Using kaon regeneration to probe the quark mixing parameter $\cos 2\beta$ in $B \to \psi K$ decays

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We suggest a novel method to determine the sign of $\cos 2\beta$ in the decays $B \to \psi K$, by creating interference between K_L and K_S final states via "regeneration," that is propagation through a matter target region to convert some K_L to K_S. The determination of this quantity resolves an ambiguity between β and $90° - \beta$ that remains after the standard measurements of $\sin 2\beta$ and may turn out to be important in resolving whether the result is in agreement with Standard Model predictions or indicates the presence of new physics. We find the measurement is feasible at a B-factory, but requires several years of high-luminosity running with a regeneration target affecting a significant fraction of the detector.

11.30.Er, 13.25.Hw, 13.20.Eb

The determination of the parameters of the quark-mixing or Cabibbo–Kobayashi–Maskawa (CKM) [1] matrix as a consistency test of the Standard Model is a priority in current high-energy physics research. The BaBar [2] and Belle [3] collaborations have announced preliminary results from the study of the CP-violating asymmetry in the decay $B^0 \to \psi K_S$ [4] which attain a precision comparable to the earlier CDF measurement [5]. Eventually, these and other experiments will provide us with an accurate and clean determination of the CKM-parameter $\sin 2\beta$. Unfortunately, knowledge of $\sin 2\beta$ only determines the angle β up to a four-fold ambiguity, meaning that new physics could be hiding under an apparent confirmation of the Standard Model [6,7]. A measurement of the sign of $\cos 2\beta$ would remove the $\beta \to 90° - \beta$ ambiguity.

The interference between the K_S and the K_L decays to $\pi\pi$ in principle provides enough information in the time-dependence of the decay chain $B \to \psi K \to \psi(\pi\pi)_K$ to extract $\cos 2\beta$. However, the CP suppression of the $K_L \to \pi\pi$ decay makes such a measurement unrealistic. Kayser has suggested the use of semileptonic kaon decays to avoid the CP suppression, but this again leads to tiny effects because of the small $K_S \to \pi\ell\nu$ branching ratio [8].

Here we propose to utilize neutral kaon regeneration to enhance the K_S–K_L interference, leading to a measurement of $\cos 2\beta$ in the decay $B \to \psi K \to \psi(\pi\pi)_K$ that does not suffer from CP suppression nor from small branching ratios. Regeneration hinges on the fact that the eigenstates in matter and in vacuum do not coincide. They are related through $K_L' \sim K_L + rK_S$ and $K_S' \sim K_S - rK_L$, where

$$r = -\frac{\pi N}{m_K} \frac{\Delta f}{\Delta\lambda} \tag{1}$$

is the regeneration parameter. Here, N is the density of scattering centers, $\Delta f = f - \bar{f}$, f ($\bar{f}$) is the elastic forward scattering amplitude of K^0 ($\overline{K^0}$), whose imaginary part is related by the optical theorem to the total cross section σ_T ($\bar{\sigma}_T$), $\Delta\lambda = \Delta m_K - i\Delta\Gamma_K/2 = \lambda_L - \lambda_S$, and $\lambda_n = m_n - i\Gamma_n/2$ are the vacuum eigenvalues corresponding to the two kaon mass eigenstates with masses m_n and widths Γ_n for $n = S$ and $n = L$. We use the approximation $m_K = (m_L + m_S)/2$.

Consider the situation depicted schematically in Fig. 1, where an initial $|B^0\rangle$ state evolves at proper t_B as

$$e^{\Gamma_B t_B/2}|B^0(t_B)\rangle = \cos\left(\Delta m_B\, t_B/2\right)|B^0\rangle$$
$$+ q_B/p_B\,\sin\left(\Delta m_B\, t_B/2\right)|\overline{B^0}\rangle. \tag{2}$$

Here Δm_B is the difference between the masses of the two B mass eigenstates B_H and B_L, Γ_B is their average width (the difference can be neglected), and q_B/p_B is defined via the relation $|B_{H,L}\rangle = p_B|B^0\rangle \pm q_B|\overline{B^0}\rangle$. At time t_B the B mixture decays into a ψ and a kaon.

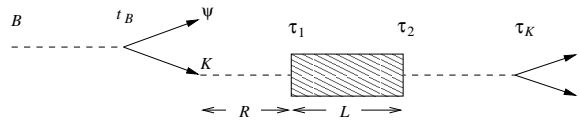FIG. 1. The diagram illustrates the $B^0 \to \psi f_K$ evolution when the kaon traverses a regenerator before decaying into f_K. The time evolutions are indicated by dashed lines and the decays by solid lines.

The resulting kaon state is given by $|K_{\mathrm{from}\ B^0(t_B)}\rangle = \langle\psi K^0|T|B^0(t_B)\rangle\ |K^0\rangle + \langle\psi\overline{K^0}|T|B^0(t_B)\rangle\ |\overline{K^0}\rangle$, which may be rewritten as $\alpha_S^0|K_S\rangle + \alpha_L^0|K_L\rangle$, with [7,8]

$$\sqrt{2}\,\alpha_{L,S}^0 = A(B^0 \to \psi K^0)\, e^{-\Gamma_B t_B/2} \times$$
$$\left[\cos\left(\Delta m_B\, t_B/2\right)e^{i\beta} \pm i\sin\left(\Delta m_B\, t_B/2\right)e^{-i\beta}\right]. \tag{3}$$

We have used Eq. (2), written the kaon states in terms of the mass eigenstates, and used $-q_B/p_B\, A(\overline{B^0} \to \psi\overline{K^0})/A(B^0 \to \psi K^0)\, p_K/q_K = \exp(-2i\beta)$ [9].

We now consider the evolution of this kaon state. The short- and long-lived components evolve independently until the kaon hits the regenerator at kaon proper time τ_1, at which time $\alpha_n(\tau_1) = \exp\left(-i\lambda_n\tau_1\right)\alpha_n^0$, with $n = S, L$. These components mix as the kaon state evolves through matter, until proper time $\tau_2 = \tau_1 + \delta\tau$ when the kaon emerges from the regenerator of length L. The kaon components at that instant may be written as [10–12],

$$\alpha_i(\tau_2) = e^{-N(\sigma_T + \bar{\sigma}_T)L/4}e^{-\Gamma_S\,\delta\tau/2}\sum_j m_{ij}\alpha_j(\tau_1), \tag{4}$$

where $i, j = S, L$. To linear order in r, we have $m_{SS} \sim 1$, $m_{SL} \sim m_{LS} \sim r\,[\exp(-i\Delta\lambda\,\delta\tau) - 1]$, and $m_{LL} \sim \exp(-i\Delta\lambda\,\delta\tau)$. Thenceforth the two components evolve independently again according to $\alpha_n(\tau) = \exp[-i\lambda_n(\tau - \tau_2)]\,\alpha_n(\tau_2)$. At proper time τ_K the kaon state decays into f_K.

Factoring out the K_S lifetime and decay rate, we obtain that the total decay rate (which we denote by $\Gamma_\beta(t_B, \tau_K)$) is given by the expected normalization factors $1/2\; e^{-\Gamma_B t_B}\;\Gamma[B^0 \to \psi K^0]\; e^{-N(\sigma_T + \bar\sigma_T)L/2}$ $e^{-\Gamma_S \tau_K}\Gamma[K_S \to f_K]$ multiplied by

$$
\begin{aligned}
&|a_{SS} + \eta a_{LS}|^2\,[1 - \sin 2\beta \sin(\Delta m_B t_B)] \\
&+ |a_{SL} + \eta a_{LL}|^2\,[1 + \sin 2\beta \sin(\Delta m_B t_B)] \\
&+ 2\mathrm{Im}\,[(a_{SS} + \eta a_{LS})(a_{SL} + \eta a_{LL})^*]\cos 2\beta \sin(\Delta m_B t_B) \\
&+ 2\mathrm{Re}\,[(a_{SS} + \eta a_{LS})(a_{SL} + \eta a_{LL})^*]\cos(\Delta m_B t_B). \quad (5)
\end{aligned}
$$

We have defined $\eta = A(K_L \to f_K)/A(K_S \to f_K)$, $a_{SS} = m_{SS}$, $a_{SL} = m_{SL}\exp(-i\Delta\lambda\,\tau_1)$, $a_{LS} = \exp[-i\Delta\lambda\,(\tau_K - \tau_2)]m_{LS}$, and $a_{LL} = \exp[-i\Delta\lambda\,(\tau_K - \tau_2)]\,m_{LS}\exp(-i\Delta\lambda\,\tau_1)$. The second (first) sub-index in a_{ij} indicates the vacuum eigenstate before the kaon state reaches (after it leaves) the regenerator. Eq. (5) and the relations between a_{ij} and m_{ij} are valid for any value of r. However, when r is large we must use the complete expressions of the m_{ij} in terms of r, which can be found in Ref. [12]. A similar expression applies for $\overline{B}$ decays, $\overline{\Gamma}_\beta(t_B, \tau_K)$, with the sign of all $\sin(\Delta m_B t_B)$ and $\cos(\Delta m_B t_B)$ terms reversed.

Kayser [8] has considered the case without a regenerator. In that case, a_{SL}, a_{LS}, L, and $\delta\tau$ vanish. The first line in Eq. (5) is proportional to $e^{-\Gamma_S \tau_K}\Gamma[K_S \to f_K]$; it arises from the decay path $B^0 \to \psi K_S \to \psi f_K$. It shows how one measures $\sin 2\beta$ in the CP-violating asymmetry of $B^0 \to \psi K_S$. The second line, proportional to $e^{-\Gamma_L \tau_K}\Gamma[K_L \to f_K]$, arises from the decay path $B^0 \to \psi K_L \to \psi f_K$ and shows that the CP-violating asymmetry in $B^0 \to \psi K_L$ has the opposite sign to the asymmetry in $B^0 \to \psi K_S$. At intermediate times these two paths interfere and one has access to the $\cos 2\beta$ term on the third line, which in vacuum is given by $2|\eta|\exp(-\Delta\Gamma_K \tau_K/2)\sin(\Delta m_K \tau_K - \arg\eta)$ [8]. This interference vanishes with η and is maximized for semileptonic decays, where $|\eta| \approx 1$. A simple estimate shows that measuring $\cos 2\beta$ in $B \to \psi(\pi\ell\nu)_K$ should require around a few hundred to a thousand times as many events as the measurement of $\sin 2\beta$ from $B \to \psi K_S$ [7,8].

One advantage of using a regenerator is that the interference term exists even when $\eta = 0$. Indeed, in that case the third line of Eq. (5) becomes proportional to $2|r|\,[\exp(-\Delta\Gamma_K \tau_2/2)\sin(\Delta m_K \tau_2 - \arg r) - \exp(-\Delta\Gamma_K \tau_1/2)\sin(\Delta m_K \tau_1 - \arg r)]$, which determines the asymmetry arising from the $\cos 2\beta$ term. Since $\tau_2 - \tau_1 = Lm_K/p_K$ (where p_K is the momentum of the kaon), this term is related to the matter absorption $\exp[-N(\sigma_T + \bar\sigma_T)L/2]$ which determines the rate. To achieve the maximum sensitivity to $\cos 2\beta$ we must find the optimal balance between the two effects.

The observable of interest is the decay rate asymmetry

$$
A_\beta(t_B, \tau_K) = \frac{\overline{\Gamma}_\beta(t_B, \tau_K) - \Gamma_\beta(t_B, \tau_K)}{\overline{\Gamma}_\beta(t_B, \tau_K) + \Gamma_\beta(t_B, \tau_K)}, \quad (6)
$$

shown in Fig. 2 for a kaon momentum of 2 GeV/c and t_B fixed at the mean B lifetime, in comparison with the asymmetry obtained with Kayser's semileptonic method, where the rate is very small.
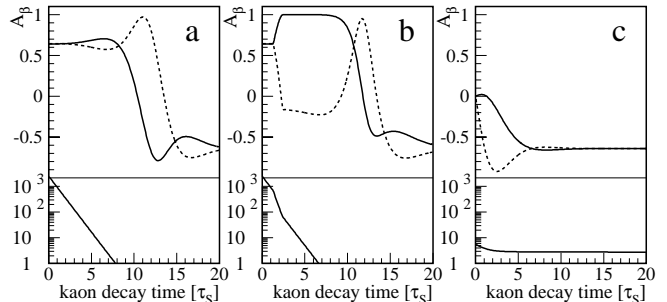

FIG. 2. $B^0 - \overline{B^0}$ decay rate asymmetries in the ψK channel as a function of the kaon decay time for various kaon decay final states: (a) 2π; (b) 2π with a tungsten regenerator 12 cm thick at a distance of 14 cm; and (c) $\pi^-\ell^+\nu$ (Kayser method). The solid (dashed) lines mark the asymmetries for $\beta = 70°$ ($20°$). A kaon momentum of 2 GeV/c is assumed. To illustrate the statistical requirements we show in the lower thirds of the figures the number of expected decays per τ_S bin for 10000 $B \to \psi K$ decays

It is convenient to integrate the decay rates behind the regenerator as the dependence on the kaon decay time contains little information on $\cos 2\beta$ compared to the regeneration effect. Neglecting the small contributions from $K_L \to 2\pi$ decays, the asymmetry simply becomes

$$
A_\beta^{\mathrm{reg}}(t_B) = \frac{\overline{\Gamma}_\beta(t_B, \tau_2) - \Gamma_\beta(t_B, \tau_2)}{\overline{\Gamma}_\beta(t_B, \tau_2) + \Gamma_\beta(t_B, \tau_2)}. \quad (7)
$$

We assume that $\sin 2\beta$ will be known with good precision by the time such an experiment will be considered. The experimental task will then consist in determining which of the two hypotheses, β or $90° - \beta$, is favored by the data. To compare the probabilities of the two hypotheses we calculate the logarithm of the likelihood ratio [13],

$$
\log\mathcal{R}(t_B)dt_B = \frac{1}{2}n_\beta(t_B)dt_B \sum_{k=\pm 1}(1 + kA_\beta^{\mathrm{reg}}(t_B))
$$

$$
\times \log\left(\frac{1 + kA_\beta^{\mathrm{reg}}(t_B)}{1 + kA_{90°-\beta}^{\mathrm{reg}}(t_B)}\right) \quad (8)
$$

where $n_\beta(t_B)dt_B$ denotes the total number of $K \to 2\pi$ decays behind the regenerator for which the K originates from a B that decayed in the time interval $[t_B, t_B + dt_B]$.

2

Integration over the B decay time yields the average likelihood ratio:

$$\langle \log \mathcal{R} \rangle = \Gamma_B \int_0^\infty e^{-\Gamma_B t_B} \log \mathcal{R}(t_B) dt_B \qquad (9)$$

The average likelihood ratio is a good measure of the expected discrimination power of an experiment and therefore may be used as a figure of merit to optimize the regenerator geometry.

To obtain a first estimate of the number of events needed to establish the sign of $\cos 2\beta$ we calculate $\langle \log \mathcal{R} \rangle$ for a tungsten regenerator of varying thickness at varying distance. We propose tungsten for this experiment because it gives large regeneration effects. Since the elastic forward scattering amplitudes f and $\bar{f}$ for K^0 and $\overline{K^0}$ in tungsten are not known, we use an empirical scaling law as determined by Gsponer and collaborators from their measurements in C, Al, Cu, Sn, and Pb for kaon momenta between 20 to 150 GeV/c. They found that $(\sigma_T + \bar{\sigma}_T)/2 \approx \sigma_T(K_L) = 23.5$ mb $(A/\text{g mol}^{-1})^{0.840}$ [14] and $|\Delta f| = 1.13$ fm $(A/\text{g mol}^{-1})^{0.758} (p_K/\text{GeV}c^{-1})^{0.386}$ [15,16]. The last result exhibits a power law momentum dependence in accordance with Regge theory [17], which also predicts that $\arg \Delta f$ should be constant and given by $-(1 + 0.386)\pi/2 = -0.693\pi$ [16]. As a result, $r = 0.033\ e^{i0.049\pi}\ (\rho/\text{g cm}^{-3})\ (A/\text{g mol}^{-1})^{-0.242} (p_K/\text{GeV}c^{-1})^{0.386}$ where ρ is the density of the material used. The power-law approximation is fairly good down to a few GeV/c momentum, where low-energy resonances set in [18]. Such resonances can enhance or degrade regeneration effects, but are not expected to alter significantly the conclusions of this work. At an asymmetric e^+e^- collider operating at the $\Upsilon(4S)$ resonance the average kaon momentum from $B \to \psi K$ decays lies around 2 GeV/c, so that $r = 0.24\ e^{i0.049\pi}$ for tungsten.

Using these numbers we find that for $\beta = 0°$ ($\sin 2\beta = 0$), $\langle \log \mathcal{R} \rangle$ has a rather broad maximum for regenerator distances around $R_{\text{opt}} = 17.3$ cm and thicknesses of about $L_{\text{opt}} = 11.9$ cm. For this case, $\langle \log \mathcal{R} \rangle$ equals 0.006

times the total number of (flavor-tagged) $B \to \psi K$ decays with the K aiming at the regenerator. This means that in order to get a likelihood ratio of 370 (corresponding to a 3σ separation) one would need about 980 such events. If $\beta = 70°$ ($\sin 2\beta = 0.7$), the optimum position and thickness are not much different, but $\langle \log \mathcal{R} \rangle$ drops by a factor of 1.7. Table 1 contains more examples at different kaon momenta and can be used to estimate the requirements at various facilities. At high momenta regeneration effects become smaller and more events are needed to probe $\cos 2\beta$.

A more realistic estimate of the statistical requirements needs to take into account experiment-specific details such as the geometry of the detector and the phase space occupied by the kaons from the B decays. As an example, we have modeled regenerator geometry as a cylinder in the BaBar detector.

At BaBar, 9 GeV electrons collide with 3.1 GeV positrons, copiously producing $B\overline{B}$ pairs via the $\Upsilon(4S)$ resonance. The tight constraints between the kaon momentum and the polar angle due to the kinematics of the decay chain $\Upsilon(4S) \to B\overline{B}$ followed by $B \to \psi K$ allow us to optimize the position and thickness of the regenerator. In this experiment one B decays to a flavor-identifying mode, or tag, and the other to ψK. In this case Eq. (9) becomes $\langle \log \mathcal{R} \rangle = \Gamma_B \int_{-\infty}^\infty e^{-\Gamma_B |t|} \log \mathcal{R}(t) dt$, where $t = t_{\psi K} - t_{\text{tag}}$ is the time between the two decays. The value $\langle \log \mathcal{R} \rangle$ is maximal for the 9 cm thick cylindrical regenerator placed at a radius of 5 cm and a polar angle of 120°. At this point each kaon aimed at the regenerator contributes 0.008 to $\langle \log \mathcal{R} \rangle$ which implies that about 600 perfectly tagged events would achieve a 3σ separation between $\beta = 0°$ and $\beta = 90°$. For a regenerator occupying 25% of the instrumented solid angle around the optimal point, we estimate that (with a merged J/ψ and $\psi(2S)$ sample) a luminosity of about 300 fb^{-1} would be needed to reach 3σ separation, where we have used the reconstruction efficiency and effective tagging efficiency currently achieved by BaBar [2] and assumed that tagging particles that hit the regenerator are lost. If $\beta = 70°$ then about 600 fb^{-1} would be needed.

We have also briefly considered the application of this method at a hadron collider experiment where hundreds of thousands of reconstructed ψK decays and an effective tagging efficiency of $\approx 10\%$ are expected. However, this statistical advantage is offset by the reduction in the size of the regeneration effect, because the difference between the K^0 and the $\overline{K^0}$ cross sections drops with momentum (see Table 1) and because the broad spectrum of kaon momentum makes it impossible to design a regenerator optimal for all decays. A regenerator optimized for 2 GeV/c averages about 38% of its optimum for momenta between 1 and 10 GeV/c. So, an experiment carried out in the central region where kaon momenta are modest might be more effective.

In passing we would like to point out that a tungsten

TABLE I. Optimal distance and thickness of a tungsten regenerator for the geometry depicted in Fig. 1, and for various kaon momenta. Also shown is the average total number of $B \to \psi K$ events (with the kaon aiming at the regenerator) needed to achieve a separation of the $\pm \cos 2\beta$ hypotheses corresponding to 3σ.

p_K GeV/c	$\beta = 0°$			$\beta = 70°$		
	R_{opt} cm	L_{opt} cm	$N(3\sigma)$	R_{opt} cm	L_{opt} cm	$N(3\sigma)$
1	7.1	9.4	680	4.9	7.7	1230
2	17.3	11.9	980	13.7	10.1	1670
5	49.5	14.4	2070	44.4	12.7	3200
10	105	15.5	4200	99	14.2	6000
50	570	16.3	26k	570	15.7	34k
100	1190	16.3	60k	1180	15.9	75k

target inside the detector would not only act as a regenerator, but also as a strangeness indicator via reactions such as $\overline{K^0}n \to K^-p$ or $\overline{K^0}p \to \Lambda\pi^+$ etc. The outgoing strange particle unambiguously identifies the strangeness of the neutral kaon at the time of the interaction in the regenerator, just as the lepton from a semileptonic decay tags the strangeness at the time of decay, allowing us to apply Kayser's method with considerably higher statistics. Comparing Figs. 2b and 2c we see that an optimized regenerator would also be in the right position for this method to be sensitive to $\cos 2\beta$. Depending on the capability of the detector to reconstruct and identify such events, combined analysis of the two effects may substantially reduce the number of events needed to determine the sign of $\cos 2\beta$.

If the measurement of the sign of $\cos 2\beta$ becomes an issue of significant interest, then the method suggested here offers an opportunity to measure it at a high-luminosity B-factory. It requires insertion of a tungsten target deep inside the detector, affecting a significant portion of the detector solid angle. Thus it is not without cost for other physics measurements, so it will be a question of competing priorities to decide whether to implement this approach. Similar considerations will apply in a hadronic B-physics experiment, but detailed simulation of the actual situation is needed before one can say which configuration of machine, target and detector offers the best choice for this measurement.

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