Supersymmetry Breaking through Transparent Extra Dimensions

D. Elazzar Kaplan et al.

Submitted to Physical Review D

Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309
Work supported by Department of Energy contract DE-AC03-76SF00515.
Supersymmetry Breaking through Transparent Extra Dimensions

D. Elazzar Kaplan\textsuperscript{a}\textsuperscript{*}, Graham D. Kribs\textsuperscript{b}\textsuperscript{†} and Martin Schmaltz\textsuperscript{c}\textsuperscript{‡}

\textsuperscript{a}Enrico Fermi Institute, Department of Physics, University of Chicago, 5640 Ellis Avenue, Chicago, IL 60637 and Argonne National Laboratory, Argonne, IL 60439

\textsuperscript{b}Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213-3890

\textsuperscript{c}Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309

Abstract

We propose a new framework for mediating supersymmetry breaking through an extra dimension. It predicts positive scalar masses and solves the supersymmetric flavor problem. Supersymmetry breaks on a “source” brane that is spatially separated from a parallel brane on which the standard model matter fields and their superpartners live. The gauge and gaugino fields propagate in the bulk, the latter receiving a supersymmetry breaking mass from direct couplings to the source brane. Scalar masses are suppressed at the high scale but are generated via the renormalization group. We briefly discuss the spectrum and collider signals for a range of compactification scales.

\textsuperscript{*}dkaplan@theory.uchicago.edu

\textsuperscript{†}kribs@cmu.edu

\textsuperscript{‡}schmaltz@slac.stanford.edu
I. INTRODUCTION

Electroweak precision data indicate that the mechanism of electroweak symmetry breaking involves a weakly coupled Higgs field. Through radiative corrections the Higgs mass is quadratically sensitive to any scale of new physics. It is therefore hard to understand why the Higgs mass is so much lower than other mass scales which we believe exist in nature, for example the Planck scale.

Low energy supersymmetry is arguably the most compelling framework for addressing this problem: in the minimal supersymmetric standard model (MSSM) one simply introduces superpartners which cancel the divergences order by order in perturbation theory. Unfortunately this solution to the hierarchy problem introduces new problems. Accidental flavor symmetries which suppress flavor changing neutral currents (FCNC) in the standard model (SM) are badly broken by the supersymmetry breaking scalar masses and A-terms in a generic version of the MSSM [1]. Experimental limits on FCNCs force us to consider only very special regions in parameter space where the squark and slepton mass matrices are nearly degenerate [2] or aligned with quark and lepton masses [3]. Two recent proposals for the communication of supersymmetry breaking which do give such degenerate squark and slepton masses are gauge mediation [4,5] and anomaly mediation [6,7].

In this article we propose a new mechanism for communicating supersymmetry breaking that leads to a distinctive spectrum of superpartner masses. It is phenomenologically viable and respects the approximate flavor symmetries of the SM. In our scenario, the matter fields of the MSSM (quarks, leptons, Higgs fields and superpartners) are localized on a $3 + 1$ dimensional brane (the "matter" brane) embedded in extra dimensions. The $SU(3) \times SU(2) \times U(1)$ gauge fields and gauginos live in the bulk of the extra dimensions [15]. Supersymmetry is broken (dynamically) on a parallel "source" brane that is separated from the matter brane in the extra dimensions [17]. Note that in contrast to hidden sector models, our source brane is not hidden at all; the SM gauge fields couple directly to both branes. This set-up leads to the following spectrum of superpartner masses at the compactification scale: gauginos obtain masses through their direct couplings to the supersymmetry breaking source and all other supersymmetry breaking masses are suppressed by the spatial separation of the source and matter branes and/or by loop factors. Thus after integrating out the extra dimensional dynamics at the compactification scale $L^{-1}$ we obtain the MSSM with the only non-negligible supersymmetry breaking being the gaugino masses. This implies that our scenario is very predictive since all supersymmetry breaking parameters can be traced to a single source.

\[1\]

\[2\]

\[3\]

\[4\]

\[5\]

\[6\]

\[7\]

\[8\]

\[9\]

\[10\]

\[11\]
It is easy to understand that this high scale boundary condition is also very attractive phenomenologically. The absence of soft scalar masses and trilinear $A$ terms implies that the only source of flavor violation is the Yukawa matrices. This solves the supersymmetric flavor problem by a super-GIM mechanism. Furthermore, gaugino masses contribute to the renormalization of the scalar masses with the correct sign to give only positive scalar squared masses. There is one subtlety in this argument which leads to successful radiative electroweak symmetry breaking. Because of their strong couplings to gluinos, the masses of colored scalars become large much faster than the supersymmetry breaking Higgs masses. As a consequence the heavy stops running in loops involving the large top Yukawa coupling eventually drive the up-type Higgs (mass)$^2$ negative. Thus radiative electroweak symmetry breaking [12] is also automatic in our framework.

For $L^{-1}$ near the Planck scale, the phenomenology of this model is similar to that of “no-scale” supergravity [13] with unified gaugino masses. However, in our scenario the compactification scale is a free parameter, so the superpartner spectrum and the associated phenomenology varies with this parameter.

In the next section we present our theoretical framework and discuss the coupling of bulk gauge fields to the two branes. In Section III we calculate the effective gaugino masses and scalar masses resulting from integrating out the higher dimensional physics for general supersymmetry breaking sectors. As an example we then present a specific model of supersymmetry breaking. In Section IV a renormalization group analysis is performed, determining the spectrum of superpartner masses at the weak scale. We find that the NLSP is nearly always the stau, and we show that current LEP bounds on charged sparticle masses already restrict a significant portion of parameter space. Finally, the collider signals are briefly mentioned. In Section V we discuss various potential solutions to the $\mu$ problem and in Section VI we conclude.

II. SUPERSYMMETRY BREAKING FROM A DISTANCE

Our underlying assumption is that all the MSSM matter fields live on a three brane in extra dimensions whereas the gauge fields live in the bulk[^2]. Furthermore we assume that supersymmetry is broken dynamically on a brane which is a distance $d$ away from the matter brane. The supersymmetry breaking “source” brane could either be a three brane or – in the case of more than one extra dimension – it could also be of higher dimension. For the explicit calculations in the next section we will assume that the two branes are at the boundaries of one extra dimension such that $d = L$.

[^2]: We could also have additional larger dimensions in which only gravity propagates [14]; such purely gravitational dimensions do not alter our framework significantly.
The basic idea is that supersymmetry breaking couples directly to the gauginos in the bulk whereas locality in the extra dimensions forbids direct couplings between matter fields and the SUSY breaking sector (see Fig. 1).

The matter superpartners receive their masses via loop contributions through the bulk. Depending on the dimensionality of the bulk and the source brane, as well as the details of the supersymmetry breaking sector, their masses are suppressed by varying powers of $d$. This additional suppression of the scalar masses relative to the gaugino masses leads to a very predictive low energy theory: after integrating out the extra dimensions at the scale $1/L$ we obtain the MSSM with – to a good approximation – only soft SUSY breaking gaugino masses.

As with gauge mediation and anomaly mediation, this framework solves the SUSY flavor problem in that the only flavor violation comes from the Yukawa couplings $[2,13]$. This is because contact terms between MSSM matter and the supersymmetry breaking sector are exponentially suppressed due to the fact that these are non-local interactions at the high scale as in the anomaly mediated scenario of [6]. The advantage of our scenario over anomaly mediation is that all scalar mass squareds (except for the up-type Higgs) receive positive contributions from renormalization group running. Also, in gauge mediation, stringent constraints must be imposed on the supersymmetry breaking sector in order to prevent negative or logarithmically enhanced scalar masses. Here the scalar masses at the compactification scale are small enough to render such concerns irrelevant. In addition, direct couplings

\[3\] We are assuming that the flavor scale, the scale at which the Yukawa couplings are generated, is at or above the compactification scale.
between fields on the source brane and matter brane are automatically forbidden by locality, while in gauge mediation, forbidding messenger-matter couplings requires a non-generic superpotential.

Before we go on to describe some specifics of the model, we would like to discuss a few general properties of the framework.

i. strong coupling: One might worry that our theory is non-renormalizable and therefore not predictive. In particular, the gauge coupling in five dimensions carries dimensions of \((\text{mass})^{-1/2}\) and the theory becomes strongly coupled at high energies. At lower energies the effects of the strong coupling are included in the unknown coefficients of higher dimensional operators. We can estimate the scale of strong coupling \(M\) in terms of the volume of the extra dimensions \(V\) by using

\[
\frac{1}{g_4^2} = \frac{V}{g_{4+n}^2} \sim \frac{VM^n}{(4\pi)^2}.
\]

Here \(g_4\) and \(g_{4+n}\) are the four- and higher-dimensional gauge couplings respectively, and we defined \(M\) as the scale where the effective dimensionless coupling constant is nonperturbative \((g_{4+n}M^{n/2} \approx 4\pi)\). For example, compactifying on a strip of length \(L\) gives \(ML = 16\pi^2/g_4^2 \sim O(100)\). Thus as long as we only consider external momenta \(\ll M\) and use \(M\) to cut off loop momenta, our effective theory is perturbative and predictive.

ii. mass scales: The relevant mass scales in our scenario are the compactification scale \(1/L\), the cutoff scale \(M\) (which – for simplicity – we set equal to the scale at which supersymmetry breaking is communicated\(^4\)), and the supersymmetry breaking VEV \(\sqrt{F}\). \(F\) is determined by the scale at which supersymmetry breaking is mediated and the weak scale by requiring that the gaugino masses \(m_\lambda\) are of order \(M_{\text{weak}}\). As shown in i., strong coupling appears at distances about 100 times shorter than \(L\), thus \(M \lesssim 100 L^{-1}\). Therefore only one scale is left undetermined. We take the compactification scale to correspond to this parameter and allow it to vary between \(10^4 - 10^{16}\) GeV. The lower limit comes from imposing fine-tuning constraints at the weak scale. We also impose \(L^{-1} \lesssim M_{\text{GUT}}\) because even higher compactification scales lead to essentially the same boundary conditions at \(M_{\text{GUT}}\): unified gaugino masses and negligible scalar masses.

iii. unification and proton decay: Our framework is fully unifiable, and even though our framework does not require it we do assume gauge unification. This assumption implies gaugino mass unification which makes our theory more predictive. Grand unification might occur at or below the compactification scale \((M_{\text{GUT}} \leq 1/L)\) in which case the running and meeting of the gauge couplings is entirely four-dimensional. However we could also have \(M_{\text{GUT}} > 1/L\) in which case the couplings will exhibit power-law running from the compact-

\(^4\) Messengers could appear on the source brane at a scale below the cutoff. In this case the messenger scale plays the role of the cutoff, although one must require \(M \gtrsim 5L^{-1}\) to suppress higher dimensional contributions to MSSM scalar masses.
ification scale to the unification scale [18]. This would lower the GUT scale, possibly all the way down to of order $10^6$ GeV. For such low scales proton decay via higher dimensional operators or $X$ and $Y$ gauge boson interactions represents a potential disaster. A solution to this problem which would be very natural in our context is to have quarks and leptons live on separate "branes" in the extra dimensions. The separation forbids direct local couplings between quarks and leptons, and proton decay via $X$ and $Y$ gauge bosons would be exponentially suppressed by the massive Yukawa propagators of $X$ and $Y$ propagating between the quark- and lepton branes [19,20].

iv. $B_\mu$ versus $\tan \beta$: Naively, our model predicts $B_\mu = 0$ at the high scale from which we can determine $\tan \beta$. However this prediction probably should not be taken very seriously because, as it stands, the framework has a $\mu$-problem. The mechanism which sets $\mu$ to the weak scale will likely also set $B_\mu$. Therefore we treat $\tan \beta$ as a free parameter in our analysis. We discuss different attempts at solving the $\mu$ problem in Section V.

To be more specific let us now specialize to the case of one extra dimension which we parameterize by the coordinate $x_5$. For convenience we choose the matter and source branes to be located at opposite ends of the the extra dimension. None of the physics we discuss depends on this choice, what is important is that the separation is greater than the short distance cut-off length scale. Coupling supersymmetric three branes to a supersymmetric bulk gauge theory is complicated by the fact that the minimal amount of supersymmetry in five dimensions corresponds to $N = 2$ supersymmetry in four dimensions. Ignoring auxiliary fields the minimal five-dimensional vector superfield contains a real scalar $\phi$, a vector $A_N$, and a four component spinor $\lambda$. They decompose as follows when reduced to four dimensions

$$
\begin{align*}
(\phi & \quad A_N \quad \lambda) \quad \rightarrow \quad (A_\mu \quad \lambda_L) \quad + \quad (\phi + iA_5 \quad \lambda_R) \\
5-d \text{ vector} & \quad 4-d \text{ vector} \quad 4-d \text{ chiral}
\end{align*}
$$

where $\lambda_{R/L} \equiv \frac{1}{2}(1 \pm \gamma^5)\lambda$. In order to break the additional supersymmetry and to give mass to the unwanted adjoint chiral superfield we compactify the fifth dimension on an orbifold. We choose a $\mathbb{Z}_2$ orbifold which acts as $x_5 \rightarrow -x_5$ on the circle $x_5 \in (-L, L]$. The $\mathbb{Z}_2$ breaks half of the supersymmetries by distinguishing the components of the vector superfield. We take it to act as

$$
\begin{align*}
(A_\mu \quad \lambda_L) (x, x_5) & \quad \rightarrow \quad (A_\mu \quad \lambda_L) (x, -x_5) \\
(\phi + iA_5 \quad \lambda_R) (x, x_5) & \quad \rightarrow \quad - (\phi + iA_5 \quad \lambda_R) (x, -x_5)
\end{align*}
$$

which allows a massless mode for the 4-d vector but not for the 4-d chiral superfield. In practice this means that we expand the fields of the vector superfield with cosine KK wave functions, whereas the chiral superfield is expanded in sine modes$^5$.

$^5$For a more detailed description of the orbifold we refer the reader to Ref. [21].
In order to write couplings between the bulk fields and brane fields we note that at the boundaries the components of the $N = 2$ fields which are non-vanishing exactly correspond to a 4-d vector multiplet. Therefore, we can couple them to boundary fields in the same way as we would couple a four-dimensional $N = 1$ vector multiplet. The action is then

$$\mathcal{L} = \int d^5 x [ \mathcal{L}_5 + \delta(x_5) \mathcal{L}_m + \delta(x_5 - L) \mathcal{L}_s ]$$

where $\mathcal{L}_5$ is the bulk Lagrangian for the SM gauge fields

$$\mathcal{L}_5 = -\frac{1}{2} \text{tr} (F_{MN})^2 + \text{tr} (\bar{\lambda} i \Gamma^M D_M \lambda) + \ldots$$

Here $M, N$ label all five dimensions, $\Gamma^\mu \equiv \gamma^\mu$ are the usual four-dimensional gamma matrices, $\Gamma^5 = i \gamma^5$, $D_M$ is the five-dimensional covariant derivative, and we suppressed all terms involving the scalar adjoint $\phi$ and auxiliary fields. Note that there is such an expression for each of the gauge multiplets in $SU(3) \times SU(2) \times U(1)$.

The supersymmetry breaking sector on the source brane at $x_5 = L$ can be quite arbitrary. It is one of the strengths of our framework that it is compatible with many different SUSY breaking sectors. The only requirement of this sector is that the gaugino masses generated are not highly suppressed compared to the scale $F/M$. If there is a singlet chiral superfield $S$ with an $F$ at or near the supersymmetry breaking scale squared, then it will give the dominant contribution to gaugino masses. Though it is possible to produce a viable spectrum even without a singlet, we will assume the singlet exists. We briefly discuss the alternative in Section VI.

A. Source brane action

The source brane action is in general very complicated and involves all the fields required to break supersymmetry dynamically as well as couplings to the bulk gauge fields. However, in order to compute the MSSM gaugino and scalar masses only a small subset of the operators are necessary. If we assume that the leading supersymmetry breaking VEV is the $F$ component of a singlet chiral superfield $S$, then we only need terms of the effective action which couple this singlet to the MSSM gauge fields. The leading superpotential term which couples $S$ to the bulk gauge fields and which contains only two field strengths $W$ is of the form

$$\mathcal{L}_s \sim \int d^2 \theta \frac{S}{M^2} WW + h.c.$$  \hspace{2cm} (2.5)

The gauge field strength superfields $W$ here are five-dimensional with mass dimension two, and the $S$ field is four-dimensional with mass dimension one. This term contributes a gluino mass $\delta(x_5 - L) F_S/M^2$ which is localized on the source brane. Terms with more powers of $S$ do not give rise to new supersymmetry breaking interactions; they only give higher order (in $S/M$) contributions to the gluino mass and are therefore irrelevant. Next we consider the
most general supersymmetry breaking Kähler potential terms with only two $W$s, arbitrary powers of $S$ and no derivatives. (Note that Lorentz invariance forbids terms of the form $WW$.) The leading non-vanishing terms contain a single $S$

$$L_s \sim \int d^2 \theta d^2 \bar{\theta} \frac{1}{M^3} S^\dagger WW (1 + \frac{S}{M} + \cdots) = \int d^2 \theta \frac{F_S^\dagger}{M^3} WW (1 + \frac{S}{M} + \cdots). \quad (2.6)$$

Equivalent terms with less suppression are already contained in the superpotential. Therefore there are no important supersymmetry breaking terms in the Kähler potential with no derivatives.

Using arguments similar to those given above and the constraint $\bar{D}_a \bar{W}^a = D^a W_a$, it is straightforward to determine all Kähler potential terms with two derivatives which give rise to new supersymmetry breaking. The most important such terms are non-supersymmetric contributions to kinetic terms such as

$$G = -\frac{1}{x^5} \frac{S^\dagger S}{M^5} WD^2 W \rightarrow \frac{F_S F_S^\dagger}{M^5} \bar{\lambda}_L \lambda_L \cdot \quad (2.7)$$

In the next section we will see that this supersymmetry breaking correction to the gaugino kinetic term gives rise to (small) scalar masses when inserted into loop diagrams.

III. THE MSSM SCALAR AND GAUGINO MASSES

In this section we compute the MSSM soft supersymmetry breaking masses that result from integrating out the extra dimensions. We always assume that loop momenta are larger than $L^{-1}$. Smaller loop momenta are more conveniently dealt with by considering the four-dimensional effective theory, as we do in Section IV.

It is straightforward to determine the gaugino masses resulting from the term eq. (2.5) on the source brane by expanding the five-dimensional gaugino fields in KK modes. The zero mode which corresponds to the light four-dimensional gaugino has an $x_5$-independent wave function, which when normalized to produce a canonical kinetic term has height $1/\sqrt{L}$. Thus the gaugino mass is

$$m_\lambda = \frac{1}{M L} \frac{F_S}{M}. \quad (3.1)$$

To calculate the scalar masses more effort is required. The leading contributions come from loop diagrams which involve both the scalars on the matter brane as well as supersymmetry violating operators on the source brane (Fig. 1). Any of the fields in the five-dimensional gauge multiplets can be exchanged. In principle, this leads to a large number of diagrams which need to be calculated. However, since we are only interested in showing that the scalar masses are small, we only compute two representative diagrams with bulk fermion exchange. The other diagrams are of comparable size and therefore also negligible.
It is most convenient to compute the five-dimensional Feynman diagrams in momentum space in four dimensions and position space in the fifth dimension. This mixed position-momentum space calculation is well adapted to the symmetries of the problem (translation invariance in four dimensions but broken translation invariance in $x_5$). The necessary propagators are obtained by partially Fourier transforming normal momentum space propagators, whereby care needs to be taken to properly take into account the orbifold boundary conditions eq. (2.2). For example, the scalar $\phi$ propagator with 4-d Euclidean momentum $q^2$ propagating from coordinate $b$ to $a$ in the $x_5$-direction is

$$\mathcal{P}_0(q^2; b, a) = \frac{2}{L} \sum_{n,m=1}^{\infty} \sin(-p_n a) \frac{\delta_{nm}}{q^2 + p_n^2} \sin(p_n b) \sim - \int_{-\infty}^{\infty} \frac{dp}{2\pi} \frac{e^{ip(b-a)}}{q^2 + p^2} = -\frac{e^{-|b-a|\sqrt{q^2}}}{2\sqrt{q^2}}.$$ (3.2)

We have implemented the orbifold boundary conditions for $\phi$ by expanding in sine modes with Fourier momenta $p_n = n\pi/L$. By approximating the sum with an integral we have assumed large volume ($L > 1/\sqrt{q^2}$). Performing the sum exactly is straightforward [22] but not necessary for our purposes.

Analogously the fermionic propagator is obtained by Fourier expanding the momentum space propagator in sine and cosine wave functions for the right and left handed components respectively

$$\mathcal{P}(q; a, b) = \frac{2}{L} \sum_{n,m=0}^{\infty} \left[ P_L \frac{\cos(p_n a)}{\sqrt{2}^{b,n}} - P_R \sin(p_n a) \right] \delta_{nm} \frac{q - i\gamma_5 p_n}{q^2 + p_n^2} \left[ P_R \frac{\cos(p_m b)}{\sqrt{2}^{b,m}} + P_L \sin(p_m b) \right].$$ (3.3)

Again $p_n = n\pi/L$, the factor of $\sqrt{2}^{b,0}$ arises from the different wave function normalization of the zero mode, and again we have Wick-rotated the four-momentum to Euclidean space. At the boundaries $x_5 = 0$ and $x_5 = L$ only the left-handed gaugino component is non-vanishing and can be coupled directly to the scalars and the supersymmetry breaking sector. The other components require $\partial_5/M$ derivatives in the couplings and are therefore subleading (after regularization and renormalization of the divergent momentum integrals). We only keep the leading cosine components of the propagator. Summing over momenta we find

$$\mathcal{P}(q; 0, L) = \frac{P_L q}{q \sinh(qL)} \sim \frac{2P_L q}{q} e^{-qL}.$$ (3.4)

Armed with this very simple formula for the 5-d gaugino propagator it is straightforward to compute the diagram with two gluino mass insertions in Fig. 1. Ignoring Casimirs and factors of 2 we find

$$g_5^2 \left( \frac{F_S}{M^2} \right)^2 \times \int \frac{d^4q}{(2\pi)^4} \text{tr} \left[ \frac{1}{q} \mathcal{P} L P(q; 0, L) C \mathcal{P}'(q; L, L) C^{-1} \mathcal{P}(q; L, 0) \right] \sim \frac{g_5^2}{16\pi^2} \left( \frac{F_S}{M^2} \right)^2 \frac{1}{L^3} = \frac{g_4^2}{16\pi^2} m_\lambda^2.$$ (3.5)
We see that the scalar masses are suppressed by three powers of the brane separation which can be absorbed into the four-dimensional gauge couplings and gaugino masses. Thus we find that the scalar mass contributions from this diagram are smaller compared to the gluino masses by a loop factor. Note that these small contributions to the scalar masses are flavor universal and do not give rise to flavor changing effects.

As a second example we compute the contribution from a supersymmetry breaking gaugino wave function renormalization insertion eq. (2.7) on the source brane. We find

\[
g_5^2 \frac{F_5^2}{M^3} \times \int \frac{d^4q}{(2\pi)^4} \text{tr} \left[ \frac{1}{q^4} P_L(q; 0, L) q^4 P(q; L, 0) \right] \approx \frac{g_5^2}{16\pi^2} \left( \frac{F_5}{M^2} \right)^2 \frac{1}{ML^3} = \frac{g_4^2}{16\pi^2} m^2 \frac{1}{ML},
\]

which is suppressed by an additional power of the separation compared to the contribution of eq. (3.5). Note that one could have obtained this result from dimensional analysis: soft scalar masses require two insertions of supersymmetry breaking $F_5$, the powers of $M$ in the denominator are determined by the dimensionality of the operators which we insert on the source brane, the exponent of the separation $L$ can then be determined by dimensional analysis. This dimensional analysis also shows that diagrams involving even higher dimensional operators (such as operators with additional $\partial_0/M$ derivatives) are suppressed by additional powers of $(ML)^{-1}$.

In summary we find that the MSSM scalar mass squareds are suppressed relative to the gaugino masses by at least a loop factor, and are therefore negligible compared to the masses which are generated from the (four-dimensional) renormalization group evolution between the compactification scale and the weak scale. This conclusion also holds for the other soft supersymmetry breaking parameters involving matter fields, the $A$-terms and $B_{\mu}$. Note that these contributions to soft parameters are flavor-diagonal and are thus irrelevant with regards to bounds on FCNCs.

### A. Example: gauge mediation with branes

Here we demonstrate the above results with an explicit model for the supersymmetry breaking sector on the source brane. We take the source brane action to be identical to the ordinary messenger sector of gauge mediation where the SM gauge fields are replaced by the boundary values of the bulk gauge fields

\[
\mathcal{L}_s = \int d^4\theta \, Q^\dagger e^{2gV[A^\mu, \lambda_L]} Q + \tilde{Q}^\dagger e^{-2gV[A^\mu, \lambda_L]} \tilde{Q} + \int d^2\theta \, SQ\tilde{Q}.
\]

Here $Q + \tilde{Q}$ are the messenger chiral superfields which we take to transform under the SM gauge interactions with the quantum numbers $5 + \bar{5}$ of $SU(5)$. The vector superfield $V[A^\mu, \lambda_L]$ contains the SM gauge fields and gauginos in the normalization appropriate for five-dimensional fields. The $S$ field has been rescaled to absorb the Yukawa coupling, and
as in ordinary gauge mediation we assume that it acquires supersymmetry preserving and violating expectation values

\[ S = M + F_s \theta^2. \]

Then the messenger fermions obtain the Dirac mass \( M \) whereas the messenger scalars in \( Q \) and \( \tilde{Q} \) acquire the (mass)\(^2\) \( M^2 \pm F_s \). Note that role of the cut-off (or new physics) scale in our more general effective theory of the source brane is played by the messenger mass in this example.

The bulk gauginos obtain a mass which is localized on the source brane from a one-loop diagram with messenger scalars and fermions in the loop as in ordinary gauge mediation. Since the messengers are stuck to the brane this calculation is entirely four-dimensional and we find the effective gaugino mass

\[
\frac{g^2}{16\pi^2 \lambda} \frac{F_s}{M}.
\]

The gauge couplings \( g_5 \) here are five-dimensional (and in the GUT normalization); they are related to four-dimensional couplings by \( g^2_5 / L = g^2_4 \). We see that our gaugino masses are identical to ordinary four dimensional gauge mediation gaugino masses.

The computation of the scalar masses is more involved. We simply quote the result obtained by Mirabelli and Peskin [21] who computed the scalar masses at two loops for arbitrary separation. Expanding to second order in \( F_s \) and to leading order in \( (LM)^{-1} < 1 \) their result reduces to

\[
m^2 = 2C \left( \frac{g^2_5}{16\pi^2 \lambda} \frac{F_s}{M} \right)^2 \frac{\zeta(3)}{(ML)^2} - m^2_4 \frac{\zeta(3)}{(ML)^2}.
\]

Here \( m^2 \) is the scalar mass in the five-dimensional theory, \( m^2_4 \) is the ordinary four-dimensional gauge mediation result, and \( C \) is a group theory factor of order one which depends on the quantum numbers of the matter and source fields. The important conclusion is that the scalar mass squareds are suppressed relative to gaugino masses by a factor of 1/(\( ML \))\(^2\). Again assuming a distance which is at least a factor of 5 larger than the messenger scale, we find that eq. (3.9) is negligible compared to the masses which are generated from four-dimensional running.

To compare this to our general analysis of the previous section note that the scalar mass squared scales as \( 1/L^4 \) when expressed in terms of five-dimensional quantities. This is in agreement with the scaling found for eq. (3.6). The scalar mass contribution scaling as \( 1/L^3 \) eq. (3.5) corresponds to a three loop diagram in the gauge mediation model. We see that the scalar mass squareds are suppressed by at least \( (ML)^{-2} \) or a loop factor.

**IV. SPECTRUM AND PHENOMENOLOGY**

To calculate the spectrum in our scenario we use the renormalization group to connect the physics near the cutoff scale with the weak scale. In particular, there are two scaling
regions that must be considered when evolving masses and couplings: between the cutoff scale and the compactification scale, and between the compactification scale and the weak scale.

Above the compactification scale the theory is five-dimensional and we need to evolve masses and couplings according to five-dimensional evolution equations. Happily this turns out to be rather straightforward. The calculations of the previous section showed that the scalar masses which are generated above the compactification scale are negligible. Therefore we do not need to evolve either scalar masses or $A$-terms in the five-dimensional theory. The gaugino mass evolution is important however. For this purpose it is most convenient to think of the theory as four-dimensional with KK excitations. Across each KK threshold, the four-dimensional gauge and gaugino beta functions are modified, and such corrections must be included to calculate the low energy spectra. However, the ratio of the gaugino mass to the gauge coupling squared is invariant to one-loop, as in the normal four-dimensional case\(^6\) (for discussion of this, see Ref. [24]). Summing over the towers of KK thresholds up to the cutoff $M_*$ can be represented by terms that resemble corrections to the renormalization group equations to both the gauge couplings [18] and gaugino masses and gives the same result [25]. Specifically, assuming gauge coupling unification, the relations

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} \quad (4.1)$$

hold at any scale to one-loop order in the $\beta$-functions. This means that we can incorporate the extra dimensional running of the gaugino masses simply by starting with the boundary condition, Eq. (4.1), at the compactification scale. Note that this relation also implies that our predictions for gaugino masses will be nearly independent of the compactification scale.

Just below the compactification scale, our theory is four-dimensional with nonzero gaugino masses, vanishing scalar masses, and vanishing trilinear scalar couplings. Scalar masses and trilinear scalar couplings are regenerated through renormalization group evolution between the compactification scale and the weak scale, and this provides the basis to calculate the spectrum and phenomenology. The parameters for the model can be chosen to be

$$L^{-1}, M_{1/2}, \tan \beta, \text{sign}(\mu).$$

Here $M_{1/2}$ is the common gaugino mass at the unification scale. For $L^{-1} < 10^{16} \text{ GeV}$ the individual gaugino masses at the compactification scale can be determined from $M_a(L^{-1})/g_a^2(L^{-1}) = M_{1/2}/g_{\text{unit}}^2$ with $g_{\text{unit}} \sim 0.7$. Imposing electroweak symmetry breaking

---

\(^6\) This can also be seen by noting that the gaugino mass is in the same supermultiplet as the holomorphic gauge coupling $\tau$ and therefore evolves in parallel. For the usual 4-d arguments [23] to go through even in the theory with KK modes, the orbifold boundary conditions must preserve 4-d $N=1$ supersymmetry as in our framework.
constraints at the weak scale determines $\mu^2$, leaving $\tan \beta$ and $\text{sign}(\mu)$ unknown. Generally, the scalar masses are proportional to $M_{1/2}$ to reasonable accuracy unless Yukawa coupling effects are large (i.e., particularly for the up-type Higgs mass), or weak interaction eigenstate mixing is important (i.e., stau masses at moderate to large $\tan \beta$).

![Graph](image)

**FIG. 2.** Evolution of several soft masses as a function of the renormalization scale with the input parameters $L^{-1} = 10^{16}$ GeV, $M_{1/2} = 350$ GeV, and $\tan \beta = 10$. The (top, middle, bottom) dashed lines correspond to $(M_3, M_2, M_1)$, while the solid lines from top to bottom correspond to $m_{\tilde{\chi}}$, $m_{H_d}$, $m_{\tau_1}$, $\text{sign}(m_{H_u}^2)|m_{H_u}^2|^{1/2}$ respectively. (The kink in the up-type Higgs mass is due to taking the square-root.)

As a first example, we take $L^{-1} = 10^{16}$ GeV, $M_{1/2} = 350$ GeV, and $\tan \beta = 10$, and show in Fig. 2 the evolution of the soft masses as a function of the renormalization scale. Several generic features are evident from the graph: Gaugino masses evolve in parallel with gauge couplings; the ratios $M_3/M_1$ and $M_3/M_2$ increase as the renormalization scale is decreased, causing larger squark masses relative to slepton and Higgs masses. Initially, $m_{H_u}^2$ runs toward positive values, but is quickly overcome by interactions with the heavy stops and runs to negative values at the weak scale. With these parameters, the stau is the lightest sparticle of the MSSM spectrum.

While $B\mu$ is what appears in the Lagrangian, we choose to parameterize our ignorance by $\tan \beta$. 

---

7While $B\mu$ is what appears in the Lagrangian, we choose to parameterize our ignorance by $\tan \beta$. 

---

13
FIG. 3. The weak scale masses for several sparticles are shown as a function of the compactification scale $L^{-1}$ with $M_{1/2} = 500$ GeV and $\tan \beta = 3$. The top dotted line is $m_\tilde{\chi}$, the top and bottom solid lines are $m_{\tilde{g}}$ and $m_{\tilde{t}_1}$, and the top and bottom dashed lines are $m_{\tilde{N}_3}$ and $m_{\tilde{N}_1}$. We emphasize that $L^{-1}$ is parameter of our model not to be confused with the renormalization scale.

The results of the previous analysis are the same as those in “no-scale” supergravity models [13]. However, in our framework, the detailed phenomenology depends on the compactification scale. Obviously the size of the scalar masses depends on the extent of evolution, proportional to $\sim M_{1/2}^2 \log(M_Z L)$, but also derived parameters such as $\mu$ are sensitive to the compactification scale. In Fig. 3 we show the weak scale masses of several MSSM fields as a function of the compactification scale for $M_{1/2} = 500$ GeV and $\tan \beta = 3$. A generic prediction of our model is that the stau is the NLSP for most compactification scales. However, we note that for very large $L^{-1} \gtrsim 10^{16}$ GeV with small $\tan \beta \lesssim 3$, the lightest neutralino $\tilde{N}_1$ becomes the NLSP (or LSP, as discussed below). The kinks in the mass contours of $\tilde{N}_1$ and $\tilde{N}_3$ in Fig. 3 indicate a “cross over” in the dominant interaction eigenstate content of the neutralinos from bino-like to Higgsino-like as the compactification scale is lowered below $L^{-1} \sim 10^5$ GeV. This suggests that, for example, a measurement of the gauge eigenstate content of the lightest neutralino is sensitive to the compactification scale.

The scaling of scalar masses proportional to $\sim M_{1/2}^2 \log(M_Z L)$ is clearly visible from Fig. 3; it affects the squarks most dramatically but is also important for sleptons, particularly the lightest (mostly right-handed) stau. Note that this allows us to extract significant limits on $M_{1/2}$ as a function of $L^{-1}$ by requiring that the stau avoids the lower bounds from the
recent LEP searches for charged sparticles. In Fig. 4 we show the lower bound on $M_{1/2}$ as a function of the compactification scale. The best bound comes from the lower limit on the stau mass, although low $\tan \beta \lesssim 3$ is also restricted by the limit on the lightest Higgs boson. In addition, notice that for large values of $\tan \beta$, the lower bound on $M_{1/2}$ is considerably strengthened. This is due to large mixing in the stau mass matrix from the off-diagonal term proportional to $m_{\tau} \tan \beta$ that reduces the mass of the lightest stau mass eigenstate.

As we implied above, the gravitino is the LSP for most of the parameter space. Assuming that $F_S$ is the largest supersymmetry breaking VEV, its mass is given by $m_{3/2} \sim F_S/M_{\text{Planck}}$. However, for very large compactification scales the mass of the stau which roughly scales as $M_{1/2} \sim F/(M^2 L)$ can become smaller than $m_{3/2}$. Then the stau could become the LSP which is probably in conflict with cosmology. The turn over occurs when $M_{\text{Planck}} \sim F/(M^2 L)$ or $L^{-1} \sim 10^{14-16}$ GeV. We find it amusing that coincidently the largest compactification scales also correspond to the regime where the lightest neutralino can be LSP, which would render the stau cosmologically safe again. The viability of this regime clearly deserves further study.

Superpartner production at colliders always results in two or more NLSPs (directly or indirectly), each of which then decays into the LSP with a decay length that is expected to be at least of order the size of the detector. If the stau is the NLSP one expects clearly visible
charged stau tracks in detectors resulting from meta-stable staus that escape the detector\textsuperscript{8}. Strategies to extract this signal from the muon background have been explored in Ref. [26], with the result that rather significant regions of parameter space can be probed. For very small compactification scales $L^{-1} \lesssim 10^5$ GeV, it is possible that the stau decay length could be measurable. In the small region of parameter space where the neutralino is the (N)LSP, the characteristic signal is missing energy, analogous to gauge-mediation models with a large messenger scale, or ordinary supergravity models.

V. THE $\mu$ TERM

As in other models of supersymmetry breaking we appear to have a $\mu$ problem in our framework. The $\mu$ term is the dimensionful superpotential coupling of $H_u$ and $H_d$ and is required to be at the weak scale in order to naturally produce electroweak symmetry breaking while maintaining agreement with experimental lower bounds on sparticle masses.

From naturalness [28], one would expect a dimensionful quantity to be of order the fundamental scale in the model, in our case $M$. However, it is well known that superpotential couplings can easily be non-generic, and $\mu$ can also be set to zero by imposing a discrete version of a Peccei-Quinn symmetry [29]. Allowing the discrete symmetry to break spontaneously with the breaking of supersymmetry, it is easy to produce a weak-scale $\mu$ term. However, it is difficult to produce soft Higgs-mass terms at the same scale (they normally come out too large). Here we present some possible solutions to the $\mu$ problem. This new framework may allow for more novel solutions and we leave these for future work.

Perhaps the most elegant possibility for a solution lies with the Next-to-Minimal Supersymmetric Standard Model (NMSSM) [30]. Inserting this mechanism into our framework means adding a gauge singlet $N$ to the matter brane and replacing the $\mu$ term in the superpotential by:

$$W_N = \lambda N H_u H_d + \frac{k}{3} N^3. \quad (5.1)$$

As the soft masses are run from the compactification scale to the weak scale, $N$ develops a scalar vacuum expectation value of order the weak scale for some range of parameters $\lambda$ and $k$. Thus an effective $\mu$ term is produced. This mechanism was thoroughly analyzed by de Gouvea, Friedland and Murayama in the context of gauge mediation with a range of messenger scales [31]. They found the NMSSM could produce a $\mu$ term but only at the expense of giving unacceptably light masses to Higgs bosons and/or sleptons. However, our boundary conditions are different and may push the results in the right direction.

A twist on this solution is to put the singlet $N$ in the bulk. The first obvious requirement is that the $F$ term of $N$ must be suppressed relative to the supersymmetry breaking scale.

\textsuperscript{8}A stau NLSP could also have interesting implications for cosmology [27].
Otherwise, \( F_N \) would generically give non-universal scalar masses. If there are fields on the source brane charged under the SM gauge group (say, extra vector-like quarks) to which \( N \) couples, then a solution may be found as suggested in [32,31]. This solution appears to be fine-tuned and the fine tuning comes from the same source as the fine tuning in the MSSM. So this mechanism could explain the dynamical origin of the \( \mu \) term, but it does not give a dynamical reason for the cancellation of large soft parameters. One could also consider more than one singlet and could place singlets in the bulk or on either of the branes. This would allow certain couplings to be small or vanish, possibly giving the right parameter values for a natural \( \mu \) term as in [33].

The suggestion of Chacko et al. [34] to put the Higgs fields in the bulk (while keeping \( S \) on the source brane) is also interesting. The \( \mu \) term could be produced on the opposite brane via the Giudice-Masiero mechanism [35]. The operators in the (5 dimensional) Lagrangian would be:

\[
\int d^4 \theta \left[ \lambda_\mu \frac{S^\dagger}{M^2} H_u H_d + \lambda_H \frac{S S^\dagger}{M^3} H_u H_d + \frac{SS^\dagger}{M^3} (\lambda_u H_u H_d^\dagger + \lambda_d H_d H_u^\dagger) + h.c. \right] \delta(x_5 - L) \tag{5.2}
\]

where the coupling constants \( \lambda_i \) are dimensionless. Thus the natural value of \( \mu \) would be \( F_S/(M^2 L) \), where as the natural value of the soft parameters \( B \mu, m_{H_u}^2 \) and \( m_{H_d}^2 \) would be \( F_S^2/(M^3 L) \sim \mu (F_S/M) \). We find the standard problem of producing soft terms which are too large. We could of course set the appropriate couplings to be small (\( \sim (ML)^{-1} \)), however we do not have a compelling theoretical reason for doing so. Also we note that placing the Higgs fields in the bulk changes the spectrum of the model significantly as their scalar masses would be generated above the compactification scale. We have found the resulting phenomenology is viable and thus a detailed analysis would be interesting.

In summary, there exist a number of ways to produce a \( \mu \) term dynamically in our scenario. However, they all appear to require small or fine-tuned parameters. Thus finding a natural origin for \( \mu \) and \( B \mu \) of the right size is still an open problem.

**VI. DISCUSSION**

We have presented a model of supersymmetry breaking in extra dimensions in which only gauginos receive soft masses at a high scale, and scalar masses come dominantly from renormalization group running. The model clearly avoids the supersymmetric flavor problem, and all scalar mass squareds (except for a Higgs) are positive at the weak scale. The model is highly predictive, depending only on three parameters and a sign (\( M_{1/2}, L^{-1}, \tan \beta, \) and the sign of \( \mu \)), and allows for compactification scales as low as \( 10^4 \) GeV.

For simplicity, we required the gaugino masses to unify at or above the compactification scale. This comes from the assumption that the theory is unified at a high scale and that threshold effects are small. By relaxing either assumption, one could impose more general boundary conditions, i.e., with split gaugino masses. As long as the gluino is heavy enough to
give squark masses larger than Higgs masses, and the bound in Fig. 4 is respected (properly
reinterpreted as a bound on $M_1$), then supersymmetry breaking through transparent extra
dimensions would still work perfectly.

The only requirement on the source brane is that there exists a singlet whose $F$ compo-

ten is comparable to the scale of supersymmetry breaking. However, even this requirement
may be relaxed. Without a singlet, the main contribution to the gaugino masses is via
anomaly mediation – a one-loop effect [6,7]. The dominant contributions to the scalars
would come from the anomaly-mediated contributions and from non-renormalizable oper-
ators inserted in loops (as in Sec. III), both of which are flavor blind. For small values of
$ML$, the latter may dominate allowing for a (different) realistic spectrum.

While the size of the compactification scale does not allow for direct detection of KK
modes, it does leave an imprint on TeV scale phenomenology. The field content of the
lightest neutralino (bino versus Higgsino) changes with $L^{-1}$ and therefore so do the couplings
to matter. In addition, while the gaugino spectrum is approximately independent of scale,
the scalar spectrum is not, thus this model is distinguishable from a minimal supergravity
model if $L^{-1} \ll M_{\text{Planck}}$. In fact, by measuring the scalar spectrum (e.g., at the NLC) one
may be able to determine the scale at which the scalar masses unify and thus the size of the
extra dimensions!

Note added: While this work was in progress we learned that similar ideas are being
pursued independently by Chacko, Luty, Nelson, and Pontón [36].

ACKNOWLEDGMENTS

Discussions with N. Arkani-Hamed, A.G. Cohen, A. de Gouvea, J.L. Feng, A. Friedland,
Z. Ligeti, H. Murayama, R. Sundrum, C.E.M. Wagner were helpful and fun. Thanks! We also
thank the theory group at Fermilab, where this work was initiated, for its kind hospitality.
DEK is supported in part by the DOE under contracts DE-FG02-90ER40560 and W-31-
109-ENG-38. GDK is supported in part by the DOE under contract DOE-ER-40682-143.
MS is supported by the DOE under contract DE-AC03-76SF00515.
REFERENCES


[34] Z. Chacko, M. Luty, I. Maksymyk, E. Ponton, [hep-ph/9905390].
