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Numerical Calculation of Coherent Synchrotron Radiation Effects Using TraFiC^{4*}

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Abstract

Coherent synchrotron radiation (CSR) occurs when short bunches travel on strongly bent trajectories. Its effects on high-quality beams can be severe and are well understood qualitatively. For quantitative results, however, one has to rely on numerical methods. There exist several simulation codes utilizing different approaches. We describe in some detail the code TraFiC^4 developed at DESY for design and analysis purposes, which approaches the problem from first principles and solves the equations of motion either perturbatively or self-consistently. We present some calculational results and comparison with experimental data. Also, we give examples of how the code can be used to design beamlines with minimal emittance growth due to CSR.

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1 Introduction

1.1 Effects of Retarded Fields on a Bend

When short bunches travel on bent trajectories, electromagnetic fields emitted by the tail of the bunch may overtake the head, influencing the collective behavior of the bunch. As opposed to space-charge effects, this effect is, for relativistic beams, completely due to geometry and thus independent of energy. On a bend with radius R, the arc length needed to allow fields to catch up a distance σ is, to first nontrivial order in L/R, $L_{OT} = \sqrt[3]{24\sigma R^2}$ [1]. The bunch then interacts collectively with itself; this mechanism is commonly referred to as coherent synchrotron radiation (CSR).

The longitudinal electric field will cause the bunch to develop a longitudinal energy gradient, which will lead to a growth of the projected emittance due to dispersion mismatch, as the energy gains and losses are induced in regions with non-vanishing dispersion. For Free-Electron Laser operation, the interesting quantity is the slice emittance, i. e., the emittance of a longitudinally small sub-ensemble of the bunch. This quantity may also suffer degradation, as a slice can pick up an energy spread due to both the transversal and – in case the slice tilts due to the beamline geometry – longitudinal variation of the CSR fields. The transverse fields will exert transverse kicks on the particles, leading to both projected and slice emittance growth, although this effect is of less importance for the cases studied so far. The transverse and longitudinal part of the CSR force are intricately related to each other, leading to a cancellation of part of the transverse force by the effective force due to the energy change in dispersive regions [2, 3, 4]. Using the usual textbook formulae for retarded fields, one easily sees that, on a curved trajectory, the transverse electric field caused by acceleration is acting instantaneously, while the longitudinal field needs some build-up length (of the order of L_{OT}) when entering or a bent region before reaching a steady-state value. The same is true for the decay to 0 after leaving a bent region. Consequently, CSR effects are not expected to play a role if $L_{bends} \ll L_{OT}$. If $L_{bends} \gg L_{OT}$, the system can be treated using analytic formulae derived for the steady-state case[1]. When evaluating systems in which $L_{OT} \approx L_{bends}$, steady-state formulae are of limited utility, as they disregard the transient effects.

2 The Code `TraFiC`^{4}

A way to handle this difficulty is a tracking code incorporating a field calculation from first principles, i. e., by numerically solving Maxwell's equations for a bunch of particles traveling through a given beamline. This can be done by using the method of retarded fields. `TraFiC`^{4} (standing for "Tracking particles in the Fields of Continuous Charges in Cartesian Coordinates") is a tracking code implementing this concept.

2.1 Cartesian Tracking

The effects of CSR on the bunch described above are nonlocal both temporally and spatially: the equations of motion are integro-differential equations. Thus, the history of the beam has to be known to determine the fields acting on it. To store the history of each particle, using a global Cartesian coordinate system instead of the usual local co-moving one is an obvious choice. Since a complete history is impossible to keep in memory, the problem is discretized. The beamline is divided into slices; on each slice, the relevant parameters of a particle entering the slice are stored by a first, "macro-tracking" run. These could, in principle, just be the usual six local phasespace coordinates, which are the complete set of initial conditions for the local differential equations of motion. Instead, we use as six independent parameters the positional vector $\boldsymbol{q}$, the normalized tangential vector $\boldsymbol{e} = \frac{\mathrm{d}}{\mathrm{d}s}\boldsymbol{q}(s)$, and the relativistic factor γ (using the velocity vector $\boldsymbol{\beta}$ would not make sense from a numerical point of view). We also store the arc length s and time t traveled by the particle from the zero-point of the trajectory. In these coordinates, the equations of motion read: $\frac{\mathrm{d}}{\mathrm{d}s}(\boldsymbol{e}, \boldsymbol{q}, \gamma) = \left(\frac{1}{\gamma - \gamma^{-1}}\boldsymbol{F}_\perp, \boldsymbol{e}, F_\parallel\right)$ where $\boldsymbol{F} = \frac{e}{m}(\beta\boldsymbol{e} \times \boldsymbol{B} + \boldsymbol{E})$ and $F_\parallel = (\boldsymbol{F} \cdot \boldsymbol{e})$, $\boldsymbol{F}_\perp = \boldsymbol{F} - F_\parallel \boldsymbol{e}$. Each slice contains a reference to the geometry and the parameters of the encompassing beamline element (such as drift, bend, and quad). If we are now to find the parameters of a particle for a given time or arc length, we can do so by a two-step process: (1) The slice n in which the particle is at that t or s is found by a binary search over the stored positions on the slices; (2) The particle is "micro-tracked" into the slice by an amount of $t - t_n$, using the initial conditions stored at the slice's entrance. By this combination of storing and calculating trajectories, we are able to efficiently refer to the history of particles in computations to any desired precision. For efficiency reasons, the routine doing the micro-tracking is controlled by a bit-field telling it which parameters to output. E. g., the local acceleration of a particle is not needed for finding the retarded position of a particle, although it is needed for the field calculation.

The usual phasespace-coordinates are regained by, for a given instant, finding t_0 for which $|\boldsymbol{p}(t) - \boldsymbol{p}_0(t + t_0)|$ becomes minimal. $\boldsymbol{p}(t) - \boldsymbol{p}_0(t + t_0)$ then is decomposed with respect to the local dreibein tangential to $p_0(t + t_0)$ whose (x, l)-plane coincides with the plane of motion in the last preceding bend.

2.2 Field Calculation

Extended charged distributions are used to model the bunch generating the field. One can not use point particles for this purpose: as it is known from the theory of synchrotron radiation, a point particle on a trajectory of bending radius ρ generates a radiation pulse of time width given by the critical frequency: $\tau \approx \frac{1}{\omega_c} \approx \rho\gamma^{-3}$. If a one-dimensional bunch of length σ_z was to be modeled by point particles, one would need $N \approx \frac{\sigma_z}{\rho}\gamma^3$ particles and $O(N^2)$ operations when using a point-to-point algorithm. Instead, `TraFiC`[4] tracks the centroids of smeared-out bunchlets; a smooth field is generated by numerically integrating over the distribution [5, 6] and summing up the fields generated by the bunchlets.

1- or 2-dimensional bunches are smeared along their trajectory or along their trajectory and perpendicular to the plane of motion, respectively. For the time being, one plane of motion has to be specified; in this regard, `TraFiC`^{4} is not fully 3-dimensional yet (a new version of the field calculation lifts this restriction[7]). The charge distribution is Gaussian in both directions.

2.3 Shielding

The fields generated by the bunch can change substantially when non-trivial boundary conditions are present. `TraFiC`^{4} can handle a very limited case of shielding, namely that of infinitely extended conductive parallel plates. The shielding effects are incorporated by summing up the fields due to the image charges of the generating bunchlets up to a certain user-defined cut-off distance. As the trajectories of the test particles cannot cross the trajectories of the images charges, no singularity will be encountered. Therefore, 1-dimensional bunchlets can be used for the image charges, substantially saving CPU time.

2.4 Perturbative Tracking

In most practical instances, one can assume that the changes of a trajectory due to self-forces is small as compared to the characteristic dimensions (bending radii, focal lengths) of the system. Thus, the fields generated by a bunch not affected by its own field will not differ too much from the fields generated by a bunch traveling under the influence of its self-generated fields. Consequently, we can find an approximate solution of the problem by (1) tracking a bunch of particles through the beamline, considering only the external guiding fields and (2) tracking a second bunch with identical initial conditions, this by solving the discretized version of 2.1 in the presence of the fields which can be calculated by referring to the unperturbed trajectory. The tracked bunch need not be an exact copy of the generating bunch. When calculating emittance dilutions for FEL applications, one is interested in the slice emittance of a slice about the length of the FEL slippage length. For this purposes, a sampling bunch can be used. It consists of point particles, can have initial parameters different from the generating bunch and does not contribute to the field calculation, but samples only the fields generated by the higher-dimensional generating bunchlets.

2.5 Self-Consistent Tracking

The approach described above will not work if the self-interaction of the bunch will generate significant deviations from the unperturbed trajectory – e. g., a bunch in a compressor chicane might collect a correlated longitudinal energy spread of the same order of magnitude as the one induced before the chicane. Then, the bunch length will not agree with the one generated by unperturbed tracking; consequently, the calculated CSR fields and resulting phase space distributions will be incorrect. As the problem is causal, we can approximate a self-consistent solution in the following way: Two bunches B_0 and B_1 with identical initial conditions are created. In turns, the bunches are tracked through the sequence of slices. While tracking bunch B_i through slice n, it is kicked by the fields generated by B_{1-i}. The deviation of the trajectories of $B_{0,1}$ gives an estimate of the deviation from the self-consistent solution; it can diminished by refining the discretization of the beamline and the bunch population. When using a separate sampling bunch with this approach, the average of the fields generated by $B_{0,1}$ is applied after they have been tracked.

2.6 Bunches

Bunches consist of particles (0-dimensional charges) or bunchlets (1- or 2-dimensional charge distributions). Particles are used to sample field distributions.

Three methods of beam population are implemented in `TraFiC`^{4}: The bunchlets can either be set on a regular lattice in six-dimensional phasespace, they can be distributed quasi-randomly or pseudo-randomly. All three methods are based on a general module for generating distributions with a given correlation matrix $\sigma_{ik} = \langle x_i x_k \rangle$.The Cholesky decomposition C, where $CC^T = \Sigma$, is used to transform a normal-distributed vector sequence into one with given M. The vector sequence can be generated by

1. enumerating a d-dimensional parallelepiped lattice
2. enumerating a d-dimensional Sobol sequence
3. generating d-tuples of Gaussian distributed pseudo-random numbers

Method (1) is used to generate the generating bunch, while (2) or (3) are used for the sampling bunch. In the `TraFiC`^{4} input file, the correlation matrix is not given directly, but in its traditional form (assuming vanishing correlations between transversal planes and longitudinal plane), namely by specifying Twiss parameters α, β, ϵ for the transversal plane and $\delta_{incoherent}, \delta_{coherent}, \sigma_z$ for the longitudinal plane.

2.7 Output Data

`TraFiC`^{4} can provide the complete information gathered in a run for further processing: the fields acting on each point and the phasespace co-ordinates of each particle. However, this huge amount of data can be suppressed. `TraFiC`^{4} also calculates the more useful collective quantities (rms values, average values, Twiss parameters, transfer matrices, non-linear transfer matrices to 2nd order, chromaticities). The emittance is the usual statistical emittance; also, the area of the convex hull of the n-σ particles is calculated. The data is written in ASCII format into a single output file, from which its parts (e.g., Twiss parameters vs. beamline position) can be extracted with an included tool for easy post-processing. For identification and debugging purposes, the complete parameters of the run, including version information on all modules, are written to the output file.

2.8 The Code

`TraFiC`^{4} is written in FORTRAN77 (field calculation) and ANSI C++ (tracking, setup and evaluation). It currently comprises about 10 000 lines of sources text. Its object-oriented approach allows for easy augmentation by new types of elements. A symbolic input language with the possibility to define beamline-valued functions makes it easy to check beamline design alterations.

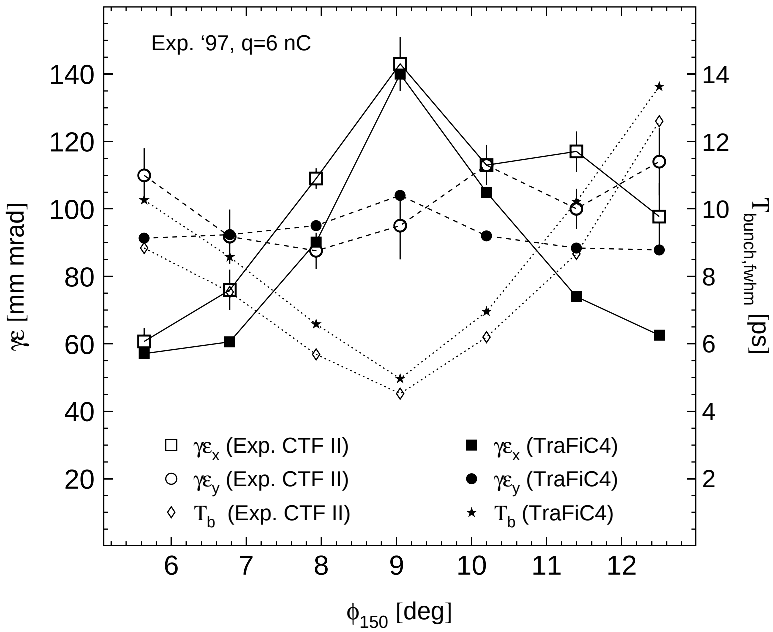

Figure 1: Emittance measurements and simulation results for the CLIC Test Facility

3 Application Examples

3.1 The CTF2 Bunch Compressor

`TraFiC`^{4} was used to simulate the emittance dilution in the CLIC Test Facility Bunch Compressor[8], which has been measured[9]. Figure 1[9, 10] shows the results for $\epsilon_{x,y}$ and σ_z. There is some underestimation of the emittance growth for bending angles left of the peak. A cause of this might be the overestimation of the bunch length, which might stem from a non-Gaussian bunch in the experiment. On the other side of the peak, the experiment shows a strongly deviating behavior in ϵ_x and ϵ_y. As ϵ_y also increases, one may conclude that other sources than CSR-induced dispersive mismatch of emittance might be responsible for that, such as the proximity of the vacuum chamber wall. However, the agreement is quite reasonable, and the signature of the experimental result is clearly reproduced.

3.2 LCLS Dogleg

Emittance growth induced by CSR and dispersion mismatch is a highly correlated process. This means that it can, in principle, be undone: one can untangle the disturbed transverse phasespace by applying to each particle the opposite dispersive kick it suffered at an earlier stage[11]. For an example, consider the optics for a proposed dogleg injector layout for the LCLS[12]. It is used to transport a 150MeV beam to a parallel offset tunnel through two bends of $\pm 38\,\text{deg}$. The bends are realised by four bending magnets, which have signature $+_1 + -_2 -$, and some quadrupole chosen such that the transfer matrix $T_{x,12} = 1$. 2 shows

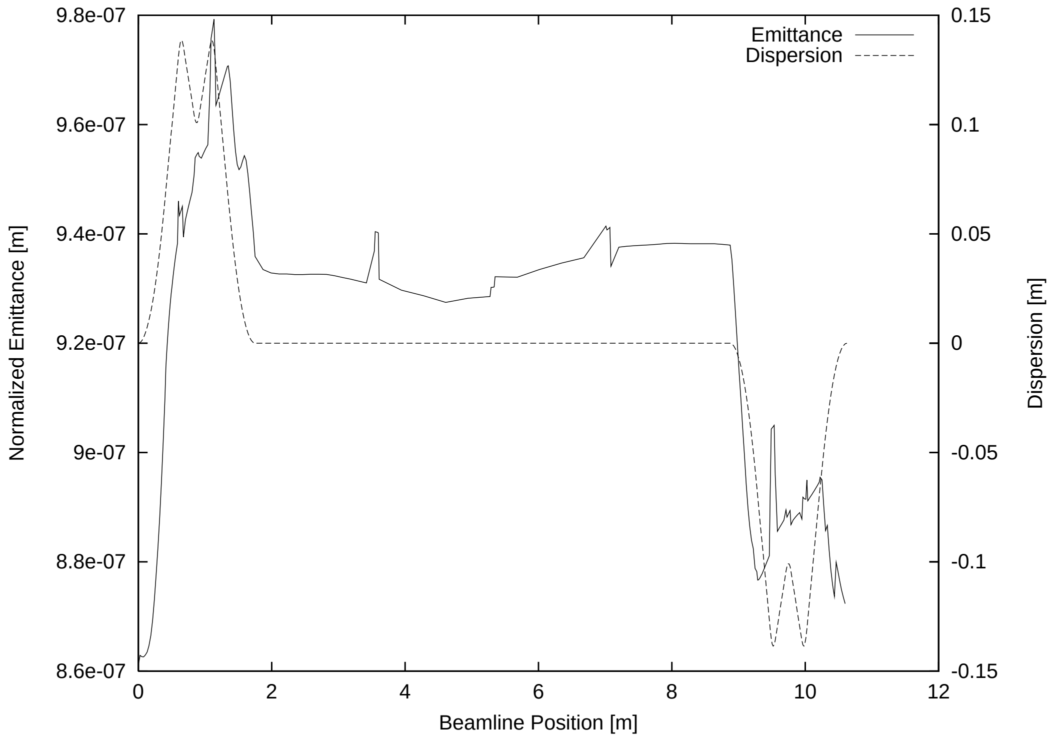

Figure 2: Emittance growth compensation along the proposed LCLS injector dogleg

the transverse projected emittance growth for this setup as calculated by `TraFiC`^{4}; the 8% growth after the first bends is almost completely canceled between the last ones. This is possible because the bunch retains its length, so the fields have the same behavior in each bend. In bunch compression chicanes, however, one has to chose a different scheme. One possibility is to use several chicanes with an 1 or -1 transfer matrix between them and to scale magnet lengths and dispersion according to the expected field strengths.

3.3 TTF Bunch Compressor

Even in the absence of adjustable optical elements there is room for optimization in terms of emittance. The TESLA Test Facility Bunch Compressor II[13] comprises four bending magnets and no quadrupoles within the dispersive section. Coherent synchrotron Radiation is a serious issue given its parameter set [14, 15, 16] However, one can use three quadrupoles upstream of the chicane to adjust the initial Twiss parameters. As the beam distortion exhibits a non-linear behavior, this can be utilized to cancel some of the induced kicks by an appropriate choice of transverse beam sizes and divergences. (A purely linear behavior would leave the emittance growth invariant under symplectic transformations of the initial phasespace). Figure 3 shows the results of a parameter scan of the initial Twiss parameters; the differences in slice emittance strongly suggest operating the compressor near the "sweet spot" around $\alpha = 1.2$, $\beta = 15\mathrm{m}$

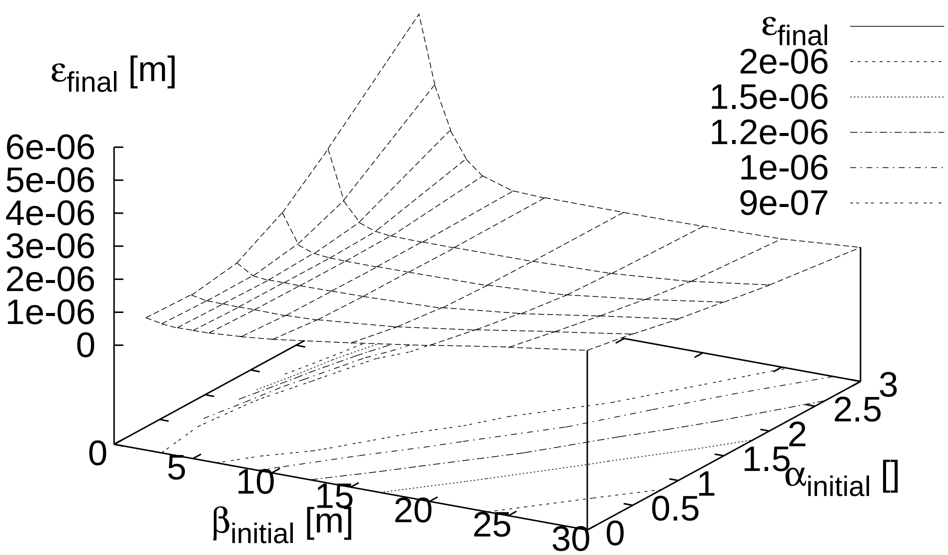

Figure 3: Slice emittance growth in the TESLA Test Facility BCII chicane as a function of initial Twiss parameters (from [17])

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