SLAC–PUB–8398
March 2000

Characterizing an Improved Broad Band Impedance*

Sam Heifets and Alex Chao
Stanford Linear Accelerator Center, Stanford Unversity, CA 94309

Abstract

A phenomenological model of broadband impedance containing two free parameters has been recently proposed. This paper attempts to assign physical charaterizations to these free parameters by relating them to the geometric dimensions of a stand-alone cavity structure.

Contributed paper at the Broadband Impedance Measurements and Modelling Workshop
SLAC, Stanford, USA
February 28 - March 2, 2000

*Work supported by Department of Energy contract DE–AC03–76SF00515.

One problem with the usual broad band resonator impedance model is that it behaves as a pure capacitance at high frequencies, i.e. $Z(\omega) \sim i/\omega$. This is not the correct behavior because at high frequencies, we expect a cavity to be described by the diffraction model, for which $Z(\omega) \sim (1+i)\omega^{-1/2}$. To overcome this problem, it has been suggested that an improved longitudinal broad band impedance model can be parametrized as[1, 2]

$$Z(\omega) = -\frac{i\omega L}{(1 - i\omega T)^{3/2}} \tag{1}$$

Compared with the usual broad band resonator model, the impedance model (1) allows for the correct behavior at high frequencies.

The attempt of this note is to connect the free parameters L and T in Eq.(1) to the geometric dimensions of a cavity, shown in Fig.1. We are aware that, depending on the various relative sizes of g, b, and d, the resulting model will have varying degrees of accuracy. However, we intend to proceed in a simplistic and straightforward manner.

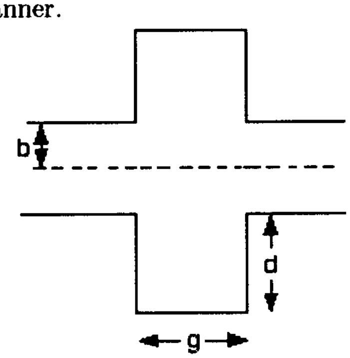

Figure 1: Cavity geometry.

We first decompose the impedance (1) into real and imaginary parts:

$$\mathrm{Re}Z(\omega) = \frac{L}{T} f(\omega T), \quad \mathrm{Im}Z(\omega) = -\frac{L}{T} g(\omega T) \tag{2}$$

where

$$\begin{aligned} f(x) &= \frac{x}{(1+x^2)^{3/4}} \sin\left(\frac{3}{2}\tan^{-1} x\right) \\ &= \frac{x\sqrt{1-\frac{1}{\sqrt{1+x^2}}}\,(2+\sqrt{1+x^2})}{\sqrt{2}\,(1+x^2)^{5/4}} \end{aligned} \tag{3}$$

and

$$\begin{aligned} g(x) &= \frac{x}{(1+x^2)^{3/4}} \cos\left(\frac{3}{2}\tan^{-1} x\right) \\ &= \frac{x\sqrt{1+\frac{1}{\sqrt{1+x^2}}}\,(2-\sqrt{1+x^2})}{\sqrt{2}\,(1+x^2)^{5/4}} \end{aligned} \tag{4}$$

The functions $f(x)$ and $g(x)$ are shown in Fig.2.

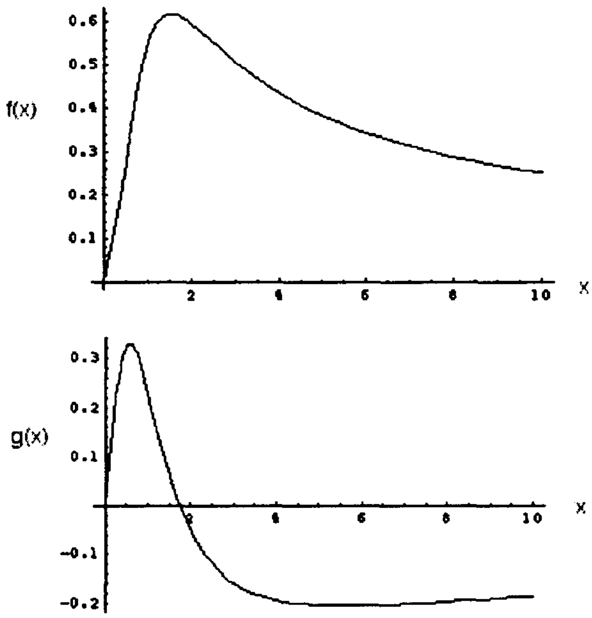

Figure 2: f(x) and g(x).

At low frequencies, we have

$$\text{Im}Z(\omega) \approx -\omega L \tag{5}$$

$$\text{Re}Z(\omega) \approx \frac{3}{2}\omega^2 LT \tag{6}$$

while at high frequencies, we have

$$Z(\omega) \approx \frac{1+i}{\sqrt{2}} \frac{L}{\omega^{1/2}T^{3/2}} \tag{7}$$

The procedure we adopt is to determine the parameters L and T by connecting Eqs.(5) and (7) with what we expect from the cavity of Fig.1. The whole impedance is then determined. We then mention a few properties of the resulting impedance and calculate the loss factor using this model.

At low frequencies, we expect the cavity of Fig.1. to behave like a pure inductance,[3]

$$Z(\omega) \approx -i\omega Z_0 \frac{gd}{2\pi bc} \tag{8}$$

where $Z_0 = 377\Omega$. Identifying this with Eq.(5) gives

$$L = Z_0 \frac{gd}{2\pi bc} \tag{9}$$

At high frequencies, the diffraction model predicts[3]

$$Z(\omega) \approx (1+i)\frac{Z_0}{2\pi^{3/2}b}\sqrt{\frac{cg}{\omega}} \tag{10}$$

Identifying this with Eq.(7), and using Eq.(9), gives

$$cT = \left(\frac{\pi g d^2}{2}\right)^{1/3} \tag{11}$$

Equations (9) and (11) complete the impedance model (1). We make a few comments on this result below:

(1) The longitudinal wake function is given by

$$W(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega\, e^{i\omega z/c} Z(\omega) = W_0 h(\frac{z}{cT}) \tag{12}$$

where[4]

$$\begin{aligned} W_0 &= \frac{L}{T^2} = \frac{Z_0 c}{2b} \left(\frac{g}{4\pi d}\right)^{1/3} \\ h(x) &= \begin{cases} 0, & x > 0 \\ \frac{1}{\sqrt{-\pi x}}(1+2x)e^{x}, & x < 0 \end{cases} \end{aligned} \tag{13}$$

The functions $h(x)$ is shown in Fig.3.

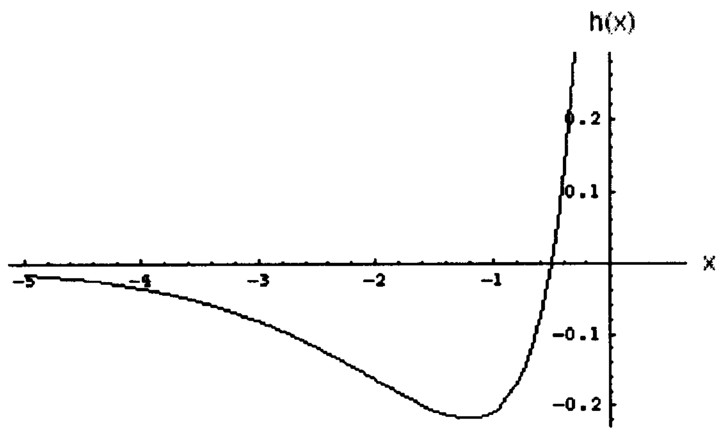

Figure 3: $h(x)$ versus x.

(2) The real part of the impedance has a maximum (see Fig.2),

$$\begin{aligned} \omega_{\text{max}} &= \frac{1.7}{T} = 1.7\, c \left(\frac{2}{\pi g d^2}\right)^{1/3} \\ \text{Re}Z_{\text{max}} &= 0.6\frac{L}{T} = 0.6\frac{Z_0}{b}\left(\frac{g^2 d}{4\pi^4}\right)^{1/3} \end{aligned} \tag{14}$$

These expressions may be identified as the "resonant frequency" and "shunt impedance" of this impedance model. Note that this resonant frequency is independent of the pipe radius b.

(3) One could perhaps also identify the "resonant frequency" as the frequency when the imaginary impedance vanishes. This gives a resonant frequency very close to that given in Eq.(14).

(4) At low frequencies, the usual broad band resonator model gives Re$Z \sim R_S\omega^2/Q^2\omega_R^2$. If we let $R_S = 0.6\,L/T$ and $\omega_R = 1.7/T$ as proposed in Eq.(14), then the low-frequency real part of the impedance is Re$Z \sim 0.2\,\omega^2 LT/Q^2$. Compared with Eq.(6), one obtains an effective "Q-value" of

$$Q_{\text{eff}} \approx 0.37 \tag{15}$$

Equations (14) and (15) are the "resonant frequency", the "shunt impedance", and the "Q-value" of the impedance model (1). The low value of Q_{eff} indicates this impedance is rather broad.

(5) For a gaussian beam bunch with rms bunch length σ_z, the loss factor is given by

$$k = W_0 H(\sigma_z/cT) \tag{16}$$

where

$$H(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} du\, e^{-u^2x^2} f(u) \tag{17}$$

where $f(u)$ is given by Eq.(3) and W_0 is given by Eq.(13). Figure 4 shows $H(x)$.

(6) As a reference, it maybe useful to consider the case when $g = d = b$. In this case, we find

$$\begin{aligned} L &= 0.16\,\frac{Z_0 b}{c} \\ T &= 1.16\,\frac{b}{c} \end{aligned}$$

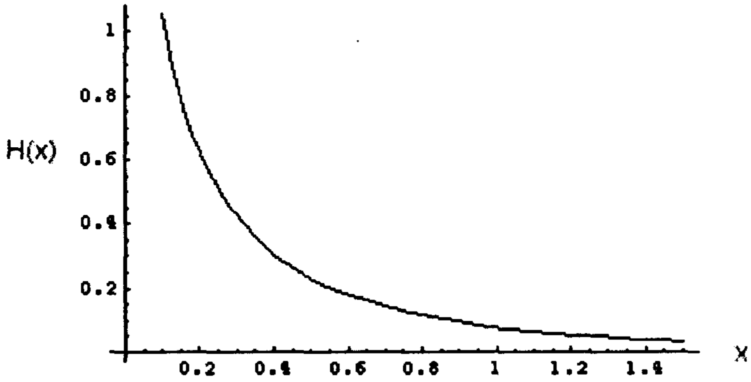

Figure 4: $H(x)$ versus x.

$$
\begin{aligned}
W_0 &= 0.22\,\frac{Z_0 c}{b} \\
\omega_{\max} &= 1.5\,\frac{c}{b} \\
\mathrm{Re}Z_{\max} &= 0.08\,Z_0 = 30\ \Omega
\end{aligned}
\tag{18}
$$

(7) As a numerical example, Fig.5 shows a comparison of the loss factors predicted using the impedance model (1) and the broadband resonator ($Q = 1$) impedance model with the loss factor predicted by the code ABCI for a cavity with $b = 2.5$ cm, $g = 1$ cm, and $d = 0.5$ cm ($cT = 0.732$ cm, $L = 0.4$ nH). For short bunches, the agreement between model (1) with ABCI is better than the agreement between the broadband resonator model with ABCI.

References

[1] S. Heifets, G. Sabbi, SLAC/AP-104 (1996).

[2] B. Zotter, this workshop.

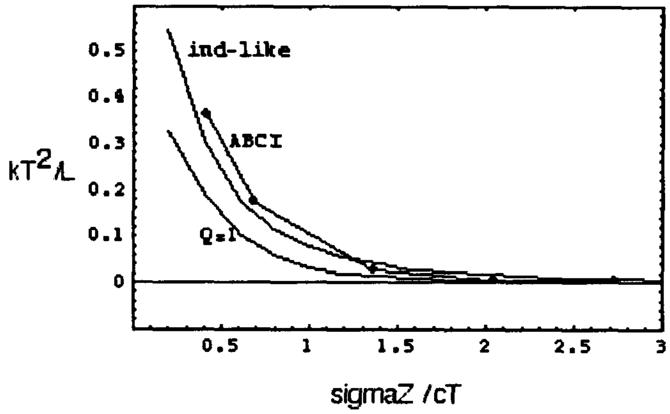

Figure 5: Comparison of the loss factors predicted using the impedance model (1), using the broadband resonator ($Q = 1$) impedance model, and using ABCI code.

[3] See for example A. Chao, "Physics of Collective Beam Instabilities in High Energy Accelerators", Wiley, 1993.

[4] W. Ng, p.204, "Handbook of Accelerator Physics and Engineering", Ed. Chao and Tigner, 1999.