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Phenomenology of non-standard Z couplings in exclusive semileptonic $b \to s$ transitions[1]

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Abstract

The rare decays $B \to K^{(*)}\ell^+\ell^-$, $B \to K^{(*)}\nu\bar{\nu}$ and $B_s \to \mu^+\mu^-$ are analyzed in a generic scenario where New Physics effects enter predominantly via Z penguin contributions. We show that this possibility is well motivated on theoretical grounds, as the $\bar{s}bZ$ vertex is particularly susceptible to non-standard dynamics. In addition, such a framework is also interesting phenomenologically since the $\bar{s}bZ$ coupling is rather poorly constrained by present data. The characteristic features of this scenario for the relevant decay rates and distributions are investigated. We emphasize that both sign and magnitude of the forward-backward asymmetry of the decay leptons in $\bar{B} \to \bar{K}^*\ell^+\ell^-$, $\mathcal{A}_{FB}^{(\bar{B})}$, carry sensitive information on New Physics. The observable $\mathcal{A}_{FB}^{(\bar{B})} + \mathcal{A}_{FB}^{(B)}$ is proposed as a useful probe of non-standard CP violation in $\bar{s}bZ$ couplings.

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1 Introduction

Despite the fact that the Cabibbo-Kobayashi-Maskawa (CKM) mechanism provides a consistent description of presently available data on quark-flavour mixing, the flavour structure of the Standard Model (SM) is not very satisfactory from the theoretical point of view, especially if compared to the elegant and economical gauge sector. On the contrary, it is natural to consider it as a phenomenological low-energy description of a more fundamental theory, able, for instance, to explain the observed hierarchy of the CKM matrix.

A special role in searching for experimental clues about non-standard flavour dynamics is provided by flavour-changing neutral-current (FCNC) processes. Within the SM these are generated only at the quantum level and are additionally suppressed by the smallness of the off-diagonal entries of the CKM matrix. On one side this makes their observation very challenging but on the other side it ensures a large sensitivity to possible non-standard effects, even if these occur at very high energy scales.

In general we can distinguish two types of FCNC processes: $\Delta F = 2$ and $\Delta F = 1$ transitions. The former has been successfully tested in $K^0 - \bar{K}^0$ and $B_d - \bar{B}_d$ systems, both via CP-conserving (ΔM_K and ΔM_{B_d}) and CP-violating observables (ε_K and $\sin 2\beta$). On the other hand, much less is known about the latter. Few $\Delta S = 1$ FCNC transitions have been observed in K decays, but most of them are affected by sizable long-distance uncertainties. The only exception is $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$ [1], which is however affected by a large experimental error. The situation is slightly better in the B sector, where the inclusive $b \to s\gamma$ rate provides a theoretically clean $\Delta B = 1$ FCNC observable [2]. Nonetheless, it is clear that a substantial improvement is necessary in order to perform more stringent tests of the SM.

In the present paper we focus on a specific class of non-standard $\Delta B = 1$ FCNC transitions: those mediated by the Z-boson exchange. As we shall discuss, these are particularly interesting for two main reasons: i) there are no stringent experimental bounds on these transitions yet; ii) it is quite natural to conceive extensions of the SM where the Z-mediated FCNC amplitudes are substantially modified, even taking into account the present constraints on $\Delta B = 2$ and $b \to s\gamma$ processes.

The simplest way to search for non-standard $\Delta B = 1$ FCNC effects mediated by the Z-boson exchange is to look for parton-level transitions of the type $b \to s(d) + \ell^+\ell^-(\nu\bar{\nu})$. None of such processes has been observed yet, but the situation will certainly improve in a short term, with the advent of new high statistics experiments at e^+e^- and hadron B-factories. In principle the theoretically cleanest observables are provided by inclusive decays, which should play an important role in the longer run. On the other hand, the exclusive variants will be more readily accessible in experiment. Despite the sizable theoretical uncertainties in the exclusive hadronic form factors, these processes could therefore give interesting first clues on deviations from what is expected in the Standard Model. This is particularly true if those happen to be large or if they show striking patterns. Since in the present study we are mainly interested in such a possibility, we shall restrict our phenomenological discussion to the exclusive three-body processes $B \to (K, K^*) + (\mu^+\mu^-, \nu\bar{\nu})$. Having branching ratios in the $10^{-6} - 10^{-5}$ range, and a relatively clear signature, these decays represent one of the primary goals of the new experiments. As we will show, forward-backward and CP asymmetries of these modes provide a powerful tool not only to search for New Physics, but also to clearly identify the interesting scenario where the dominant source of non-standard dynamics can be encoded in effective

FCNC couplings of the Z boson.

The paper is organized as follows. In Section 2 the general features characterizing the FCNC couplings of the Z boson beyond the SM are discussed; we further introduce a general parameterization of these effects, both for $b \to s$ and $b \to d$ transitions, in terms of the complex couplings $Z_{qb}^{L,R}$ ($q = s,\ d$) and evaluate their model-independent constraints. In Section 3 we present various estimates for these couplings in specific extensions of the Standard Model. Notations and general formulae for the phenomenological analysis are introduced in Section 4. In Section 5 and Section 6 we discuss how the non-standard FCNC couplings of the Z would manifest themselves and how they could possibly be isolated in $B \to (K, K^*) + \nu\bar{\nu}$ and $B \to (K, K^*) + \mu^+\mu^-$ decays, respectively. Implications for $B_s \to \mu^+\mu^-$ are briefly described in Section 7. A summary of the results can be found in Section 8.

2 General features of FCNC couplings of the Z boson

In a generic extension of the Standard Model where new particles appear only above some high scale $M_X > M_Z$, we can always integrate out the new degrees of freedom and generate a series of local FCNC operators already at the electroweak scale. Those relevant for $b \to s(d) + \ell^+\ell^-(\nu\bar{\nu})$ transitions can be divided into three wide classes:

- *Four-fermion operators.* The local four-fermion operators obtained by integrating out the new particles necessarily have dimension greater or equal to six. These could be generated either at the tree level (e.g. by leptoquark exchange) or at one loop (e.g. by SUSY box diagrams) but in both cases, due to dimensional arguments, their Wilson coefficients are expected to be suppressed at least by two inverse powers of the New Physics scale M_X.

- *Magnetic operators.* The integration of the heavy degrees of freedom can also lead to operators with dimension lower that six, creating an effective FCNC coupling between quarks and SM gauge fields. In the case of the photon field, the unbroken electromagnetic gauge invariance implies that the lowest dimensional coupling is provided by the so-called "magnetic" operators $\sim \bar{b}\sigma^{\mu\nu} s F_{\mu\nu}$. Having dimension five, their Wilson coefficients are expected to be suppressed at least by one inverse power of M_X.

- *FCNC Z couplings.* Due to the spontaneous breaking of $SU(2)_L \times U(1)_Y$ we are allowed, in the case of the Z boson, to build an effective FCNC coupling of dimension four: $\bar{b}_{L(R)}\gamma^\mu s_{L(R)} Z_\mu$. The coefficient of this operator must be proportional to some symmetry-breaking term but, for dimensional reasons, it does not need to contain any explicit $1/M_X$ suppression.

Given the above discussion, the effective FCNC couplings of the Z boson appear particularly interesting and worth to be studied independently of the other effects: in a generic model with additional sources of $SU(2)_L \times U(1)_Y$ breaking, these are the only $\Delta F = 1$ FCNC couplings that do not necessarily decouple by dimensional arguments in the limit $M_X/M_Z \gg 1$. It should be noticed that the requirement of naturalness in the size of the $SU(2)_L \times U(1)_Y$ breaking terms suggests that also the adimensional couplings of the non-standard Z-mediated

FCNC amplitudes decouple in the limit $M_X/M_Z \to \infty$. However, the above naive dimensional argument remains a strong indication of an independent behaviour of these couplings with respect to the other FCNC amplitudes [3, 4]. As we will illustrate in Section 3, this independent behaviour is indeed realized within various extensions of the SM.

Interestingly, FCNC couplings of the Z represent also the least constrained class among those listed above: magnetic operators are bounded by $b \to s\gamma$ and, within most models, dimension-six operators are strongly correlated to those entering $B - \bar{B}$ mixing. The scenario where the dominant non-standard contribution to $b \to s(d) + \ell^+\ell^-(\nu\bar{\nu})$ transitions is mediated by a $Z\bar{b}s(d)$ coupling is therefore particularly appealing also from a purely phenomenological point of view.

2.1 Effective Lagrangian and model-independent constraints

The effective FCNC couplings of the Z, relevant for the $b \to s$ transition, can be described by means of the following effective Lagrangian

$$\mathcal{L}^Z_{FC} = \frac{G_F}{\sqrt{2}} \frac{e}{\pi^2} M_Z^2 \frac{\cos\Theta_W}{\sin\Theta_W} Z^\mu \left(Z^L_{sb}\, \bar{b}_L \gamma_\mu s_L + Z^R_{sb}\, \bar{b}_R \gamma_\mu s_R \right) + \text{h.c.} , \tag{1}$$

where $Z^{L,R}_{sb}$ are complex couplings and the overall normalization has been chosen in analogy to the $s \to d$ case discussed in [3, 5]. For later convenience we also define $Z^{L,R}_{bs} = (Z^{L,R}_{sb})^*$. The SM contribution to $Z^{L,R}_{sb}$, evaluated in the 't Hooft-Feynman gauge, can be written as[2]

$$Z^R_{sb}|_{\text{SM}} = 0 , \qquad Z^L_{sb}|_{\text{SM}} = V^*_{tb} V_{ts} C_0(x_t) , \tag{2}$$

where V_{ij} denote the CKM matrix elements, $x_t = m_t^2/m_W^2$ and the function $C_0(x)$ can be found in [6].

At present the cleanest model-independent constraints on $|Z^{L,R}_{sb}|$ can be obtained from the experimental upper bounds on $\mathcal{B}(B \to X_s \ell^+\ell^-)$. Normalizing the inclusive rate for $B \to X_s \ell^+\ell^-$ to the well known $\Gamma(B \to X_c e^+ \nu_e)$ and assuming that all contributions to the former but those generated by $\mathcal{L}^Z_{FC}$ are negligible, we can write

$$\frac{\Gamma(B \to X_s \ell^+\ell^-)}{\Gamma(B \to X_c e^+ \nu_e)} = \frac{\alpha^2}{\pi^2 \sin^4\Theta_W} \frac{\left|Z^L_{sb}\right|^2 + \left|Z^R_{sb}\right|^2}{|V_{cb}|^2 f(m_c/m_b)} \left[\left(a^\ell_L\right)^2 + \left(a^\ell_R\right)^2 \right] , \tag{3}$$

where $f(z) = (1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln z)$ is the phase space factor due to the non-vanishing charm mass and, for consistency, we have neglected the small QCD correction factor in $\Gamma(B \to X_c e^+ \nu_e)$. Here $a^\ell_{L(R)}$ denotes the left(right)-handed coupling of the lepton to the Z, namely $a^\ell_L = \sin^2\Theta_W - 1/2$ and $a^\ell_R = \sin^2\Theta_W$ for $\ell = e$ or μ, whereas $a^\nu_L = 1/2$ and $a^\nu_R = 0$ for the neutrino case. Using $\mathcal{B}(B \to X_c e^+ \nu_e) = 0.105$, $\sin^2\Theta_W = 0.23$, $\alpha^{-1} = 129$, $|V_{cb}| = 0.04$ and $f(m_c/m_b) = 0.54$, we find

$$\mathcal{B}(B \to X_s \ell^+\ell^-) = 1.76 \times 10^{-3} \left(\left|Z^L_{sb}\right|^2 + \left|Z^R_{sb}\right|^2 \right) , \tag{4}$$

$$\mathcal{B}(B \to X_s \nu\bar{\nu}) = 1.05 \times 10^{-2} \left(\left|Z^L_{sb}\right|^2 + \left|Z^R_{sb}\right|^2 \right) , \tag{5}$$

[2] As it is well known, the SM contribution to FCNC Z penguins is not gauge invariant. We recall, however, that the leading contribution to both $b \to s(d)\ell^+\ell^-$ and $b \to s(d)\nu\bar{\nu}$ amplitudes in the limit $x_t \to \infty$ is gauge independent and is indeed generated by the Z penguin ($C_0(x_t) \to x_t/8$ for $x_t \to \infty$).

where in the neutrino mode we have summed over the three lepton families. Experimental upper bounds exist both for $\mathcal{B}(B \to X_s \ell^+ \ell^-)$ and $\mathcal{B}(B \to X_s \nu\bar{\nu})$, leading to

$$\left(\left|Z_{sb}^L\right|^2 + \left|Z_{sb}^R\right|^2\right)^{1/2} \lesssim 0.15 \;, \qquad \text{from} \qquad \mathcal{B}(B \to X_s \ell^+ \ell^-) < 4.2 \times 10^{-5} \; [7] \;, \tag{6}$$

$$\left(\left|Z_{sb}^L\right|^2 + \left|Z_{sb}^R\right|^2\right)^{1/2} \lesssim 0.27 \;, \qquad \text{from} \qquad \mathcal{B}(B \to X_s \nu\bar{\nu}) < 7.7 \times 10^{-4} \; [8] \;. \tag{7}$$

The strongest bound is presently imposed by $\mathcal{B}(B \to X_s \ell^+ \ell^-)$, since the larger sensitivity of $\mathcal{B}(B \to X_s \nu\bar{\nu})$ is compensated by its more difficult experimental determination.[3] The limits in (6–7) have been derived assuming that all the non-Z-mediated contributions are negligible, which is a reasonable approximation in view of the present experimental sensitivities. On the other hand, if the experimental bounds were much closer to the SM expectations, we stress that the neutrino mode would definitely be preferable from the theoretical point of view due to the absence of electromagnetic and long-distance contributions [10, 11].

Employing the Wolfenstein expansion of the CKM matrix in powers of $\lambda = 0.22$ [12] and recalling that $C_0(x_t) \sim \mathcal{O}(1)$, the SM contribution to Z_{sb}^L turns out to be of $\mathcal{O}(\lambda^2) \sim 0.04$ (see Eq. (2)), therefore much below the bound (6). As we will show later, more severe constraints on $|Z_{sb}^{L,R}|$ can be obtained by the experimental bound on the exclusive branching ratio $\mathcal{B}(B \to K^* \mu^+ \mu^-)$. These are however subject to stronger theoretical uncertainties, related to the assumptions on the form factors, and require a detailed discussion that we postpone to Section 6.2.

Additional model-independent information on these couplings could in principle be obtained by the direct constraints on $\mathcal{B}(Z \to b\bar{s})$ and by $B_s - \bar{B}_s$ mixing, but in both cases these are not very significant. Concerning the first case, we find

$$\mathcal{B}(Z \to b\bar{s}) \;=\; \frac{G_F^2 M_Z^5 \alpha \cos^2\theta_W}{4\pi^4 \Gamma_Z \sin^2\theta_W} \left(\left|Z_{sb}^L\right|^2 + \left|Z_{sb}^R\right|^2\right) \;=\; 2.3 \times 10^{-5} \;\left(\left|Z_{sb}^L\right|^2 + \left|Z_{sb}^R\right|^2\right) \;, \tag{8}$$

which is quite far from the present experimental sensitivity at LEP of $\mathcal{O}(10^{-3})$ [13], even for $|Z_{sb}^{L,R}| \sim \mathcal{O}(1)$. Using the bound (6) in Eq. (8) leads to

$$\mathcal{B}(Z \to b\bar{s}) \;\lesssim\; 5 \times 10^{-7} \;, \tag{9}$$

to be compared with the SM expectation $\mathcal{B}(Z \to b\bar{s})|_{\rm SM} \simeq 1.4 \times 10^{-8}$ [14].

Concerning $B_s - \bar{B}_s$ mixing, assuming for simplicity $Z_{sb}^R = 0$ and employing the notations of [6], we find

$$\mathcal{M}(B_s - \bar{B}_s)^Z \;=\; \frac{\alpha G_F^2 M_W^2}{3\pi^3 \sin^2\theta_W} B_{B_s} f_{B_s}^2 M_{B_s} \eta_B \left(Z_{sb}^L\right)^2 \tag{10}$$

$$\;=\; \frac{4\alpha \mathcal{M}(B_s - \bar{B}_s)^{SM}}{\pi \sin^2\theta_W S_0(x_t)} \left(\frac{Z_{sb}^L}{V_{tb}^* V_{ts}}\right)^2 \;. \tag{11}$$

At the moment we cannot extract any interesting information from (11) due to the lack of a significant upper bound on $|\mathcal{M}(B_s - \bar{B}_s)|$. If in the future we were able to exclude that $|\mathcal{M}(B_s - \bar{B}_s)^Z|$ is larger than $|\mathcal{M}(B_s - \bar{B}_s)^{SM}|$, then we would obtain

$$\left|Z_{sb}^L\right| < 7.6\, |V_{tb}^* V_{ts}| \sim 0.3 \;. \tag{12}$$

[3] A result similar to the one in (6) has recently been presented also in Ref. [9].

Performing the exchange $s \to d$ in Eq. (1-2) we can define, analogously to $Z_{sb}^{L,R}$, the couplings $Z_{db}^{L,R}$ relevant for the $b \to d$ transition. The upper bound (6) would be valid also for these couplings if we could assume $\mathcal{B}(B \to X_d \mu^+\mu^-) \leq \mathcal{B}(B \to X_s \mu^+\mu^-)$, but in the $b \to d$ case more stringent constraints can be derived from $B_d - \bar{B}_d$ mixing. The SM contribution to $\mathcal{M}(B_d - \bar{B}_d)$ can account for the observed value of ΔM_{B_d}, nevertheless, due to the theoretical uncertainty on $B_{B_d} f_{B_d}^2$, non-standard contributions of comparable size cannot be excluded at present. Imposing for instance $|\mathcal{M}(B_d - \bar{B}_d)^Z| < |\mathcal{M}(B_d - \bar{B}_d)^{SM}|$ and replacing $s \to d$ in Eq. (11) we obtain

$$\left| Z_{db}^L \right| < 7.6\, |V_{tb}^* V_{td}| \sim 0.06 \ , \tag{13}$$

which is still substantially larger than the SM contribution: $Z_{db}^L|_{\rm SM} = \mathcal{O}(\lambda^3) \sim 0.01$.

3 Model-dependent expectations for $Z_{qb}^{L,R}$

In the previous section we have seen that sizable non-standard contributions to the FCNC couplings of the Z are allowed, at least from a purely phenomenological point of view, both for $b \to s$ and $b \to d$ transitions. In the following we shall analyze the expectations for the $Z_{qb}^{L,R}$ couplings in a few specific theoretical frameworks. Moreover, we will show various consistent models where it is a good approximation to encode all the non-standard FCNC effects in the couplings of $\mathcal{L}_{FC}^Z$.

3.1 Fourth generation

A simple extension of the SM, particularly useful as a toy model for more complicated scenarios, is obtained by adding a sequential fourth generation of quarks and leptons. This is allowed by Tevatron and LEP data provided all the new fermions, neutrinos included, are sufficiently heavy ($m_{t'} \gtrsim 200$ GeV) and the splitting among the weak isospin doublets is very small ($|m_{t'} - m_{b'}|/m_{t'} \lesssim 0.1$) (see e.g. [15] and references therein).

This model exhibits a typical non-decoupling effect in the Z_{qb} coupling. Indeed, denoting by $V_{t'q}$ the mixing angles of the new up-type quark with the light generations, the dominant non-standard contribution to the $Z_{qb}^{L,R}$ coupling is given by

$$Z_{qb}^R|_{4^{\rm th}} = 0 \ , \qquad Z_{qb}^L|_{4^{\rm th}} = V_{t'b}^* V_{t'q} C_0(x_{t'}) \simeq \frac{x_{t'}}{8} V_{t'b}^* V_{t'q} \ , \tag{14}$$

where $x_{t'} = m_{t'}^2/m_W^2$. In the limit

$$V_{t'b}^* V_{t'q} \to 0 \ , \qquad m_{t'}^2 \to \infty \ , \qquad V_{t'b}^* V_{t'q} m_{t'}^2 \to \text{const.}, \tag{15}$$

this is the only non-standard effect surviving in $b \to s(d) + \ell^+\ell^-(\nu\bar{\nu})$ transitions. Choosing sufficiently small mixing angles one can therefore easily evade the experimental constraint on $V_{t'b}^* V_{t'q}$ and, by raising the value of $m_{t'}$, still obtain sizable effects in Z_{qb}^L.

In the case of $b \to s$ transitions the dominant constraint on the combination $V_{t'b}^* V_{t's}$ is imposed by $b \to s\gamma$. Indeed the bounds from $K - \bar{K}$ mixing and K decays can always be evaded assuming $V_{t'd} = 0$, whereas the constraint from $B_s - \bar{B}_s$ mixing is very loose. Barring accidental cancellations in the $b \to s\gamma$ amplitude, namely assuming that the dominant contribution to the latter is the SM one, leads to $|V^*_{t'b}V_{t's}| \lesssim \lambda^3$, almost independently of the value of m'_t. Even employing this stringent constraint,[4] however, one could still have $|Z^L_{sb}|_{4^{\rm th}}| \sim |Z^L_{sb}|_{\rm SM}|$ provided $m_{t'} \gtrsim 400$ GeV.

3.2 Generic SUSY models

Due to the large number of new particles carrying flavor quantum numbers, sizable modifications of FCNC amplitudes are naturally expected within low-energy supersymmetric extensions of the SM with generic flavour couplings [17, 18]. Assuming R parity conservation and minimal particle content, FCNC amplitudes involving external quark fields turn out to be generated only at the quantum level, like in the SM. However, in addition to the standard penguin and box diagrams, also their corresponding superpartners, generated by gaugino/higgsino-squark loops, play an important role. These contributions to inclusive and exclusive $b \to s\ell^+\ell^-$ transitions have been widely discussed in the literature (see e.g. [19, 20, 21, 22, 23] for a recent discussion and a complete list of references), employing different assumptions for the soft-breaking terms. In the following we will emphasize the role of the Z penguins in the context of the mass-insertion approximation [18].

Similarly to the $Z\bar{s}d$ case, extensively discussed in [3, 24], the potentially dominant non-SM effect in the effective $Z\bar{b}q$ vertex is generated by chargino-up-squark diagrams [19, 21]. Indeed sizable $SU(2)_L$ breaking effects can be expected only in the up sector due to the large Yukawa coupling of the third generation. Moreover, since terms involving external right-handed quarks are suppressed by the corresponding down-type Yukawa couplings, also within this framework Z^R_{qb} turns out to be negligible.

Employing the notations of [3], the full chargino-up-squark contribution to Z^L_{sb} can be written as

$$Z^L_{sb}|_{\rm SUSY} = \frac{1}{8} A^s_{jl} \bar{A}^b_{ik} F_{jilk} \ , \tag{16}$$

where

$$\begin{aligned}
A^s_{jl} &= \hat{H}_{ls_L}\hat{V}^\dagger_{1j} - g_t V_{ts} \hat{H}_{lt_R}\hat{V}^\dagger_{2j} \ , & (17)\\
\bar{A}^b_{ik} &= \hat{H}^\dagger_{b_L k}\hat{V}_{i1} - g_t V^*_{tb} \hat{H}^\dagger_{t_R k}\hat{V}_{i2} \ , & (18)\\
F_{jilk} &= \hat{V}_{j1}\hat{V}^\dagger_{1i}\ \delta_{lk}\ k(x_{ik}, x_{jk}) - 2\hat{U}_{i1}\hat{U}^\dagger_{1j}\ \delta_{lk}\ \sqrt{x_{ik}x_{jk}} j(x_{ik}, x_{jk}) \\
&\quad -\delta_{ij}\ \hat{H}_{kq_L}\hat{H}^\dagger_{q_L l}\ k(x_{ik}, x_{lk}) \ . & (19)
\end{aligned}$$

Here $g_t = m_t/(\sqrt{2} m_W \sin\beta)$ is the top Yukawa coupling; V is the CKM matrix; $\hat{V}$ and $\hat{U}$ are the unitary matrices that diagonalize the chargino mass matrix ($\hat{U}^* M_\chi \hat{V}^\dagger = {\rm diag}(M_{\chi_1}, M_{\chi_2})$) and $\hat{H}$ is the one that diagonalizes the up-squark mass matrix (written in the basis where the $d^i_L - \tilde{u}^j_L - \chi_n$ coupling is family diagonal and the $d^i_L - \tilde{u}^j_R - \chi_n$ one is ruled by the CKM matrix). The explicit expressions of $k(x,y)$ and $j(x,y)$ can be found in [3, 24] and, as usual, x_{ij} denote ratios of squared masses.

The product of A^s_{jl} and $\bar{A}^b_{ik}$ in (16) generates four independent terms, proportional to $g^2_t V^*_{tb} V_{ts}$, $g_t V_{ts}$, $g_t V^*_{tb}$ and 1, respectively. As a first approximation we can neglect those proportional to V_{ts}, which are clearly suppressed with respect to the SM contribution. A further

[4] Substantially larger values of $|V^*_{t'b}V_{t's}|$ are possible assuming that the contribution of the fourth generation changes the sign of the $b \to s\gamma$ amplitude. See Ref. [16] for a recent discussion of this point.

simplification can be obtained employing the so-called mass-insertion approximation, i.e. expanding the up-squark mass matrix around its diagonal. In this way it can been shown that the potentially dominant contribution is the one generated to the first order by the $\tilde{t}_R - \tilde{u}^s_L$ mixing [19], namely

$$\begin{aligned} Z^L_{sb}|^{\rm RL}_{\rm SUSY} \;=\; & -\frac{1}{8} g_t V^*_{tb} \, \frac{(M^2_U)_{t_R s_L}}{M^2_{\tilde{u}_L}} \; \hat{V}^\dagger_{1j} \Big[\hat{V}_{j1} \hat{V}^\dagger_{1i} k(x_{iu_L}, x_{ju_L}, x_{t_R u_L}) \\ & -\delta_{ij} k(x_{iu_L}, x_{t_R u_L}, 1) - 2 \hat{U}_{i1} \hat{U}^\dagger_{1j} \sqrt{x_{iu_L} x_{ju_L}} \, j(x_{iu_L}, x_{ju_L}, x_{t_R u_L}) \Big] \hat{V}_{i2} \; . \end{aligned} \qquad (20)$$

Notice that, contrary to the Z^L_{ds} case, here the CKM factor V^*_{tb} does not imply any additional suppression and therefore the double left-right mixing discussed in [3] represents only a subleading correction. In $Z^L_{sb}|^{\rm RL}_{\rm SUSY}$ the necessary $SU(2)_L$ breaking ($\Delta I_W = 1$) is equally shared by the left-right mixing of the squarks and by the chargino-higgsino mixing (shown by the mismatch of $\hat{V}$ indices), carrying both $\Delta I_W = 1/2$.

For a numerical evaluation, varying the SUSY parameters entering (20) in the allowed ranges, we find

$$\left| Z^L_{sb}|^{\rm RL}_{\rm SUSY} \right| \lesssim 0.1 \left| \frac{(M^2_U)_{t_R s_L}}{M^2_{\tilde{u}_L}} \right| = 0.1 \left| (\delta^U_{RL})_{32} \right| \; , \qquad (21)$$

in agreement with the results of [19]. The factor $(\delta^U_{RL})_{32}$, which represents the analog of V_{ts} in the SM case, is not very constrained at present [19, 24] and can be of $\mathcal{O}(1)$, with an arbitrary CP-violating phase [21].

Eq. (16-21) can simply be extended to the $b \to d$ case with the replacement $s \to d$. Similarly to $(\delta^U_{RL})_{32}$, also $(\delta^U_{RL})_{31}$ is essentially unconstrained at present.

As it can be checked by the detailed analysis of [19], in the interesting limit where the left-right mixing of the squarks is the only non-standard source of flavour mixing, the Z-penguin terms discussed above are largely dominant with respect to supersymmetric box and γ-penguin contributions to $b \to s \ell^+ \ell^-$. On the other hand, we note that in processes of the type $b \to s q \bar{q}$ these true penguin terms could easily compete in size with the so-called trojan-penguin amplitudes discussed in [9].

3.3 Strong electroweak symmetry breaking

The natural alternative to low-energy supersymmetry is the scenario where the Higgs field is not elementary and the electroweak symmetry breaking is generated by some new strong dynamics appearing at a scale $\Lambda \sim 1$ TeV. Without a detailed knowledge of the new dynamics, and of the new degrees of freedom associated with it, a convenient way to describe this scenario is obtained by considering the most general effective Lagrangian written in terms of fermions and gauge fields of the SM, as well as the Nambu-Goldstone bosons associated with the spontaneous breaking of $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$ [25]. In this way, imposing the custodial $SU(2)$ symmetry on the Nambu-Goldstone boson sector, the lowest order terms in the Lagrangian are completely determined, corresponding to the SM case in the limit of infinite Higgs mass. On the other hand, the effect of the new dynamics is encoded in the Wilson coefficients of higher-order operators, suppressed by appropriate inverse powers of Λ.

A conservative assumption, usually employed to reduce the number of free parameters, is that the higher-order operators do not involve directly the fermionic sector. In other words, it is assumed that the new dynamics involves only the interactions of electroweak gauge fields and Nambu-Goldstone bosons [25]. Under this assumption most of the coefficients of the allowed dimension-four operators (appearing at the next-to-leading order) are strongly constrained by electroweak precision data. However, as pointed out in [26, 27, 28], some of them naturally escape these bounds and could show up in sizable modifications of FCNC amplitudes. Interestingly, this happens despite the intrinsic flavor-conserving nature of these terms. It occurs at the loop level, either via modifications of the trilinear gauge-boson couplings [28] or via corrections to the Nambu-Goldstone boson propagators [26, 27].

Also within this context the FCNC couplings of the Z play a special role. As an example, we consider here the effect of the anomalous WWZ coupling. Following the work of Ref. [28], this can be written as

$$\begin{aligned} Z^L_{qb}|_{WWZ} &= \alpha_3 g^2 V^*_{tb} V_{tq} \frac{3x_t}{8} \log\left(\frac{M_W^2}{\Lambda^2}\right) + \ldots \\ &\sim \mathcal{O}(1) \times V^*_{tb} V_{tq} \frac{g^2 m_t^2}{\Lambda^2} \log\left(\frac{M_W^2}{\Lambda^2}\right) \ . \end{aligned} \tag{22}$$

where g is the usual $SU(2)_L$ coupling and the dots denote additional finite terms (i.e. not logarithmically enhanced). The adimensional coupling α_3 is one of the unknown coefficients appearing in the next-to-leading order Lagrangian of Ref. [25]. This is essentially unconstrained by other processes (unless further assumptions are employed) and is expect to be of $\mathcal{O}(M_W^2/\Lambda^2)$ by dimensional arguments. The relative shift of Z^L_{qb} with respect to the SM case can thus be up to 50%. Interestingly, the same relative shift would be present in Z^L_{ds}, leading to interesting correlations between rare B and K decays [28]. It is worthwhile to point out that this is the only non-standard FCNC effect due to anomalous gauge-boson couplings which is logarithmically divergent, which can be taken as an indication of a particular sensitivity of Z^L_{ds} to the new dynamics [28]. We finally note that also within this context Z^R_{qb} remains unaffected: this is clearly due to the chiral nature of the SM gauge group and indeed it remains valid also if we consider the effects due to modified Nambu-Goldstone boson propagators [27].

If the conservative assumption that higher-order operators do not involve directly the fermionic sector is relaxed, the freedom in generating new FCNC effects is clearly enhanced. The first natural step is to include only higher-order operators which involve the quarks of the third generation, as for instance done in [29]. However, the most general scenario is obtained by considering all generations. In this latter option one could generate FCNC transitions already at the tree-level [30] and, by restricting the attention to the lowest-dimensional operators, one would recover the general case described by Eq. (1). The predictivity of this scenario is obviously very limited, but still, only on dimensional arguments, one can conclude that the FCNC couplings of the Z could play a very special role. The natural suppression of FCNC would then suggest $Z^{L,R}_{qb} \sim \mathcal{O}(m_t^2/\Lambda^2) \times V^*_{tb} V_{tq}$, leaving open the possibility of $\mathcal{O}(1)$ corrections with respect to the SM case.

3.4 Tree-level Z-mediated FCNC couplings

FCNC couplings of the Z can be generated already at the tree level in various exotic scenarios. Two popular examples discussed in the literature are the models with addition of non-sequential generations of quarks (see e.g. [31] and references therein) and those with an extra $U(1)$

symmetry (see e.g. [32] and references therein). In the former case, adding a different number of up- and down-type quarks, the pseudo CKM matrix needed to diagonalize the charged currents is no more unitary and this leads to tree-level FCNC couplings. On the other hand, in the case of an extra $U(1)$ symmetry the FCNC couplings of the Z are induced by $Z - Z'$ mixing, provided the SM quarks have family non-universal charges under the new $U(1)$ group. Interestingly these two possibilities (i.e. the extra $U(1)$ and the non-sequential quarks) are often linked in many consistent extensions of the SM [33]. Here we will not discuss any of such model in detail. We simply note, however, that for our purposes these could be well described by the effective Lagrangian in (1), provided the contribution of the Z' exchange is negligible or the couplings of the Z' to light charged leptons and neutrinos are proportional to the SM ones.

4 Generalities of exclusive $b \to s\ell^+\ell^-(\nu\bar{\nu})$ decays

4.1 Effective Hamiltonian

The starting point for the analysis of $b \to s\ell^+\ell^-(\nu\bar{\nu})$ transitions is the determination of the low-energy effective Hamiltonian, obtained by integrating out the heavy degrees of freedom of the theory, renormalized at a scale $\mu = \mathcal{O}(m_b)$. In our framework this can be written as

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left(\sum_{i=1}^{10} [C_i Q_i + C_i' Q_i'] + C_L^\nu Q_L^\nu + C_R^\nu Q_R^\nu \right) + \text{ h.c. }, \tag{23}$$

where Q_i denotes the Standard Model basis of operators relevant to $b \to s\ell^+\ell^-$ [6] and $\mathcal{O}_i'$ their helicity flipped counter parts. In particular, we recall that $Q_i \sim (\bar{s}b)(\bar{c}c)$, for $i = 1 \dots 6$, $Q_8 \sim m_b \bar{s}(\sigma \cdot G)b$, whereas the only operators with a tree-level non-vanishing matrix element in $b \to s\ell^+\ell^-$ are given by

$$\begin{aligned}
Q_7 &= \frac{e}{4\pi^2} \bar{s}_L \sigma_{\mu\nu} m_b b_R F^{\mu\nu} , & Q_7' &= \frac{e}{4\pi^2} \bar{s}_R \sigma_{\mu\nu} m_b b_L F^{\mu\nu} , \\
Q_9 &= \frac{e^2}{4\pi^2} \bar{s}_L \gamma^\mu b_L \bar{\ell} \gamma_\mu \ell , & Q_9' &= \frac{e^2}{4\pi^2} \bar{s}_R \gamma^\mu b_R \bar{\ell} \gamma_\mu \ell , \\
Q_{10} &= \frac{e^2}{4\pi^2} \bar{s}_L \gamma^\mu b_L \bar{\ell} \gamma_\mu \gamma_5 \ell , & Q_{10}' &= \frac{e^2}{4\pi^2} \bar{s}_R \gamma^\mu b_R \bar{\ell} \gamma_\mu \gamma_5 \ell .
\end{aligned} \tag{24}$$

The last two operators in $\mathcal{H}_{eff}$ are defined as

$$Q_{L,R}^\nu = \frac{e^2}{4\pi^2} \bar{s}_{L,R} \gamma_\mu b_{L,R} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu \tag{25}$$

and constitute the complete basis relevant to $b \to s\nu\bar{\nu}$.

Due to the absence of flavour-changing right-handed currents, within the Standard Model one has

$$C_{1-10}'|_{\text{SM}} = C_R^\nu|_{\text{SM}} = 0 . \tag{26}$$

whereas the remaining non-vanishing coefficients are known at the next-to-leading order [6, 34, 35]. The coefficients of Q_{10} and Q_L^ν are scale independent and are completely dominated by short-distance dynamics associated with top quark exchange. Their values are therefore well approximated by the leading order results, given by ($\bar{m}_t(m_t) = 166\,\mathrm{GeV}$)[5]

$$C_L^\nu|_{\rm SM} = \frac{4B_0(x_t) - C_0(x_t)}{\sin^2\Theta_W} = -6.6\ , \qquad C_{10}|_{\rm SM} = \frac{B_0(x_t) - C_0(x_t)}{\sin^2\Theta_W} = -4.2\ , \tag{27}$$

where the contribution proportional to $C_0(x_t)$ is the one induced by $Z^L_{sb}|_{\rm SM}$ in Eq. (2) once the Z field has been integrated out (the full expression for $B_0(x)$ can be found in [6]). The difference among the two numerical values in (27) can be taken as an indication of the size of the non-Z-induced contributions to these coefficients within the SM. On the other hand, in the generic non-standard scenario described by $\mathcal{L}^Z_{FC}$ we can write

$$C_L^\nu - C_L^\nu|_{\rm SM} = C_{10} - C_{10}|_{\rm SM} = -\frac{Z^L_{bs} - Z^L_{bs}|_{\rm SM}}{V^*_{ts}V_{tb}\sin^2\Theta_W}\ , \qquad C_R^\nu = C'_{10} = -\frac{Z^R_{bs}}{V^*_{ts}V_{tb}\sin^2\Theta_W}\ . \tag{28}$$

In principle the coefficients C_9 and C'_9 are also sensitive to Z^L_{bs} and Z^R_{bs}. In this case, however, the contribution of $\mathcal{L}^Z_{FC}$ is suppressed by the smallness of the vector coupling of the Z to charged leptons ($|a^e_V/a^e_A| = |4\sin^2\Theta_W - 1| \simeq 0.08$) and as a first approximation can be neglected. Given the above considerations, we will assume in the following that all the Wilson coefficients but those in (28) coincide with their SM expressions.

4.2 Kinematics and form factors

In the following sections we shall discuss integrated observables and distributions in the invariant mass of the dilepton system, q^2, for the three-body decays $B \to H\ell\bar{\ell}$, with $H = K$, K^* and $\ell = \mu,\ \nu$. The kinematical range of q^2 is given by $4m_\ell^2 \simeq 0 \le q^2 \le (m_B - m_H)^2$. In the neutrino case q^2 is not directly measurable but is related to the kaon energy in the B meson rest frame, varying in the interval $m_H \le E_H \le (m_B^2 + m_H^2)/(2m_B)$ by the relation $q^2 = m_B^2 + m_H^2 - 2m_B E_H$. For convenience we define also the dimensionless variables $s = q^2/m_B^2$ and $r_H = m_H^2/m_B^2$, and the function

$$\lambda_H(s) = 1 + r_H^2 + s^2 - 2s - 2r_H - 2r_H s\ . \tag{29}$$

In the case $H = K$ the hadronic matrix elements needed for our analysis can be written as

$$\langle \bar{K}(p_K)|\bar{s}\gamma_\mu b|\bar{B}(p)\rangle = f_+(q^2)(p+p_K)_\mu + f_-(q^2)q_\mu\ , \tag{30}$$

$$q^\nu\langle \bar{K}(p_K)|\bar{s}\sigma_{\mu\nu} b|\bar{B}(p)\rangle = i\frac{f_T(q^2)}{m_B + m_K}\left[q^2(p+p_K)_\mu - (m_B^2 - m_K^2)q_\mu\right]\ , \tag{31}$$

where $q^\mu = p^\mu - p^\mu_K$. Up to small isospin breaking effects, which we shall neglect, the same set of form factors describes both charged ($B^- \to K^-$) and neutral ($\bar{B}^0 \to \bar{K}^0$) transitions.

[5] Here and in the following we employ the running ($\overline{MS}$) mass for the top quark, $\bar{m}_t(m_t)$. For $b \to s\ell^+\ell^-$ the distinction between the pole mass and the running mass enters, strictly speaking, only beyond the next-to-leading order we are working in [36]. However, the short-distance $\overline{MS}$-mass is the more appropriate definition for FCNC processes involving virtual top quarks, and the higher order corrections are generally better behaved. This is true in particular for the transitions $b \to s\nu\bar{\nu}$ and $B_s \to \mu^+\mu^-$, where the use of the running mass in the known next-to-leading order expressions is entirely well defined and leads indeed to a small size of the NLO QCD corrections.

Similarly, in the case $H = K^*$ we can write ($\epsilon^{0123} = +1$)

$$\begin{aligned}
\langle \bar{K}^*(p_{_K}, \varepsilon)|\bar{s}\gamma_\mu\gamma_5 b|\bar{B}(p)\rangle &= 2m_{K^*}A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q_\mu + (m_B + m_{K^*})A_1(q^2)\left[\varepsilon^*_\mu - \frac{\varepsilon^*\cdot q}{q^2}q_\mu\right] \\
&\quad -A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B + m_{K^*}}\left[(p+p_{_K})_\mu - \frac{m_B^2 - m_{K^*}^2}{q^2}q_\mu\right] , \qquad (32)\\
\langle \bar{K}^*(p_{_K}, \varepsilon)|\bar{s}\gamma_\mu b|\bar{B}(p)\rangle &= i\frac{2V(q^2)}{m_B + m_{K^*}}\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho p^\sigma_{_K} , \qquad (33)\\
q^\nu\langle \bar{K}^*(p_{_K}, \varepsilon)|\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b|\bar{B}(p)\rangle &= -2T_1(q^2)\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho p^\sigma_{_K} \\
&\quad -iT_2(q^2)\left[\varepsilon^*_\mu(m_B^2 - m_{K^*}^2) - (\varepsilon^*\cdot q)(p+p_{_K})_\mu\right] - iT_3(q^2)(\varepsilon^*\cdot q)\left[q_\mu - \frac{q^2(p+p_{_K})_\mu}{m_B^2 - m_{K^*}^2}\right] \qquad (34)
\end{aligned}$$

Here we have used the phase conventions of [37]. In particular, all form factors are real and positive. We remark that the large-energy limit discussed in [37] is especially useful to fix the relative sign of the various form factors in a model independent way.
The form factors f_T, T_1, T_2 and T_3 depend on the renormalization scale, which here and in the following is understood to be $\mu = m_b$. There is no need to further specify the renormalization scheme for the tensor operator $\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b$, since the issue of a non-trivial scheme dependence enters only beyond the next-to-leading logarithmic approximation in $b \to s\ell^+\ell^-$

For the numerical evaluations of $f_i(q^2)$, $A_i(q^2)$, $T_i(q^2)$ and $V(q^2)$ we refer to the recent analysis of Ref. [20], performed in the framework of light-cone sum rules.

5 $B \to (K, K^*)\nu\bar{\nu}$

From a theoretical point of view the neutrino channels are certainly much cleaner compared to the charged lepton ones due to the absence of long-distance effects of electromagnetic origin. Moreover the smaller number of operators involved (only two) simplifies their description. Finally the branching fractions are enhanced by the summation over the three neutrino flavours. All these virtues, however, are partially compensated by the difficult experimental signature.

5.1 $B \to K\nu\bar{\nu}$

The dilepton spectrum of this mode is particularly simple and is sensitive only to the combination $|C_L^\nu + C_R^\nu|$ [38]:

$$\frac{d\Gamma(B \to K\nu\bar{\nu})}{ds} = \frac{G_F^2\alpha^2 m_B^5}{256\pi^5}\,|V_{ts}^* V_{tb}|^2\,\lambda_K^{3/2}(s) f_+^2(s)|C_L^\nu + C_R^\nu|^2 \qquad (35)$$

The differential branching ratio computed within the SM is plotted in Fig. 1, showing the uncertainty due to the form factors. Note that in the neutral modes the strangeness eigenstates of the kaons do not coincide with the mass eigenstates, which are experimentally detected. Therefore, neglecting isospin-breaking and $\Delta S = 2$ CP-violating effects, we can write

$$\Gamma(B \to K\nu\bar{\nu}) \equiv \Gamma(B^+ \to K^+\nu\bar{\nu}) = 2\Gamma(B^0 \to K_{L,S}\nu\bar{\nu}) \ . \qquad (36)$$

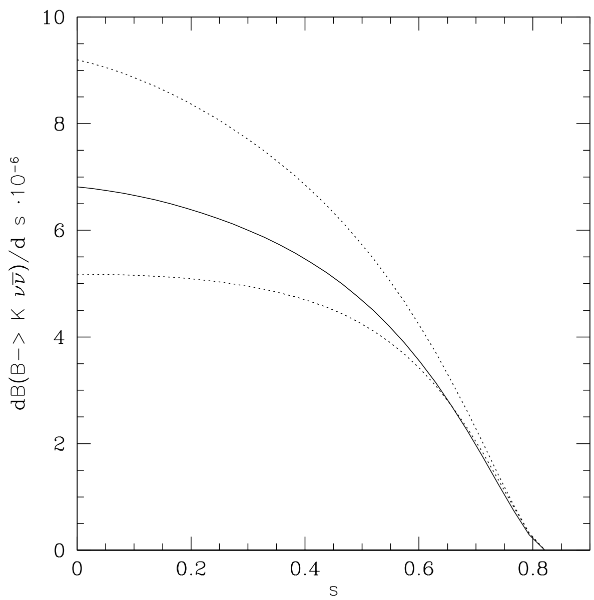

Figure 1: *Dilepton invariant mass distribution for $\mathcal{B}(B \to K\nu\bar{\nu})$ within the SM. The three lines correspond to the central, minimal and maximal values of $f_+(s)$ from [20].*

The absence of absorptive final-state interactions in this process also leads to $\Gamma(B \to K\nu\bar{\nu}) = \Gamma(\bar{B} \to \bar{K}\nu\bar{\nu})$, preventing the observation of any direct-CP-violating effect.

Integrating Eq. (35) over the full range of s leads to

$$\begin{aligned} \mathcal{B}(B \to K\nu\bar{\nu}) &= (3.8^{+1.2}_{-0.6}) \times 10^{-6} \left| \frac{C_L^\nu + C_R^\nu}{C_L|^\nu_{SM}} \right|^2 \\ &\approx 4 \times 10^{-6} \left| 1 - \frac{(Z^L_{bs} - Z^L_{bs}|_{SM}) + Z^R_{bs}}{0.06} \right|^2 , \end{aligned} \tag{37}$$

where the error in the first equality is due to the uncertainty in the form factors and the second relation has been obtained by means of Eq. (28). Given the constraint (6), without further assumptions we find $\mathcal{B}(B \to K\nu\bar{\nu}) \lesssim 5 \times 10^{-5}$. This bound sets the level below which an experimental constraint on this mode starts to provide significant information. On the other hand, in most of the scenarios discussed in Section 3, where $Z^R_{bs} = 0$ and $|Z^L_{bs}| \lesssim 0.1$, we find

$$\mathcal{B}(B \to K\nu\bar{\nu}) \lesssim 2 \times 10^{-5} . \tag{38}$$

If the experimental sensitivity on $\mathcal{B}(B \to K\nu\bar{\nu})$ reached the 10^{-6} level, then the uncertainty due the form factors would prevent a precise extraction of $|C_L^\nu + C_R^\nu|$ from (37). This problem can be substantially reduced by relating the differential distribution of $B \to K\nu\bar{\nu}$ to the one of $B \to \pi e \nu_e$ [39, 40]:

$$\frac{d\Gamma(B \to K\nu\bar{\nu})/ds}{d\Gamma(B^0 \to \pi^- e^+ \nu_e)/ds} = \frac{3\alpha^2}{4\pi^2} \left| \frac{V^*_{ts} V_{tb}}{V_{ub}} \right|^2 \left(\frac{\lambda_K(s)}{\lambda_\pi(s)} \right)^{3/2} \left| \frac{f_+^K(s)}{f_+^\pi(s)} \right|^2 |C_L^\nu + C_R^\nu|^2 . \tag{39}$$

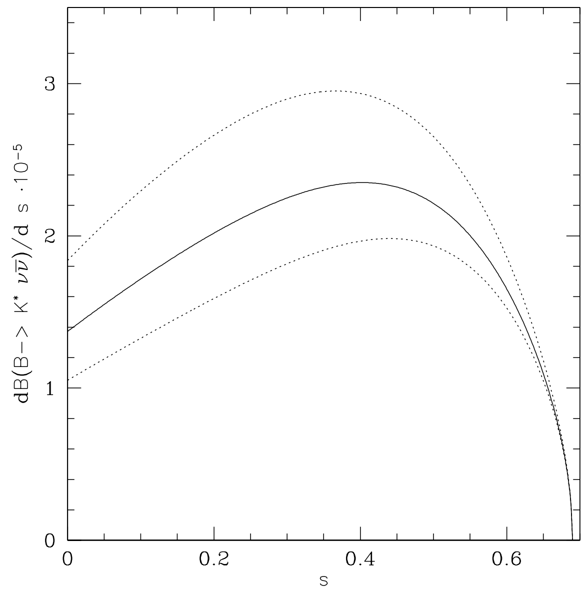

Figure 2: *Dilepton invariant mass distribution for $\mathcal{B}(B \to K^* \nu\bar{\nu})$ within the SM. The three lines correspond to the central, minimal and maximal values, as obtained by varying the form factors within the ranges quoted in [20].*

Indeed $f_+^K(s)$ and $f_+^\pi(s)$ coincide up to $SU(3)$ breaking effects, which are expected to be small, especially far from the endpoint region. An additional uncertainty in (39) is induced by the CKM ratio $|V_{ts}^* V_{tb}|^2/|V_{ub}|^2$ which, however, can independently be determined from other processes.

5.2 $B \to K^* \nu\bar{\nu}$

The dilepton invariant mass spectrum of $B \to K^* \nu\bar{\nu}$ decays is sensitive to both combinations $|C_L^\nu - C_R^\nu|$ and $|C_L^\nu + C_R^\nu|$ [38, 41]:

$$
\begin{aligned}
\frac{d\Gamma(B \to K^* \nu\bar{\nu})}{ds} &= \frac{G_F^2 \alpha^2 m_B^5}{1024\pi^5} \, |V_{ts}^* V_{tb}|^2 \, \lambda_{K^*}^{1/2}(s) \Bigg\{ \frac{8s\lambda_{K^*}(s)V^2(s)}{(1+\sqrt{r_{K^*}})^2} \, |C_L^\nu + C_R^\nu|^2 \\
&+ \frac{1}{r_{K^*}} \Bigg[(1+\sqrt{r_{K^*}})^2 \, (\lambda_{K^*}(s) + 12 r_{K^*} s) \, A_1^2(s) + \frac{\lambda_{K^*}^2(s) A_2^2(s)}{(1+\sqrt{r_{K^*}})^2} \\
&- 2\lambda_{K^*}(s)(1 - r_{K^*} - s) A_1(s) A_2(s) \Bigg] \, |C_L^\nu - C_R^\nu|^2 \Bigg\} . \qquad (40)
\end{aligned}
$$

The branching fraction obtained within the SM is shown in Fig. 2.

Integrating Eq. (40) over the full range of s leads to

$$
\begin{aligned}
\mathcal{B}(B \to K^* \nu\bar{\nu}) &= (2.4^{+1.0}_{-0.5}) \times 10^{-6} \, \left| \frac{C_L^\nu + C_R^\nu}{C_L|_{SM}^\nu} \right|^2 + (1.1^{+0.3}_{-0.2}) \times 10^{-5} \, \left| \frac{C_L^\nu - C_R^\nu}{C_L|_{SM}^\nu} \right|^2 , \qquad (41) \\
\mathcal{B}(B \to K^* \nu\bar{\nu})\Big|_{\rm SM} &= (1.3^{+0.4}_{-0.3}) \times 10^{-5} \, . \qquad (42)
\end{aligned}
$$

Similarly to the case of $\mathcal{B}(B \to K\nu\bar{\nu})$, the bound (6) leaves open the possibility of enhancements of $\mathcal{B}(B \to K^*\nu\bar{\nu})$ up to one order of magnitude with respect to the SM case. Whereas if $Z_{bs}^R = 0$ and $|Z_{bs}^L| \lesssim 0.1$, we find the constraint

$$\mathcal{B}(B \to K^*\nu\bar{\nu}) \lesssim 10^{-4} \ , \tag{43}$$

which is almost one order of magnitude below the present experimental sensitivity [42].

A reduction of the error induced by the poor knowledge of the form factors can be obtained by normalizing the dilepton distributions of $B \to K^*\nu\bar{\nu}$ to the one of $B \to \rho e \nu_e$ [43, 40]. This is particularly effective in the limit $s \to 0$, where the contribution proportional to $|C_L^\nu + C_R^\nu|$ (vector current) drops out:

$$\begin{aligned} \left.\frac{d\Gamma(B \to K^*\nu\bar{\nu})/ds}{d\Gamma(B^0 \to \rho^- e^+ \nu_e)/ds}\right|_{s=0} &= \frac{3\alpha^2}{4\pi^2}\left|\frac{V_{ts}^* V_{tb}}{V_{ub}}\right|^2 \left(\frac{1-r_{K^*}}{1-r_\rho}\right)^3 \frac{r_\rho}{r_{K^*}} |C_L^\nu - C_R^\nu|^2 \\ &\times \left|\frac{A_1^{K^*}(0)(1+\sqrt{r_{K^*}}) - A_2^{K^*}(0)(1-r_{K^*})/(1+\sqrt{r_{K^*}})}{A_1^\rho(0)(1+\sqrt{r_\rho}) - A_2^\rho(0)(1-r_\rho)/(1+\sqrt{r_\rho})}\right|^2 \end{aligned} \tag{44}$$

Similarly to the ratio $f_+^K(s)/f_+^\pi(s)$ in (39), also the last term in (44) is equal to one up to $SU(3)$-breaking corrections.

6 $B \to (K, K^*)\ell^+\ell^-$

The possibility to detect the leptons not only provides a clear experimental signature for $B \to (K, K^*)\ell^+\ell^-$ decays, it also allows to consider interesting observables in addition to the decay distribution, like the forward-backward asymmetry. Moreover, the non-vanishing absorptive contributions lead to potentially large direct-CP-violating effects.

The problem of these modes is the uncertainty in the non-perturbative contributions generated by the operators Q_{1-6} in $\mathcal{H}_{eff}$. Indeed these induce transitions of the type $b \to s(c\bar{c}) \to s\ell^+\ell^-$ that can be handled in perturbation theory only within specific regions of the dilepton spectrum.

In the following we shall restrict our attention to the transitions with a $\mu^+\mu^-$ pair in the final state, which have the clearest experimental signature, however the whole discussion is equally applicable to the e^+e^- case.

6.1 Non-perturbative $c\bar{c}$ corrections and $C_9^{\rm eff}$

In the kinematic region of large dilepton invariant mass, above the Ψ' peak, the light quark fields (u, d, s, c) appearing in $\mathcal{H}_{eff}$ may be integrated out explicitly since they enter loop diagrams with a hard external scale ($q^2 \sim m_b^2$) [43, 44]. The endpoint effective Hamiltonian thus derived, valid at the next-to-leading order in QCD, can be obtained from the one in (23) setting to zero the coefficients of Q_{1-6} and replacing C_9 with

$$C_9^{\rm EP}(s) = C_9 + h\left(\frac{m_c}{m_b}, \frac{m_B^2}{m_b^2}s\right)(3C_1 + C_2) + \mathcal{O}\left(C_{3-6}\right) \ , \tag{45}$$

where the function $h(x,y)$ and the numerically small $\mathcal{O}(C_{3-6})$ terms can be found in [45] (we recall that to the next-to-leading order accuracy, only the leading order values of C_{1-6} are need in $C_9^{\rm EP}$). Note that the coefficient function in (45) differs from the effective coupling of Q_9 usually introduced to describe inclusive decays [6], since it does not include the QCD correction to the matrix element of the $\bar{s}_L\gamma^\mu b_L$ current. Indeed the latter has to be included in the corresponding hadronic matrix elements, assuming they are computed in full QCD and appropriately normalized at $\mu = \mathcal{O}(m_b)$.

In the region of large q^2 one still expects non-perturbative corrections induced by intermediate $c\bar{c}$ states. Although in principle power suppressed ($\sim \Lambda_{QCD}/m_b$), locally these are likely to produce sizable modifications to the dilepton spectrum. The relative importance of these non-perturbative effects, however, can be diminished by integrating over sufficiently large ranges of q^2.

Far from the endpoint region it is not possible, in principle, to safely integrate out the light quark fields in $\mathcal{H}_{eff}$ and one should estimate separately the matrix elements of Q_{1-6}. In general this is a very complicated task that has so far been treated only with the help of some non-rigorous simplifying assumptions. For instance, assuming that the matrix elements of Q_{1-6} can be factorized as

$$\langle H\mu^+\mu^-|Q_i|\bar{B}\rangle \propto \langle H|\bar{s}_L\gamma^\mu b_L|\bar{B}\rangle \times \langle \mu^+\mu^-|\bar{c}\gamma^\mu c|0\rangle\ , \tag{46}$$

one can employ the Krüger-Sehgal (KS) approach [46] and estimate $\langle \mu^+\mu^-|\bar{c}\gamma^\mu c|0\rangle$ by means of $\sigma(e^+e^- \to c\bar{c})$ data. This approach has the advantage of avoiding double-counting and to provide a rigorous non-perturbative estimate of $\langle \mu^+\mu^-|\bar{c}\gamma^\mu c|0\rangle$. Other recipes to evaluate the contributions of $\langle Q_{1-6}\rangle$ can be found e.g. in [47] and [48]. In all cases, in analogy with (45), these contributions are encoded via an effective coupling for the operator Q_9 of the type

$$C_9^{\rm eff}(s) = C_9 + Y(s)\ . \tag{47}$$

Due to the real intermediate $c\bar{c}$ states, $Y(s)$ develops an imaginary part that plays a crucial role in determining the size of direct-CP-violating observables. A comparison of the different approaches to compute ${\rm Im}C_9^{\rm eff}(s)$ is shown in Fig. 3.

In the following we shall compare results obtained by identifying $C_9^{\rm eff}(s)$ with $C_9^{\rm EP}(s)$ or, alternatively, by employing the KS approach.

6.2 Branching ratios and dilepton spectra

Neglecting the lepton mass, the q^2 distributions of $\bar{B} \to \bar{K}\mu^+\mu^-$ and $\bar{B} \to \bar{K}^*\mu^+\mu^-$ decays, computed with the effective Hamiltonian of Section 4.1, can be written as

$$\begin{aligned}\frac{d\Gamma(\bar{B}\to\bar{K}\mu^+\mu^-)}{ds} &= \frac{G_F^2\alpha^2 m_B^5}{1536\pi^5}|V_{tb}V_{ts}|^2\lambda_K^{3/2}(s)\left\{f_+^2(s)\left(|C_9^{\rm eff}(s)|^2+|C_{10}+C_{10}'|^2\right)\right.\\ &\left.+\frac{4m_b^2 f_T^2(s)}{(m_B+m_K)^2}|C_7|^2+\frac{4m_b f_T(s)f_+(s)}{m_B+m_K}{\rm Re}\left(C_9^{\rm eff}(s)C_7^*\right)\right\},\end{aligned} \tag{48}$$

$$\frac{d\Gamma(\bar{B}\to\bar{K}^*\mu^+\mu^-)}{ds} = \left.\frac{d\Gamma(\bar{B}\to\bar{K}^*\mu^+\mu^-)}{ds}\right|_{\rm SM}$$

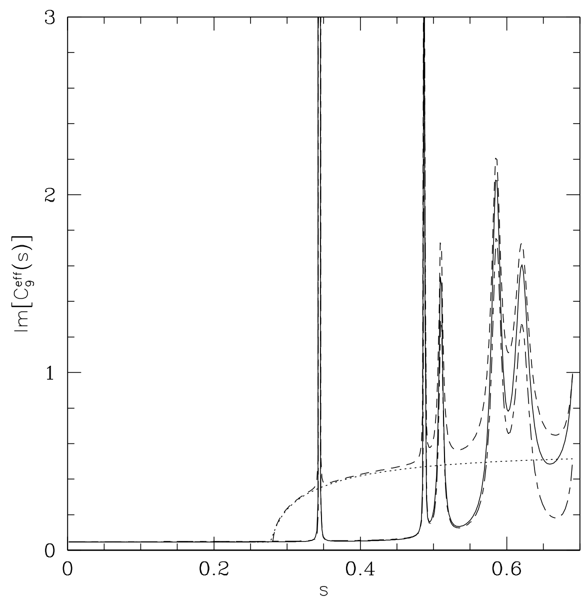

Figure 3: *The imaginary part of* $C_9^{\rm eff}$ *as a function of s:* $\mathrm{Im}C_9^{\rm eff}(s) = \mathrm{Im}C_9^{\rm EP}(s)$ *as in (45) (dotted); KS prescription [46] (solid); Ref. [48] (dot-dashed). For comparison we have also included the approach of Ref. [47] (dashed), where Breit-Wigner resonances are naively added to the partonic calculation. (This procedure is disfavoured since it has a manifest problem of double-counting.)*

$$+\frac{G_F^2\alpha^2 m_B^5}{1024\pi^5}|V_{tb}V_{ts}|^2\lambda_{K^*}^{1/2}(s)\Bigg\{\frac{4s\lambda_{K^*}(s)V^2(s)}{3(1+\sqrt{r_{K^*}})^2}\left(|C_{10}+C'_{10}|^2-|C_{10}|_{\rm SM}|^2\right)$$
$$+\Bigg[\frac{\lambda_{K^*}(s)+12r_{K^*}s}{6r_{K^*}}(1+\sqrt{r_{K^*}})^2A_1^2(s)-\frac{\lambda_{K^*}(s)}{3r_{K^*}}(1-r_{K^*}-s)A_1(s)A_2(s)$$
$$+\frac{\lambda_{K^*}^2(s)A_2^2(s)}{6r_{K^*}(1+\sqrt{r_{K^*}})^2}\Bigg]\left(|C_{10}-C'_{10}|^2-|C_{10}|_{\rm SM}|^2\right)\Bigg\}\,. \qquad (49)$$

The SM expression of $d\Gamma(\bar{B}\to\bar{K}^*\mu^+\mu^-)/ds$ is given by

$$\frac{d\Gamma(\bar{B}\to\bar{K}^*\mu^+\mu^-)}{ds} = \frac{G_F^2\alpha^2 m_B^5}{1024\pi^5}\left|V_{ts}^*V_{tb}\right|^2\lambda_{K^*}^{1/2}(s) \qquad (50)$$
$$\times\left\{R_9\left(|C_9^{\rm eff}(s)|^2+|C_{10}|^2\right)+R_7\frac{m_b^2}{m_B^2}|C_7|^2+R_{97}\frac{m_b}{m_B}\mathrm{Re}C_9^{\rm eff}(s)C_7^*\right\},$$

where

$$R_9 = \frac{4s\lambda_{K^*}(s)V^2(s)}{3(1+\sqrt{r_{K^*}})^2}+\frac{(1+\sqrt{r_{K^*}})^2}{6r_{K^*}}(\lambda_{K^*}(s)+12r_{K^*}s)A_1^2(s)+\frac{\lambda_{K^*}^2(s)}{6r_{K^*}}\frac{A_2^2(s)}{(1+\sqrt{r_{K^*}})^2}$$
$$-\frac{\lambda_{K^*}(s)(1-r_{K^*}-s)}{3r_{K^*}}A_1(s)A_2(s) \qquad (51)$$

$$
\begin{aligned}
R_7 \;=\;& \frac{16\lambda_{K^*}(s)T_1^2(s)}{3s} + \frac{2(1-r_{K^*})^2}{3r_{K^*}s^2}(\lambda_{K^*}(s)+12r_{K^*}s)T_2^2(s) + \frac{2\lambda_{K^*}^2(s)}{3r_{K^*}(1-r_{K^*})^2}T_4^2(s) \\
& -\frac{4\lambda_{K^*}(s)(1-r_{K^*}-s)}{3r_{K^*}s}T_2(s)T_4(s)
\end{aligned}
\tag{52}
$$

$$
\begin{aligned}
R_{97} \;=\;& \frac{16\lambda_{K^*}(s)V(s)T_1(s)}{3(1+\sqrt{r_{K^*}})} + \frac{2(1-r_{K^*})(1+\sqrt{r_{K^*}})}{3r_{K^*}s}(\lambda_{K^*}(s)+12r_{K^*}s)A_1(s)T_2(s) \\
& +\frac{2\lambda_{K^*}^2(s)(1-\sqrt{r_{K^*}})}{3r_{K^*}(1-r_{K^*})^2}A_2(s)T_4(s) \\
& -\frac{2\lambda_{K^*}(s)(1-r_{K^*}-s)}{3r_{K^*}}\left(\frac{1-\sqrt{r_{K^*}}}{s}A_2(s)T_2(s) + \frac{1}{1-\sqrt{r_{K^*}}}A_1(s)T_4(s)\right)
\end{aligned}
\tag{53}
$$

and we have defined

$$
T_4(s) \equiv T_3(s) + \frac{1-r_{K^*}}{s}T_2(s) \tag{54}
$$

Here we have again neglected the lepton mass, which is an excellent approximation for $\ell = e,\, \mu$ if $s \gg 4m_\ell^2/m_B^2$. The full m_ℓ dependence can be found for instance in [20].

As it can be noticed, the coefficients C_{10} and C_{10}', which could have a potentially large CP-violating phase induced by $Z_{bs}^{L,R}$, do not interfere with $C_9^{\rm eff}(s)$, which has a non-vanishing CP-conserving phase. As a consequence, similarly to the SM case, also within our generic non-standard scenario we do not expect to observe any sizable (i.e. above the 10^{-2} level) CP asymmetry in the dilepton invariant mass distribution of both decay modes. In the remaining part of this subsection we will therefore not distinguish between B and $\bar{B}$ states.

The integration over the full range of s with $C_9^{\rm eff}(s) \equiv C_9^{\rm EP}(s)$ (non-resonant branching ratio) and the SM Wilson coefficients leads to $\mathcal{B}(B\to K^*\mu^+\mu^-)^{\rm n.r.}|_{\rm SM} = 1.9^{+0.5}_{-0.3}\times 10^{-6}$ and $\mathcal{B}(B\to K\mu^+\mu^-)^{\rm n.r.}|_{\rm SM} = 5.7^{+1.6}_{-1.0}\times 10^{-7}$ [20], where the error is mainly determined by the uncertainty on the form factors. Interestingly $\mathcal{B}(B\to K^*\mu^+\mu^-)^{\rm n.r.}|_{\rm SM}$ is quite close to the experimental limit

$$
\mathcal{B}(B^0\to K^{*0}\mu^+\mu^-)^{\rm n.r.} < 4.0\times 10^{-6} \tag{55}
$$

recently obtained by CDF [49], whereas for $\mathcal{B}(B\to K\mu^+\mu^-)^{\rm n.r.}$ the best bound-to-SM ratio is around 9 [49]. Thus the K^* mode provides a powerful tool to constraint $|C_{10}|$ and $|C_{10}'|$, or $|Z_{bs}^{L,R}|$, via the relation

$$
\begin{aligned}
\mathcal{B}(B\to K^*\mu^+\mu^-)^{\rm n.r.} \;=\;& \mathcal{B}(B\to K^*\mu^+\mu^-)^{\rm n.r.}|_{\rm SM} \\
& + \left(4.1^{+1.0}_{-0.7}\right)\times 10^{-8}\left(|C_{10}-C_{10}'|^2 - |C_{10}|_{\rm SM}|^2\right) \\
& + \left(0.9^{+0.4}_{-0.2}\right)\times 10^{-8}\left(|C_{10}+C_{10}'|^2 - |C_{10}|_{\rm SM}|^2\right)\ ,
\end{aligned}
\tag{56}
$$

obtained by integrating (49). Using the bound (55) and setting $C_{10}'=0$ we recover the result of [20] $|C_{10}| \lesssim 10$, that in turn implies

$$
\left|Z_{bs}^L\right| \lesssim 0.10\ . \tag{57}
$$

Note that, since C_{10} is basically dominated by the Z penguin already within the SM, the maximal allowed value for $|Z^L_{bs}|$ is to a good approximation independent of the sign of Z^L_{bs}. On the other hand, if we allow also C'_{10} to be different from zero we find the relation

$$|C_{10}|^2 + |C'_{10}|^2 - (1.25 \pm 0.05) \times \mathrm{Re}\,(C^*_{10}C'_{10}) \lesssim 100 \ , \tag{58}$$

where the coefficient of $\mathrm{Re}(C^*_{10}C'_{10})$ is quite stable with respect to variations of the form factors. Varying $\arg(C_{10}/C'_{10})$ over 2π we find $|C_{10}|$, $|C'_{10}| \lesssim 13$, leading to

$$\left|Z^{L,R}_{bs}\right| \lesssim 0.13 \ . \tag{59}$$

Due to the uncertainties in the form factors and the assumptions on the non-perturbative non-resonant contributions, the bounds derived from Eq. (56) could appear less clean, from a theoretical point of view, than those derived from the inclusive rates. We stress, however, that even doubling the errors on the form factors the constraints in (57) and (59) do not increase by more than 10%.

Though still at the border of most of the model predictions discussed in Section 3, the bound (57) starts to provide a significant information. For instance, it strengthens the model-independent character of the bounds (38) and (43) for the neutrino modes. As already discussed in Section 5, if the experiments reached the SM sensitivity on $B \to K^*\mu^+\mu^-$, more precise information on C_{10} and C'_{10} could be obtained by relating the from factors of this mode to those of its $SU(3)$ partner $B \to \rho e \nu_e$.

6.3 Forward-backward asymmetry in $B \to K^*\mu^+\mu^-$

As anticipated, the possibility of detecting the leptons in the final state allows us to study interesting asymmetries in the decay distribution of $B \to H\mu^+\mu^-$ modes. The (lepton) forward-backward asymmetry of $\bar{B} \to \bar{K}^*\mu^+\mu^-$ can be defined as

$$\mathcal{A}^{(\bar{B})}_{FB}(s) = \frac{1}{d\Gamma(\bar{B} \to \bar{K}^*\mu^+\mu^-)/ds} \int_{-1}^{1} d\cos\theta \ \frac{d^2\Gamma(\bar{B} \to \bar{K}^*\mu^+\mu^-)}{ds \ d\cos\theta} \mathrm{sgn}(\cos\theta) \ , \tag{60}$$

where θ is the angle between the momenta of μ^+ and $\bar{B}$ in the dilepton center-of-mass frame. Given the vector or axial-vector structure of the leptonic current generated by $\mathcal{H}_{eff}$, this asymmetry can be different from zero only if the final hadronic system has a non-vanishing angular momentum and therefore it is identically zero in the case of $B(\bar{B}) \to K(\bar{K})\mu^+\mu^-$.

The explicit expression for $\mathcal{A}^{(\bar{B})}_{FB}(s)$ in terms of Wilson coefficients and form factors can be written as

$$\begin{aligned}\mathcal{A}^{(\bar{B})}_{FB}(s) \ &= \ -\frac{G_F^2 \alpha^2 m_B^5 \left|V^*_{ts}V_{tb}\right|^2}{256\pi^5 d\Gamma(\bar{B} \to \bar{K}^*\mu^+\mu^-)/ds} \ \lambda_{K^*}(s)\left|V(s)A_1(s)\right| \\ &\quad \times \mathrm{Re}\left\{C^*_{10}\left[s\ C_9^{\mathrm{eff}}(s) + \alpha_+(s)\frac{m_b C_7}{m_B} + \alpha_-(s)\frac{m_b C_7 C'^*_{10}}{m_B C^*_{10}}\right]\right\} \ ,\end{aligned} \tag{61}$$

where

$$\alpha_\pm(s) = \frac{T_2(s)}{A_1(s)}(1 - \sqrt{r_{K^*}}) \pm \frac{T_1(s)}{V(s)}(1 + \sqrt{r_{K^*}}) \tag{62}$$

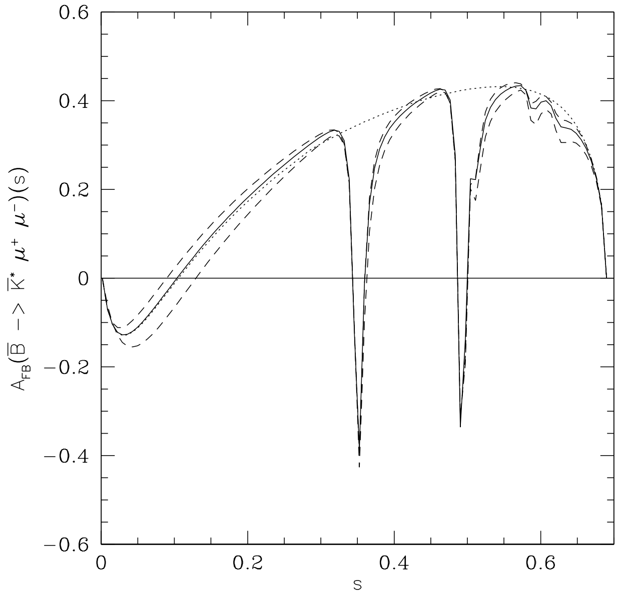

Figure 4: *Forward-backward asymmetry for $\bar{B} \to \bar{K}^{*}\mu^{+}\mu^{-}$, defined as in (60). The solid (dotted) curves have been obtained employing the Krüger-Sehgal approach (using $C_9^{\rm eff}(s) \equiv C_9^{\rm EP}(s)$). The dashed lines show the effect of varying the renormalization scale of the Wilson Coefficients between $m_b/2$ and $2m_b$, within the Krüger-Sehgal approach.*

and we have used the model-independent relation between the signs of $V(s)$ and $A_1(s)$, discussed in Section 4.2, to elucidate the overall sign of $\mathcal{A}_{FB}^{(\bar{B})}(s)$.

The ratios of form factors in (62) can be determined to a good accuracy by means of those entering $B \to \rho e \nu$ decays, leading to a precise determination of the point s_0 where $\mathcal{A}_{FB}^{(\bar{B})}(s_0) = 0$ [50]. The interest in the zero of $\mathcal{A}_{FB}^{(\bar{B})}(s)$ is further reinforced by the fact that most of the intrinsic hadronic uncertainties affecting $T_{1,2}$, A_1 and V cancel in $\alpha_\pm(s)$ [20, 50], an observation that can be justified in the large-energy expansion of heavy-to-light form factors [37]. In this limit it is also easy to realize that $|\alpha_-(s)/\alpha_+(s)| = r_{K^*}/(1-s) \ll 1$, so that the term proportional to C'_{10} in (61) is to a good approximation negligible. Since the position of s_0 does not depend on magnitude or sign of C_{10} (assuming $C_{10} \neq 0$) we conclude that within our New Physics scenario the zero of $\mathcal{A}_{FB}^{(\bar{B})}(s)$ remains unchanged with respect to the SM case ($s_0|_{\rm SM} = 0.10^{+0.02}_{-0.01}$ [20]).

Contrary to s_0, magnitude and sign of the forward-backward asymmetry can be very much affected by possible non-standard contributions to C_{10}. The sign, in particular, is of great interest being related in a model-independent way to the relative signs of the Wilson coefficients. This relation deserves a clarifying discussion, as there is apparently some confusion on this issue in the recent literature.

- First of all we stress that the sign is different for B and $\bar{B}$ decays. In fact, in the limit of CP conservation one expects

$$\mathcal{A}_{FB}^{(\bar{B})}(s) = -\mathcal{A}_{FB}^{(B)}(s) \ . \qquad (63)$$

This can be easily understood by noting that CP conjugation requires not only the exchange $b \leftrightarrow \bar{b}$ but also the one of $\mu^+ \leftrightarrow \mu^-$. Since the two leptons are emitted back to back in the dilepton center-of-mass frame, the asymmetry defined in terms of the direction of the positive charged lepton (both for B and $\bar{B}$), changes sign under CP conjugation.

- The sign in (61) implies that within the SM, where $\mathrm{Re}(C_{10}^* C_9) < 0$, $\mathcal{A}_{FB}^{(\bar{B})}(s)$ is *positive* for $s > s_0$ (see Fig. 4). This coincides with the SM behavior of the inclusive forward-backward asymmetry of the process $b \to s\mu^+\mu^-$ (see e.g. [44]) and indeed it has a simple partonic interpretation (we recall that we denote by $\bar{B}$ the meson with a valence b quark). At sufficiently large values of q^2 the contribution of C_7 can be neglected and, within the SM, the decay is almost a pure $(V-A)\times(V-A)$ interaction ($C_{10}|_{SM} \approx -C_9$). In the $\bar{B}$ rest frame the emitted s quark tends to be left-handed polarized and, when its spin is combined with the one of the spectator, this leads to a $\bar{K}^*$ meson with helicity -1 or 0. Since the initial $\bar{B}$ meson has spin 0, the total helicity of the recoiling lepton pair must also be -1 or 0, respectively. If it is zero then there is no forward-backward asymmetry, as in the $\bar{B} \to \bar{K}\mu^+\mu^-$ case. On the other hand, if the polarization of the lepton pair is -1, then the positive lepton prefers to travel backward with respect to the total momentum of the dilepton system, or in the direction of the K^* meson. This configuration corresponds to a positive $\cos\theta$, leading to a positive $\mathcal{A}_{FB}^{(\bar{B})}(s)$.

Having firmly established the sign of $\mathcal{A}_{FB}^{(\bar{B})}(s)$ within the SM, a striking signal of New Physics could clearly be observed if $\mathrm{sgn}(\mathrm{Re}C_{10}) = -\mathrm{sgn}(\mathrm{Re}C_{10}|_{\mathrm{SM}})$. In this case $\mathcal{A}_{FB}^{(\bar{B})}(s)$ would be positive for $s < s_0$ and negative for $s > s_0$, opposite to the SM expectation. Similarly, a clear signal of non-standard dynamics would occur if $\mathrm{Re}C_{10}$ was purely imaginary, so that $\mathcal{A}_{FB}^{(\bar{B})}(s)$ would be very much suppressed with respect to the SM case. Note that in both of these examples one could still have an absolute value of C_{10} close to its SM expectation, hiding these New Physics effects in branching ratios and dilepton spectra.

6.3.1 Forward-backward CP asymmetry

More generally, a powerful tool to probe a possible CP-violating phase in C_{10} is provided by the sum of the forward-backward asymmetries of $\bar{B}$ and B decays, which is expected to vanish in the absence of CP violation.[6] For this purpose we introduce the *forward-backward CP asymmetry*, defined as

$$\mathcal{A}_{FB}^{CP}(s) = \frac{\mathcal{A}_{FB}^{(\bar{B})}(s) + \mathcal{A}_{FB}^{(B)}(s)}{\mathcal{A}_{FB}^{(\bar{B})}(s) - \mathcal{A}_{FB}^{(B)}(s)} \,. \tag{64}$$

This observable is very small within the SM, where the CP-violating phases of the relevant Wilson coefficients are suppressed by the factor $\mathrm{Im}(V_{ub}V_{us}^*/V_{tb}V_{ts}^*) \sim \mathcal{O}(\eta\lambda^2) \sim 0.01$. The explicit calculation of $\mathcal{A}_{FB}^{CP}(s)$ within the SM requires to keep the small $u\bar{u}$ contribution in $C_9^{\mathrm{eff}}(s)$ (see e.g. [52]), which we have so far neglected. Employing the partonic calculation for

[6] This effect has already been pointed out in Ref. [51] in the context of $b \to d\ell^+\ell^-$ transitions.

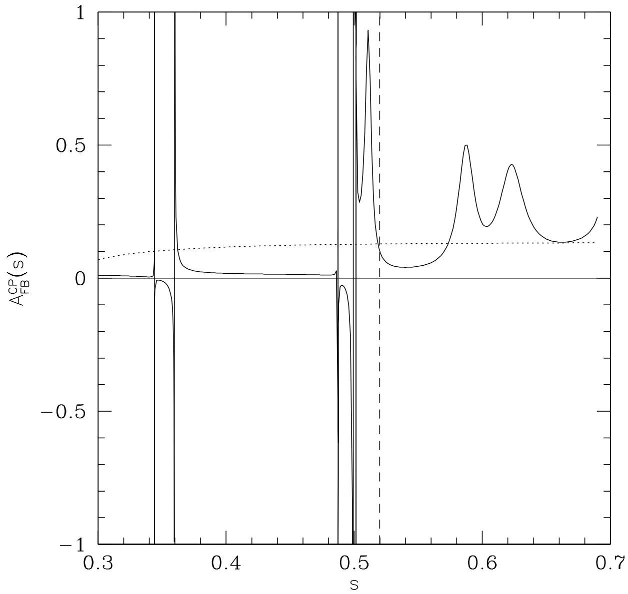

Figure 5: *The forward-backward CP asymmetry defined in (64), in units of* $\mathrm{Im}C_{10}/\mathrm{Re}C_{10}$, *as a function of s. Solid and dotted lines correspond to the Krüger-Sehgal approach and to the choice $C_9^{\rm eff}(s) \equiv C_9^{\rm EP}(s)$, respectively. The vertical dashed line denotes the lower limit of the integration range in (67).*

both $u\bar{u}$ and $c\bar{c}$ loops we find

$$\mathcal{A}_{FB}^{CP}(s)\Big|_{SM} = \frac{\mathrm{Im}(V_{ub}V_{us}^*)}{\mathrm{Re}(V_{cb}V_{cs}^*)} \frac{\mathrm{Im}\left[h\left(\frac{m_c}{m_b}, \frac{m_B^2}{m_b^2}s\right) - h\left(0, \frac{m_B^2}{m_b^2}s\right)\right](3C_1 + C_2)}{\mathrm{Re}C_9^{\rm eff}(s)} \times \left[1 + \frac{\alpha_+(s)}{s} \frac{m_b C_7}{m_B \mathrm{Re}C_9^{\rm eff}(s)}\right]^{-1}, \tag{65}$$

which in the region above the Ψ' peak leads to an integrated asymmetry below 10^{-3}.

On the other hand, if we allow C_{10} to have a large CP-violating phase and neglect those of C_7 and C_9, as expected within our generic non-standard framework, we find

$$\mathcal{A}_{FB}^{CP}(s) = \frac{\mathrm{Im}C_{10}}{\mathrm{Re}C_{10}} \frac{\mathrm{Im}C_9^{\rm eff}(s)}{\mathrm{Re}C_9^{\rm eff}(s)} \left[1 + \frac{\alpha_+(s)}{s} \frac{m_b C_7}{m_B \mathrm{Re}C_9^{\rm eff}(s)}\right]^{-1}, \tag{66}$$

which can be substantially different from zero above the $c\bar{c}$ threshold if $\mathrm{Im}C_{10}/\mathrm{Re}C_{10} \sim \mathcal{O}(1)$. Note that the expression (66) is almost free from uncertainties in the form factors, since for large s (where $\mathrm{Im}C_9^{\rm eff}(s) \neq 0$) the term proportional to C_7 is rather small. Unfortunately this virtue is somewhat compensated by the uncertainties in $\mathrm{Im}C_9^{\rm eff}(s)$ discussed in Section 6.1. A plot of $\mathcal{A}_{FB}^{CP}(s)$, in units of $\mathrm{Im}C_{10}/\mathrm{Re}C_{10}$, in the interesting region above the Ψ peak is shown in Fig. 5.

To decrease the effect of the non-perturbative uncertainties in $\mathrm{Im}C_9^{\rm eff}(s)$ it is convenient to integrate $\mathcal{A}_{FB}^{CP}(s)$ over a large interval of q^2. To avoid the uncontrollable errors associated with the narrow Ψ and Ψ' peaks, as well as with the $D-\bar{D}$ threshold, we consider a safe integration region

$$q^2_{\rm min} = 14.5~{\rm GeV}^2 \leq q^2 < (m_B - m_{K^*})^2 = q^2_{\rm max}~, \tag{67}$$

where we find

$$\Delta\mathcal{A}^{CP}_{FB} = \int_{s_{\rm min}}^{s_{\rm max}} ds \mathcal{A}^{CP}_{FB}(s) = (0.03 \pm 0.01) \times \frac{{\rm Im}C_{10}}{{\rm Re}C_{10}}~. \tag{68}$$

The central value in (68) has been obtained within the Krüger-Sehgal approach, whereas the error has been estimated by comparing this result with the one obtained by identifying $C_9^{\rm eff}(s)$ with $C_9^{\rm EP}(s)$. Here and in Fig. 5 we did not use any phenomenological correction factors for the resonance contributions in applying the KS method, that is we put $\kappa_V = 1$ (notation of [46]).

Unfortunately the numerical coefficient of ${\rm Im}C_{10}/{\rm Re}C_{10}$ in $\Delta\mathcal{A}^{CP}_{FB}$ is rather suppressed, however it leaves open the possibility of $\mathcal{O}(10\%)$ effects. These would naturally occur if the non-standard contributions to Z^L_{bs} had the same magnitude as the SM term and a CP-violating phase of $\mathcal{O}(1)$, a scenario that is allowed in most of the specific models discussed in Section 3.

7 $B_s \to \mu^+\mu^-$

The constraint (58) implies also an upper bound for $\mathcal{B}(B_s \to \mu^+\mu^-)$ in our generic non-standard scenario. Introducing the B_s decay constant, f_{B_s}, the decay rate for this process can be written as

$$\Gamma(B_s \to \mu^+\mu^-) \;=\; \frac{G_F^2\alpha^2}{16\pi^3} f^2_{B_s} |V^*_{ts}V_{tb}|^2 m_{B_s} m^2_\mu \left(1 - \frac{4m^2_\mu}{m^2_{B_s}}\right)^{1/2} |C_{10} - C'_{10}|^2~, \tag{69}$$

implying

$$\mathcal{B}(B_s \to \mu^+\mu^-) = \mathcal{B}(B_s \to \mu^+\mu^-)|_{\rm SM} \times \left|\frac{C_{10} - C'_{10}}{C_{10}|_{SM}}\right|^2~. \tag{70}$$

Using the constraint (58) we then find a maximal enhancement of a factor 7 for $\mathcal{B}(B_s \to \mu^+\mu^-)$ with respect to the SM value.

Employing the full next-to-leading order expression for $C_{10}|_{\rm SM}$ [6, 34, 35] one has

$$\mathcal{B}(B_s \to \mu^+\mu^-)\Big|_{\rm SM} = 3.4 \times 10^{-9} \left(\frac{f_{B_s}}{0.210{\rm GeV}}\right)^2 \left(\frac{|V_{ts}|}{0.040}\right)^2 \left(\frac{\tau_{B_s}}{1.6{\rm ps}}\right) \left(\frac{\overline{m}_t(m_t)}{170{\rm GeV}}\right)^{3.12}~. \tag{71}$$

Allowing for the maximal enhancement in (70) and adopting conservative upper bounds for the ratios in (71) we finally obtain

$$\mathcal{B}(B_s \to \mu^+\mu^-) < 3.4 \times 10^{-8}~, \tag{72}$$

which is about two orders of magnitude below the current best limit from CDF [53]: $\mathcal{B}(B_s \to \mu^+\mu^-) < 2.6 \times 10^{-6}$ (95% C.L.).

8 Summary and conclusions

We have presented a study of the rare decay modes $B \to K^{(*)}\nu\bar{\nu}$, $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \mu^+\mu^-$, which are mediated by $b \to s$ FCNC transitions. These processes have long been recognized as very interesting probes of the flavour sector where New Physics effects could modify considerably the Standard Model expectations.

In this paper we have pursued the idea that the largest deviations from the Standard Model could arise in the FCNC couplings of the Z boson. We have thus investigated a scenario where new dynamics determines the $\bar{s}_{L,R}\gamma^\mu b_{L,R}Z_\mu$ interactions, while the contributions of a different origin (boxes, photonic penguins) are still, to a good approximation, given by their Standard Model values. As we have shown, this scenario is both phenomenologically and theoretically well motivated. Indeed, contrary to other FCNC amplitudes, the $\bar{s}bZ$ couplings are not yet very well constrained by experimental data and considerable room for substantial modifications still exists. On the other hand, also on theoretical grounds these couplings play a special role and are potentially dominant in the presence of a high scale of New Physics. It has also been shown that such a generic scenario could naturally arise in specific and consistent extensions of the SM, as for instance in the framework of Supersymmetry.

Within the Standard Model the following branching ratios are expected, listed here in comparison with the current experimental limits:

$$\begin{array}{rcll} \mathcal{B}(B \to K\nu\bar{\nu}) & \approx & 4 \times 10^{-6} & (< 7.7 \times 10^{-4} \quad [42]) \\ \mathcal{B}(B \to K^*\nu\bar{\nu}) & \approx & 1.3 \times 10^{-5} & (< 7.7 \times 10^{-4} \quad [42]) \\ \mathcal{B}(B \to K\mu^+\mu^-)^{n.r.} & \approx & 6 \times 10^{-7} & (< 5.2 \times 10^{-6} \quad [49]) \\ \mathcal{B}(B \to K^*\mu^+\mu^-)^{n.r.} & \approx & 2 \times 10^{-6} & (< 4 \times 10^{-6} \quad [49]) \\ \mathcal{B}(B_s \to \mu^+\mu^-) & \approx & 3 \times 10^{-9} & (< 2.6 \times 10^{-6} \quad [53]) \end{array} \tag{73}$$

The Standard Model estimates have at present hadronic uncertainties of typically $\pm 30\%$. Our generic New Physics scenario still allows for substantial enhancements that could saturate the experimental bounds for $B \to K^*\mu^+\mu^-$ and increase the remaining branching fractions by factors of 5 to 10.

An observable of particular interest is the forward-backward asymmetry $\mathcal{A}_{FB}^{(\bar{B})}$ in $\bar{B} \to \bar{K}^*\mu^+\mu^-$ decay. This quantity is complementary to rate measurements and can reveal non-standard flavourdynamics that might remain invisible from the decay rates alone. We have clarified the sign of the asymmetry within the Standard Model. The sign (as a function of the dilepton mass) has the same behaviour in the exclusive channel $\bar{B} \to \bar{K}^*\mu^+\mu^-$ as in the inclusive decay $b \to s\mu^+\mu^-$. As we have shown, even for the hadronic process $\bar{B} \to \bar{K}^*\mu^+\mu^-$ the sign of $\mathcal{A}_{FB}^{(\bar{B})}$ can be fixed in a model-independent way. This property provides us with an important Standard Model test. The "wrong" sign of the experimentally measured $\mathcal{A}_{FB}^{(\bar{B})}$ would be a striking manifestation of New Physics. Such a test is comparable, and complementary, to determining the position of the $\mathcal{A}_{FB}$ zero, whose usefulness as a clean probe of New Physics has been stressed in the literature. An interesting observation is that within our scenario of non-standard Z couplings the asymmetry $\mathcal{A}_{FB}^{(\bar{B})}$ is likely to be affected, possibly including even a change of sign, while this class of New Physics would leave the $\mathcal{A}_{FB}^{(\bar{B})}$ zero essentially unchanged.

Finally, we have emphasized that the CP violating forward-backward asymmetry $\mathcal{A}_{FB}^{CP}$ is an interesting probe of non-standard CP violation in the $\bar{s}bZ$ couplings. Potential effects are of order 10%, compared to an entirely negligible Standard Model asymmetry of about 10^{-3}.

Similar observables can also be studied with inclusive modes such as $b \to s\mu^+\mu^-$, which are theoretically cleaner and could play an important role for precision tests in the future. Nevertheless, on a shorter term the exclusive channels are more accessible experimentally, in particular at hadron machines. As we have seen, exciting possibilities for tests of the flavour sector exist also in this case in spite of, in general, larger hadronic uncertainties. The pursuit of these opportunities in rare B decays will certainly contribute to a deeper understanding of flavour physics in the Standard Model and beyond.

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