

**Suppression of Beam Breakup Instability in a Long Train by
Introducing Energy Spread Between the Bunches**

By G. V. Stupakov

Presented at 20th International Linac Conference, 8/21/2000–8/25/2000,
Monterey, CA, USA

Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

Work supported by Department of Energy contract DE AC03 76SF00515.

SUPPRESSION OF BEAM BREAKUP INSTABILITY IN A LONG TRAIN BY INTRODUCING ENERGY SPREAD BETWEEN THE BUNCHES*

G. V. Stupakov, SLAC, Stanford, CA 94309, USA

Abstract

Interaction between the bunches in the NLC main linac via long range wakefields can cause a beam breakup instability. Although the magnitude of the long range wakefields for an ideal NLC structure is below the instability threshold, the wake increases when structure manufacturing errors are taken into account. In case of large errors, the developing instability can result in the projected emittance dilution of the bunch train. To suppress the instability, we propose to introduce an energy spread between the bunches, similar to the BNS energy spread for damping of the beam breakup within a single bunch [1]. Based on simple estimates, we show that the energy spread of order of 1-2% should be enough for suppression of the instability. The results of computer simulations with the simulation code LIAR confirm theoretical estimates and indicate that both the tolerances for structure misalignments and the incoming beam jitter can be considerably loosened by introducing the energy spread within the beam.

1 INTRODUCTION

Interaction between the bunches in the NLC linac via the long range wakefields can cause a beam breakup instability [2], similar to the beam breakup within a single bunch that is caused by short range wakes [3]. This instability, if severe, imposes tight tolerances on the beam injection errors as well as structure misalignments in the lattice.

An example of the the long-range wake for the NLC structures is shown in Fig. 1. This wake was calculated assuming a random frequency error of the dipole mode in the structure with the rms spread of 3 MHz [4]. Note that in contrast to the short-range wakefield, that can usually be approximated by a linear function, the long-range wake is a complicated oscillating function of the bunch position.

One can try to estimate the projected normalized emittance growth $\Delta\epsilon$ of a train of bunches caused by randomly misaligned structures in the linac using the result of Ref. [5] for the expectation value of $\Delta\epsilon$

$$\langle\Delta\epsilon\rangle = \delta y^2 r_e^2 N^2 \bar{\beta}_0 N_s L_s^2 \langle\Delta S^2\rangle \frac{1 - (\gamma_0/\gamma_f)^{1/2}}{\gamma_0^{1/2} \gamma_f^{3/2}}, \quad (1)$$

where δy is the rms structures offset in the linac, r_e is the classical electron radius, N is the number of particles in the bunch, $\bar{\beta}_0$ is the average value of the beta function at the beginning of the linac, N_s is the number of structures in the linac, L_s is the length of the structure, γ_0 and γ_f are the initial and final relativistic factors of the beam, and S

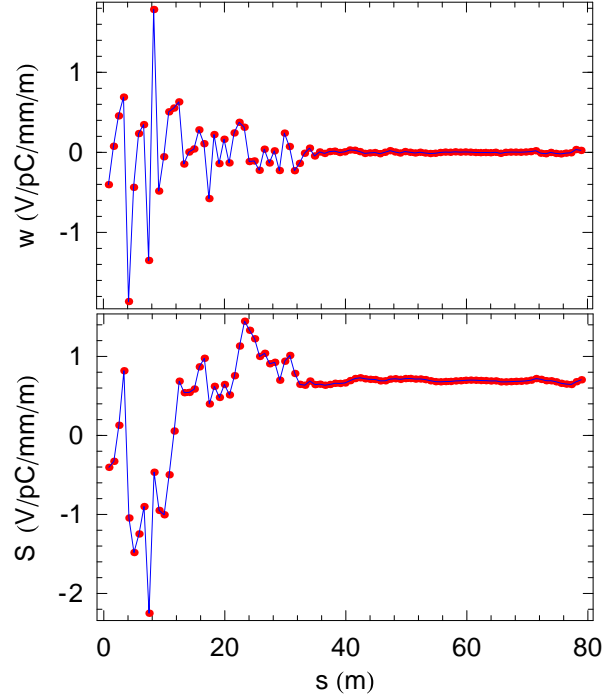


Figure 1: Transverse wake w and the corresponding sum wake S for NLC linac with 3 MHz rms frequency error in the dipole modes. Each point corresponds to a single bunch. The total number of bunches in the train is 95 and the spacing between the bunches is 83 cm.

is the sum wake. The k -th component of S is defined as a sum of the transverse wakes w_i generated by all bunches preceding the bunch number k (with $S_1 = w_1 = 0$),

$$S_k = \sum_{i=1}^k w_i. \quad (2)$$

The quantity ΔS is the difference between S and the average value $\langle S \rangle$, $\Delta S_k = S_k - \langle S \rangle$, with

$$\langle S \rangle = \frac{1}{N_b} \sum_{k=1}^{N_b} S_k, \quad \langle \Delta S^2 \rangle = \frac{1}{N_b} \sum_{k=1}^{N_b} \Delta S_k^2, \quad (3)$$

where N_b is the number of bunches. Eq. (2) is derived assuming a lattice with the beta function smoothly increasing along the linac as $\bar{\beta} \sim E^{1/2}$. Note that Eq. (2) is only valid if there is no beam break up instability, and hence gives a minimal emittance growth for given misalignment.

For the wake shown in Fig. 1, $\langle \Delta S^2 \rangle^{1/2} = 0.59$ V/pC/m/mm. Using the nominal NLC parameters: $N = 1.1 \cdot 10^{10}$, $\bar{\beta} = 7$ m, $N_s = 4969$, $L_s = 1.8$ m, $E_0 = 10$

* Work supported by DOE contract DE-AC03-76SF00515.

GeV, $E_f = 500$ GeV, and assuming $\delta y = 10 \mu\text{m}$ one finds $\langle \Delta \epsilon \rangle = 2.9 \cdot 10^{-10}$ m, which is about 1% of the nominal beam emittance at the beginning of the linac, $\epsilon = 3 \cdot 10^{-8}$ m.

However, computer simulations for this case with the wake shown in Fig. 1 demonstrate the projected emittance growth in the train of order of 50%, (see Fig. 2), that is much larger than the above estimate. The reason for such large emittance dilution is the development of the beam breakup instability due to the long-range wakefields in the bunch train reflected in quasi-exponential growth in Fig. 2. To suppress the instability, one can try to introduce an en-

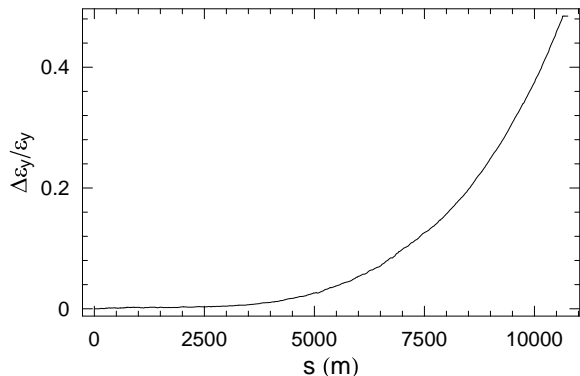


Figure 2: Relative emittance growth along the linac in the train of bunches, for $10 \mu\text{m}$ rms structure misalignments, when all bunches have the same energy. The bunches are simulated as single particles, so that there is no emittance growth within the bunch.

ergy spread in the train of bunches [6], similar to the energy spread within a bunch that is routinely use for the BNS damping of a single bunch beam breakup.

2 ESTIMATE OF REQUIRED ENERGY SPREAD

To obtain a rough estimate of the energy spread required to suppress the long range beam break up instability, we use the autophasing condition for the BNS damping in the two-particle model for a FODO lattice [7, 8]

$$\delta = \frac{N e^2 w l_{cell}^2}{24E} \left(1 + \frac{3}{2} \text{ctn}^2 \frac{\mu}{2} \right), \quad (4)$$

where δ is the relative energy difference between the particles, N is the number of particles in the macroparticle, w is the transverse wake, E is the beam energy, μ is the betatron phase advance per cell in the FODO lattice, and l_{cell} is the cell length. The value of δ given by Eq. (4) would be enough for suppression of the beam break up instability between the two macroparticles with the interaction characterized by the wake w .

To apply Eq. (4) for a train of bunches, we use the rms value of the wake shown in Fig. 1,

$w_{\text{rms}} = 0.36 \text{ V/pC/m/mm}$. The quantity $F \equiv l_{cell}^2 \left(1 + \frac{3}{2} \text{ctn}^2 \frac{\mu}{2} \right) / (24E)$ from Eq. (4) was calculated for the NLC lattice as a function of distance s , see Fig. 3. For the estimate, we use the average value of $F_{av} = 0.54 \text{ m}^2/\text{GeV}$. Finally, we need to relate the quantity δ to the

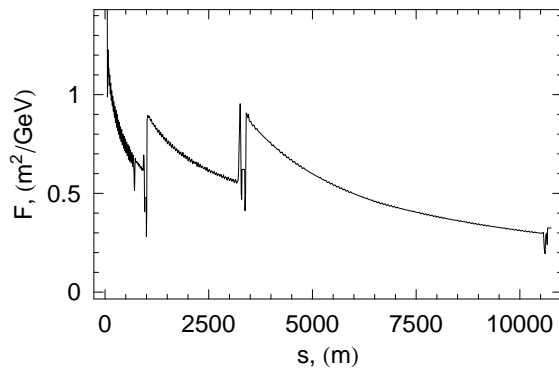


Figure 3: Function $F(s)$ for the NLC lattice.

energy spread in the train. Since Eq. (4) was derived for two macroparticles, one can expect that δ refers to the energy difference between the adjacent bunches in the train. Hence, the required energy spread in the train for suppression of the instability is equal to δ multiplied by the total number of bunches in the train, $\delta_{\text{train}} \approx N_b \delta$. We can now estimate δ_{train} as $N_b N e^2 w_{\text{rms}} F_{av}$, which gives

$$\delta_{\text{train}} \approx 0.03. \quad (5)$$

This should be compared with the energy spread of the order of 1% within the bunch introduced for the BNS damping of the short-range beam breakup instability.

A more detailed theory of the beam breakup instability in a train with the energy chirp can be found in Ref. [9].

3 COMPUTER SIMULATIONS

We carried out computer simulations using LIAR code [10] with variable energy spread in the train for the wake shown in Fig. 1. In these simulations, the bunches were treated as macroparticles to suppress the effect of short range wakefields. We used two types of energy profiles in the train. In the first case, the energy in the train varied linearly so that $\Delta E_k = -e(s)k$, where k is the bunch number in the train, and $e(s)$ is the energy slope. The rms energy spread in the train ΔE in this case is $\Delta E(s) = |e(s)| N_b / \sqrt{12}$. An example of the profile of $\Delta E(s)$ along the linac is shown in Fig. 4 — the energy spread was generated at the beginning of the linac, and taken out at the end, so that the final energy spread was close to zero. We also used an exponential energy profile in the train $\Delta E_k = e(s)[1 - \exp(-3k/N_b)]$ with the same functional form of the rms $\Delta E(s)$ as shown in Fig. 4.

In the first set of simulations, all structures were randomly misaligned with rms value of $10 \mu\text{m}$. The energy spread in the train varied by scaling the profile shown in

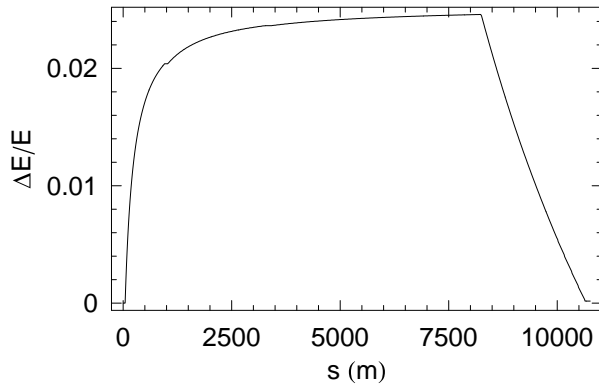


Figure 4: Variation of the relative energy spread $\Delta E/E$ along the linac.

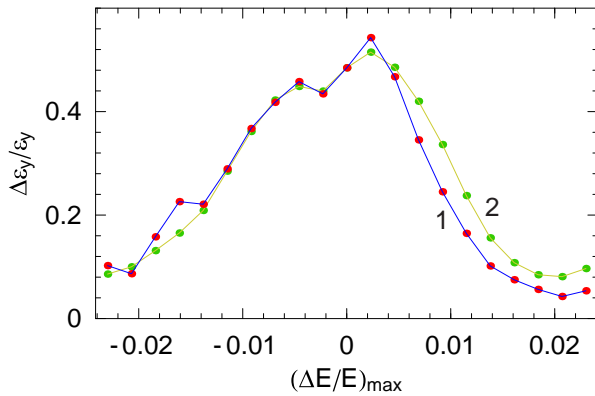


Figure 5: Relative emittance increase in the train of bunches as a function of the maximum rms energy spread in the train. Curve 1 corresponds to a linear variation of the energy, and curve 2 — to exponential function ΔE_k .

Fig. 4 and the final effective emittance of the train was calculated as a function of the maximum rms energy spread $(\Delta E/E)_{\max}$ (approximately at $s \approx 8000$ m). The result is shown in Fig. 5. Positive values of $(\Delta E/E)_{\max}$ correspond to the BNS-like energy profile, when the energy decreases toward the tail of the bunch and negative values of $(\Delta E/E)_{\max}$ correspond to the opposite slope of the energy profile. As we see, the positive values of $(\Delta E/E)_{\max}$ are more effective in suppression of the emittance growth.

In the second set of simulations, the train was initially offset by $1 \mu\text{m}$ (with all structures and quadrupoles perfectly aligned). The resulting emittance growth of the beam as a function of $(\Delta E/E)_{\max}$ is shown in Fig. 6. Again, we see that the energy spread of the order of 1% results in much smaller emittance growth then for the bunches with the same energy.

4 DISCUSSION

An energy spread between the bunches in the NLC is naturally generated in the linac by the beam loading effect. A

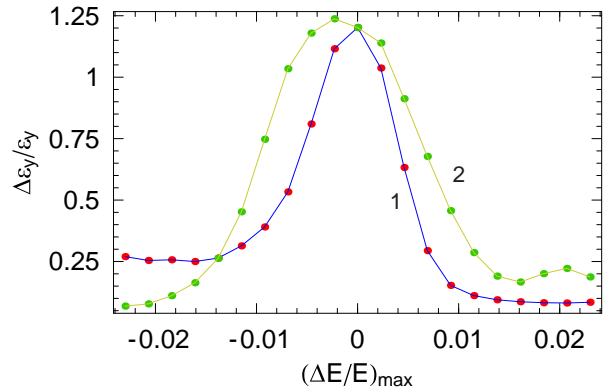


Figure 6: Relative emittance increase as a function of the energy spread for initial train offset of $1 \mu\text{m}$. Curve 1 corresponds to a linear variation of the energy, and curve 2 — to exponential function ΔE_k .

special compensation scheme in the present NLC design will correct the energy spread to a minimal value. With a slight modification of the compensation scheme, it should also be able to introduce a small controllable energy spread required for the long-range BNS damping, as studied in this paper. Another option of generating the energy spread between the bunches is using RFQ magnets.

5 ACKNOWLEDGEMENTS

The author thanks T. Raubenheimer for useful discussions.

6 REFERENCES

- [1] V. E. Balakin, A. V. Novokhatsky and V. P. Smirnov. *Proc. International Conference on High-Energy Accelerators, Batavia, 1983*, pp. 119-120, 1984.
- [2] K. Thompson and R. D. Ruth, *Phys. Rev.*, **D41**, 964 (1990).
- [3] A. W. Chao, B. Richter and C. Y. Yao, *Nucl. Instrum. Meth.*, **178**, 1 (1980).
- [4] Wakefield for RDDS1 C Incl HOM Ref and 4 Cells Decoupled computed by R. Jones.
- [5] K. L. Bane, C. Adolphsen, K. Kubo and K. A. Thompson, SLAC-PUB-6581, Presented at *4th European Particle Accelerator Conference (EPAC 94)*, London, England, 27 Jun - 1 Jul 1994.
- [6] G. V. Stupakov, NLC note LCC-0027, SLAC, September 1999.
- [7] A. W. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators* (Wiley, New York, year 1993).
- [8] NLC ZDR Design Group and NLC Physics Working Group. *Physics and technology of the next linear collider: a report submitted to Snowmass '96*, SLAC-R-0485, 1996.
- [9] C. L. Bohn and K.-Y. Ng. Paper MOA01, these Proceedings.
- [10] R. Assmann et al. SLAC-AP-103, SLAC, April 1997.