

# Hadronic Light-Cone Wavefunctions and the Unification of QCD Bound-State Phenomena\*

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## Abstract

The light-cone Fock representation encodes the bound-state quark and gluon properties of hadrons, including their helicity and flavor correlations, in terms of universal process-independent and frame-independent wavefunctions. It also provides a physical factorization scheme for separating hard and soft contributions in both exclusive and inclusive hard processes. A new type of jet production reaction, “self-resolving diffractive interactions” can provide direct information on the light-cone wavefunctions of hadrons in terms of their QCD degrees of freedom as well as the composition of nuclei in terms of their nucleon and mesonic degrees of freedom.

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# 1 Introduction

Ever since the discovery of the Bjorken scaling of deep inelastic lepton-proton scattering in 1967, [1] high energy lepton scattering experiments have provided increasingly detailed information on the flavor, momentum, and helicity distributions of the quarks and gluons in hadrons. The results are represented in the form of leading-twist light-cone momentum and helicity distributions  $q(x, \lambda, Q)$ ,  $\bar{q}(x, \lambda, Q)$  and  $g(x, \lambda, Q)$  at the resolution of  $Q$ . However, such distributions represent single-particle probabilities and thus do not contain information on the transverse momentum, spin, and flavor correlations of the bound quarks and gluons. In particular, structure functions cannot specify the phases needed to understand QCD processes at the amplitude level, the physics which underlies form factors, exclusive and diffractive scattering processes, and the hadronic decay amplitudes of heavy hadrons. The polarized beam and polarized target experiments now in progress and planned at Jefferson Laboratory, HERMES at DESY, BNL, CERN, and SLAC, and measurements of rare exclusive channels and their polarization correlations in  $e^+e^-$  and  $\gamma\gamma$  annihilation at the high luminosity  $B$  factories promise a new level of precision in testing QCD and determining fundamental properties of hadrons. A global unified interpretation of such inclusive and exclusive experiments is a challenging theoretical problem, mixing issues involving non-perturbative and perturbative dynamics.

Ideally, one wants to have a frame-independent, quantum-mechanical description of hadrons at the amplitude level capable of encoding all possible quark and gluon momentum, helicity, and flavor correlations in the form of universal process-independent hadron wavefunctions for each particle number configuration. Remarkably, the light-cone Fock expansion allows just such a unifying representation. Moreover, the light-cone formalism provides a physical factorization scheme which conveniently separates and factorizes soft non-perturbative physics from hard perturbative dynamics in both exclusive and inclusive reactions.

Formally, the light-cone expansion is constructed by quantizing QCD at fixed light-cone time[2]  $\tau = t + z/c$  and forming the invariant light-cone Hamiltonian:  $H_{LC}^{QCD} = P^+P^- - P_\perp^2$  where  $P^\pm = P^0 \pm P^z$ . [3] The momentum generators  $P^+$  and  $P_\perp$  are kinematic; *i.e.*, they are independent of the interactions. The generator  $P^- = i\frac{d}{d\tau}$  generates light-cone time translations, and the eigen-spectrum of the Lorentz scalar  $H_{LC}^{QCD}$  gives the mass spectrum of the color-singlet hadron states in QCD together with their respective light-cone wavefunctions. For example, the proton state satisfies:  $H_{LC}^{QCD}|\psi_p\rangle = M_p^2|\psi_p\rangle$ . The expansion of the proton eigensolution  $|\psi_p\rangle$  on the color-singlet  $B = 1, Q = 1$  eigen states  $\{|n\rangle\}$  of the free Hamiltonian  $H_{LC}^{QCD}(g = 0)$  gives the light-cone Fock expansion:  $|\psi_p(P^+, P_\perp)\rangle = \sum_n \psi_n(x_i, k_{\perp i}, \lambda_i)|n; x_i P^+, x_i P_\perp + k_{\perp i}, \lambda_i\rangle$ .

The light-cone momentum fractions  $x_i = k_i^+/P^+$  with  $\sum_{i=1}^n x_i = 1$  and  $k_{\perp i}$  with  $\sum_{i=1}^n k_{\perp i} = 0_\perp$  represent the relative momentum coordinates of the QCD constituents. The physical transverse momenta are  $p_{\perp i} = x_i P_\perp + k_{\perp i}$ . The  $\lambda_i$  label the light-cone spin  $S_z$  projections of the quarks and gluons along the quantization  $z$  direction. The

physical gluon polarization vectors  $\epsilon^\mu(k, \lambda = \pm 1)$  are specified in light cone gauge  $k \cdot \epsilon = 0, \eta \cdot \epsilon = \epsilon^+ = 0$ . Light-cone quantization is most conveniently carried out in the physical ghost-free light-cone gauge  $A^+ = 0$ ; however, light-cone quantization in Feynman gauge also has a number of attractive features, including manifest covariance and a straightforward passage to the Coulomb limit in the case of heavy static quarks.[4]

The solutions of  $H_{LC}^{QCD} |\psi_p\rangle = M_p^2 |\psi_p\rangle$  are independent of  $P^+$  and  $P_\perp$ ; thus given the eigensolution Fock projections  $\langle n; x_i, k_{\perp i}, \lambda_i | p \rangle = \psi_n(x_i, k_{\perp i}, \lambda_i)$ , the wavefunction of the proton is determined in any frame.[5] In contrast, in equal-time quantization, a Lorentz boost always mixes dynamically with the interactions, so that computing a wavefunction in a new frame requires solving a nonperturbative problem as complicated as the Hamiltonian eigenvalue problem itself.

The LC wavefunctions  $\psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)$  are universal, process independent, and thus control all hadronic reactions. Given the light-cone wavefunctions, one can compute the moments of the helicity and transversity distributions measurable in polarized deep inelastic experiments. Similarly, the matrix elements of the currents as integrated squares of the LC wavefunctions. [5] For example, the polarized quark distributions at resolution  $\Lambda$  correspond to

$$q_{\lambda_q/\Lambda_p}(x, \Lambda) = \sum_{n, q_a} \int \prod_{j=1}^n dx_j d^2 k_{\perp j} \sum_{\lambda_i} |\psi_{n/H}^{(\Lambda)}(x_i, \vec{k}_{\perp i}, \lambda_i)|^2 \quad (1)$$

$$\times \delta\left(1 - \sum_i x_i\right) \delta^{(2)}\left(\sum_i \vec{k}_{\perp i}\right) \delta(x - x_q) \delta_{\lambda_a, \lambda_q} \Theta(\Lambda^2 - \mathcal{M}_n^2)$$

where the sum is over all quarks  $q_a$  which match the quantum numbers, light-cone momentum fraction  $x$ , and helicity of the struck quark. Similarly, moments of transversity distributions and other off-diagonal helicity convolutions are defined as a density matrix of the light-cone wavefunctions. The light-cone wavefunctions also specify the multi-quark and gluon correlations of the hadron. For example, the distribution of spectator particles in the final state which could be measured in the proton fragmentation region in deep inelastic scattering at an electron-proton collider are in principle encoded in the light-cone wavefunctions.

The effective lifetime of each configuration in the laboratory frame is  $2P_{\text{lab}} / (\mathcal{M}_n^2 - M_p^2)$  where  $\mathcal{M}_n^2 = \sum_{i=1}^n (k_{\perp i}^2 + m_i^2) / x_i < \Lambda^2$  is the off-shell invariant mass and  $\Lambda$  is a global ultraviolet regulator. The light-cone momentum integrals are thus limited by requiring that the invariant mass squared of the constituents of each Fock state is less than the resolution scale  $\Lambda$ . As I discuss below, this cutoff serves to define a factorization scheme for separating hard and soft regimes in both exclusive and inclusive hard scattering reactions.[5]

The ensemble  $\psi_{n/H}$  of light-cone Fock wavefunctions is a key concept for hadronic physics, providing the interpolation between physical hadrons (and also nuclei) and their fundamental quark and gluon degrees of freedom. Each Fock state interacts distinctly; *e.g.* Fock states with small particle number and small impact separation

have small color dipole moments and can traverse a nucleus with minimal interactions. This is the basis for the predictions for “color transparency”. [6]

Given the  $\psi_{n/H}^{(\Lambda)}$ , one can construct any spacelike electromagnetic or electroweak form factor or local operator product matrix element from the diagonal overlap of the LC wavefunctions.[7] Similar results hold for the matrix elements which occur in deeply virtual Compton scattering. Exclusive semi-leptonic  $B$ -decay amplitudes such as  $B \rightarrow A\ell\bar{\nu}$  can also be evaluated exactly.[8] In this case, the timelike decay matrix elements require the computation of both the diagonal matrix element  $n \rightarrow n$  where parton number is conserved and the off-diagonal  $n+1 \rightarrow n-1$  convolution such that the current operator annihilates a  $q\bar{q}$  pair in the initial  $B$  wavefunction. This term is a consequence of the fact that the time-like decay  $q^2 = (p_\ell + p_{\bar{\nu}})^2 > 0$  requires a positive light-cone momentum fraction  $q^+ > 0$ . Conversely for space-like currents, one can choose  $q^+ = 0$ , as in the Drell-Yan-West representation of the space-like electromagnetic form factors.[7] However, as can be seen from the explicit analysis of timelike form factors in a perturbative model, the off-diagonal convolution can yield a nonzero  $q^+/q^+$  limiting form as  $q^+ \rightarrow 0$ . This extra term appears specifically in the case of “bad” currents such as  $J^-$  in which the coupling to  $q\bar{q}$  fluctuations in the light-cone wavefunctions are favored. In effect, the  $q^+ \rightarrow 0$  limit generates  $\delta(x)$  contributions as residues of the  $n+1 \rightarrow n-1$  contributions. The necessity for such “zero mode”  $\delta(x)$  terms has been noted by Chang, Root and Yan [9], Burkardt [10], and Ji and Choi.[11]

The off-diagonal  $n+1 \rightarrow n-1$  contributions give a new perspective for the physics of  $B$ -decays. A semi-leptonic decay involves not only matrix elements where a quark changes flavor, but also a contribution where the leptonic pair is created from the annihilation of a  $q\bar{q}$  pair within the Fock states of the initial  $B$  wavefunction. The semi-leptonic decay thus can occur from the annihilation of a nonvalence quark-antiquark pair in the initial hadron. This feature carries over to exclusive hadronic  $B$ -decays, such as  $B^0 \rightarrow \pi^- D^+$ . In this case the pion can be produced from the coalescence of a  $d\bar{u}$  pair emerging from the initial higher particle number Fock wavefunction of the  $B$ . The  $D$  meson is then formed from the remaining quarks after the internal exchange of a  $W$  boson.

Light-cone Fock state wavefunctions thus encode all of the bound state quark and gluon properties of hadrons such as spin and flavor correlations in the form of universal process- and frame- independent amplitudes. Is there any hope of computing these wavefunctions from first principles? In the discretized light-cone quantization method (DLCQ), [12] periodic boundary conditions are introduced in  $b_\perp$  and  $x^-$  so that the momenta  $k_{\perp i} = n_\perp \pi / L_\perp$  and  $x_i^+ = n_i / K$  are discrete. A global cutoff in invariant mass of the partons in the Fock expansion is also introduced. Solving the quantum field theory then reduces to the problem of diagonalizing the finite-dimensional hermitian matrix  $H_{LC}$  on a finite discrete Fock basis. The DLCQ method has now become a standard tool for solving both the spectrum and light-cone wavefunctions of one-space one-time theories. Virtually any 1+1 quantum field theory, including “reduced

QCD” (which has both quark and gluonic degrees of freedom) can be completely solved using DLCQ.[13, 14] Hiller, McCartor, and I [15, 16] have recently shown that the use of covariant Pauli-Villars regularization with discrete light-cone quantization allows one to obtain the spectrum and light-cone wavefunctions of simplified theories in physical space-time dimensions, such as (3+1) Yukawa theory. Dalley *et al.* have also showed how one can use DLCQ with a transverse lattice to solve gluonic QCD.[17] Remarkably, the spectrum obtained for gluonium states is in remarkable agreement with lattice gauge theory results, but with a huge reduction of numerical effort. One can also formulate DLCQ so that supersymmetry is exactly preserved in the discrete approximation, thus combining the power of DLCQ with the beauty of supersymmetry.[18, 19] The “SDLCQ” method has been applied to several interesting supersymmetric theories, to the analysis of zero modes, vacuum degeneracy, massless states, mass gaps, and theories in higher dimensions, and even tests of the Maldacena conjecture.[20] Broken supersymmetry is interesting in DLCQ, since it may serve as a method for regulating non-Abelian theories. [16]

Another remarkable advantage of light-cone quantization is that the vacuum state  $|0\rangle$  of the full QCD Hamiltonian coincides with the free vacuum. For example, as discussed by Bassetto,[21] the computation of the spectrum of  $QCD(1+1)$  in equal time quantization requires constructing the full spectrum of non perturbative contributions (instantons). However, light-cone methods such as DLCQ, give the correct result immediately, without any need for vacuum related contributions.

It is also possible to model the light-cone wavefunctions. For example one can find simple forms for the three valence quark wavefunctions  $\psi_{qqq/N}^{LC}(x_i, k_{\perp i}, \lambda_i)$  satisfying  $SU(6)$  spin-flavor symmetry which can account for the “static” properties of the baryons: their magnetic moments, axial couplings  $g_A$ , and charged radii. [22, 23] Such LC models satisfy the rigorous constraint that the magnetic moment of a composite spin-half state must approach its Dirac moment  $\mu = e/2M$  in the pointlike limit  $R \rightarrow 0$  with  $M$  fixed, where  $R^2 = dF_1(q^2)/dq^2|_{q^2 \rightarrow 0}$ . In addition, the LC model predicts that the quark chirality measures  $\Delta q$ ,  $\Delta \Sigma$ , and  $g_A$  vanish in the same pointlike limit. For the physical proton, their values are approximately 0.75 of the nonrelativistic values. Physically, this reduction occurs because the quark chirality, (which can be identified with quark helicity  $\vec{S}_q \cdot \hat{p}$  in the massless limit) fluctuates strongly as the bound state becomes pointlike. Thus one cannot identify the chirality measures which appear in the Bjorken and Ellis-Jaffe-Gourdin sum rules in a relativistic theory with the spin projection of the equal-time wavefunction in the hadron rest-frame. One can also construct exact models based on the perturbative structure of the QED calculation of the anomalous moment [7] using Pauli-Villars spectra.[24] As discussed by Bo-Qiang Ma in these proceedings, the chirality sum rules are effectively measures of the light-cone spin projections, not the usual equal-time spin.[25]

## 2 Intrinsic versus Extrinsic Sea

The deep inelastic scattering data show that the nonperturbative structure of nucleons is more complex than a simple three quark bound state. For example, if the sea quarks were generated solely by perturbative QCD evolution via gluon splitting, the anti-quark distributions would be approximately isospin symmetric. However, the  $\bar{u}(x)$  and  $\bar{d}(x)$  antiquark distributions of the proton at  $Q^2 \sim 10 \text{ GeV}^2$  are found to be quite different in shape [26] and thus must reflect dynamics intrinsic to the proton's structure. Evidence for a difference between the  $\bar{s}(x)$  and  $s(x)$  distributions has also been claimed.[27] There have also been surprises associated with the chirality distributions  $\Delta q = q_{\uparrow/\uparrow} - q_{\downarrow/\uparrow}$  of the valence quarks which show that a simple valence quark approximation to nucleon spin structure functions is far from the actual dynamical situation.[28]

It is helpful to categorize the parton distributions as “intrinsic”—pertaining to the long-time scale composition of the target hadron, and “extrinsic”—reflecting the short-time substructure of the individual quarks and gluons themselves. Gluons carry a significant fraction of the proton's spin as well as its momentum. Since gluon exchange between valence quarks contributes to the  $p - \Delta$  mass splitting, it follows that the gluon distributions cannot be solely accounted for by gluon bremsstrahlung from individual quarks, the process responsible for DGLAP evolutions of the structure functions. Similarly, in the case of heavy quarks,  $s\bar{s}$ ,  $c\bar{c}$ ,  $b\bar{b}$ , the diagrams in which the sea quarks are multi-connected to the valence quarks are intrinsic to the proton structure itself.[29]

The higher Fock state of the proton  $|uud s\bar{s}\rangle$  should resemble a  $|K\Lambda\rangle$  intermediate state, since this minimizes its invariant mass  $\mathcal{M}$ . In such a state, the strange quark has a higher mean momentum fraction  $x$  than the  $\bar{s}$ . [30, 31, 32] Similarly, the helicity intrinsic strange quark in this configuration will be anti-aligned with the helicity of the nucleon.[30, 32] This  $Q \leftrightarrow \bar{Q}$  asymmetry is a striking feature of the intrinsic heavy-quark sea.

In a recent paper, Merino, Rathsman, and I have shown that the asymmetry in the fractional energy of charm versus anticharm jets produced in high energy diffractive photoproduction is sensitive to the interference of the Odderon ( $C = -$ ) and Pomeron ( $C = +$ ) exchange amplitudes in QCD. We can predict the dynamical shape of the asymmetry in a simple model and have estimated its magnitude to be of the order 15% using an Odderon coupling to the proton which saturates constraints from proton-proton vs. proton-antiproton elastic scattering. Measurements of this asymmetry at HERA could provide the first evidence for the presence of Odderon exchange in the high energy limit of strong interactions.

The main features of the heavy sea quark-pair contributions of the Fock state expansion of light hadrons can be derived from perturbative QCD, since  $\mathcal{M}_n^2$  grows with  $m_Q^2$ . One identifies two contributions to the heavy quark sea, the “extrinsic” contributions which correspond to ordinary gluon splitting, and the “intrinsic” sea which is multi-connected via gluons to the valence quarks. The intrinsic sea is thus

sensitive to the hadronic bound state structure.[29] The maximal contribution of the intrinsic heavy quark occurs at  $x_Q \simeq m_{\perp Q} / \sum_i m_{\perp}$  where  $m_{\perp} = \sqrt{m^2 + k_{\perp}^2}$ ; *i.e.* at large  $x_Q$ , since this minimizes the invariant mass  $\mathcal{M}_n^2$ . The measurements of the charm structure function by the EMC experiment are consistent with intrinsic charm at large  $x$  in the nucleon with a probability of order  $0.6 \pm 0.3\%$ . [33] Similarly, one can distinguish intrinsic gluons which are associated with multi-quark interactions and extrinsic gluon contributions associated with quark substructure. [34] One can also use this framework to isolate the physics of the anomaly contribution to the Ellis-Jaffe sum rule. [35] Thus neither gluons nor sea quarks are solely generated by DGLAP evolution, and one cannot define a resolution scale  $Q_0$  where the sea or gluon degrees of freedom can be neglected.

Light-cone wavefunctions are the natural quantities to encode hadron properties and to bridge the gap between empirical constraints and theoretical predictions for the bound state solutions. We can thus envision a program to construct the  $\Psi_n^P(x_i, k_{\perp i}, \lambda_i)$  using not only data, but theoretical constraints such as

(1) Since the state is far off shell at large invariant mass  $\mathcal{M}$ , one can derive rigorous limits on the  $x \rightarrow 1$ , high  $k_{\perp}$ , and high  $\mathcal{M}_n^2$  behavior of the wavefunctions in the perturbative domain. [5, 36]

(2) Ladder relations connecting state of different particle number follow from the QCD equation of motion and lead to Regge behavior of the quark and gluon distributions at  $x \rightarrow 0$ . QED provides a constraint at  $N_C \rightarrow 0$ . [37]

(3) One can obtain guides to the exact behavior of LC wavefunctions in QCD from analytic or DLCQ solutions to toy models such as “reduced”  $QCD(1+1)$ . [14]

(4) QCD sum rules, lattice gauge theory moments, and QCD inspired models such as the bag model, chiral theories, provide important constraints.

(5) Since the LC formalism is valid at all scales, one can utilize empirical constraints such as the measurements of magnetic moments, axial couplings, form factors, and distribution amplitudes.

(6) In the nonrelativistic limit, the light-cone and many-body Schrödinger theory formalisms must match.

### 3 The Light-Cone Factorization Scheme

Factorization theorems for hard exclusive, semi-exclusive, and diffractive processes allow a rigorous separation of soft non-perturbative dynamics of the bound state hadrons from the hard dynamics of a perturbatively-calculable quark-gluon scattering amplitude.

Roughly, the direct proofs of factorization in the light-cone scheme proceed as follows: [5] In hard inclusive reactions all intermediate states are divided according to  $\mathcal{M}_n^2 < \Lambda^2$  and  $\mathcal{M}_n^2 > \Lambda^2$  domains. The lower region is associated with the quark and gluon distributions defined from the absolute squares of the LC wavefunctions in the light cone factorization scheme. In the high invariant mass regime, intrinsic

transverse momenta can be ignored, so that the structure of the process at leading power has the form of hard scattering on collinear quark and gluon constituents, as in the parton model. The attachment of gluons from the LC wavefunction to a propagator in the hard subprocess is power-law suppressed in LC gauge, so that the minimal  $2 \rightarrow 2$  quark-gluon subprocesses dominate. The higher order loop corrections lead to the DGLAP evolution equations, as well as the higher order in  $\alpha_s$  corrections to the hard amplitude.

It is important to note that the effective starting point for the PQCD evolution of the structure functions cannot be taken as a constant  $Q_0^2$  since as  $x \rightarrow 1$  the invariant mass  $\mathcal{M}_n$  exceeds the resolution scale  $\Lambda$ . Thus in effect, evolution is quenched at  $x \rightarrow 1$ . [5, 38, 39]

One of the most interesting aspects of deep inelastic lepton-proton scattering is the contribution to the  $g_1^p$  spin-dependent structure function from photon-gluon fusion subprocesses  $\gamma^*(q)g(p) \rightarrow q\bar{q}$ . [40, 35] Naively, one would expect zero contributions from light mass  $q\bar{q}$  pairs to the first moment  $\int_0^1 dx g_1^p(x, Q^2)$  since the  $q$  and  $\bar{q}$  have opposite helicities. In fact, this is not the case if the quark mass  $m_q$  is small compared to a scale set by the spacelike gluon virtuality  $p^2$ . This is the origin of the so-called anomalous correction  $-3\frac{\alpha_s}{2\pi} \Delta g$  to the Ellis-Jaffe sum rule for isospin zero targets assuming three light flavors. Here  $\Delta g$  is the helicity carried by gluons in the hadron target,  $\Delta g(Q) = \int_0^1 dx [g_\uparrow(x, Q) - g_\downarrow(x, Q)]$ , at the factorization scale  $Q$ . If the sea quark mass is heavy compared to the gluon virtuality  $4m_q^2 \gg P^2 = -p^2$ , the photon-gluon fusion contribution to  $\int_0^1 dx g_1(x, Q^2)$  vanishes to leading order in  $\alpha_s(Q^2)$ . This result follows from a general theorem based on the Drell-Hearn-Gerasimov sum rule which states that the integral

$$\int_{\nu_\pi}^{\infty} \frac{d\nu}{\nu} \sigma_{\gamma a \rightarrow bc}(\nu) = 0(\alpha^3) ; \quad (2)$$

*i. e.*, vanishes at order  $\alpha^2$  for any  $2 \rightarrow 2$  Standard Model process. [41, 42] In the present case the gluon (for  $p^2 = 0$ ) takes the role of the target  $a$ . For large  $Q^2$ , the DHG integral evolves to the first moment of the helicity-dependent structure function  $g_1(x, Q^2)$  for any photon virtuality. Thus the fusion  $\gamma^*g \rightarrow q\bar{q}$  contribution to  $\int_0^1 dx g_1(x, Q^2)$  vanishes for small gluon virtuality  $P^2 \ll 4m_q^2$ ,  $P^2 \ll Q^2$ . This virtuality can be interpreted directly in the light-cone factorization scheme. If the off-shellness of the state is larger than the quark pair mass, one obtains the usual anomaly contribution. [40, 35] The specific contribution of a given sea quark pair  $q\bar{q}$  thus depends not only on  $Q^2$ , but more critically on the ratio of scales  $p^2/4m_q^2$ . The spectrum  $N(p^2)$  of gluon virtuality in the target nucleon in turn depends in detail on the physics of the nucleon light-cone wavefunction. Bass, Schmidt and I [35] have discussed specific forms which allow one to estimate the effect of extrinsic and intrinsic  $s$  and  $c$  quarks on the anomaly. The application of the DHG theorem to photoabsorption is more general than leading twist. [43] The fusion contribution to the DHG moment vanishes even if  $Q^2 < 4m_q^2$ , as long as the gluon virtuality can



be neglected. The result also holds for the weak as well as electromagnetic current probes.[42, 44]

In exclusive amplitudes, the LC wavefunctions are the interpolating functions between the quark and gluon states and the hadronic states. In an exclusive amplitude involving a hard scale  $Q^2$ , the intermediate states can again be divided in invariant mass domains. The high invariant mass contributions to the amplitude has the structure of a hard scattering process  $T_H$  in which the hadrons are replaced by their respective (collinear) quarks and gluons. In light-cone gauge only the minimal Fock states contribute to the leading power-law fall-off of the exclusive amplitude. The wavefunctions in the lower invariant mass domain can be integrated up to the invariant mass cutoff  $\Lambda$  and replaced by the gauge invariant distribution amplitudes,  $\phi_H(x_i, \Lambda)$ . Final-state and initial-state corrections from gluon attachments to lines connected to the color-singlet distribution amplitudes cancel at leading twist. Thus the key non-perturbative input for exclusive processes is the gauge and frame independent hadron distribution amplitude [5] defined as the integral of the valence (lowest particle number) Fock wavefunction; *e.g.* for the pion

$$\phi_\pi(x_i, \Lambda) \equiv \int d^2 k_\perp \psi_{q\bar{q}/\pi}^{(\Lambda)}(x_i, \vec{k}_\perp, \lambda) \quad (3)$$

where the global cutoff  $\Lambda$  is identified with the resolution  $Q$ . The distribution amplitude controls leading-twist exclusive amplitudes at high momentum transfer, and it can be related to the gauge-invariant Bethe-Salpeter wavefunction at equal light-cone time. The logarithmic evolution of hadron distribution amplitudes  $\phi_H(x_i, Q)$  can be derived from the perturbatively-computable tail of the valence light-cone wavefunction in the high transverse momentum regime.[5]

The features of exclusive processes to leading power in the transferred momenta are well known:

(1) The leading power fall-off is given by dimensional counting rules for the hard-scattering amplitude:  $T_H \sim 1/Q^{n-1}$ , where  $n$  is the total number of fields (quarks, leptons, or gauge fields) participating in the hard scattering.[45, 46] Thus the reaction is dominated by subprocesses and Fock states involving the minimum number of interacting fields. The hadronic amplitude follows this fall-off modulo logarithmic corrections from the running of the QCD coupling, and the evolution of the hadron distribution amplitudes. In some cases, such as large angle  $pp \rightarrow pp$  scattering, pinch contributions from multiple hard-scattering processes must also be included.[47] The general success of dimensional counting rules implies that the effective coupling  $\alpha_V(Q^*)$  controlling the gluon exchange propagators in  $T_H$  are frozen in the infrared, *i.e.*, have an infrared fixed point, since the effective momentum transfers  $Q^*$  exchanged by the gluons are often a small fraction of the overall momentum transfer.[48] The pinch contributions are then suppressed by a factor decreasing faster than a fixed power.[45]

(2) The leading power dependence is given by hard-scattering amplitudes  $T_H$  which conserve quark helicity.[49, 50] Since the convolution of  $T_H$  with the light-

cone wavefunctions projects out states with  $L_z = 0$ , the leading hadron amplitudes conserve hadron helicity; *i.e.*, the sum of initial and final hadron helicities are conserved. Hadron helicity conservation thus follows from the underlying chiral structure of QCD. For example, hadron helicity conservation predicts the suppression of vector meson states produced with  $J_z = \pm 1$  in  $e^+e^-$  annihilation to vector-pseudoscalar final states. However,  $J/\psi \rightarrow \rho\pi$  appears to occur copiously whereas  $\psi' \rightarrow \rho\pi$  has never been conserved. The PQCD analysis assumes that a heavy quarkonium state such as the  $J/\psi$  always decays to light hadrons via the annihilation of its heavy quark constituents to gluons. However, as Karliner and I [51] have shown, the transition  $J/\psi \rightarrow \rho\pi$  can also occur by the rearrangement of the  $c\bar{c}$  from the  $J/\psi$  into the  $|q\bar{q}c\bar{c}\rangle$  intrinsic charm Fock state of the  $\rho$  or  $\pi$ . On the other hand, the overlap rearrangement integral in the decay  $\psi' \rightarrow \rho\pi$  will be suppressed since the intrinsic charm Fock state radial wavefunction of the light hadrons will evidently not have nodes in its radial wavefunction. This observation can provide a natural explanation of the long-standing puzzle why the  $J/\psi$  decays prominently to two-body pseudoscalar-vector final states, whereas the  $\psi'$  does not.

I will mention here several other applications of the light-cone formalism and factorization scheme:

*Diffraction vector meson photoproduction.* The light-cone Fock wavefunction representation of hadronic amplitudes allows a simple eikonal analysis of diffractive high energy processes, such as  $\gamma^*(Q^2)p \rightarrow \rho p$ , in terms of the virtual photon and the vector meson Fock state light-cone wavefunctions convoluted with the  $gp \rightarrow gp$  near-forward matrix element.[52] One can easily show that only small transverse size  $b_\perp \sim 1/Q$  of the vector meson distribution amplitude is involved. The hadronic interactions are minimal, and thus the  $\gamma^*(Q^2)N \rightarrow \rho N$  reaction can occur coherently throughout a nuclear target in reactions without absorption or shadowing. The  $\gamma^*A \rightarrow VA$  process is thus a laboratory for testing QCD color transparency.[6]

*Regge behavior of structure functions.* The light-cone wavefunctions  $\psi_{n/H}$  of a hadron are not independent of each other, but rather are coupled via the equations of motion. The constraint of finite “mechanical” kinetic energy allows one to derive “ladder relations” which interrelate the light-cone wavefunctions of states differing by one or two gluons.[37] We can then use these relations to derive the Regge behavior of both the polarized and unpolarized structure functions at  $x \rightarrow 0$ , extending Mueller’s derivation of the BFKL hard QCD pomeron using the properties of heavy quarkonium light-cone wavefunctions at large  $N_C$  QCD.[53]

*Structure functions at large  $x_{bj}$ .* The behavior of structure functions where one quark has the entire momentum requires the knowledge of LC wavefunctions with  $x \rightarrow 1$  for the struck quark and  $x \rightarrow 0$  for the spectators. This is a highly off-shell configuration, and thus one can rigorously derive quark-counting and helicity-retention rules for the power-law behavior of the polarized and unpolarized quark and gluon distributions in the  $x \rightarrow 1$  endpoint domain. Modulo DGLAP evolution, the counting rule for finding parton  $a$  in hadron  $a$  at large  $x \sim 1$   $G_{a/A}(x, Q) \propto (1-x)^{2n_{\text{spect}}-1+2|\Delta S_z|}$  where  $n_{\text{spect}}$  is the minimum number of partons left behind

when parton  $a$  is removed from  $A$ , and  $\Delta S_z$  is the difference of the  $a$  and  $A$  helicities. This predicts  $(1-x)^3$  behavior for valence quarks aligned in helicity with the proton helicity, and  $(1-x)^3$  behavior for anti-aligned quarks. As noted above, DGLAP evolution is quenched in the large  $x$  limit in the fixed  $W^2$  domain. Burkardt, Schmidt, and I have discussed the phenomenological implications of this rule for gluon and sea distributions.

*Materialization of far-off-shell configurations.* In a high energy hadronic collisions, the highly-virtual states of a hadron can be materialized into physical hadrons simply by the soft interaction of any of the constituents.[54] Thus a proton state with intrinsic charm  $|uud\bar{c}c\rangle$  can be materialized by the interaction of a light-quark in the target, producing a  $J/\psi$  at large  $x_F$ . The production occurs on the front-surface of a target nucleus, implying an  $A^{2/3}$   $J/\psi$  production cross section at large  $x_F$ , which is consistent with experiment, such as Fermilab experiments E772 and E866.

*Comover phenomena.* Light-cone wavefunctions describe not only the partons that interact in a hard subprocess but also the associated partons freed from the projectile. The projectile partons which are comoving (*i.e.*, which have similar rapidity) with the final state quarks and gluons can interact strongly producing (a) leading particle effects, such as those seen in open charm hadroproduction; (b) suppression of quarkonium [55] in favor of open heavy hadron production, as seen in the E772 experiment; (c) changes in color configurations and selection rules in quarkonium hadroproduction, as has been emphasized by Hoyer and Peigne.[56] Further, more than one parton from the projectile can enter the hard subprocess, producing dynamical higher twist contributions, as seen for example in Drell-Yan experiments.[57, 58]

*Jet hadronization in light-cone QCD.* One of the goals of nonperturbative analysis in QCD is to compute jet hadronization from first principles. The DLCQ solutions provide a possible method to accomplish this. By inverting the DLCQ solutions, we can write the “bare” quark state of the free theory as  $|q_0\rangle = \sum |n\rangle \langle n|q_0\rangle$  where now  $\{|n\rangle\}$  are the exact DLCQ eigen states of  $H_{LC}$ , and  $\langle n|q_0\rangle$  are the DLCQ projections of the eigen-solutions. The expansion is automatically infrared and ultraviolet regulated if we impose global cutoffs on the DLCQ basis:  $\lambda^2 < \Delta\mathcal{M}_n^2 < \Lambda^2$  where  $\Delta\mathcal{M}_n^2 = \mathcal{M}_n^2 - (\Sigma\mathcal{M}_i)^2$ . It would be interesting to study jet hadronization at the amplitude level for the existing DLCQ solutions to QCD (1+1) and collinear QCD.

*Hidden Color.* The deuteron form factor at high  $Q^2$  is sensitive to wavefunction configurations where all six quarks overlap within an impact separation  $b_{\perp i} < \mathcal{O}(1/Q)$ ; the leading power-law fall off predicted by QCD is  $F_d(Q^2) = f(\alpha_s(Q^2))/(Q^2)^5$ , where, asymptotically,  $f(\alpha_s(Q^2)) \propto \alpha_s(Q^2)^{5+2\gamma}$ . [59] The derivation of the evolution equation for the deuteron distribution amplitude and its leading anomalous dimension  $\gamma$  is given by Ji, Lepage, and myself. [60] In general, the six-quark wavefunction of a deuteron is a mixture of five different color-singlet states. The dominant color configuration at large distances corresponds to the usual proton-neutron bound state. However at small impact space separation, all five Fock color-singlet components eventually acquire equal weight, *i.e.*, the deuteron wavefunction evolves to 80% “hidden color.” The relatively large normalization of the deuteron form factor observed

at large  $Q^2$  points to sizable hidden color contributions.[61] Hidden color components can play a predominant role in the reaction  $\gamma d \rightarrow J/\psi pn$  at threshold if it is dominated by the multi-fusion process  $\gamma gg \rightarrow J/\psi$ .

*Spin-Spin Correlations and the Charm Threshold.* One of the most striking anomalies in elastic proton-proton scattering is the large spin correlation  $A_{NN}$  observed at large angles.[62] At  $\sqrt{s} \simeq 5$  GeV, the rate for scattering with incident proton spins parallel and normal to the scattering plane is four times larger than that for scattering with anti-parallel polarization. This strong polarization correlation can be attributed to the onset of charm production in the intermediate state at this energy.[63, 64] A resonant intermediate state  $|uuduudc\bar{c}\rangle$  has odd intrinsic parity and can thus couple to the  $J = L = S = 1$  initial state, thus strongly enhancing scattering when the incident projectile and target protons have their spins parallel and normal to the scattering plane. The charm threshold can also explain the anomalous change in color transparency observed at the same energy in quasi-elastic  $pp$  scattering. A crucial test is the observation of open charm production near threshold with a cross section of order of  $1\mu b$ . Analogous strong spin effects should also appear at the strangeness threshold and in exclusive photon-proton reactions such as large angle Compton scattering and pion photoproduction near the strangeness and charm thresholds.

## 4 Self-Resolved Diffractive Reactions and Light Cone Wavefunctions

Diffractive multi-jet production in heavy nuclei provides a novel way to measure the shape of the LC Fock state wavefunctions and test color transparency. For example, consider the reaction [65, 66, 67]  $\pi A \rightarrow \text{Jet}_1 + \text{Jet}_2 + A'$  at high energy where the nucleus  $A'$  is left intact in its ground state. The transverse momenta of the jets have to balance so that  $\vec{k}_{\perp 1} + \vec{k}_{\perp 2} = \vec{q}_{\perp} < R_A^{-1}$ , and the light-cone longitudinal momentum fractions have to add to  $x_1 + x_2 \sim 1$  so that  $\Delta p_L < R_A^{-1}$ . The process can then occur coherently in the nucleus. Because of color transparency, *i.e.*, the cancelation of color interactions in a small-size color-singlet hadron, the valence wavefunction of the pion with small impact separation, will penetrate the nucleus with minimal interactions, diffracting into jet pairs.[65] The  $x_1 = x$ ,  $x_2 = 1 - x$  dependence of the di-jet distributions will thus reflect the shape of the pion distribution amplitude; the  $\vec{k}_{\perp 1} - \vec{k}_{\perp 2}$  relative transverse momenta of the jets also gives key information on the underlying shape of the valence pion wavefunction.[66, 67] The QCD analysis can be confirmed by the observation that the diffractive nuclear amplitude extrapolated to  $t = 0$  is linear in nuclear number  $A$ , as predicted by QCD color transparency. The integrated diffractive rate should scale as  $A^2/R_A^2 \sim A^{4/3}$ . A diffractive dissociation experiment of this type, E791, is now in progress at Fermilab using 500 GeV incident pions on nuclear targets.[68] The preliminary results from E791 appear to be consistent with color transparency. The momentum fraction distribution of the jets

is consistent with a valence light-cone wavefunction of the pion consistent with the shape of the asymptotic distribution amplitude,  $\phi_\pi^{\text{asympt}}(x) = \sqrt{3}f_\pi x(1-x)$ . Data from CLEO [69] for the  $\gamma\gamma^* \rightarrow \pi^0$  transition form factor also favor a form for the pion distribution amplitude close to the asymptotic solution [5] to the perturbative QCD evolution equation.[70, 71, 48, 72, 73] It will also be interesting to study diffractive tri-jet production using proton beams  $pA \rightarrow \text{Jet}_1 + \text{Jet}_2 + \text{Jet}_3 + A'$  to determine the fundamental shape of the 3-quark structure of the valence light-cone wavefunction of the nucleon at small transverse separation.[66] One interesting possibility is that the distribution amplitude of the  $\Delta(1232)$  for  $J_z = 1/2, 3/2$  is close to the asymptotic form  $x_1x_2x_3$ , but that the proton distribution amplitude is more complex. This would explain why the  $p \rightarrow \Delta$  transition form factor appears to fall faster at large  $Q^2$  than the elastic  $p \rightarrow p$  and the other  $p \rightarrow N^*$  transition form factors.[74] Conversely, one can use incident real and virtual photons:  $\gamma^*A \rightarrow \text{Jet}_1 + \text{Jet}_2 + A'$  to confirm the shape of the calculable light-cone wavefunction for transversely-polarized and longitudinally-polarized virtual photons. Such experiments will open up a direct window on the amplitude structure of hadrons at short distances.

The diffractive dissociation of a hadron or nucleus can also occur via the Coulomb dissociation of a beam particle on an electron beam (*e.g.* at HERA or eRHIC) or on the strong Coulomb field of a heavy nucleus (*e.g.* at RHIC or nuclear collisions at the LHC).[75] The amplitude for Coulomb exchange at small momentum transfer is proportional to the first derivative  $\sum_i e_i \frac{\partial}{\partial k_{Ti}} \psi$  of the light-cone wavefunction, summed over the charged constituents. The Coulomb exchange reactions fall off less fast at high transverse momentum compared to pomeron exchange reactions since the light-cone wavefunction is effectively differentiated twice in two-gluon exchange reactions.

For example, consider the Coulomb dissociation of a high energy proton at HERA. The proton can dissociate into three jets corresponding to the three-quark structure of the valence light-cone wavefunction. We can demand that the produced hadrons all fall outside of an “exclusion cone” of opening angle  $\theta$  in the proton’s fragmentation region. Effectively all of the light-cone momentum  $\sum_j x_j \simeq 1$  of the proton’s fragments will thus be produced outside the exclusion cone. This requirement then limits the invariant mass of the Fock state  $\mathcal{M}_n^2 > \Lambda^2 = P^{+2} \sin^2 \theta / 4$  from below, so that perturbative QCD counting rules can predict the fall-off in the jet system invariant mass  $\mathcal{M}$ . At large invariant mass one expects the three-quark valence Fock state of the proton to dominate. The segmentation of the forward detector in azimuthal angle  $\phi$  can be used to identify structure and correlations associated with the three-quark light-cone wavefunction. A further discussion is in progress. [75]

The light-cone formalism is also applicable to the description of nuclei in terms of their nucleonic and mesonic degrees of freedom.[76] Self-resolving diffractive jet reactions in high energy electron-nucleus collisions and hadron-nucleus collisions at moderate momentum transfers can thus be used to resolve the light-cone wavefunctions of nuclei.

## 5 Semi-Exclusive Processes: New Probes of Hadron Structure

A new class of hard “semi-exclusive” processes of the form  $A + B \rightarrow C + Y$ , have been proposed as new probes of QCD.[77, 78, 79] These processes are characterized by a large momentum transfer  $t = (p_A - p_C)^2$  and a large rapidity gap between the final state particle  $C$  and the inclusive system  $Y$ . Here  $A, B$  and  $C$  can be hadrons or (real or virtual) photons. The cross sections for such processes factorize in terms of the distribution amplitudes of  $A$  and  $C$  and the parton distributions in the target  $B$ . Because of this factorization, semi-exclusive reactions provide a novel array of generalized currents, [79] which not only give insight into the dynamics of hard scattering QCD processes, but also allow experimental access to new combinations of the universal quark and gluon distributions.

## 6 Summary

In this talk I have discussed how universal, process-independent and frame-independent light-cone Fock-state wavefunctions can be used to encode the properties of a hadron in terms of its fundamental quark and gluon degrees of freedom. Given the proton’s light-cone wavefunctions, one can compute not only the moments of the quark and gluon distributions measured in deep inelastic lepton-proton scattering, but also the multi-parton correlations which control the distribution of particles in the proton fragmentation region and dynamical higher twist effects. Light-cone wavefunctions also provide a systematic framework for evaluating exclusive hadronic matrix elements, including time-like heavy hadron decay amplitudes and form factors. The formalism also provides a physical factorization scheme for separating hard and soft contributions in both exclusive and inclusive hard processes. A new type of jet production reaction, “self-resolving diffractive interactions” can provide direct information on the light-cone wavefunctions of hadrons in terms of their QCD degrees of freedom, as well as the composition of nuclei in terms of their nucleon and mesonic degrees of freedom. Progress in QCD is driven by experiment, and we are fortunate that there are new experimental facilities such as Jefferson laboratory, new studies of exclusive processes  $e^+e^-$  and  $\gamma\gamma$  processes at the high luminosity  $B$  factories, as well as the new accelerators and colliders now being planned to further advance the study of QCD phenomena.

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