 0$.

We performed computer simulations for the two-mode acceleration scheme assuming that the modes have the same frequency, equal Rayleigh lengths and σ_z, with the energy $\mathcal{E}_0 = 1\,\mathrm{J}$ (Gaussian beam), $\mathcal{E}_1 = 1.3\,\mathrm{J}$ (second mode), $\lambda = 0.8\,\mu\mathrm{m}$ and $\sigma_z/c = 21\,\mathrm{fs}$ (50 fs FWHM). A collection of particles was initially uniformly distributed near the laser focus and the particle orbits were tracked using the averaged equations of motion. The results presented below refer to the case when initially electrons occupied the volume $r < r_0$ and $z_1 < z < z_2$ with $r_0 = 0.8\ \mu$m, and $z_1 = 1\ \mu$m, $z_2 = 10\ \mu$m and initial zero velocity. We found that the transverse motion in this case was stable, and the average over particles γ factor was very close to the smooth curve shown in Fig. 2. The relative energy spread after acceleration (rms) was $\Delta\gamma/\gamma \approx 3\%$. The normalized transverse emittance, of the accelerated electrons defined by $\epsilon_x = \gamma \left[\langle (x - \langle x \rangle)^2 \rangle \langle (x' - \langle x' \rangle)^2 \rangle - \langle (x - \langle x \rangle)(x' - \langle x' \rangle) \rangle^2 \right]^{1/2}$, with the averaging over the ensemble of particles was $8 \cdot 10^{-7}$ cm.

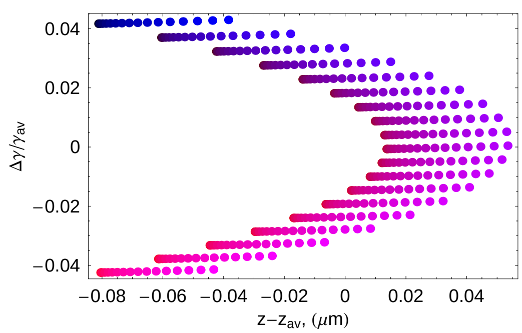

FIG. 4. Longitudinal phase space

We also observed an extremely small longitudinal emittance of the bunch. The longitudinal phase space at the point of the minimum bunch length is shown in Fig. 4. The minimal RMS bunch length is about 0.03 μm, or 100 attoseconds, with the normalized longitudinal emittance $\epsilon_z = 10^{-6}$ cm.

The maximum number of electrons that can be accelerated in such bunch will be limited by the space charge effect and it can be estimated as follows. The transverse electric field in the bunch is $E_\perp \sim eN\gamma/\sigma_\perp^2$, where N is the number of electrons. Due to the relativistic cancellation of the magnetic and electric forces the net transverse force has an additional factor γ^{-2}

$$F_\perp \sim \frac{eE_\perp}{\gamma^2} \sim \frac{e^2 N}{\gamma \sigma_\perp^2}. \tag{17}$$

After acceleration, the transverse momentum of electrons $p_\perp$ is will result in the radial expansion of the beam on the time scale $t \sim \sigma_\perp/v_\perp \sim m\gamma\sigma_\perp/p_\perp$. The contribution of the space charge (SC) force to $p_\perp$ during expansion is $\Delta p_\perp^{(SC)} \sim F_\perp t \sim m^2 c^2 r_e N/\sigma_\perp p_\perp$. To avoid the emittance dilution due to the space charge, we require that $\Delta p_\perp^{(SC)} < p_\perp$ yielding

$$N < \frac{\sigma_\perp}{r_e} \left(\frac{p_\perp}{mc} \right)^2. \tag{18}$$

For our parameters this gives $N \lesssim 2 \cdot 10^5 - 10^6$.

To illustrate the extraordinary quality of such beam, we calculate its *brightness* B defined as the number of particles divided by the product of the normalized emittances:

$$B = \frac{N}{\epsilon_x \epsilon_y \epsilon_z}. \tag{19}$$

For the parameters of the beam described above, $B = 3 \cdot 10^{23} - 1.5 \cdot 10^{24}$ cm^{-3} which is 4-5 orders of magnitude higher than in the best electron sources available today. A beam with such brightness opens new opportunities for generation of coherent x-ray radiation as well as extremely short (attosecond) electromagnetic pulses.

In summary, we used averaged equations of motion for a relativistic electron to study the laser acceleration in vacuum in the regime that avoids scattering of accelerating electron in the radial direction. A simple analytical formula for the electron energy is found, and it was shown that both effective acceleration and focusing can be achieved with the use of the radial profiling of the laser beam. The resulting electron beam is characterized by extremely small longitudinal and the transverse emittances.

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