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CHIRAL BOSON THEORY ON THE LIGHT-FRONT *

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Abstract

The *front form* framework for describing the quantized theory of chiral boson is discussed. It avoids the conflict with the requirement of the principle of microcausality as is found in the conventional *instant form* treatment. The discussion of the Floreanini-Jackiw model and its modified version for describing the chiral boson becomes transparent on the light-front.

Keywords: Chiral boson, Selfdual fields, Light-front quantization

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1- There is a wide interest in the chiral bosons, also called self-dual scalar fields [1]. They are, for example, among the basic ingredients in the formulation of heterotic string theory [2], in the description [3] of boundary excitations of the quantum Hall state, in a number of two-dimensional statistical systems [4, 5] which are related to the Coulomb-gas model, and in the context of W-gravities. Although apparently simple, the quantization of chiral bosons presents intriguing and instructive features. The canonical quantization of Siegel's Lagrangian [6] model requires an additional Wess-Zumino term to take care of the anomaly. The resulting theory does not describe pure chiral boson but rather their coupling to gravity [7]. The model which employs the Lagrange multiplier field to impose the chiral constraint linearly [8] has received criticism [9]. An improved version [10] of it, however, is found equivalent at the quantum level to the much studied model of chiral boson, proposed [11] earlier by Floreanini and Jackiw (FJ). The rather extensive literature, which employs the conventional equal-time framework in its study, reveals its polemic character. It is thus worthwhile to study its quantization on the light-front (LF) which, as shown below, does throw some new light on the problem. If we take into consideration the requirement of the principle of microcausality, the *front form* framework is seen to be more appropriate for discussing the quantized theory of the chiral boson. We will study in some detail the FJ model modified by the introduction of an additional parameter in it.

2- Half a centuary ago, Dirac [12] discussed the unification, in a relativistic theory, of the principles of the quantization and the special relativity theory which were by then firmly established. The Light-Front (LF) quantization which studies the relativistic quantum dynamics of physical system on the hyperplanes : $x^0 + x^3 \equiv \sqrt{2}x^+ = const.$, called the *front form* theory, was also proposed there and some of its advantages pointed out. The *instant form* or the conventional equal-time theory on the contrary uses the $x^0 = const.$ hyperplanes. The LF coordinates $x^\mu : (x^+, x^-, x^\perp)$, where $x^\pm = (x^0 \pm x^3)/\sqrt{2} = x_\mp$ and $x^\perp = (x^1, x^2) = (-x_1, -x_2)$, are convenient to use in the *front form* theory. They are *not related by a finite Lorentz transformation* to the coordinates $(x^0 \equiv t, x^1, x^2, x^3)$ employed in the *instant form* theory. The descriptions of the same physical content in a dynamical theory on the LF, which studies the evolution of the system in x^+ in place of x^0, may thus come out to be different from that given in the conventional treatment. This is found, for example, to be the case in the description of the spontaneous symmetry breaking (SSB) mechanism [13] and in the studies [14] of some soluble two-dimensional gauge theory models, where it was also demonstrated that LF quantization is very economical in displaying the relevant degrees of freedom, leading directly to the physical Hilbert space.

The interest in the *front form* theory has been revived [15, 16, 13] due to the difficulties encountered in the computation, in the conventional framework, of the nonperturbative effects in the context of QCD and the problem of the relativistic bound states of fermions [15, 16] in the presence of the complicated vacuum. LF variables have found applications in several contexts, for example, in the quantization of (super-) string theory and M-theory [17], in the nonabelian bosonization [18] of the field theory of N free Majorana fermions, in the study of the vacuum structures [14] in the Schwinger model (SM) and the Chiral SM among many others. The LF quantized QCD in covariant gauges has also been studied [19] recently in the context of the Dyson-Wick perturbation theory, where it is shown that the apparent lack of manifest covariance usually encountered in such calculations becomes tractable thanks to the introduction of a useful construction of the LF spinor. The applications in the context of the Bethe-Salpeter dynamics have also been considered [20, 21] recently.

3- We will make the *convention* to regard $x^+ \equiv \tau$ as the LF-time coordinate while x^- as the *longitudinal spatial* coordinate. The temporal evolution in x^0 or x^+ of the system is generated by the Hamiltonians which are different in the two *forms* of the theory.

Consider [14] the invariant distance between two spacetime points : $(x-y)^2 = (x^0 - y^0)^2 - (\vec{x}-\vec{y})^2 = 2(x^+ - y^+)(x^- - y^-) - (x^\perp - y^\perp)^2$. On an equal $x^0 = y^0 = const.$ hyperplane the points have spacelike separation except for if they are *coincident* when it becomes lightlike one. On the LF with $x^+ = y^+ = const.$ the distance becomes *independent of* $(x^- - y^-)$ and the seperation is again spacelike; it becomes lightlike one when $x^\perp = y^\perp$ but with the difference that now the points need *not* necessarily be coincident along the longitudinal direction. The LF field theory hence *need not necessarily be local* in x^-, even if the corresponding *instant form* theory is formulated as a local one. For example, the commutator $[A(x^+, x^-, x^\perp), B(0, 0, 0^\perp)]_{x^+=0}$ of two scalar observables would vanish on the grounds of microcausality principle in relativistic field theory for $x^\perp \neq 0$ when $x^2|_{x^+=0}$ is spacelike. Its value would hence be proportional to $\delta^2(x^\perp)$ and a finite number of its derivatives, implying locality only in $x^\perp$ but not necessarily so in x^-. Similar arguments in the *instant form* theory lead to the locality in all the three spatial coordinates. In view of the microcausality [22] both of the commutators $[A(x), B(0)]_{x^+=0}$ and $[A(x), B(0)]_{x^0=0}$ are nonvanishing only on the light-cone.

An important advantge of the *front form* theory pointed out by Dirac is that here seven out of the ten Poincaré generators are *kinematical* while there are only six such ones in the the conventional theory. Moreover, based on the general considerations it can be argued [13] that the LF hyperplane is equally valid and appropriate as the conventional equal-time one for the canonical quantization of relativisitic theory.

4- The massless two dimensional free scalar theory has, at the classical level, the chiral boson solutions satisfying $\partial_0\phi = \pm\partial_1\phi$. We would like to construct the corresponding Lagrangian formulation which describes, in the quantized theory, the excitations of, say, a right-moving massless particle.

The FJ model is based on the following *manifestly non-covariant Lagrangian*

$$\begin{aligned}\mathcal{L} &= (\partial_0\phi - \partial_1\phi)\partial_1\phi \\ &= \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(\partial_0\phi - \partial_1\phi)^2. \end{aligned} \tag{1}$$

where ϕ is a real scalar field and $\eta^{00} = -\eta^{11} = 1,\ \eta^{01} = \eta^{10} = 0$. In the *instant form* frame work its canonical quantization results in [11, 23] the following equal-time commutator

$$\Big[\phi(x^0, x^1), \phi(x^0, y^1)\Big] = \frac{-i}{4}\epsilon(x^1 - y^1). \tag{2}$$

It is nonlocal and nonvanishing for spacelike distances, e.g., it *violates* the microcausality principle, contrary to what we encounter [22] normally in the conventional theory framework. These objections are found below to *disappear* if we regard the theory under discussion as being considered in the *front form* framework.

The FJ Lagrangian (1) may, in fact, be rewritten in terms of the LF coordinates as $\mathcal{L} = (\partial_+\phi - \partial_-\phi)\ \partial_-\phi$. We will consider instead the following modified form of the Lagrangian for describing the chiral boson

$$\begin{aligned}\mathcal{L} &= (\partial_+\phi - \frac{1}{\alpha}\partial_-\phi)\ \partial_-\phi \\ &= \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{\alpha}(\partial_-\phi)^2, \end{aligned} \tag{3}$$

where $\eta^{+-} = \eta^{-+} = 1, \eta^{++} = \eta^{--} = 0,\ \mu,\nu = \pm$, and α is a fixed parameter. The canonical momentum following from (3) is $\pi = \partial_-\phi$, which indicates that we are dealing with a constrained dynamical system. The Dirac procedure [24] or the Faddeev and Jackiw method [25] may be followed to construct the Hamiltonian framework which in its turn may be quantized canonically. The discussion in our case is straightforward and follows closely the one given in refs. [11, 23] and we are lead to the following LF Hamiltonian

$$H^{lf} = \int dx^-\ \frac{1}{\alpha}(\partial_-\phi)^2. \tag{4}$$

The equal-τ commutator is derived to be

$$\Big[\phi(\tau, x^-), \phi(\tau, y^-)\Big] = \frac{-i}{4}\epsilon(x^- - y^-) \tag{5}$$

while π gets eliminated from the theory. The LF commutator (5) is nonlocal in x^- and nonvanishing only on the light-cone. It does *not* conflict with the microcausality principle unlike the equal-time commutator (2) in the *instant form* theory. The Heisenberg equation of motion for the field operator is found to be

$$\partial_+\phi = \frac{1}{i}\Big[\phi, H^{lf}\Big] = \frac{1}{\alpha}\partial_-\phi \tag{6}$$

and the Lagrange equation

$$\partial_- \left[\partial_+\phi - \frac{1}{\alpha}\partial_-\phi\right] = 0. \tag{7}$$

is recovered.

The commutator (5) can be realized in momentum space through the following Fourier transform of the field

$$\phi(\tau, x^-) = \frac{1}{\sqrt{2\pi}}\int dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}}\left[a(\tau,k^+)e^{-ik^+x^-} + a^\dagger(\tau,k^+)e^{ik^+x^-}\right], \tag{8}$$

if the creation and annihilation operators $a^\dagger$ and a are assumed to satisfy the equal-τ canonical commutation relations, with the nonvanishing one given by $\left[a(\tau,k^+), a^\dagger(\tau,p^+)\right] = \delta(k^+ - p^+)$. On using the equation of motion (6) we derive easily

$$a(x^+,k^+) = e^{-ik^-x^+}a(k^+), \qquad a^\dagger(x^+,k^+) = e^{ik^-x^+}a^\dagger(k^+), \tag{9}$$

where

$$k^- = \frac{1}{\alpha}k^+, \qquad \text{implying} \qquad 2k^+k^- = \frac{2}{\alpha}(k^+)^2. \tag{10}$$

The Fourier transform of the field then assumes the form

$$\phi(x^+, x^-) = \frac{1}{\sqrt{2\pi}}\int dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}}\left[a(k^+)e^{-ik\cdot x} + a^\dagger(k^+)e^{ik\cdot x}\right]. \tag{11}$$

where $k\cdot x \equiv k^-x^+ + k^+x^- = k^+(x^- + x^+/\alpha)$ and the nonvanishing commutator satisfies $\left[a(k^+), a^\dagger(p^+)\right] = \delta(k^+ - p^+)$.

We recall now that on the LF the dispersion relation associated with the free massive particle is $2p^+p^- = (p^\perp p^\perp + m^2) > 0$. It has no square root, like in the conventional case $p^0 = \pm\sqrt{\vec{p}^2 + m^2}$. The signs of p^+ and p^- are thus *correlated* in view of $p^+p^- > 0$. For massless particles the correlation ceases to exist at the point $p^\perp \to 0$ when $2p^+p^- = p^\perp p^\perp \to 0$. On the other hand, for finite values of α, the dispersion relation (10) obtained above is different from that of a free massless particle. Only in the limit when $|\alpha| \to \infty$ does $2k^+k^- \to 0$ and, according to (11), $\phi \to \phi_R(x^-)$, which describes a right (moving) chiral boson.

Consider next the components of the classical canonical energy-momentum tensor $T^{\mu\nu}$. We find

$$\begin{aligned} T^{+-} = -T^{-+} = \frac{1}{\alpha}T^{++} &= \frac{1}{\alpha}(\partial_-\phi)^2, \\ T^{--} &= (\partial_+\phi)^2 - \frac{2}{\alpha}(\partial_+\phi)(\partial_-\phi). \end{aligned} \tag{12}$$

They obey the on shell conservation equations

$$\partial_\mu T^{\mu\pm} = 2(\partial_\mp\phi)\,\partial_-\left[\partial_+\phi - \frac{1}{\alpha}\partial_-\phi\right] = 0 \tag{13}$$

as may be easily checked. We may thus define, if the surface integrals can be ignored, the following conserved translation generators

$$P^+ = \int dx^- \; :T^{++}: \, = \int dx^- \; :(\partial_- \phi)^2: \, = \int dk^+ \theta(k^+)\; N(k^+)\; (k^+) \tag{14}$$

and

$$P^- \equiv H^{lf} = \int dx^- \; :T^{+-}: \, = \frac{1}{\alpha}\; P^+, \qquad \text{implying} \qquad 2P^+P^- = \frac{2}{\alpha}(P^+)^2. \tag{15}$$

Here $N(k^+) = a^\dagger(k^+)a(k^+)$ is the number operator and $:\;:$ indicates the normal ordering.

From (13) and in virtue of $(T^{+-} + T^{-+}) = 0$, following from (12), we derive

$$\partial_+ \left[x^- T^{++} + x^+ T^{+-} \right] + \partial_- \left[x^- T^{-+} + x^+ T^{--} \right] = 0, \tag{16}$$

which is valid on shell. It hence allows us to define the following conserved symmetry generator

$$M = x^+ P^- + \int dx^- \; x^- T^{++}. \tag{17}$$

The generators M, P^+, P^- are shown to form a closed algebra : $[M, P^+] = -iP^+$, $[M, P^-] = -iP^-$, and $[P^+, P^-] = 0$. The operator M thus generates the scale (boost) transformations on $P^\pm$ by the same amount which leaves P^+/P^- invariant. The *mass operator* $2P^+P^-$, however, does get scaled and is *not* invariant under the transformations generated by M. The Lagrange equation is easily shown to be form invariant under the infinitesimal symmetry transformation $\phi \to \phi + \epsilon(x^- + x^+/\alpha)\partial_- \phi$ generated by M. The operator M clearly resembles the (kinematical) Lorentz boost generator, *viz*, $M^{+-} \equiv -x^+P^- + \int dx^- \; x^- T^{++}$ which, as seen from (12) and (13), however, is *not conserved* in the manifestly noncovariant model under consideration.

In the limit when $|\alpha| \to \infty$ we find $\phi \to \phi_R(x^-)$ while $H^{lf} \to 0$, like in the case of the LF Hamiltonian of the free massless scalar theory. The field commutator of ϕ_R is found to be : $[\phi_R(x^-), \phi_R(y^-)] = -i\epsilon(x^- - y^-)/4$. The limiting case is thus shown to describe a right (moving) chiral boson theory with the Lagrangian density given in (3).

An alternative form of the Lagrangian density is also possible. We recall that in the quantization of gauge theory in 3+1 dimensions it is found useful to introduce an auxiliary Nakanishi-Lautrup field $B(x)$ of canonical mass dimension two and add to the Lagrangian density $(B\partial_\mu A^\mu + \alpha B^2)$ as the gauge-fixing term. In the two dimensional theory under consideration it is also possible to follow this procedure, since the corresponding $B(x)$ field here carries the canonical mass dimension one. The discussion parallel to the one given above may be based [26] equally well on the following *front form* Lagrangian density

$$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \sqrt{2}B(x)(\partial_-\phi) + \frac{\alpha}{2}B(x)^2. \tag{18}$$

If we eliminate in it the auxiliary field B by using its equation of motion it leads back to (3). The conclusions following from the one or the other are the same.

We make only brief comments on the other models. Siegel's [6] theory which employs

$$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + B(x)(\partial_0\phi - \partial_1\phi)^2 \tag{19}$$

is afflicted by anomaly which is to be eliminated by the addition of a Wess-Zumino term. The resulting theory does not describe [7] pure chiral bosons since they are coupled to the gravity. In this model the auxiliary field carries vanishing canonical dimension and, for example, a B^2 term cannot be added to it without introducing the dimensionful parameters. The model based on the idea of implementing the chiral constraint through a linear constraint [8, 26],

$$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + B_\mu(\eta^{\mu\nu} - \epsilon^{\mu\nu})\partial_\nu\phi, \tag{20}$$

where B_μ is Lagrange multiplier field, does not seem to exhibit physical excitations [9]. We note that the field B_μ carries dimension one and that this is the usual procedure in the classical theory which, however, breaks down in the quantized theory.

Conclusions

The *front form* quantized theory of chiral boson is straightforward to construct. It may be based on the modified FJ model as described by the Lagrangian density (3) or in its alternative form (18). The discussion becomes transparent on the LF and it also avoids the conflict with the requirement of the principle of microcausality, in contrast to what found in the corresponding *instant form* theory.

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