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Closed Form Expression for the Geometric Effect of a Beam Scraper on the Transverse Beam Distribution*

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Abstract

The geometric effect of a beam scraper on the transverse and longitudinal properties of a charged bunch has been extensively investigated. Analytical expressions have been derived for various asymptotic limits such as small offsets of the beam through the scraper and/or very short or very long bunch lengths compared to the scraper gap.

A more general, semi-analytical approach that is valid in the asymptotic limits. and for the intermediate cases is presented in this paper. The relative expressions are compact and easy to evaluate.

A comparison is made with numerical simulation results from MAFIA. In addition, estimates of the effect on the higher order moments of the beam distributions are provided for typical SLC parameters.

Geometric wakes analytic approximation

The problem of wake fields generated when a beam passes from a round collimator of radius a to a tube of radius b has been extensively studied. Bane and Morton [1] derived an expression for the relative transverse kick as a function of the longitudinal bunch distribution for small y beam offset from the center and long bunch length ($\sigma_z > a$):

$$\Delta y'(z) \approx \frac{4Nr_e}{a\gamma} y \cdot \rho(z); \quad \rho(z) = \frac{e^{-\frac{z^2}{2\sigma_z^2}}}{\sqrt{2\pi}\sigma_z}$$

where N denotes the number of particles in the bunch, r_e the classical electron radius and γ the relativistic Lorentz factor.

Gianfelice and Palumbo [2] derived an expression for very short bunch lengths ($\sigma_z << a$):

$$\Delta y'(z) \approx \frac{4Nr_e}{\gamma}\left(\frac{1}{a^2} - \frac{1}{b^2}\right) y \int_{-\infty}^{z} \rho(t)dt$$

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From Zimmermann [3] using a different approach, it is possible to generalize the result for any offset:

$$\Delta y'(z) \approx \frac{4Nr_e}{\gamma}\left(\frac{1}{a^2 - y^2} - \frac{1}{b^2 - y^2}\right) y \int_{-\infty}^{z} \rho(t)dt$$

The y dependence of this equation can be easily derived in a "naive" model, assuming that the transverse wake field is generated by the image charge that travels with the beam in the smaller beam pipe and that it is accumulated at the edge of the transition.

From basic electrodynamics, the image charge generated by a charge q, displaced by y in a beam pipe of radius a, is equivalent to a charge q placed at a distance a^2/y from the center of the beam pipe. When the driving charge reaches the transition, the image charge stops following it and generates an *electrostatic* field on the particlesthat follow which is proportional to their distance:

$$E \propto \frac{1}{\frac{a^2}{y} - y} = \frac{y}{a^2 - y^2}$$

Moreover, as in equation [2], on the other side of the transition there will be a *deficit* of image charges to compensate for the magnetic field, leading to an overall effect proportional to y:

$$E \propto \frac{y}{a^2 - y^2} - \frac{y}{b^2 - y^2}$$

In the general case of not negligible bunch length, we can argue that the image charge will dilute somewhat in time, so its effect on the later particles will diminish. Since the charge cannot disappear, the only possible effect is that its *equivalent distance* from the center of the pipe becomes greater in time according to the equation:

$$d = \frac{a^2}{y} f(z - z_0)$$

where z-z_0 is the distance between the leading and the test particle.

$f(z)$ must be a monotonicly growing function of z for z>0. Moreover, it satisfies the following constraints:

$$f(0)=1; \qquad f(z)=0 \ \ for \ z<0; \quad f(z \to \infty) \to \infty$$

Hence, the transverse kick can be approximated as:

$$\Delta y'(z) \approx \frac{4Nr_e}{\gamma} y \int_{-\infty}^{z} \left(\frac{1}{a^2 f(z-t) - y^2} - \frac{1}{b^2 f(z-t) - y^2} \right) \rho(t) dt$$

This expression is a solution of Maxwell's equations for an axially symmetric geometry. In terms of a multipole field expansion, the function f is related to the m^{th} moment of the longitudinal wake function W_m according to the equation:

$$W_m(z) = \left(\frac{1}{a^{2m}} - \frac{1}{b^{2m}} \right) f(z)^{-m}$$

A simple function that meets the requirement for $f(z)$ is:

$$f(z) = e^{z/a} \quad z>0; \quad f(z)=0 \quad z<0;$$

With this solution, the asymptotic limits for both very long and very short bunch lengths are satisfied.

Moreover, for a very long bunch length, the kick for any offset will become:

$$\Delta y'(z) \approx \frac{4Nr_e}{\gamma} \left[\frac{a}{y} \ln\left(\frac{a^2}{a^2 - y^2} \right) - \frac{b}{y} \ln\left(\frac{b^2}{b^2 - y^2} \right) \right] \cdot \rho(z)$$

showing a much weaker singularity for $y \dashrightarrow a$.

Comparison with numerical simulations.

We have compared the predicted kick from this model with numerical simulations obtained by using MAFIA.

Fig. 1 compares the predicted maximum wake field kicks as function of the beam offset for different bunch lengths as computed by MAFIA.

Fig. 2 compares the predicted kick as function of z for two different offsets from this model and from MAFIA results.

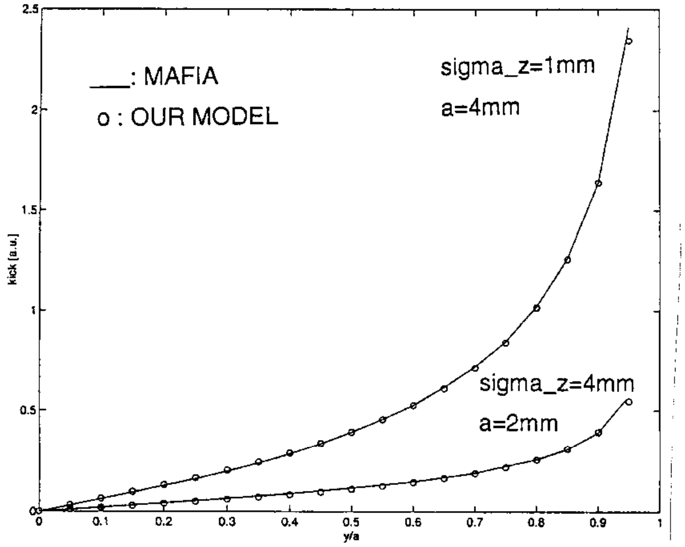

Fig.1 Maximum wake field kicks as a function of the offset for two different bunch lengths and scraper gaps.

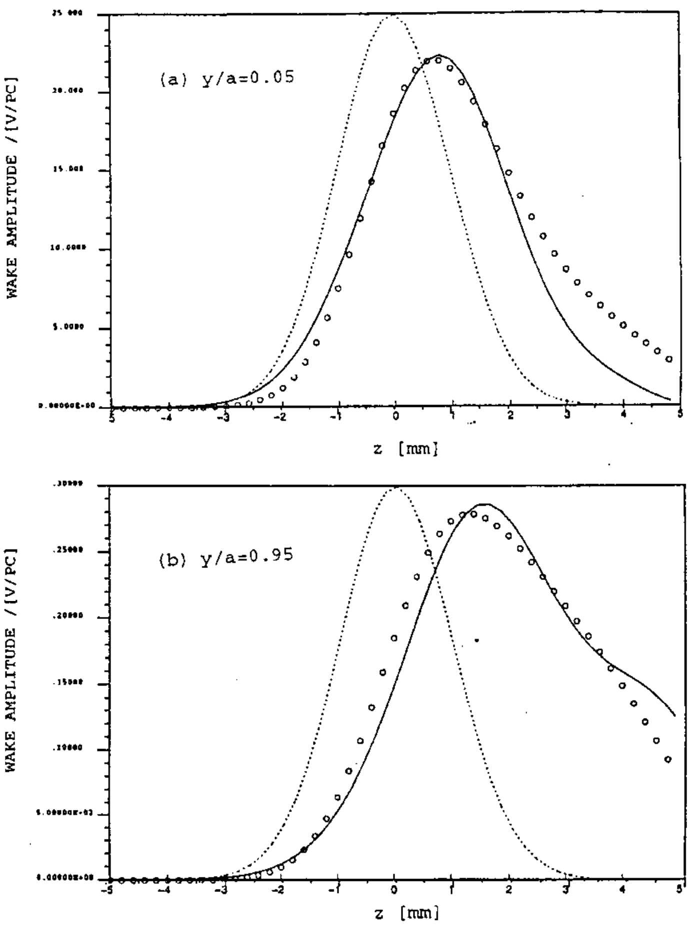

Fig.2 Transverse kick vs z for two different offsets.
a) y/a=0.05 b) y/a=0.95
σ_z=4mm a=1mm
The dotted line is the bunch distribution

In all the examples the agreement is excellent, indicating that this model fits all cases to better than 10%.

We have upgraded the tracking code TURTLE [4] to evaluate the effect on the beam due to collimators in the SLC using this model. The simulation shows that even assuming that the beam is on average centered through the scrapers, the typical beam jitter (30% of the beam spot size) generates enough dipole wakes to increase the vertical spot size by about 10%.

Conclusions

We have developed a simple model that allows us to estimate with good accuracy the effect of collimators for all typical SLC and NLC parameters. This is an important and useful tool for understanding and minimizing the luminosity dilution due to wake fields.

Bibliography

[1] K.L.F.Bane, P.L.Morton: SLAC-PUB-3983 (1986)

[2] E.Gianfelice, L.Palumbo, IEEE Tr. N.S., 37, 2, 1084 (1990).

[3] F.Zimmermann, K.L.F.Bane, C.K.Ng: EPAC-96 pg.504-506

[4] D.C.Carey, SLAC-246 UC-28