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Experimental Signatures of Split Fermions in Extra Dimensions

Yuval Grossman

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Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

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Yuval Grossman†
Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309

The smallness and hierarchy of the fermion parameters could be explained in theories with extra dimensions where doublets and singlets are localized at slightly separated points. Scattering cross sections for collisions of such fermions vanish exponentially at energies high enough to probe the separation distance. This is because the separation puts a lower bound on the attainable impact parameter in the collision. The NLC, and in particular the combination of the e^+e^- and e^-e^- modes, can probe this scenario, even if the inverse fermion separation is of order tens of TeVs.

1. Introduction

The smallness and hierarchy of the fermion masses and mixing angles are the most puzzling features of fermion parameters. They suggest that there exist a more fundamental theory that generate these properties in a natural way. Traditionally, (spontaneously broken) flavor symmetries were assumed. Recently, with the developments in constructing models based on compact "large" dimensions, new solutions which exploits the new space in the extra dimensions has been proposed.[2]

One particularly interesting framework is due to Arkani-Hamed and Schmaltz (AS).[3] The idea is to separate the various fermion fields in the extra dimension. Consider for example a model where the SM gauge and Higgs fields live in the bulk of one extra compact dimension of radius of order TeV^{-1} while the SM fermions are localized at different positions with narrow wavefunctions in the extra dimension. (The gauge fields may also be confined to a brane of thickness of about an inverse TeV in much larger extra dimensions. Then the fermions would be stuck to thin parallel "layers" within the brane.) This separation of the fermion fields suppresses the Yukawa couplings. The reason is that the Yukawa couplings are proportional to the direct couplings between the two fermion fields (e.g., e_L and e_R for the electron Yukawa coupling). When fermion fields are separated the direct couplings between them is exponentially suppressed by the overlap of their wavefunctions. Actually, any interaction in this setup is proportional to the overlap of the wavefunctions of the fields involved. For example, higher dimensional operators such as $QQQL$ which lead to proton decay can be suppressed to safety by separating the quarks and lepton fields.

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†Based on work with N. Arkani-Hamed and M. Schmaltz.[1]

In this talk we present model independent experimental signature of this scenario which follows simply from locality in the extra dimensions: At energies above a TeV, the large angle scattering cross section for fermions which are separated in the extra dimensions falls off exponentially with energy. This is understood from the fact that the fermion separation in the extra dimension implies a minimum impact parameter of order TeV^{-1}. At energies corresponding to shorter distances the large angle cross section falls off exponentially because the particles "miss" each other. The amplitude involves a Yukawa propagator for the exchanged gauge boson where the four dimensional momentum transfer acts as the mass in the exponential. More precisely, any t and u channel scattering of split fermions has an exponential suppression. However, s channel exchange is time-like, and therefore the fermion separation in space does not force an exponential suppression. Nevertheless, s channel processes also lead to interesting signatures as the interference of the SM amplitude with Kaluza Klein (KK) exchange diagrams depends on the fermion separation.

2. The AS Framework

In this section we briefly describe the AS framework.[3] Our starting point is the observation that simple compactifications of higher dimensional theories typically do not lead to chiral fermions. The known mechanisms which do lead to chiral spectra usually break translation invariance in the extra dimensions and the chiral fermions are localized at special points in the compact space. Examples include twisted sector fermions stuck at orbifold fixed points in string theory, chiral states from intersecting D-branes, or zero modes trapped to defects in field theory. Given that fermions generically are localized at special points in the extra dimensions we are motivated to consider the possibility of having different locations for the different SM fermions. In such a scenario locality in the higher dimensions forbids direct couplings between fermions which live at different places.

To be specific we concentrate on the field theoretical model of AS. Consider a theory with one infinite spatial dimension. We introduce a scalar field (Φ) where we assume its expectation value to have the shape of a domain wall transverse to the extra dimension centered at $x_5 = 0$. Moreover, we use the linear approximation for the vev in the vicinity of zero $\langle\Phi(x_5)\rangle = 2\mu^2 x_5$. The action for a five dimensional fermion Ψ coupled to the background scalar is then

$$S = \int \mathrm{d}^4\mathbf{x}\ \mathrm{d}x_5\, \overline{\Psi}[i\partial\!\!\!/_4 + i\gamma^5\partial_5 + \Phi(x_5)]\Psi\ . \tag{1}$$

Performing the KK reduction one find the left handed fermion zero mode wavefunction

$$\psi_L(x_5) = N e^{-\mu^2 {x_5}^2}\,, \qquad N^2 = \frac{\mu}{\sqrt{\pi/2)}}\,. \tag{2}$$

One also finds a right handed fermion zero mode, but for an infinite x_5 this mode cannot be normalizable.

We can easily generalize Eq. (1) to the case of several fermion fields. We simply couple all 5-d Dirac fields to the same scalar Φ

$$S = \int \mathrm{d}^5 x \sum_{i,j} \bar{\Psi}_i [i \not{\partial}_5 + \lambda \Phi(x_5) - m]_{ij} \Psi_j \,. \tag{3}$$

Here we allowed for general Yukawa couplings λ_{ij} and also included masses m_{ij} for the fermion fields. Mass terms for the five-dimensional fields are allowed by all the symmetries and should therefore be present in the Lagrangian. In the case that we will eventually be interested in – the standard model – the fermions carry gauge charges. This forces the couplings λ_{ij} and m_{ij} to be block-diagonal, with mixing only between fields with identical gauge quantum numbers.

When we set $\lambda_{ij} = \delta_{ij}$, m_{ij} can be diagonalized with eigenvalues m_i, and the resulting wave functions are Gaussians with offset centers

$$\psi_L^i(x_5) = N e^{-\mu^2 (x_5 - x_5^i)^2} \,, \qquad x_5^i = \frac{m_i}{2\mu^2} \,. \tag{4}$$

When λ_{ij} is not proportional to the unit matrix, m_{ij} and λ_{ij} cannot be diagonalized simultaneously. Nevertheless, even in that case the picture is not dramatically altered. While the wave functions are no longer Gaussians, they keep the important property to our purpose, namely, they are still narrowly peaked at different points. Actually, the wave functions can be approximated by Gaussians with slowly varying x_5^i. Near the peaks they resemble Gaussians with $x_5^i = m_i/2\mu^2$ where m_i are the eigenvalues of m_{ij}. At the tails, they look like Gaussians with $x_5^i = m_{ii}/2\mu^2$ where m_{ii} are the diagonal values of m_{ij} in the basis where λ_{ij} is diagonal.

Eventually, we need to be working with a finite x_5. While most of the above results hold also for finite volume, there are some problems. First, in the simplest compactification scenarios an anti-domain wall has to be introduced. This is a problem since the domain wall and the anti-domain wall will prefer to annihilate each other. Namely, the ground state is flat. Without getting into details, we only mention that more complicated models can be constructed where anti-domain wall does not have to be introduced,[5] or when the domain wall/anti-domain wall configuration is stable.[4] The second problem is that for finite volume the right handed zero modes are normalizable and they are localized at the anti-domain wall or at the boundaries of the extra dimension. Clearly, since we did not observe them, such "mirror" fermions have to be much heavier then the SM quarks and leptons. Moreover, the electroweak precision measurements,[6] and in particular the bound from the S parameter, disfavor mirror families. This constrain can be avoided if there are extra, negative contribution to the S parameter. While the exact new contribution form the KK modes in the AS model has not been calculated, some of them do have a negative contribution.[3]

Now we are in the position to calculate the Yukawa coupling. We demonstrate it for the electron case. In the five dimensional theory the Yukawa interaction is giving by

$$S_Y = \int d^5 x \kappa H \bar{L} E \,. \tag{5}$$

We assume that the relevant massless zero mode from L is localized at $x_5 = 0$ and that from E at $x_5 = r$. For simplicity, we will assume that the Higgs is delocalized inside the wall.

We now determine what effective four-dimensional interactions between the light fields results from the Yukawa coupling in eq. (5). To this end we replace L, E and the Higgs field H by their lowest Kaluza-Klein modes. We obtain for the Yukawa coupling

$$S_Y = \int \mathrm{d}^4\mathbf{x}\ \kappa\, h(\mathbf{x}) l(\mathbf{x}) e(\mathbf{x}) \ \int \mathrm{d}x_5\ \psi_l(x_5)\ \psi_e(x_5)\ . \tag{6}$$

The zero-mode wave functions for the lepton doublet and singlet, $\psi_l(x_5)$ and $\psi_e(x_5)$, are Gaussian centered at $x_5 = 0$ and $x_5 = r$ respectively. Overlap of Gaussians is itself a Gaussian and we find

$$\int \mathrm{d}x_5\ \psi_l(x_5)\ \psi_e(x_5) = \frac{\sqrt{2}\mu}{\sqrt{\pi}} \int \mathrm{d}x_5\ e^{-\mu^2 x_5^2} e^{-\mu^2 (x_5 - r)^2} = e^{-\mu^2 r^2/2}\ . \tag{7}$$

We found that the Yukawa coupling is highly suppressed once r is not much smaller than μ^{-1}. We finally note that a concrete model that reproduced the observed quark and lepton parameters has been worked out in.[7]

3. Scattering of fermions localized at different places

We now discuss scattering of fermions localized at different places at the extra dimension.[1] Let us imagine colliding fermions which are localized at two different places in a circular extra dimension of radius R. For concreteness we consider the scattering of right handed electrons on left handed electrons. In the context of our model there are three potentially relevant mass scales for this collision: the momentum transfer of the t-channel scattering $\sqrt{-t}$, the inverse of the quark-lepton separation d^{-1} which we take to be of order of the inverse thickness R^{-1} of the extra dimension, and the inverse width of the fermion wave functions σ^{-1}. However, we will approximate the fermion wave functions by delta functions for the calculation. The corrections which arise from the finite width of the wave functions were calculated and found to be negligible for practical purposes.[1]

To calculate the scattering through intermediate bulk gauge fields we need the five-dimensional propagator. In momentum space it is $(t - p_5^2 - m^2)^{-1}$ where we separated out the five dimensional momentum transfer p_5. As we are interested in propagation between definite positions in the fifth dimension it is convenient to Fourier transform in the fifth coordinate

$$P_d(t) = \sum_{n=-\infty}^{\infty} \frac{e^{ind/R}}{t - (n/R)^2 - m^2}\ , \tag{8}$$

where $d = x_L - x_R$ and $x_L(x_R)$ is the location of $e_L(e_R)$ in the extra dimension. The Fourier transform is a sum and not an integral since momenta in the fifth coordinate are quantized in units of $1/R$. This propagator can also be understood in the four dimensional (4d) language as arising from exchange of the 4d gauge boson and its infinite tower of KK excitations.[1] This propagator can be simplified by performing the sum. We find[1]

$$P_d(t) = -\frac{\pi R}{\sqrt{-t+m^2}}\,\frac{\cosh[(d-\pi R)\sqrt{-t+m^2}]}{\sinh[\pi R\sqrt{-t+m^2}]}\,. \tag{9}$$

The Feynman rules for diagrams involving exchange of bulk gauge fields are now identical to the usual four dimensional SM Feynman rules except for the replacement of 4d gauge boson propagators by the corresponding 5d propagators. Before we proceed with calculating cross sections we note a few properties of the above propagator.

It is easy to understand the two limits $\sqrt{-t} \gg R^{-1}$ and $\sqrt{-t} \ll R^{-1}$. In the former case we obtain

$$P_d(t) \simeq -\frac{\pi R}{\sqrt{-t}}\; e^{-\sqrt{-t}\,d}\,, \tag{10}$$

which vanishes exponentially with the momentum transfer in the process as we anticipated from five dimensional locality. In the limit of small momentum transfer we obtain

$$P_d(t) \simeq \frac{1}{t-m^2} - R^2\left(\frac{d^2}{2R^2} - \frac{d\pi}{R} + \frac{\pi^2}{3}\right)\,, \tag{11}$$

which is the four dimensional t-channel propagator plus a correction term whose sign and magnitude depends on the fermion separation. For small separation $d < \pi R\,(1-1/\sqrt{3})$ the correction enhances the magnitude of the amplitude, while for larger separation it reduces it.

It is also instructive to expand the propagator in exponentials (ignoring the mass m)

$$P_d(t) = -\frac{\pi R}{\sqrt{-t}}\left(e^{-\sqrt{-t}d} + e^{\sqrt{-t}(d-2\pi R)}\right)\left(1 + e^{-\sqrt{-t}2\pi R} + e^{-\sqrt{-t}4\pi R} + \ldots\right)\,, \tag{12}$$

which can be understood as a sum of contributions from five dimensional propagators. The two terms in the first parenthesis correspond to propagation from x_L to x_R in clockwise and counter–clockwise directions, and the series in the other parenthesis adds the possibility of also propagating an arbitrary number of times around the circle.

The expression for the u-channel KK-tower propagator $P_d(u)$ is identical to eq. (9) with the obvious replacement $t \to u$, and $P_d(s)$ is obtained by analytic continuation

$$P_d(s) = \frac{\pi R}{\sqrt{s-m^2}}\frac{\cos[(d-\pi R)\sqrt{s-m^2}]}{\sin[R\pi\sqrt{s-m^2}]}\;. \tag{13}$$

The poles at $\sqrt{s-m^2} = n/R$ are not physical and can be avoided by including a finite width.

Armed with this propagator it is easy to evaluate any KK boson exchange diagram in terms of its SM counterpart. For example, a pure t channel exchange diagram becomes

$$\mathcal{M} = (t-m^2)P_d(t) \times \mathcal{M}\big|_{SM}\;, \tag{14}$$

where $\mathcal{M}\big|_{SM}$ is the SM amplitude and the factor $(t-m^2)P_d(t)$ replaces the SM gauge boson propagator $1/(t-m^2)$ by the 5d propagator $P_d(t)$.

4. Collider signatures

Having calculated the 5d propagator, the calculation of differential cross sections is a simple generalization of SM results.[8] We start by considering the predictions of our model for high energy e^+e^- or $\mu^+\mu^-$ machines. We assume that the separation of left and right handed fields is responsible for at least part of the suppression of the muon and electron Yukawa couplings. Therefore, we consider a case where the doublet and singlet components of the charged leptons are split by a distance d in the extra dimensions. We start by looking into a pure t channel exchange which is (in principle) possible at a lepton collider with polarizable beams $l_L^+ l_R^- \longrightarrow l_L^+ l_R^-$. To compute the differential cross section we sum over contributions from neutral current exchange (photon and Z plus KK towers). In the formulae in this section we neglect m_Z. It is easy to reintroduce it, and in our numerical plots we keep it. Happily, each term in the sum is simply equal to the SM term times $tP_d(t)$ which can be factored so that our final expression for the differential cross section becomes

$$r_\sigma^t \equiv \frac{d\sigma/dt}{d\sigma/dt\,|_{\mathrm{SM}}} = \left|t\, P_d(t)\right|^2 \,, \tag{15}$$

where $P_d(t)$ is given in eq. (9). The effect of the KK tower would be seen as a dramatic reduction of the cross section at large $|t|$. To illustrate this point in Fig. 1 we plot the ratio r_σ^t of eq. (15) as a function of t for $R = 1$ TeV^{-1} and representative values of d.

Experimental Signatures of Split Fermions in Extra Dimensions We can get more information on the values on d and R by combining the above with the processes $e_N^+ e_N^- \to \mu_N^+ \mu_N^-$ ($N = L$ or R). (The same considerations also apply to scattering into quark pairs, but this case is more difficult to study experimentally.) This process is a pure s channel between unseparated fermions so that

$$r_\sigma^{sN} \equiv \frac{d\sigma/dt}{d\sigma/dt\,|_{\mathrm{SM}}} = \left|s\, P_0(s)\right|^2 \,. \tag{16}$$

For $\sqrt{s}$ small compared to the inverse size of the extra dimension the cross section is reduced independently of d. An extra dimensional theory without fermion separation predicts $r_\sigma^{sN} < 1$ and $r_\sigma^t > 1$. Thus, a measurement of $r_\sigma^{sN} < 1$ together with $r_\sigma^t < 1$ would be evidence for fermion separation in the extra dimension.

Another interesting probe of d using s channel has been suggested by Rizzo.[9] Suppose that the first KK mode has been produced and its mass $1/R$ measured. The case of $d = 0$ can be distinguished from $d \neq 0$ by looking at the cross-section at lower energies. In particular, for $d = 0$, the first KK exchange exactly cancels the SM amplitude at $\sqrt{s} = 1/(\sqrt{2}R)$, whereas for $d \neq 0$ the cross-section can still be large. Therefore, a beam scan at energies beneath the first resonance can be an efficient probe of d.

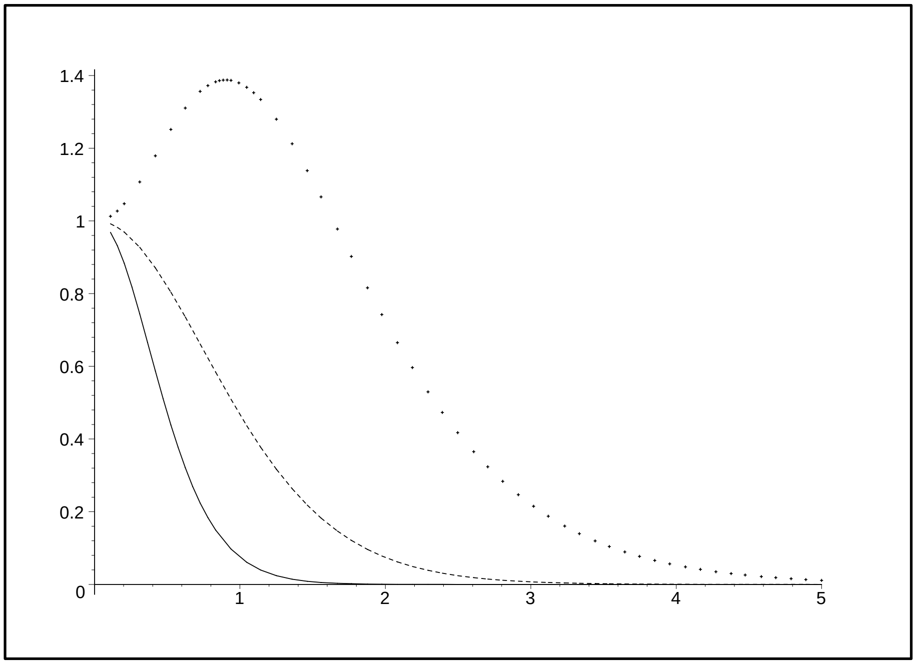

Figure 1: r_σ^t (the cross section for t channel exchange in the 5d theory normalized by the corresponding SM cross section) as a function of $\sqrt{-t}$ in units of TeV. We assume $R^{-1} = 1\,\mathrm{TeV}$. The dotted, dashed and solid curves are for separations of $d/R = 1$, $\pi/2$ and π respectively.

Even if beam polarization is not available, one can still probe the nature of the extra dimensions by looking at several processes and using angular information. Consider an unpolarized $e^+e^- \to \ell^+\ell^-$ scattering. (The same holds for incoming muons.) We get the tree level cross section

$$\frac{d\sigma}{dt} = \frac{\pi\alpha^2}{s^2}\left[\left(1+\frac{1}{16\sin^4\theta_w}\right)\frac{u^2(P_0(s)+P_0(t))^2}{\cos^4\theta_w} + \frac{t^2P_d^2(s)+s^2P_d^2(t)}{2\cos^4\theta_w}\right]. \quad (17)$$

When $\ell = e$ both s and t channels are possible, while for $\ell \neq e$ only the s channel is present, and in the above formula one should set $P_d(t) = P_0(t) = 0$. We also define, as before, the ratio of the 5d cross section to the SM one as r_σ^s (r_σ^{st}) for the $e^+e^- \to \mu^+\mu^-$ ($e^+e^- \to e^+e^-$) reaction. In Fig. 2 we presented r_σ^{st} as a function of the scattering angle. As we can see, the cross sections depend in a non trivial way on the separation. This is because the helicity changing amplitude depends on d, while the helicity conserving one does not. By looking at angular distributions, one can separate the different contributions, and extract both R and d.

Another interesting collider mode which allows a very clean measurement of fermion separations is e^-e^- scattering. The advantage of this mode is that both

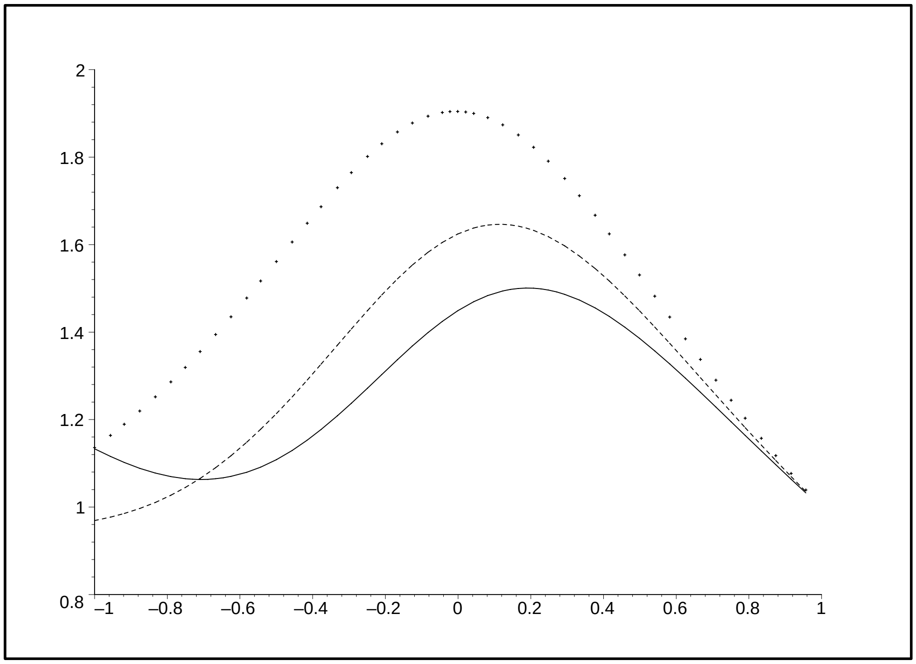

Figure 2: r_σ^s (the cross section for $e^+e^- \to e^+e^-$, in the 5d theory normalized by the corresponding SM cross section) as a function of the scattering angle, $\cos\theta$. We assume $R^{-1} = 4\,\mathrm{TeV}$ and $\sqrt{s} = 1.5\,\mathrm{TeV}$. The dotted, dashed and solid curves are for separation of $d/R = 0$, 1 and π respectively.

beams can be polarized to a high degree which allows for a clean separation of the interesting t and u channels from s channel. We find for $e_L^- e_R^-$ scattering to e^-e^- (summed over final polarizations)

$$r_\sigma^{+-} \equiv \frac{d\sigma/dt}{d\sigma/dt\,|_{\mathrm{SM}}} = \frac{u^2|P_d(t)|^2 + t^2|P_d(u)|^2}{u^2/t^2 + t^2/u^2}\,. \tag{18}$$

Higher sensitivity to d can be achieved by changing the electron polarization. Consider e^-e^- scattering where one electron if left handed and the other has polarization p which can vary between -1 and 1. We find

$$r_\sigma^{p-} - 1 \approx 2R^2(|t| + |u|)\left[\frac{1+p}{2}\left(\frac{d^2}{R^2} - \frac{2\pi d}{R}\right) + \frac{2\pi^2}{3}\right] \tag{19}$$

Varying p one could, in principle, determined both d and R.

While an exponential suppression of the cross section would be an unambiguous signal of fermion separation in the extra dimension, we can still probe d if a small deviation of r_σ from unity is found. The sensitivity can estimated from eq. (11). Assuming maximum separation, $d = \pi R$, there is a reduction in the cross-section

($r^t_\sigma < 1$), and we obtain a sensitivity

$$R \leq \sqrt{\frac{3\,\Delta r^a_\sigma}{\pi^2 Q^2}}\,, \tag{20}$$

where Δr^a_σ is the combined theoretical and experimental error on r^a_σ. For $d = 0$ one should find $r^t_\sigma > 1$ with a factor of $\sqrt{2}$ higher sensitivity. Assuming $\Delta r_\sigma \approx 1\%$ and using eq. (20) we conclude that we get sensitivity down to $R \approx (27\,\mathrm{TeV})^{-1}$ at a 1.5 TeV linear collider.

5. Conclusion

Fermion separation in extra dimension is a useful model building tool. It can explain the smallness and hierarchy of the Yukawa couplings in the SM and suppress proton decay in models with low fundamental scale. A model independent prediction of this framework is that the space-like exchange amplitudes between split fermions falls off exponentially. If the inverse size of the extra dimension is not much larger then 10 TeV, we can see this fall off. The NLC, and in particular the combination of the e^+e^- and e^-e^- options, is very promising in this respect.

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