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QCD Factorization for $B \to \pi\pi$ Decays: Strong Phases and CP Violation in the Heavy Quark Limit*

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Abstract

We show that, in the heavy quark limit, the hadronic matrix elements that enter B meson decays into two light mesons can be computed from first principles, including 'non-factorizable' strong interaction corrections, and expressed in terms of form factors and meson light-cone distribution amplitudes. The conventional factorization result follows in the limit when both power corrections in $1/m_b$ and radiative corrections in α_s are neglected. We compute the order-α_s corrections to the decays $B_d \to \pi^+\pi^-$, $B_d \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ in the heavy quark limit and briefly discuss the phenomenological implications for the branching ratios, strong phases and CP violation.

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We show that, in the heavy quark limit, the hadronic matrix elements that enter B meson decays into two light mesons can be computed from first principles, including 'non-factorizable' strong interaction corrections, and expressed in terms of form factors and meson light-cone distribution amplitudes. The conventional factorization result follows in the limit when both power corrections in $1/m_b$ and radiative corrections in α_s are neglected. We compute the order-α_s corrections to the decays $B_d \to \pi^+\pi^-$, $B_d \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ in the heavy quark limit and briefly discuss the phenomenological implications for the branching ratios, strong phases and CP violation.

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The detailed study of B meson decays is a key source of information for understanding CP violation and the physics of flavour. The interest in this field is reinforced by the numerous upcoming experiments that will test crucial aspects of B decay properties with unprecedented scope and precision. Among the large number of B decay channels, two-body non-leptonic modes, such as $B \to \pi\pi$, $B \to \pi K$ etc., open a particularly rich field of phenomenological investigation. A theoretical treatment, however, is generally complicated owing to the non-trivial QCD dynamics related to the all-hadronic final state.

In this Letter we describe important simplifications that occur in the limit $m_b \gg \Lambda_{QCD}$, when the b quark mass is large compared to the strong interaction scale Λ_{QCD}. We find that in this limit the hadronic matrix elements for, say, $\bar{B} \to \pi\pi$ can be represented in the form

$$\langle\pi\pi|Q|\bar{B}\rangle = \langle\pi|j_1|\bar{B}\rangle\langle\pi|j_2|0\rangle \cdot \cdot\Big[1 + \sum r_n \alpha_s^n + \mathcal{O}(\Lambda_{QCD}/m_b)\Big], \qquad (1)$$

where Q is a local operator in the weak effective Hamiltonian and $j_{1,2}$ are bilinear quark currents. Neglecting power corrections in Λ_{QCD} *and* radiative corrections in α_s, the original matrix element factorizes into a form factor times a decay constant (we call this conventional factorization). At higher order in α_s this simple factorization is broken, but the corrections can be calculated systematically in terms of short-distance coefficients and meson light-cone distribution amplitudes. This is similar in spirit to the well-known framework of perturbative factorization for exclusive processes in QCD at large momentum transfer [1], as applied, for example, to the electromagnetic form factor of the pion. An interesting consequence of (1) is that strong interaction phases are formally of order α_s or Λ_{QCD}/m_b in the heavy quark limit. If this limit works well, the approach discussed here allows us to calculate these phases systematically; CP violating weak phases can then be disentangled. Here we present a numerical analysis of $B \to \pi\pi$ decay amplitudes based on the heavy quark limit. We also briefly discuss important power corrections, which should eventually be estimated in order to obtain a satisfactory phenomenology at realistic b quark masses. Details of the argument that leads to the factorization formula (3) below will be explained in a forthcoming paper.

The effective weak Hamiltonian describing $\bar{B}$ decays is given by [2]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \Big[C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3\ldots6,8} C_i Q_i\Big], \qquad (2)$$

where $\lambda_p = V_{pd}^* V_{pb}$. The Q_i are local $\Delta B = 1$, $\Delta S = 0$ operators, and C_i the corresponding short-distance Wilson coefficients. We neglect electroweak penguin operators and all terms not relevant to $\bar{B} \to \pi\pi$ decays.

The essential theoretical problem for obtaining the $\bar{B} \to \pi\pi$ amplitudes is the evaluation of the hadronic matrix elements $\langle\pi\pi|Q_i|\bar{B}\rangle$. Let π_1 denote the pion that picks up the light spectator quark in the $\bar{B}$ meson, and π_2 the pion whose valence partons are supplied by the weak decay of the b quark. In the heavy quark limit both pions emerge with large energy $m_B/2$ (in the $\bar{B}$ rest frame). Power counting based on the asymptotic form of the leading-twist pion distribution amplitude shows that a leading-power contribution to the $\langle\pi\pi|Q_i|\bar{B}\rangle$ matrix element requires both valence quarks of π_2 to carry energy of order m_b. The $q\bar{q}$ pair is ejected from the weak interaction region as a small-size colour singlet object. As a consequence soft gluons with momentum of order Λ_{QCD} decouple at leading order in Λ_{QCD}/m_b, and π_2 can be represented by its leading-twist light-cone distribution amplitude. On the other hand, the spectator quark in the $\bar{B}$ meson carries momentum of order Λ_{QCD} and is transferred as a soft quark to π_1, unless it undergoes a hard interaction. The endpoint suppression of the pion wave function is not sufficient to ensure the dominance of hard interactions. [We adopt the point of view that for realistic b quark masses perturbative Sudakov suppres-

sion does not cut off soft contributions efficiently enough before one enters the non-perturbative regime.] Therefore π_1 cannot always be represented by its light-cone distribution amplitude. At leading power in Λ_{QCD}/m_b, we find that the soft interactions can be absorbed into the $B \to \pi_1$ form factor. On the other hand, any interaction of the spectator quark with the quarks of π_2 is hard at leading power and can be written as a convolution of three light-cone distribution amplitudes. This discussion can be summarized by the factorization formula

$$\langle \pi(p')\pi(q)|Q_i|\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx\, T_i^I(x)\Phi_\pi(x) + \int_0^1 d\xi dx dy\, T_i^{II}(\xi,x,y)\Phi_B(\xi)\Phi_\pi(x)\Phi_\pi(y), \qquad (3)$$

which is valid up to corrections of relative order Λ_{QCD}/m_b. Here $f^{B\to\pi}(q^2)$ is a $B \to \pi$ form factor evaluated at $q^2 = m_\pi^2 \approx 0$, and Φ_π (Φ_B) are leading-twist light-cone distribution amplitudes of the pion (B meson), normalized to 1. The $T_i^{I,II}$ denote hard-scattering kernels, which are calculable in perturbation theory. T_i^I starts at $\mathcal{O}(\alpha_s^0)$; at higher order in α_s it contains 'non-factorizable' gluon exchange, including penguin topologies, see the first two rows of Fig. 1 for the corrections at order α_s. Hard, 'non-factorizable' interactions involving the spectator quark are part of T_i^{II} (last row of Fig. 1). Annihilation topologies also exist, but are power-suppressed in Λ_{QCD}/m_b. The significance of the factorization formula is that all the non-perturbative effects in the $B \to \pi\pi$ amplitudes can be absorbed into the form factor and the light-cone wave functions.

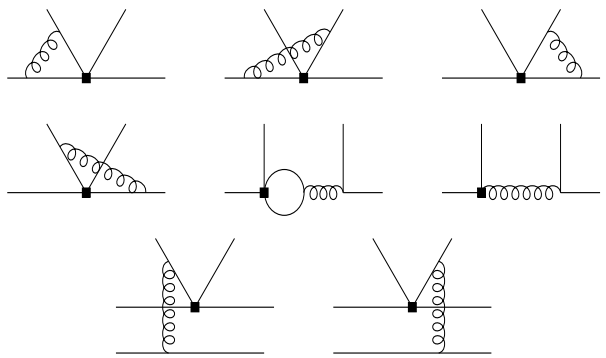

FIG. 1. Order α_s corrections to the hard scattering kernels T_i^I (first two rows) and T_i^{II} (last row). In the case of T_i^I, the spectator quark does not participate in the hard interaction and is not drawn. The two lines directed upwards represent the two quarks that make up π_2.

The following comments are in order:

(i) When α_s corrections are neglected T_i^{II} is zero and T_i^I is an x-independent constant. Conventional factorization in terms of the form factor and the pion decay constant is then recovered as a rigorous prediction in the infinite quark mass limit. The perturbative corrections are process-dependent, but calculable. Their inclusion cancels the scale-dependence of the leading-order factorization result.

(ii) The infrared finiteness of the hard scattering amplitude follows because the infrared divergences in the first four diagrams of Fig. 1 cancel in their sum. This cancellation is the technical manifestation of Bjorken's colour transparency argument [3]. Colour transparency does not apply to hard gluon interactions. These, however, are suppressed by α_s and are calculable.

(iii) The hard scattering contribution to the $B \to \pi$ form factor is suppressed by one power of α_s relative to the soft contribution, in which the B meson spectator undergoes no hard interaction. As a consequence the assumption that $B \to \pi\pi$ can be treated entirely in the hard scattering picture of [1] would miss the leading contribution in the heavy quark limit.

(iv) The decay amplitude acquires an imaginary part through the hard scattering kernels. In the heavy quark limit, the strong interaction phases can therefore be computed as expansions in α_s. In terms of hadronic intermediate states that saturate the unitarity relation this implies systematic cancellations among many intermediate states with potentially large individual rescattering phases. An estimate of rescattering effects on the basis of Regge theory is not compatible with the picture that emerges in the heavy quark limit.

(v) The factorization formula (3) generalizes to the decays into a heavy-light final state, if the heavy particle absorbs the B meson spectator quark. In this case the second line in (3) is power-suppressed and only the form factor term survives. An expression of this form has been used by Politzer and Wise to compute the 1-loop corrections to the decay rate ratio $\Gamma(\bar{B} \to D^*\pi)/\Gamma(\bar{B} \to D\pi)$ [4]. The factorization formula does *not* hold for heavy-light final states, in which the light meson absorbs the B meson spectator quark, or for a heavy-heavy final state. In this case, conventional factorization can also not be justified.

The result of an explicit calculation of the $\bar{B} \to \pi\pi$ decay amplitudes at order α_s can be compactly expressed as $\langle\pi\pi|\mathcal{H}_{eff}|\bar{B}\rangle = G_F/\sqrt{2}\sum_{p=u,c}\lambda_p\langle\pi\pi|\mathcal{T}_p|\bar{B}\rangle$, where

$$\begin{aligned}\mathcal{T}_p &= a_1^p(\pi\pi)\,(\bar{u}b)_{V-A}\otimes(\bar{d}u)_{V-A}\\ &+ a_2^p(\pi\pi)\,(\bar{d}b)_{V-A}\otimes(\bar{u}u)_{V-A}\\ &+ a_3(\pi\pi)\,(\bar{d}b)_{V-A}\otimes(\bar{q}q)_{V-A}\\ &+ a_4^p(\pi\pi)\,(\bar{q}b)_{V-A}\otimes(\bar{d}q)_{V-A}\\ &+ a_5(\pi\pi)\,(\bar{d}b)_{V-A}\otimes(\bar{q}q)_{V+A}\\ &+ a_6^p(\pi\pi)\,(-2)(\bar{q}b)_{S-P}\otimes(\bar{d}q)_{S+P}. \qquad (4)\end{aligned}$$

The symbol $\otimes$ is defined through $\langle\pi\pi|j_1\otimes j_2|\bar{B}\rangle \equiv \langle\pi|j_1|\bar{B}\rangle\langle\pi|j_2|0\rangle$. A summation over $q = u, d$ is implied. Note that the term proportional to $a_6^p(\pi\pi)$ results in a power correction that should be dropped in the heavy

quark limit. We will comment further on this term below.

Together with $a_1^c(\pi\pi) = a_2^c(\pi\pi) = 0$ and the leading-order coefficient $a_6^p(\pi\pi) = C_6 + C_5/N$, the QCD coefficients $a_i^p(\pi\pi)$ read at next-to-leading order (NLO)

$$a_1^u(\pi\pi) = C_1 + \frac{1}{N}C_2 + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_2\,F, \tag{5}$$

$$a_2^u(\pi\pi) = C_2 + \frac{1}{N}C_1 + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_1\,F, \tag{6}$$

$$a_3(\pi\pi) = C_3 + \frac{1}{N}C_4 + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_4\,F, \tag{7}$$

$$\begin{aligned} a_4^p(\pi\pi) = {} & C_4 + \frac{1}{N}C_3 - \frac{\alpha_s}{4\pi}\frac{C_F}{N}\left[\left(\frac{4}{3}C_1 + \frac{44}{3}C_3\right.\right. \\ & \left. + \frac{4f}{3}(C_4 + C_6)\right)\ln\frac{\mu}{m_b} + \left(G_\pi(s_p) - \frac{2}{3}\right)C_1 \\ & + \left(G_\pi(0) + G_\pi(1) - f_\pi^I - f_\pi^{II} + \frac{50}{3}\right)C_3 \\ & \left. + \Big(3G_\pi(0) + G_\pi(s_c) + G_\pi(1)\Big)(C_4 + C_6) + G_{\pi,8}C_8\right], \end{aligned} \tag{8}$$

$$a_5(\pi\pi) = C_5 + \frac{1}{N}C_6 + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_6\,(-F - 12). \tag{9}$$

Here $C_F = (N^2-1)/(2N)$ and $N = 3$ ($f = 5$) is the number of colours (flavours). [Note that our definition of C_1 and C_2 differs from the convention of [2], where the labels 1 and 2 are interchanged.] The internal quark mass in penguin diagrams enters as s_p, where $s_u = 0$ and $s_c = m_c^2/m_b^2$. In addition we have used ($\bar{x} \equiv 1 - x$)

$$F = -12\ln\frac{\mu}{m_b} - 18 + f_\pi^I + f_\pi^{II}, \tag{10}$$

$$f_\pi^I = \int_0^1 dx\, g(x)\Phi_\pi(x), \quad G_{\pi,8} = \int_0^1 dx\, G_8(x)\Phi_\pi(x), \tag{11}$$

$$G_\pi(s) = \int_0^1 dx\, G(s,x)\Phi_\pi(x), \tag{12}$$

with the hard-scattering functions

$$g(x) = 3\frac{1-2x}{1-x}\ln x - 3i\pi, \quad G_8(x) = \frac{2}{\bar{x}}, \tag{13}$$

$$G(s,x) = -4\int_0^1 du\, u(1-u)\ln(s - u(1-u)\bar{x} - i\epsilon). \tag{14}$$

The hard spectator scattering contribution is given by

$$f_\pi^{II} = \frac{4\pi^2}{N}\frac{f_\pi f_B}{f_+(0)m_B^2}\int_0^1 d\xi\,\frac{\Phi_B(\xi)}{\xi}\left[\int_0^1 dx\,\frac{\Phi_\pi(x)}{x}\right]^2, \tag{15}$$

where f_π (f_B) is the pion (B meson) decay constant, m_B the B meson mass, $f_+(0)$ the $B \to \pi$ form factor at zero momentum transfer, and ξ the light-cone momentum fraction of the spectator in the B meson. f_π^{II} depends on the wave function Φ_B through the integral $\int_0^1 d\xi\,\Phi_B(\xi)/\xi \equiv m_B/\lambda_B$. This introduces one new hadronic parameter λ_B. Since $\Phi_B(\xi)$ has support only for ξ of order Λ_{QCD}/m_B, λ_B is of order Λ_{QCD}.

Writing the transition operator $\mathcal{T}_p$ in terms of the QCD coefficients $a_i^p(\pi\pi)$ is a convenient notation for phenomenological applications. The notation generalizes the conventional parameters $a_{1,2}$ [5], which are seen to be process-dependent beyond leading order. We emphasize that in the present context the $a_i^p(\pi\pi)$ are not phenomenological parameters, but genuine predictions of QCD in the heavy quark limit. The Wilson coefficients C_i entering the $a_i^p(\pi\pi)$ are to be taken at NLO [2], where we consistently drop terms of $\mathcal{O}(\alpha_s^2)$ in (5)–(9). The physical amplitudes derived from (4) are independent of the renormalization scale (μ) and scheme through $\mathcal{O}(\alpha_s)$. The coefficients $a_1(\pi\pi)$–$a_5(\pi\pi)$ multiply scale and scheme independent matrix elements of (axial-)vector currents. Accordingly for $a_1(\pi\pi)$–$a_5(\pi\pi)$ the scale and scheme dependence in the Wilson coefficients C_i is canceled by the $\mathcal{O}(\alpha_s)$ corrections in the hard-scattering amplitudes. In the case of $a_6^p(\pi\pi)$, a scale and scheme dependence remains, which is precisely the one needed to cancel the corresponding dependence in the matrix elements of the (pseudo-)scalar currents, multiplying $a_6^p(\pi\pi)$ in (4). Besides the $\ln(\mu/m_b)$ terms the hard-scattering amplitudes contain a scheme dependent constant, which we have obtained in the NDR scheme as defined in [6]. This fixes the scheme to be used for the NLO coefficients C_i.

At NLO the factorization coefficients $a_i^p(\pi\pi)$ acquire complex phases, entering through the functions $g(x)$ and $G(s,x)$ in (13) and (14). Being of order α_s, these phases are generically small, except in cases where the lowest order contribution is numerically suppressed. This happens *e.g.* for $a_2^u(\pi\pi)$. Physically, the phases arise from final state rescattering, which is due to hard gluon exchange, and hence perturbative, in the heavy quark limit. The generation of strong interaction phases through the penguin function $G(s,x)$ has been discussed many years ago [7] and is commonly referred to as the Bander–Silverman–Soni (BSS) mechanism. In the present approach, the gluon virtuality $k^2 = \bar{x}m_B^2$ in the penguin diagram, which has usually been treated as a free phenomenological parameter, has a well-defined meaning. The x-dependence of $G(s,x)$ is convoluted with the pion wave function $\Phi_\pi(x)$, leaving no ambiguity as to the value of k^2. In addition we identify a further source of rescattering phases, represented by the function $g(x)$. This effect corresponds to hard gluon exchange between the two outgoing pions. Together with the BSS mechanism, it accounts for the complete asymptotic rescattering phases in $\bar{B} \to \pi\pi$ in the heavy quark limit.

Another novel result is the existence of the contribution from hard scattering involving the spectator quark in the B meson, expressed by f_π^{II} in (15). This mechanism is completely missed in phenomenological models of factorization. It is particularly important for the small coefficient $a_2^u(\pi\pi)$, where it leads to a sizable en-

hancement. Using $f_\pi = 131\,\mathrm{MeV}$, $f_B = (180 \pm 20)\,\mathrm{MeV}$, $f_+(0) = 0.275 \pm 0.025$, $\lambda_B = 0.3\,\mathrm{GeV}$ and the asymptotic pion wave function $\Phi_\pi(x) = 6x\bar{x}$, we find $f_\pi^{II} \approx 6.4$. The poor knowledge of the B meson parameter λ_B makes this number rather uncertain.

Numerical values for the $a_i^p(\pi\pi)$ are shown in Table I, using the pole masses $m_b = 4.8\,\mathrm{GeV}$, $m_c = 1.4$ GeV, the $\overline{\mathrm{MS}}$ masses $\bar{m}_t(\bar{m}_t) = 167\,\mathrm{GeV}$, $(\bar{m}_u + \bar{m}_d)(2\,\mathrm{GeV}) = 9\,\mathrm{MeV}$ and $\Lambda_{\overline{MS}}^{(5)} = 225\,\mathrm{MeV}$ as input parameters. $a_6^p(\pi\pi)$ multiplies the Λ_{QCD}/m_b-suppressed, but chirally enhanced combination

$$r_\chi = \frac{2m_{\pi^+}^2}{\bar{m}_b(\mu)(\bar{m}_u(\mu) + \bar{m}_d(\mu))} \approx 1.18 \quad [\text{at } \mu = m_b]. \tag{16}$$

In the following analysis, we give two results, one neglecting $a_6^p(\pi\pi)$ as formally power-suppressed, the other keeping the leading-order expression for $a_6^p(\pi\pi)$.

It is now straightforward to evaluate the decay amplitudes and branching ratios. The latter are given by $Br(\bar{B} \to \pi\pi) = \tau_B/(16\pi m_B) \cdot |A(\bar{B} \to \pi\pi)|^2 S$, where $S = 1$ for $\pi\pi = \pi^+\pi^-$, $\pi^-\pi^0$ and $S = 1/2$ for $\pi\pi = \pi^0\pi^0$. τ_B denotes the B meson lifetimes: $\tau(B^+) = 1.65\,\mathrm{ps}$, $\tau(B_d) = 1.56\,\mathrm{ps}$. The decay amplitudes read

$$\begin{aligned} A(\bar{B}_d \to \pi^+\pi^-) &= i\frac{G_F}{\sqrt{2}} m_B^2 f_+(0) f_\pi |\lambda_c| \cdot \\ &\cdot \big[R_b e^{-i\gamma} \left(a_1^u(\pi\pi) + a_4^u(\pi\pi) + a_6^u(\pi\pi) r_\chi\right) \\ &\quad - \left(a_4^c(\pi\pi) + a_6^c(\pi\pi) r_\chi\right)\big], \end{aligned} \tag{17}$$

$$\begin{aligned} A(\bar{B}_d \to \pi^0\pi^0) &= i\frac{G_F}{\sqrt{2}} m_B^2 f_+(0) f_\pi |\lambda_c| \cdot \\ &\cdot \big[R_b e^{-i\gamma} \left(-a_2^u(\pi\pi) + a_4^u(\pi\pi) + a_6^u(\pi\pi) r_\chi\right) \\ &\quad - \left(a_4^c(\pi\pi) + a_6^c(\pi\pi) r_\chi\right)\big], \end{aligned} \tag{18}$$

$$\begin{aligned} A(B^- \to \pi^-\pi^0) &= i\frac{G_F}{\sqrt{2}} m_B^2 f_+(0) f_\pi |\lambda_c| \cdot \\ &\cdot R_b/\sqrt{2} \cdot e^{-i\gamma} \left(a_1^u(\pi\pi) + a_2^u(\pi\pi)\right). \end{aligned} \tag{19}$$

Here $R_b = (1 - \lambda^2/2)|V_{ub}/V_{cb}|/\lambda$, where $\lambda = 0.22$ is the sine of the Cabibbo angle, γ is the phase of V_{ub}^*, and we will use $|V_{cb}| = 0.039 \pm 0.002$, $|V_{ub}/V_{cb}| = 0.085 \pm 0.020$. We find the branching fractions

$$\begin{aligned} &Br(\bar{B}_d \to \pi^+\pi^-) = \\ &\quad 6.5\,[6.1] \cdot 10^{-6} \left| e^{-i\gamma} + 0.09\,[0.18]\, e^{i\cdot 12.7\,[6.7]^\circ} \right|^2, \end{aligned} \tag{20}$$

$$\begin{aligned} &Br(\bar{B}_d \to \pi^0\pi^0) = \\ &\quad 5.2\,[7.7] \cdot 10^{-8} \left| e^{-i\gamma} + 0.73\,[1.11]\, e^{-i\cdot 137\,[149]^\circ} \right|^2, \end{aligned} \tag{21}$$

$$Br(B^- \to \pi^-\pi^0) = 4.3\,[4.3] \cdot 10^{-6}, \tag{22}$$

where the default values correspond to neglecting $a_6^p(\pi\pi)$ and the values in brackets use $a_6^p(\pi\pi)$ at leading order. While the predictions for the $\pi^+\pi^-$ and $\pi^-\pi^0$ final states are relatively robust, with errors on the order of $\pm 30\%$ due to the input parameters, the decay into $\pi^0\pi^0$ depends very sensitively on the parameter λ_B that controls the hard spectator scattering. If it is significantly smaller than $0.3\,\mathrm{GeV}$, a branching fraction of order 10^{-6} cannot be excluded. Eq. (20) can be converted into a result for the time-dependent CP asymmetry as a function of the CKM angle α. The direct CP asymmetry in the $\pi^+\pi^-$ mode is approximately $4\% \cdot \sin\gamma$.

The approach discussed here allows us to formulate, for the first time, rigorous predictions of QCD for exclusive non-leptonic B decays in the heavy quark limit. On the other hand, as the dependence on the formally power-suppressed coefficient $a_6^p(\pi\pi)$ demonstrates, the asymptotic limit may be problematic at $m_b \approx 5\,\mathrm{GeV}$ and the applicability of the theory has to be decided on a case-by-case basis. The most important power corrections are those that depend on the chirally enhanced combination (16). The α_s corrections to all such terms can in fact be identified. However, the factorization formula breaks down in this case, because the relevant twist-3 wave functions do not fall off fast enough at the endpoints. A detailed discussion of this point, and of its phenomenological consequences, will be given elsewhere.

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TABLE I. The QCD coefficients $a_i^p(\pi\pi)$ at NLO for three different renormalization scales μ. Leading order values are shown in parenthesis for comparison.

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2m_b$
$a_1^u(\pi\pi)$	$1.047 + 0.033i$	$1.038 + 0.018i$	$1.027 + 0.010i$
	(1.038)	(1.020)	(1.010)
$a_2^u(\pi\pi)$	$0.061 - 0.106i$	$0.082 - 0.080i$	$0.108 - 0.064i$
	(0.066)	(0.140)	(0.200)
$a_3(\pi\pi)$	$0.005 + 0.003i$	$0.004 + 0.002i$	$0.003 + 0.001i$
	(0.004)	(0.002)	(0.001)
$a_4^u(\pi\pi)$	$-0.030 - 0.019i$	$-0.029 - 0.015i$	$-0.026 - 0.013i$
	(-0.027)	(-0.020)	(-0.014)
$a_4^c(\pi\pi)$	$-0.038 - 0.009i$	$-0.034 - 0.008i$	$-0.031 - 0.007i$
	(-0.027)	(-0.020)	(-0.014)
$a_5(\pi\pi)$	$-0.006 - 0.004i$	$-0.005 - 0.002i$	$-0.003 - 0.001i$
	(-0.005)	(-0.002)	(-0.001)
$a_6^p(\pi\pi) r_\chi$	–	–	–
	(-0.036)	(-0.030)	(-0.024)