Conventional Beams or Neutrino Factories: The Next Generation of Accelerator-Based Neutrino Experiments *

Burton Richter

Stanford Linear Accelerator Center
Stanford, California 94309-0450

Abstract

The purpose of this paper is to provoke a discussion about the right next step in accelerator-based neutrino physics. In the next five years many experiments will be done to determine the neutrino mixing parameters. However, the small parameters θ_{13}, Δm_{21}^2, and the CP violating phase are unlikely to be well determined. Here, I look at the potential of high-intensity, low-energy, narrow-band conventional neutrino beams to determine these parameters. I find, after roughly estimating the possible intensity and purity of conventional neutrino and anti-neutrino beam, that $\sin^2 \theta_{13}$ can be measured if greater than a few parts in ten thousand, Δm_{21}^2 can be measured if it is greater than 4×10^{-5} (eV)^2, and the CP violating phase can be measured if it is greater than $20°$ and the other parameters are not at their lower bounds. If these conclusions stand up to more detailed analysis, these experiments can be done long before a muon storage ring source could be built, and at much less cost.

Neutrino physics has become one of the central studies of current high-energy physics. Experiments on neutrinos from the sun have shown that not enough of them reach the earth to be consistent with the suns energy output. Experiments on neutrinos produced in the atmosphere have shown that muon-type neutrinos seem to be depleted while passing through the earth compared to those incident on detectors directly from above. Neutrinos no longer seem to be the zero-mass passive participants in the sub-nuclear world that they were thought to be only a few years ago. Instead, the picture that is emerging shows a system of three, or possibly four, neutrino-mass eigenstates that mix in different amounts to form the familiar flavor eigenstates, the electron, muon and tau neutrino types. What is produced in a reaction is a flavor eigenstate which evolves as it travels because of a change in relative phases of the different mass eigenstates arising from their relative mass differences.

Each year brings new information that is beginning to fill in a still incomplete picture. The continuation of several current experiments and a host of soon to be begun experiments will further fill in the picture. However, enough is known now to indicate that certain of the interesting parameters, in particular the mixing angle known as θ_{13}, the mass difference Δm_{21}^2, the magnitude if any of CP violation in the neutrino sector, and the ordering of the neutrino masses cannot be well determined, if determined at all, from the current crop of running and planned experiments. Thus attention has begun to turn toward high-intensity accelerator sources to get further information.

Much of this attention has been focused on the potential of high-energy (20 GeV and above) muon storage rings as candidate sources. [1] These devices require considerable R&D to determine whether they are indeed feasible, and, if feasible, will be very expensive. In this paper I will discuss the potential of high-intensity, conventional neutrino beams to get at these small parameters. My conclusion is that low-energy, conventional muon-neutrino beams of attainable intensity are serious candidates for the next generation systems.

Before analyzing what can be done in the real world, it is useful to look at a primitive, two-neutrino, model to illustrate why low energy may be better than high energy in untangling the puzzle. Consider two species with a small mixing amplitude between them. The "signal-to-noise" ratio in an experiment looking for the appearance of neutrino type two in a beam of neutrino type one is given by

$$\frac{P(\nu_1 \to \nu_2)}{P(\nu_1 \to \nu_1)} = \frac{A^2 \sin^2(\Delta m^2 L/4E)}{1 - A^2 \sin^2(\Delta m^2 L/4E)} \tag{1}$$

where A is the mixing amplitude, Δm^2 is the difference of the squares of the masses, L is the distance from the source to the detector, and E is the beam energy. The optimum signal-to-noise ratio comes when the sine term is equal to one, $i.e.$, $\Delta m^2 L/4E$ is equal to an odd integer multiple of $\pi/2$. However, all of the muon storage ring designs have high energy, making this factor small with the known mass difference. Hence, a very sophisticated background rejection is required in the detector.

It is also sometimes argued that, because neutrino cross sections increase with energy and the flux of neutrinos increases with the square of the energy of the parent particle, high-energy beams are better than low energy ones. However, in looking for the small terms that are unlikely to be determined by the present round of experiments, the appearance of a neutrino species different from the primary species gives the most sensitive tests. This probability is proportional to E^{-2}, leaving only a single power of the energy as a potential advantage for the high-energy beams. Thus, this rationale for the choice of high energy is not as overwhelming as it might appear.

In addition, in the real three-neutrino world, there is a further complication from high-energy beams, tau-lepton production. In the scenarios using a storage-ring source, tau-lepton production is a potentially serious complication in determining the small parameters, since its leptonic and semi-leptonic decays can make a tau event look like an electron or a muon event. It is proposed to solve this problem by determining, for example, the sign of electrons produced by muon neutrinos [2] which would be effective but is very difficult in a large detector. It is a simplification if one can stay below the tau-lepton production threshold.

Now I turn to the standard three-neutrino formulation (recent results from Super Kamiokande appear to give 95% confidence in this formulation). Equation (3) includes the small terms that are often dropped in oscillation analyses. In Eq. (3), C_{ij} means $\cos \theta_{ij}$, S_{ij} means $\sin \theta_{ij}$, Δm_{ij}^2 means $m_i^2 - m_j^2$, δ is the CP violating phase, L is the distance from the source to the detector, and E is the neutrino energy. The matter effect is given by

$$a = 2\sqrt{2}\, G_F n_e E = 7.6 \times 10^{-5}\, \rho(gm/cm^3)\, E(GeV) \qquad (eV)^2 \qquad (2)$$

where G_F is the Fermi weak-coupling constant, and n_e is the electron density. Equation(3) is exact when the matter term of Eq. (2) is zero; otherwise $a/\Delta m_{31}^2$ is assumed to be small.[3]

$$
\begin{aligned}
P(\nu_\mu \to \nu_e) =\ & 4C_{13}^2 S_{13}^2 S_{23}^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E} \times \left(1 + \frac{2a}{\Delta m_{31}^2}\left(1 - 2S_{13}^2\right)\right) \\
& + 8C_{13}^2 S_{12} S_{13} S_{23} (C_{12} C_{23} \cos \delta - S_{12} S_{13} S_{23}) \cos \frac{\Delta m_{32}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \sin \frac{\Delta m_{21}^2 L}{4E} \\
& - 8C_{13}^2 S_{13}^2 S_{23}^2 \cos \frac{\Delta m_{32}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \frac{aL}{4E}\left(1 - 2S_{13}^2\right) \\
& - 8C_{13}^2 C_{12} C_{23} S_{12} S_{13} S_{23} \sin \delta \sin \frac{\Delta m_{32}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \sin \frac{\Delta m_{21}^2 L}{4E} \\
& + 4S_{12}^2 C_{13}^2 \left\{ C_{12}^2 C_{23}^2 + S_{12}^2 S_{23}^2 S_{13}^2 - 2C_{12} C_{23} S_{12} S_{23} S_{13} \cos \delta \right\} \sin^2 \frac{\Delta m_{21}^2 L}{4E} \ .
\end{aligned}
\qquad (3)
$$

It is not my purpose here to do an exhaustive analysis of all of the possibilities, but to show some examples of the potential, and in what follows I will assume that $\theta_{23} = \theta_{12} = \pi/4$. The Super Kamiokande atmospheric data favors this for θ_{23} and their solar data favor the large mixing-angle solution for the solar neutrino deficit. I

will also take $\Delta m_{32}^2 = 3 \times 10^{-3}\,(eV)^2$ which is the central value of the latest Super-K data, [4] $\Delta m_{31}^2 \approx \Delta m_{32}^2$, $S_{13}^2 \ll 1$, and the matter term $(a) = 0$.

Now, look at the results of four experiments; $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ at two values of L/E such that $\Delta m_{32}^2 L/4E$ equals $\pi/2$ and π. The result when the Δm_{32}^2 term is $\pi/2$ is,

$$P_+(\pi/2) = P(\nu_\mu \rightarrow \nu_e) + P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = 4S_{13}^2 + \sin^2 \frac{\Delta m_{21}^2 L}{4E} \tag{4}$$

$$P_-(\pi/2) = P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = -4S_{13}\sin\delta \sin \frac{\Delta m_{21}^2 L}{4E} \tag{5}$$

and, when the mass term is π, is

$$P_+(\pi) = P(\nu_\mu \rightarrow \nu_e) + P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \sin^2 \frac{\Delta m_{21}^2 L}{4E} \tag{6}$$

$$P_-(\pi) = P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = 0 \ . \tag{7}$$

The next question is, are there enough neutrinos to do useful experiments. It will require a detailed design of a beam to get a precise answer, but we can get a rough idea by extrapolating from work already done at FNAL on the MINOS beam design and from the FNAL storage ring source study.[1] The present FNAL proton beam power for the MINOS beam is about 400 kW. The beam intensity is limited by space-charge at injection into the 8-GeV Booster and at injection into the 100–GeV Main Injector. If the FNAL linacs energy was increased to about 1 GeV and the Boosters energy increased to about 16 GeV, the FNAL beam on target could be increased to about 4MW. [5] The improvements to the injector chain are straightforward. The proton target is more difficult than the present system and will certainly require more shielding. Target R&D is being intensively pursued for the Spallation Neutron Source and should present no fundamental problems for neutrino production. The target for a conventional beam is somewhat easier per unit beam power than that for a muon storage ring source. The conventional beam is designed to produce pions of multi-GeV energy, while the muon source uses pions of low energy. For maximum production, the conventional source is thus thinner and absorbs less power than the storage ring source.

The comparative yields from a high-intensity, conventional, wide-band beam and a muon storage ring can be estimated by scaling the numbers given in the FNAL report. Table 1 shows the numbers. Using those numbers, a 20 kiloton detector at 732 km would see about 9×10^4 events per year from the low-energy, wide-band, conventional beam.

At 732 km, the neutrino beam energies required to have $(\Delta m_{32}^2 L/4E)$ equal to $\pi/2$ and π would be about 2 GeV and 1 GeV respectively (π mesons of about 9 and 4.5 GeV). There is no design available for a narrow band beam, and so I use the spectrum of the MINOS low-energy beam to make a rough estimate. The spectrum has a low-energy peak and a high-energy tail, each generating about equal number of events [6] Much improved focusing can be achieved in a narrow band beam, which

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Table 1: Muon-neutrino charged-current events per kiloton year, assuming no oscillation, for a 4–MW upgraded FNAL wide-band conventional source, and for a 20-GeV muon storage ring giving 10^{20} decays per year. The numbers are scaled from Ref. [1].

Source Type	Mean ν Energy (GeV)	Baseline (km)	N CC-events/kT-year
Conventional	3	732	4.6×10^3
Conventional	6	732	1.4×10^4
Conventional	12	732	3.2×10^4
Storage Ring	15	732	1.2×10^4

increases the flux. The peak in the MINOS spectrum has a width of about $\pm$ 30%, while it is desirable for background rejection to have a narrower width, thus reducing the flux. I will assume that the spectrum has a width of ($\pm$ 10–20% and that the focusing and width effects balance out in determining the beam intensity. Finally, I will give up another factor of two because this estimate may be too optimistic, or the FNAL Main Injector upgrade may not be capable of reaching the 4 MW power level. The result for neutrinos in the 20-kiloton detector at 732 km from the source is

$$Y_{cc}(2\ GeV) = 2.5 \times 10^4 \qquad \text{events per year.} \tag{8}$$

A 1–GeV beam will have a lower rate. There will be more pions in the decay channel, but the neutrino beam divergence will increase by a factor of two and the charged-current neutrino cross-section will decrease by a factor of two resulting in a reduction of about a factor of four in event rate

$$Y_{cc}(1\ GeV) = 6 \times 10^3 \qquad \text{events per year.} \tag{9}$$

Note that antineutrino cross-sections are about half of those for neutrinos.

Electron neutrinos in the beam come from the chain $\pi \rightarrow \mu \rightarrow e$ in the decay channel. They amount to about 0.1% total at the far detector, and amount to $(1 - 2) \times 10^{-4}$ in the muon neutrino energy range. However, a more serious background will be neutral-current interactions initiated by muon and tau neutrinos in the detector. These can give π^0 mesons, and highly asymmetric conversions of the π^0 decay y's can be confused with electron-neutrino-charged current events.[7] These have to be controlled by detector design. Here I will assume that they give a background of 0.1% of the "unoscillated" muon-neutrino flux.

The optimum data-taking strategy depends on "a priori" knowledge. If, for example, KamLAND determines Δm_{21}^2, there is no reason to take data at 1 GeV. Here, I will simply assume that data equivalent to 10^4 charged-current events (in the absence of oscillation) is collected for ν_μ and $\bar{\nu}_\mu$ at both 2 GeV and 1 GeV. I also ignore the effect of the finite energy spread in the beam.

From Eq. (6), the minimum detectable value of Δm_{21}^2 different from zero at the three standard-deviation level is

$$\left(\Delta m_{21}^2\right)_{\min} = 4.2 \times 10^{-5} \ (eV)^2 \ . \tag{10}$$

This is independent of θ_{13} as long as C_{13} is near one. It is near the lower end of the solar Large Mixing Angle solution allowed region and, if this large, might be measurable by the KamLAND experiment.

Equation (4) couples Δm_{21}^2 and S_{13}^2. Table 2 gives the 3σ lower bound for S_{13}^2 for various values of Δm_{21}^2 ranging from the minimum detectable in this proposed experiment to the maximum in the solar LMA solution. What is happening, as the usually ignored Δm_{21}^2 term increases, is that more events have to come from the S_{13}^2 term to meet the three standard deviation requirement.

Table 2: Three standard deviation lower bound on the determination of S_{13}^2 for various values of Δm_{21}^2.

$\Delta m_{21}^2 \ (eV)^2$	S_{13}^2
4×10^{-5}	5×10^{-4}
2×10^{-4}	9×10^{-4}
1×10^{-3}	4×10^{-3}

The last item to look at is the sensitivity to the CP violating phase δ. This is given in Eq.(5) and depends on both S_{13} and Δm_{21}^2 (note also that the sign of Δm_{21}^2 comes in). If either of these is very small, CP violation becomes impossible to measure in this or any other experiment. To give an idea of the sensitivity, I will take $\Delta m_{21}^2 = 2 \times 10^{-4}$, the center of its range, and take S_{13}^2 at the 3σ limit for this Δm_{21}^2, *i.e.* 9×10^{-4}. The minimum detectable value for the phase that differs from zero by three σ is,

$$|\sin\delta|_{\min} = 0.35 \tag{11}$$

where Eq. (4) has been used to determine the error on the non-CP violating yield.

The determination of δ is most sensitive to the matter term. With Satos [3] first order analysis, the matter term is 30% of the CP violating term for the parameters used above. I believe the matter effect needs to be taken to higher order. It is interesting to note that as the neutrino beam energy is increased, the matter term gets bigger while the CP violating term gets smaller, another argument for low-energy beams.

In summary, I have analyzed here the potential of high-intensity, low-energy, narrow-band conventional neutrino beams. The "gedanken" experiments outlined here give interesting limits on the measurement of the small terms among the neutrino-mixing parameters. These limits are better than those from the entry level storage

ring neutrino factory. [8] It is well worth the time of the experts to see if my assumptions on potential beam intensity and purity, and background rejection are reasonable. If they are, these experiments can be carried out sooner, and at less cost than those with a muon storage ring source.

References

[1] Fermilab-FN-692, May 10, 2000.

[2] *Op cit.* FNAL report.

[3] I thank João Silva of Instituto Superior de Engenharia de Lisboa, Portugal, for showing me the exact expression in the absence of matter effect. The first order matter effects come from J. Sato, hep-ph/0006127, 13 June 2000. In determining the small parameters it may not be consistent to keep first order terms in the matter effect while keeping all of the other terms. In working out examples I have set the matter effect equal to zero. It will eventually have to be put in consistently.

[4] The latest Super Kamiokande data is not yet published, but is available in the talks of Y. Takeuchi and T. Toshito in session PA-086 at the XXX International Conference on High-Energy Physics, Osaka, July 2000, `http://ichep2000.hep.sci.osaka-u.ac.jp/`.

[5] There are many other things that would need doing, but the Linac and Booster work are the costly ones.

[6] MINOS Technical Design Report, Figure 3.3, `http://www.hep.anl.gov/ndk/hypertext/minostdr.html`. The figure shows the interaction energy spectrum for the low, medium, and high energy beams. The figure also shows the limit for "perfect focusing" of all energies. This limit is much easier to approach in a narrow-band beam than in a wide-band beam.

[7] This was the limiting factor in BNL Experiment 776 [see Phys. Rev. Lett. **62**, 2237 (1989)] .

[8] See, for example, V. Barger *et al.*, hep-ph/0003184 v2, July 2000.