Bulk Gauge Fields in the Randall-Sundrum Model [*]

H. Davoudiasl, J.L. Hewett and T.G. Rizzo

Stanford Linear Accelerator Center

Stanford CA 94309, USA

Abstract

We explore the consequences of placing the Standard Model gauge fields in the bulk of the recently proposed localized gravity model of Randall and Sundrum. We find that the Kaluza Klein excitations of these fields are necessarily strongly coupled and we demonstrate that current precision electroweak data constrain the lowest states to lie above $\simeq 23$ TeV. Taking the weak scale to be ~ 1 TeV, the resulting implications on the model parameters force the bulk curvature to be larger than the higher dimensional Planck scale, violating the consistency of the theory. Hence we conclude that it is disfavored to place the Standard Model gauge fields in the bulk of this model as it is presently formulated.

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1 Introduction

The possibility of extra space-like dimensions with accessible physics near the TeV scale[1] has opened a new avenue for explaining the gauge hierarchy. The models which address the hierarchy make use of our ignorance about gravity, in particular, the fact that gravity has yet to be probed at energy scales much above 10^{-3} eV in laboratory experiments. The prototype scenario in this class of theories is due to Arkani-Hamed, Dimopoulos and Dvali[2] who use the volume associated with large extra dimensions, which may be as sizable as a fraction of a millimeter, to bring the D-dimensional Planck scale down to a few TeV. Here, the gauge hierarchy problem is recast into the issue of stabilizing the rather large ratio between the TeV Planck scale and the compactification scale of the extra dimensions. Nonetheless, the phenomenological[3] and astrophysical[4] implications of this model have been examined by a large number of authors.

More recently, Randall and Sundrum(RS)[5] have proposed an alternative scenario wherein the hierarchy is generated by an exponential function of the compactification radius, called a warp factor. Unlike the model of Arkani-Hamed *et al.*, they assume a 5-dimensional non-factorizable geometry, based on a slice of AdS_5 spacetime. Two 3-branes, one being 'visible' with the other being 'hidden', with opposite tensions rigidly reside at S_1/Z_2 orbifold fixed points, taken to be $\phi = 0, \pi$, where ϕ is the angular coordinate parameterizing the extra dimension. It is assumed that the extra-dimensional bulk is only populated by gravity, and that the SM lies on the brane with negative tension at $\phi = \pi$. Gravity is localized on the Planck brane at $\phi = 0$. The solution to Einstein's equations for this configuration, maintaining 4-dimensional Poincare invariance, is given by the 5-dimensional metric

$$ds^2 = e^{-2\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2 \,, \tag{1}$$

where the Greek indices run over ordinary 4-dimensional spacetime, $\sigma(\phi) = kr_c|\phi|$ with r_c

being the compactification radius of the extra dimension, and $0 \leq |\phi| \leq \pi$. Here k is a scale of order the Planck scale and relates the 5-dimensional Planck scale M to the bulk cosmological constant. Similar configurations have also been found to arise in M/string-theory[6]. All calculations are performed with the assumption $k < M$ with $M \sim \overline{M}_{Pl}$ (where $\overline{M}_{Pl} \simeq 2.44 \times 10^{18}$ GeV is the reduced Planck mass) so that the 5-dimensional curvature is small compared to M and that this solution for the bulk metric can be trusted[5]. If $k > M$ then higher order terms in the curvature would need to be kept in the initial action to maintain self-consistency.

Examination of the action in the 4-dimensional effective theory in the RS scenario yields[5]

$$\overline{M}_{Pl}^2 = \frac{M^3}{k}(1 - e^{-2kr_c\pi}) \tag{2}$$

for the reduced effective 4-D Planck scale. A field on the SM brane with the fundamental mass parameter m_0 will appear to have the physical mass $m = e^{-kr_c\pi}m_0$. TeV scales are thus generated from fundamental scales of order $\overline{M}_{Pl}$ via a geometrical exponential factor and the observed scale hierarchy is reproduced if $kr_c \simeq 11 - 12$. Due to the exponential nature of the warp factor, no additional large hierarchies are generated. In fact, it has been demonstrated[7] that the magnitude of $1/r_c$ in this scenario can be stabilized without the fine tuning of parameters. This model thus provides an interesting interpretation of the electroweak scale.

In our recent analysis[8], we examined the phenomenological implications and constraints on the RS model that arise from the direct resonant production and exchange of weak scale Kaluza-Klein (KK) towers of gravitons. In this work we consider adding the SM gauge fields to the RS bulk under the assumption that they make little contribution to the bulk energy density so that the solution of Einstein's equations remains valid, i.e., the stress

energy tensor due to SM gauge fields in the bulk is far smaller than the size of the bulk cosmological constant. The possibility that the SM gauge fields may appear in the bulk of models with flat, factorizable geometries has been examined in detail[1, 9, 10] for a wide variety of reasons, including the attainment of low energy coupling constant unification[11]. Here, we will demonstrate that the spectra and couplings of the bulk gauge field KK towers are qualitatively different in the RS model of localized gravity than in the case with factorizable geometry. In addition, we will show that the resulting phenomenological constraints on the model parameters lead to a potential internal inconsistency within the theory and thus gauge fields cannot exist in the bulk without some modification to the theory.

We remind the reader that in the case with a factorizable metric and one extra dimension compactified on S^1/Z_2, (i) the masses of the KK excitations are equally spaced, given simply by the relation $m_n = n/R$, with R being the compactification radius, (ii) the SM chiral fermions are assumed to naturally remain on the SM brane at the orbifold fixed point since they live in the "twisted" sector of string theory, and (iii) the ratio of the couplings to wall fermions of the excited KK states to that of the zero mode is simply $\sqrt{2}$ for all n. While we retain the second assumption below, we will see that the other results will be quite different in the RS model. We also note that we do not need to specify whether the Higgs scalar is also a bulk field, but if it does reside in the bulk, it must be Z_2 even in order to obtain the zero-mode Higgs on the SM brane. In the remainder of the paper we first derive the KK spectrum of the gauge fields and their couplings to fermions, and then examine the phenomenological consequences of their contributions to electroweak radiative corrections. We summarize our results and their implications on the theory in the conclusions.

2 The Gauge Field KK Spectrum

In what follows we derive the KK spectrum of a $U(1)$ bulk gauge field A_M (where the upper case Roman indices extend over all 5 dimensions) in the effective 4-dimensional theory. The extension to the case of non-Abelian fields is straightforward. Here, we assume that the A_μ (where the Greek indices run over ordinary 4-dimensional spacetime) are Z_2-even and that A_4 is Z_2-odd with respect to the extra dimension x^4. This choice of Z_2 parity preserves the gauge-fermion interactions and ensures that A_4 does not have a zero mode in the effective 4-dimensional theory. The 5-dimensional action S_A for a pure $U(1)$ gauge field is given by

$$S_A = -\frac{1}{4} \int d^5x \sqrt{-G}\, G^{MK} G^{NL} F_{KL} F_{MN} \, , \tag{3}$$

where $\sqrt{-G} \equiv det\left(G^{MN}\right) = e^{-4\sigma}$ and F_{MN} is the 5-dimensional field strength tensor given by

$$F_{MN} = \partial_M A_N - \partial_N A_M \, . \tag{4}$$

Note that this definition does not involve the affine connection terms due to the antisymmetry of F_{MN}. After an integration by parts, Eq. (3) yields

$$S_A = -\frac{1}{4} \int d^5x \left[\eta^{\mu\kappa} \eta^{\nu\lambda} F_{\kappa\lambda} F_{\mu\nu} - 2\, \eta^{\nu\lambda} A_\lambda\, \partial_4 \left(e^{-2\sigma} \partial_4 A_\nu \right) \right] \, , \tag{5}$$

where we have used gauge freedom to choose $A_4 = 0$. This is consistent with the gauge invariant equation $\oint dx^4 A_4 = 0$, which results from our assumption that A_4 is a Z_2-odd function of the extra dimension. This choice eliminates A_4 from the theory on the 3-brane, but it will not disturb the gauge invariance of the action in the effective 4-dimensional theory, as we will see below.

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Let the KK expansion of A_μ be given by

$$A_\mu(x, \phi) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x) \frac{\chi^{(n)}(\phi)}{\sqrt{r_c}},\qquad (6)$$

with $x^4 = r_c\phi$. Using this expansion in Eq. (5) and integrating over ϕ gives

$$S_A = \int d^4x \sum_{n=0}^{\infty} \left[-\frac{1}{4}\eta^{\mu\kappa}\eta^{\nu\lambda}F_{\kappa\lambda}^{(n)}F_{\mu\nu}^{(n)} - \frac{1}{2}m_n^2\eta^{\nu\lambda}A_\nu^{(n)}A_\lambda^{(n)} \right],\qquad (7)$$

where $F_{\mu\nu}^{(n)} = \partial_\mu A_\nu^{(n)} - \partial_\nu A_\mu^{(n)}$, and we have required that the ϕ-dependent wavefunctions satisfy the orthonormality condition

$$\int_{-\pi}^{\pi} d\phi\, \chi^{(m)}\chi^{(n)} = \delta^{mn}\qquad (8)$$

and the differential equation

$$\frac{-1}{r_c^2}\frac{d}{d\phi}\left(e^{-2\sigma}\frac{d}{d\phi}\chi^{(n)} \right) = m_n^2\,\chi^{(n)}.\qquad (9)$$

The expression in Eq. (7) is the action for gauge fields $A_\mu^{(n)}$ of mass m_n in 4-dimensional Minkowski space and, as mentioned above, for the zero mode (with $m_n = 0$), S_A has 4-dimensional gauge invariance.

Here we note that we could have also derived the above differential equation from examining the $M = \mu$ components of the 5-dimensional Maxwell's equation

$$\frac{1}{\sqrt{-G}}\left(\sqrt{-G}\,F^{MN} \right)_{,N} = 0,\qquad (10)$$

resulting from the action S_A of the full theory in Eq. (3). Inserting the KK expansion in (6) into the $M = 4$ component of Maxwell's equation yields

$$\eta^{\mu\nu}\sum_{n=0}^{\infty}\partial_\mu A_\nu^{(n)}\frac{d}{d\phi}\chi^{(n)} = 0.\qquad (11)$$

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For $n = 0$, we have $d\chi^{(0)}/d\phi = 0$ and thus a 4-dimensional condition is not imposed on the zero mode $A_\nu^{(0)}$; this is consistent with the gauge invariance of the 4-dimensional $U(1)$ theory. However, for the excited modes, $d\chi^{(n)}/d\phi \neq 0$ and hence we must demand

$$\eta^{\mu\nu}\partial_\mu A_\nu^{(n)} = 0 , \tag{12}$$

as required for massive vector particles in 4-dimensional Minkowski space.

Defining $z_n \equiv (m_n/k)e^\sigma$ and $f^{(n)} \equiv e^{-\sigma}\chi^{(n)}$ we see that Eq. (9) can be written in the form

$$\left[z_n^2 \frac{d^2}{dz_n^2} + z_n \frac{d}{dz_n} + (z_n^2 - 1) \right] f^{(n)} = 0 , \tag{13}$$

which is the Bessel equation of order 1. Therefore, the solutions for $\chi^{(n)}$ are

$$\chi^{(n)} = \frac{e^\sigma}{N_n} \left[J_1(z_n) + \alpha_n Y_1(z_n) \right] , \tag{14}$$

where N_n are the wavefunction normalizations, J_1 and Y_1 are Bessel functions of order 1, and α_n are constant coefficients. Note that this differs from the case of gravitons[8], where the solutions involved the second order Bessel functions J_2 and Y_2. Hermiticity of the differential operator in Eq. (9) requires that the first derivative of $\chi^{(n)}$ be continuous at the orbifold fixed points $\phi = 0$ and $\phi = \pm\pi$. In the limit $e^{-kr_c\pi} \ll 1$, continuity of $d\chi^{(n)}/d\phi$ at $\phi = 0$ yields the relation

$$\alpha_n \approx -\frac{\pi}{2 \left[\ln(x_n/2) - kr_c\pi + \gamma + 1/2 \right]} , \tag{15}$$

and at $\phi = \pm\pi$ we obtain the following differential equation

$$J_1(x_n) + x_n J_1'(x_n) + \alpha_n \left[Y_1(x_n) + x_n Y_1'(x_n) \right] = 0 , \tag{16}$$

where $x_n \equiv (m_n/k)e^{kr_c\pi}$, $\gamma \approx 0.577$ is Euler's constant, and we have assumed that $m_n \ll k$. From these equations, we see that the solutions for x_n depend on the value of the model

parameter kr_c. To estimate this parameter we note that the weak scale Λ_π is related to $\overline{M}_{Pl}$ by $\Lambda_\pi = \overline{M}_{Pl}\, e^{-kr_c\pi}$, and hence to have 100 GeV $< \Lambda_\pi <$ 1000 GeV, we need $11 < kr_c < 12$. For the low lying modes, varying kr_c within this range will not significantly change the values of x_n (the results are only modified by a few percent) and for definiteness we take $\Lambda_\pi = 1000$ GeV, corresponding to $kr_c \approx 11.27$. A numerical solution of Eq. (16) then yields $x_1 \approx 2.45, x_2 \approx 5.57, x_3 \approx 8.70$, and $x_4 \approx 11.84$, for the first 4 massive KK modes $A_\mu^{(n)}$ with $m_n = kx_n e^{-kr_c\pi}$.

It is important to contrast the gauge field KK spectrum with the corresponding KK states for gravitons[8]. For gravitons we found that the KK masses are given by $M_n = k\tilde{x}_n e^{-kr_c\pi}$, where the $\tilde{x}_n$ are roots of the J_1 Bessel function, *i.e.*, $J_1(\tilde{x}_n) = 0$, with $\tilde{x}_n = 3.83\,, 7.02\,, 10.17$, and 13.32 for the first few states. Comparison of the values of the roots x_n with $\tilde{x}_n$ shows that level by level, the KK excitations of the gauge bosons are significantly lighter than those of the corresponding graviton excitations.

3   KK Couplings to Fermions

We now consider the coupling of the gauge KK modes to fermions on the 3-brane corresponding to the visible universe. The fermion kinetic and gauge interaction terms are given by

$$S_\psi = i \int d^4x \int d\phi \, [det(V)] \, \overline{\psi}\gamma^\alpha \, V_\alpha^M \, (\partial_\mu + ig_5 A_\mu)\psi \, \delta_M^\mu \, \delta(\phi - \pi) \,, \tag{17}$$

where V_α^M is the vierbein given by

$$G_{MN} = V_M^\alpha \, V_N^\beta \, \eta_{\alpha\beta} \tag{18}$$

with

$$V_4^4 = 1 \; ; \; V_\mu^\alpha = e^{-\sigma}\delta_\mu^\alpha \; ; \; det(V) = e^{-4\sigma} \,. \tag{19}$$

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Here, γ^α are the Minkowski space Dirac γ-matrices, and g_5 is the 5-dimensional $U(1)$ coupling strength. Upon integration over $\phi \in [-\pi, \pi]$ and using the KK expansion in (6), we obtain for the gauge-fermion interaction term

$$S_\psi = - \int d^4x\, g_5\, \overline{\psi} \gamma^\mu \left(\sum_{n=0}^{\infty} A_\mu^{(n)}(x) \frac{\chi^{(n)}(\pi)}{\sqrt{r_c}} \right) \psi \,, \tag{20}$$

where we have employed the redefinition $\psi \to e^{3\sigma(\pi)/2}\, \psi$.

In order to derive the effective 4-dimensional coupling, we need to know the normalization N_n of $\chi^{(n)}(\phi)$. We note that the wavefunction for the zero mode is a constant and that the orthonormality condition (8) yields

$$\chi^{(0)} = \frac{1}{\sqrt{2\pi}}\,. \tag{21}$$

For the excited modes with $n \neq 0$, we see that Eq. (15) gives $\alpha_n \sim 10^{-2}$ for the low lying states. Thus, within a few percent error, the Y_1 term, which is proportional to α_n, can be neglected in the solution for $\chi^{(n)}(\phi)$. Using the orthonormality condition we then find

$$N_n \approx \frac{e^{kr_c\pi}}{\sqrt{kr_c}}\, J_1(x_n)\,. \tag{22}$$

Defining $g \equiv g_5/\sqrt{2\pi r_c}$, where g is the effective 4-dimensional $U(1)$ coupling constant, this yields

$$S_\psi \approx - \int d^4x\, g\, \overline{\psi} \gamma^\mu \left(A_\mu^{(0)}(x) + \sqrt{2\pi kr_c} \sum_{n=1}^{\infty} A_\mu^{(n)}(x) \right) \psi \tag{23}$$

for the gauge-fermion interaction term. Taking $kr_c \approx 11.27$, we obtain $\sqrt{2\pi kr_c} \approx 8.4$. Therefore, the excited KK modes couple to the 3-brane fermions about 8 times more strongly than the zero mode, which is identified with the usual 'photon' of the 4-dimensional Minkowski

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space. It is clear that by following the same procedure as above for the non-abelian gauge fields[9] we will find that the KK excitations of all the SM fields are universally more strongly coupled than the zero mode by the factor $\sqrt{2\pi k r_c}$. This fact has significant phenomenological implications that will be discussed in the next section.

4   Phenomenological Constraints

We are now ready to explore the phenomenological consequences of the gauge KK towers. In particular, we examine the influence of these KK states on electroweak precision data, assuming that the KK fields are the only source of new physics that perturb the SM predictions for these variables.

To begin this analysis, we first realize that the above discussion regarding $U(1)$ fields in the RS bulk can be immediately generalized to the case of non-Abelian gauge groups as is appropriate for the SM. In particular we note that the mass spectra of the excited states of the W, Z and γ towers will be given by the roots of Eq. (16) plus small corrections due to the appropriate zero mode masses. In addition, the couplings of all the excitations of the SM gauge fields to the fermions on the brane will be enhanced relative to their zero modes by the same amount, $\sqrt{2\pi k r_c}$. Except for the excitation mass spectrum and the precise value of the relative coupling enhancement, we see that this situation very closely resembles the physics of the more conventional scenario of placing SM gauge fields in the 5-dimensional bulk of a factorizable geometry. Such a scenario has been studied in some detail by many authors in order to obtain a bound on the mass of the lightest KK state[9, 10]. Below, we follow closely the analysis as presented in Ref. [10] but employ the more recent precision electroweak data as presented at the summer 1999 conferences[12]. We assume that even though the gauge field couplings are large, a leading order estimate will yield qualitatively

correct results.

We consider the limit where the KK tower exchanges can be described as a set of contact interactions by integrating out the tower fields. In this case, the tower exchanges lead to new dimension-six operators whose coefficients are proportional to a single fixed dimensionless quantity

$$V = \sum_{n=1}^{\infty} \frac{g_n^2}{g_0^2} \frac{M_W^2}{m_n^2} \,. \tag{24}$$

Although the couplings are large, we treat V as a small parameter since M_W/m_n is small enough to compensate for the couplings. The effects of KK exchanges on the electroweak observables, calculated to leading order in V, are delineated in Ref. [10]. These corrections include the contributions from tree-level KK interactions with the zero modes in addition to the usual loop corrections from the zero mode states, or SM fields. It is assumed that loop corrections involving the KK states are higher order and that tree-level contributions from exchanged KK states can be neglected on the Z-pole. A second parameter, s_ϕ, is also required in this analysis to describe whether or not the SM Higgs field is in the bulk or on the wall. We let this parameter vary over its entire allowed range in the analysis below, but as we will see, it will have little influence on our final result.

The electroweak observables used in our global analysis are the leptonic width of the Z, M_W, $\sin^2 \theta_w^{\text{eff}}$ as given by a combined determination of all the electroweak asymmetries, A_b, A_c, R_b, R_c, Q_W - the weak charge of atomic parity violation, and $\sin^2 \theta_w^{\nu N}$ as measured in deep inelastic neutrino scattering. The SM loop corrections involving the light zero-mode states were computed numerically with ZFITTER6.21[13]. Performing a χ^2 fit to the most recent data set[12] and assuming only that the Higgs boson mass is ≥ 100 GeV[12] yields the constraint

$$V \leq 0.0010 - 0.0013 \tag{25}$$

at 95% C.L, where the range results from varying the parameter s_ϕ. We simply assume the weaker bound, $V < 0.0013$, in what follows. We note that this bound allows for variations in both the input values of the top quark mass, $\alpha(M_Z)$, and $\alpha_s(M_Z)$, as well as systematic effects as described in [10].

Given the ratio of coupling strengths derived in the above section, *i.e.*, $g_n/g_0 = \sqrt{2\pi k r_c} \approx 8.4$, and using

$$\sum_{n=1}^{\infty} \frac{x_1^2}{x_n^2} \approx 1.5 \,, \tag{26}$$

implies that the mass of the first gauge boson excited state is bounded by $m_1 \gtrsim 23$ TeV. It is interesting to note that this bound implies a corresponding constraint of $M_1 \gtrsim 36$ TeV on the mass of the first KK graviton resonance. Since both of these lower bounds on the first excitation mass are about a factor of 100 or more larger than the SM Higgs vacuum expectation value, one may worry that we are in danger of forming another hierarchy. Since $m_n = k x_n e^{-k r_c \pi}$, with x_n given above, this yields the constraint $k e^{-k r_c \pi} \gtrsim 9.4$ TeV. Taking the conservative value $\Lambda_\pi = 1$ TeV for the weak scale and folding in the explicit definition of Λ_π as well as the relationship in Eq. (2), we finally arrive at the constraint on the RS model parameters of

$$\frac{k}{M} \gtrsim 4.5 \,. \tag{27}$$

This implies that the magnitude of the bulk curvature violates the initial assumption of the theory that $k < M$. Note that if we had taken a smaller value for Λ_π and/or the tighter constraint on V the above bound on this ratio of RS parameters would have been stronger by as much as a factor of 4.

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5 Conclusions

In this paper we have explored the phenomenological viability of placing gauge fields in the bulk of the Randall-Sundrum model of localized gravity. We derived the gauge field KK spectrum from examination of the action of the theory and also from analyzing the 5-dimensional Maxwell's equation. We then computed the gauge-fermion interactions on the SM 3-brane and found that the excited KK states couple ~ 8 times more strongly than the zero-modes. The influence of these strongly-coupled gauge KK states on electroweak precision data was investigated with the resulting constraint on the mass of the first excited state of $m_1 \gtrsim 23$ TeV. Assuming $\Lambda_\pi \sim 1$ TeV, this in turn implies a bound on the model parameters of $k/M \gtrsim 4.5$, which suggests that the bulk curvature is too large to trust the RS metric (1) as a solution to Einstein's equations. Hence the model as presently formulated is inconsistent with gauge fields existing in the bulk. The effects of higher order curvature terms must be examined in order to determine the robustness of the theory.

Acknowledgements:

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