

# **QCD Factorization for $B \rightarrow \pi\pi$ Decays: Strong Phases and CP Violation in the Heavy Quark Limit\***

M. BENEKE<sup>a</sup>, G. BUCHALLA<sup>a</sup>, M. NEUBERT<sup>b</sup>, C.T. SACHRAJDA<sup>c</sup>

<sup>a</sup>THEORY DIVISION, CERN, CH-1211 GENEVA 23, SWITZERLAND

<sup>b</sup>STANFORD LINEAR ACCELERATOR CENTER, STANFORD UNIVERSITY, STANFORD, CA 94309, USA

<sup>c</sup>DEPARTMENT OF PHYSICS AND ASTRONOMY, UNIVERSITY OF SOUTHAMPTON,  
 SOUTHAMPTON SO17 1BJ, UK

## **Abstract**

We show that, in the heavy quark limit, the hadronic matrix elements that enter  $B$  meson decays into two light mesons can be computed from first principles, including ‘non-factorizable’ strong interaction corrections, and expressed in terms of form factors and meson light-cone distribution amplitudes. The conventional factorization result follows in the limit when both power corrections in  $1/m_b$  and radiative corrections in  $\alpha_s$  are neglected. We compute the order- $\alpha_s$  corrections to the decays  $B_d \rightarrow \pi^+\pi^-$ ,  $B_d \rightarrow \pi^0\pi^0$  and  $B^+ \rightarrow \pi^+\pi^0$  in the heavy quark limit and briefly discuss the phenomenological implications for the branching ratios, strong phases and CP violation.

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# QCD Factorization for $B \rightarrow \pi\pi$ Decays: Strong Phases and CP Violation in the Heavy Quark Limit

M. Beneke<sup>a</sup>, G. Buchalla<sup>a</sup>, M. Neubert<sup>b</sup>, C.T. Sachrajda<sup>c</sup>

<sup>a</sup>*Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

<sup>b</sup>*Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*

<sup>c</sup>*Department of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK*  
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We show that, in the heavy quark limit, the hadronic matrix elements that enter  $B$  meson decays into two light mesons can be computed from first principles, including ‘non-factorizable’ strong interaction corrections, and expressed in terms of form factors and meson light-cone distribution amplitudes. The conventional factorization result follows in the limit when both power corrections in  $1/m_b$  and radiative corrections in  $\alpha_s$  are neglected. We compute the order- $\alpha_s$  corrections to the decays  $B_d \rightarrow \pi^+\pi^-$ ,  $B_d \rightarrow \pi^0\pi^0$  and  $B^+ \rightarrow \pi^+\pi^0$  in the heavy quark limit and briefly discuss the phenomenological implications for the branching ratios, strong phases and CP violation.

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The detailed study of  $B$  meson decays is a key source of information for understanding CP violation and the physics of flavour. The interest in this field is reinforced by the numerous upcoming experiments that will test crucial aspects of  $B$  decay properties with unprecedented scope and precision. Among the large number of  $B$  decay channels, two-body non-leptonic modes, such as  $B \rightarrow \pi\pi$ ,  $B \rightarrow \pi K$  etc., open a particularly rich field of phenomenological investigation. A theoretical treatment, however, is generally complicated owing to the non-trivial QCD dynamics related to the all-hadronic final state.

In this Letter we describe important simplifications that occur in the limit  $m_b \gg \Lambda_{QCD}$ , when the  $b$  quark mass is large compared to the strong interaction scale  $\Lambda_{QCD}$ . We find that in this limit the hadronic matrix elements for, say,  $\bar{B} \rightarrow \pi\pi$  can be represented in the form

$$\langle \pi\pi | Q | \bar{B} \rangle = \langle \pi | j_1 | \bar{B} \rangle \langle \pi | j_2 | 0 \rangle \cdot \left[ 1 + \sum r_n \alpha_s^n + \mathcal{O}(\Lambda_{QCD}/m_b) \right], \quad (1)$$

where  $Q$  is a local operator in the weak effective Hamiltonian and  $j_{1,2}$  are bilinear quark currents. Neglecting power corrections in  $\Lambda_{QCD}$  and radiative corrections in  $\alpha_s$ , the original matrix element factorizes into a form factor times a decay constant (we call this conventional factorization). At higher order in  $\alpha_s$  this simple factorization is broken, but the corrections can be calculated systematically in terms of short-distance coefficients and meson light-cone distribution amplitudes. This is similar in spirit to the well-known framework of perturbative factorization for exclusive processes in QCD at large momentum transfer [1], as applied, for example, to the electromagnetic form factor of the pion. An interesting consequence of (1) is that strong interaction phases are formally of order  $\alpha_s$  or  $\Lambda_{QCD}/m_b$  in the heavy quark limit. If this limit works well, the approach discussed here allows us to calculate these phases systematically; CP violating weak phases can then be disentangled. Here we present a numerical analysis of  $B \rightarrow \pi\pi$  decay amplitudes

based on the heavy quark limit. We also briefly discuss important power corrections, which should eventually be estimated in order to obtain a satisfactory phenomenology at realistic  $b$  quark masses. Details of the argument that leads to the factorization formula (3) below will be explained in a forthcoming paper.

The effective weak Hamiltonian describing  $\bar{B}$  decays is given by [2]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[ C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3\dots 6,8} C_i Q_i \right], \quad (2)$$

where  $\lambda_p = V_{pd}^* V_{pb}$ . The  $Q_i$  are local  $\Delta B = 1$ ,  $\Delta S = 0$  operators, and  $C_i$  the corresponding short-distance Wilson coefficients. We neglect electroweak penguin operators and all terms not relevant to  $\bar{B} \rightarrow \pi\pi$  decays.

The essential theoretical problem for obtaining the  $\bar{B} \rightarrow \pi\pi$  amplitudes is the evaluation of the hadronic matrix elements  $\langle \pi\pi | Q_i | \bar{B} \rangle$ . Let  $\pi_1$  denote the pion that picks up the light spectator quark in the  $\bar{B}$  meson, and  $\pi_2$  the pion whose valence partons are supplied by the weak decay of the  $b$  quark. In the heavy quark limit both pions emerge with large energy  $m_B/2$  (in the  $\bar{B}$  rest frame). Power counting based on the asymptotic form of the leading-twist pion distribution amplitude shows that a leading-power contribution to the  $\langle \pi\pi | Q_i | \bar{B} \rangle$  matrix element requires both valence quarks of  $\pi_2$  to carry energy of order  $m_b$ . The  $q\bar{q}$  pair is ejected from the weak interaction region as a small-size colour singlet object. As a consequence soft gluons with momentum of order  $\Lambda_{QCD}$  decouple at leading order in  $\Lambda_{QCD}/m_b$ , and  $\pi_2$  can be represented by its leading-twist light-cone distribution amplitude. On the other hand, the spectator quark in the  $\bar{B}$  meson carries momentum of order  $\Lambda_{QCD}$  and is transferred as a soft quark to  $\pi_1$ , unless it undergoes a hard interaction. The endpoint suppression of the pion wave function is not sufficient to ensure the dominance of hard interactions. [We adopt the point of view that for realistic  $b$  quark masses perturbative Sudakov suppres-

sion does not cut off soft contributions efficiently enough before one enters the non-perturbative regime.] Therefore  $\pi_1$  cannot always be represented by its light-cone distribution amplitude. At leading power in  $\Lambda_{QCD}/m_b$ , we find that the soft interactions can be absorbed into the  $B \rightarrow \pi_1$  form factor. On the other hand, any interaction of the spectator quark with the quarks of  $\pi_2$  is hard at leading power and can be written as a convolution of three light-cone distribution amplitudes. This discussion can be summarized by the factorization formula

$$\begin{aligned} \langle \pi(p') \pi(q) | Q_i | \bar{B}(p) \rangle &= f^{B \rightarrow \pi}(q^2) \int_0^1 dx T_i^I(x) \Phi_\pi(x) \\ &+ \int_0^1 d\xi dx dy T_i^{II}(\xi, x, y) \Phi_B(\xi) \Phi_\pi(x) \Phi_\pi(y), \end{aligned} \quad (3)$$

which is valid up to corrections of relative order  $\Lambda_{QCD}/m_b$ . Here  $f^{B \rightarrow \pi}(q^2)$  is a  $B \rightarrow \pi$  form factor evaluated at  $q^2 = m_\pi^2 \approx 0$ , and  $\Phi_\pi$  ( $\Phi_B$ ) are leading-twist light-cone distribution amplitudes of the pion ( $B$  meson), normalized to 1. The  $T_i^{I,II}$  denote hard-scattering kernels, which are calculable in perturbation theory.  $T_i^I$  starts at  $\mathcal{O}(\alpha_s^0)$ ; at higher order in  $\alpha_s$  it contains ‘non-factorizable’ gluon exchange, including penguin topologies, see the first two rows of Fig. 1 for the corrections at order  $\alpha_s$ . Hard, ‘non-factorizable’ interactions involving the spectator quark are part of  $T_i^{II}$  (last row of Fig. 1). Annihilation topologies also exist, but are power-suppressed in  $\Lambda_{QCD}/m_b$ . The significance of the factorization formula is that all the non-perturbative effects in the  $B \rightarrow \pi\pi$  amplitudes can be absorbed into the form factor and the light-cone wave functions.

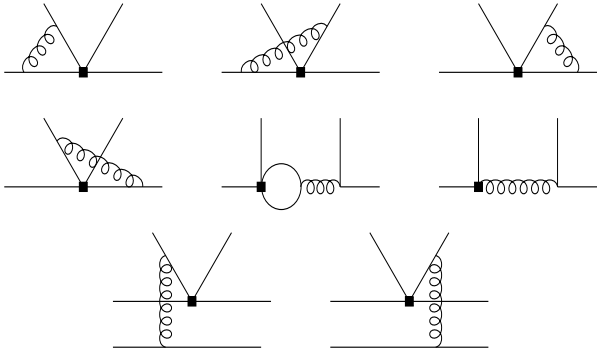


FIG. 1. Order  $\alpha_s$  corrections to the hard scattering kernels  $T_i^I$  (first two rows) and  $T_i^{II}$  (last row). In the case of  $T_i^I$ , the spectator quark does not participate in the hard interaction and is not drawn. The two lines directed upwards represent the two quarks that make up  $\pi_2$ .

The following comments are in order:

(i) When  $\alpha_s$  corrections are neglected  $T_i^{II}$  is zero and  $T_i^I$  is an  $x$ -independent constant. Conventional factorization in terms of the form factor and the pion decay constant is then recovered as a rigorous prediction in the

infinite quark mass limit. The perturbative corrections are process-dependent, but calculable. Their inclusion cancels the scale-dependence of the leading-order factorization result.

(ii) The infrared finiteness of the hard scattering amplitude follows because the infrared divergences in the first four diagrams of Fig. 1 cancel in their sum. This cancellation is the technical manifestation of Bjorken’s colour transparency argument [3]. Colour transparency does not apply to hard gluon interactions. These, however, are suppressed by  $\alpha_s$  and are calculable.

(iii) The hard scattering contribution to the  $B \rightarrow \pi$  form factor is suppressed by one power of  $\alpha_s$  relative to the soft contribution, in which the  $B$  meson spectator undergoes no hard interaction. As a consequence the assumption that  $B \rightarrow \pi\pi$  can be treated entirely in the hard scattering picture of [1] would miss the leading contribution in the heavy quark limit.

(iv) The decay amplitude acquires an imaginary part through the hard scattering kernels. In the heavy quark limit, the strong interaction phases can therefore be computed as expansions in  $\alpha_s$ . In terms of hadronic intermediate states that saturate the unitarity relation this implies systematic cancellations among many intermediate states with potentially large individual rescattering phases. An estimate of rescattering effects on the basis of Regge theory is not compatible with the picture that emerges in the heavy quark limit.

(v) The factorization formula (3) generalizes to the decays into a heavy-light final state, if the heavy particle absorbs the  $B$  meson spectator quark. In this case the second line in (3) is power-suppressed and only the form factor term survives. An expression of this form has been used by Politzer and Wise to compute the 1-loop corrections to the decay rate ratio  $\Gamma(\bar{B} \rightarrow D^* \pi)/\Gamma(\bar{B} \rightarrow D \pi)$  [4]. The factorization formula does *not* hold for heavy-light final states, in which the light meson absorbs the  $B$  meson spectator quark, or for a heavy-heavy final state. In this case, conventional factorization can also not be justified.

The result of an explicit calculation of the  $\bar{B} \rightarrow \pi\pi$  decay amplitudes at order  $\alpha_s$  can be compactly expressed as  $\langle \pi\pi | \mathcal{H}_{eff} | \bar{B} \rangle = G_F/\sqrt{2} \sum_{p=u,c} \lambda_p \langle \pi\pi | \mathcal{T}_p | \bar{B} \rangle$ , where

$$\begin{aligned} \mathcal{T}_p &= a_1^p(\pi\pi) (\bar{u}b)_{V-A} \otimes (\bar{d}u)_{V-A} \\ &+ a_2^p(\pi\pi) (\bar{d}b)_{V-A} \otimes (\bar{u}u)_{V-A} \\ &+ a_3(\pi\pi) (\bar{d}b)_{V-A} \otimes (\bar{q}q)_{V-A} \\ &+ a_4^p(\pi\pi) (\bar{q}b)_{V-A} \otimes (\bar{d}q)_{V-A} \\ &+ a_5(\pi\pi) (\bar{d}b)_{V-A} \otimes (\bar{q}q)_{V+A} \\ &+ a_6^p(\pi\pi) (-2)(\bar{q}b)_{S-P} \otimes (\bar{d}q)_{S+P}. \end{aligned} \quad (4)$$

The symbol  $\otimes$  is defined through  $\langle \pi\pi | j_1 \otimes j_2 | \bar{B} \rangle \equiv \langle \pi | j_1 | \bar{B} \rangle \langle \pi | j_2 | 0 \rangle$ . A summation over  $q = u, d$  is implied. Note that the term proportional to  $a_6^p(\pi\pi)$  results in a power correction that should be dropped in the heavy

quark limit. We will comment further on this term below.

Together with  $a_1^c(\pi\pi) = a_2^c(\pi\pi) = 0$  and the leading-order coefficient  $a_6^p(\pi\pi) = C_6 + C_5/N$ , the QCD coefficients  $a_i^p(\pi\pi)$  read at next-to-leading order (NLO)

$$a_1^u(\pi\pi) = C_1 + \frac{1}{N}C_2 + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_2 F, \quad (5)$$

$$a_2^u(\pi\pi) = C_2 + \frac{1}{N}C_1 + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_1 F, \quad (6)$$

$$a_3(\pi\pi) = C_3 + \frac{1}{N}C_4 + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_4 F, \quad (7)$$

$$a_4^p(\pi\pi) = C_4 + \frac{1}{N}C_3 - \frac{\alpha_s}{4\pi} \frac{C_F}{N} \left[ \left( \frac{4}{3}C_1 + \frac{44}{3}C_3 + \frac{4f}{3}(C_4 + C_6) \right) \ln \frac{\mu}{m_b} + \left( G_\pi(s_p) - \frac{2}{3} \right) C_1 + \left( G_\pi(0) + G_\pi(1) - f_\pi^I - f_\pi^{II} + \frac{50}{3} \right) C_3 + \left( 3G_\pi(0) + G_\pi(s_c) + G_\pi(1) \right) (C_4 + C_6) + G_{\pi,8}C_8 \right], \quad (8)$$

$$a_5(\pi\pi) = C_5 + \frac{1}{N}C_6 + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_6 (-F - 12). \quad (9)$$

Here  $C_F = (N^2 - 1)/(2N)$  and  $N = 3$  ( $f = 5$ ) is the number of colours (flavours). [Note that our definition of  $C_1$  and  $C_2$  differs from the convention of [2], where the labels 1 and 2 are interchanged.] The internal quark mass in penguin diagrams enters as  $s_p$ , where  $s_u = 0$  and  $s_c = m_c^2/m_b^2$ . In addition we have used ( $\bar{x} \equiv 1 - x$ )

$$F = -12 \ln \frac{\mu}{m_b} - 18 + f_\pi^I + f_\pi^{II}, \quad (10)$$

$$f_\pi^I = \int_0^1 dx g(x) \Phi_\pi(x), \quad G_{\pi,8} = \int_0^1 dx G_8(x) \Phi_\pi(x), \quad (11)$$

$$G_\pi(s) = \int_0^1 dx G(s, x) \Phi_\pi(x), \quad (12)$$

with the hard-scattering functions

$$g(x) = 3 \frac{1-2x}{1-x} \ln x - 3i\pi, \quad G_8(x) = \frac{2}{\bar{x}}, \quad (13)$$

$$G(s, x) = -4 \int_0^1 du u(1-u) \ln(s - u(1-u)\bar{x} - i\epsilon). \quad (14)$$

The hard spectator scattering contribution is given by

$$f_\pi^{II} = \frac{4\pi^2}{N} \frac{f_\pi f_B}{f_+(0)m_B^2} \int_0^1 d\xi \frac{\Phi_B(\xi)}{\xi} \left[ \int_0^1 dx \frac{\Phi_\pi(x)}{x} \right]^2, \quad (15)$$

where  $f_\pi$  ( $f_B$ ) is the pion ( $B$  meson) decay constant,  $m_B$  the  $B$  meson mass,  $f_+(0)$  the  $B \rightarrow \pi$  form factor at zero momentum transfer, and  $\xi$  the light-cone momentum fraction of the spectator in the  $B$  meson.  $f_\pi^{II}$  depends on the wave function  $\Phi_B$  through the integral  $\int_0^1 d\xi \Phi_B(\xi)/\xi \equiv m_B/\lambda_B$ . This introduces one new

hadronic parameter  $\lambda_B$ . Since  $\Phi_B(\xi)$  has support only for  $\xi$  of order  $\Lambda_{QCD}/m_B$ ,  $\lambda_B$  is of order  $\Lambda_{QCD}$ .

Writing the transition operator  $\mathcal{T}_p$  in terms of the QCD coefficients  $a_i^p(\pi\pi)$  is a convenient notation for phenomenological applications. The notation generalizes the conventional parameters  $a_{1,2}$  [5], which are seen to be process-dependent beyond leading order. We emphasize that in the present context the  $a_i^p(\pi\pi)$  are not phenomenological parameters, but genuine predictions of QCD in the heavy quark limit. The Wilson coefficients  $C_i$  entering the  $a_i^p(\pi\pi)$  are to be taken at NLO [2], where we consistently drop terms of  $\mathcal{O}(\alpha_s^2)$  in (5)–(9). The physical amplitudes derived from (4) are independent of the renormalization scale ( $\mu$ ) and scheme through  $\mathcal{O}(\alpha_s)$ . The coefficients  $a_1(\pi\pi)$ – $a_5(\pi\pi)$  multiply scale and scheme independent matrix elements of (axial-)vector currents. Accordingly for  $a_1(\pi\pi)$ – $a_5(\pi\pi)$  the scale and scheme dependence in the Wilson coefficients  $C_i$  is canceled by the  $\mathcal{O}(\alpha_s)$  corrections in the hard-scattering amplitudes. In the case of  $a_6^p(\pi\pi)$ , a scale and scheme dependence remains, which is precisely the one needed to cancel the corresponding dependence in the matrix elements of the (pseudo-)scalar currents, multiplying  $a_6^p(\pi\pi)$  in (4). Besides the  $\ln(\mu/m_b)$  terms the hard-scattering amplitudes contain a scheme dependent constant, which we have obtained in the NDR scheme as defined in [6]. This fixes the scheme to be used for the NLO coefficients  $C_i$ .

At NLO the factorization coefficients  $a_i^p(\pi\pi)$  acquire complex phases, entering through the functions  $g(x)$  and  $G(s, x)$  in (13) and (14). Being of order  $\alpha_s$ , these phases are generically small, except in cases where the lowest order contribution is numerically suppressed. This happens *e.g.* for  $a_2^u(\pi\pi)$ . Physically, the phases arise from final state rescattering, which is due to hard gluon exchange, and hence perturbative, in the heavy quark limit. The generation of strong interaction phases through the penguin function  $G(s, x)$  has been discussed many years ago [7] and is commonly referred to as the Bander–Silverman–Soni (BSS) mechanism. In the present approach, the gluon virtuality  $k^2 = \bar{x}m_B^2$  in the penguin diagram, which has usually been treated as a free phenomenological parameter, has a well-defined meaning. The  $x$ -dependence of  $G(s, x)$  is convoluted with the pion wave function  $\Phi_\pi(x)$ , leaving no ambiguity as to the value of  $k^2$ . In addition we identify a further source of rescattering phases, represented by the function  $g(x)$ . This effect corresponds to hard gluon exchange between the two outgoing pions. Together with the BSS mechanism, it accounts for the complete asymptotic rescattering phases in  $\bar{B} \rightarrow \pi\pi$  in the heavy quark limit.

Another novel result is the existence of the contribution from hard scattering involving the spectator quark in the  $B$  meson, expressed by  $f_\pi^{II}$  in (15). This mechanism is completely missed in phenomenological models of factorization. It is particularly important for the small coefficient  $a_2^u(\pi\pi)$ , where it leads to a sizable en-

hancement. Using  $f_\pi = 131$  MeV,  $f_B = (180 \pm 20)$  MeV,  $f_+(0) = 0.275 \pm 0.025$ ,  $\lambda_B = 0.3$  GeV and the asymptotic pion wave function  $\Phi_\pi(x) = 6x\bar{x}$ , we find  $f_\pi^{II} \approx 6.4$ . The poor knowledge of the  $B$  meson parameter  $\lambda_B$  makes this number rather uncertain.

Numerical values for the  $a_i^p(\pi\pi)$  are shown in Table I, using the pole masses  $m_b = 4.8$  GeV,  $m_c = 1.4$  GeV, the  $\overline{\text{MS}}$  masses  $\bar{m}_t(\bar{m}_t) = 167$  GeV,  $(\bar{m}_u + \bar{m}_d)(2 \text{ GeV}) = 9$  MeV and  $\Lambda_{\overline{\text{MS}}}^{(5)} = 225$  MeV as input parameters.  $a_6^p(\pi\pi)$  multiplies the  $\Lambda_{QCD}/m_b$ -suppressed, but chirally enhanced combination

$$r_\chi = \frac{2m_{\pi^+}^2}{\bar{m}_b(\mu)(\bar{m}_u(\mu) + \bar{m}_d(\mu))} \approx 1.18 \quad [\text{at } \mu = m_b]. \quad (16)$$

In the following analysis, we give two results, one neglecting  $a_6^p(\pi\pi)$  as formally power-suppressed, the other keeping the leading-order expression for  $a_6^p(\pi\pi)$ .

It is now straightforward to evaluate the decay amplitudes and branching ratios. The latter are given by  $Br(\bar{B} \rightarrow \pi\pi) = \tau_B/(16\pi m_B) \cdot |A(\bar{B} \rightarrow \pi\pi)|^2 S$ , where  $S = 1$  for  $\pi\pi = \pi^+\pi^-$ ,  $\pi^-\pi^0$  and  $S = 1/2$  for  $\pi\pi = \pi^0\pi^0$ .  $\tau_B$  denotes the  $B$  meson lifetimes:  $\tau(B^+) = 1.65$  ps,  $\tau(B_d) = 1.56$  ps. The decay amplitudes read

$$A(\bar{B}_d \rightarrow \pi^+\pi^-) = i \frac{G_F}{\sqrt{2}} m_B^2 f_+(0) f_\pi |\lambda_c| \cdot \quad (17)$$

$$\cdot [R_b e^{-i\gamma} (a_1^u(\pi\pi) + a_4^u(\pi\pi) + a_6^u(\pi\pi) r_\chi) - (a_4^c(\pi\pi) + a_6^c(\pi\pi) r_\chi)],$$

$$A(\bar{B}_d \rightarrow \pi^0\pi^0) = i \frac{G_F}{\sqrt{2}} m_B^2 f_+(0) f_\pi |\lambda_c| \cdot \quad (18)$$

$$\cdot [R_b e^{-i\gamma} (-a_2^u(\pi\pi) + a_4^u(\pi\pi) + a_6^u(\pi\pi) r_\chi) - (a_4^c(\pi\pi) + a_6^c(\pi\pi) r_\chi)],$$

$$A(B^- \rightarrow \pi^-\pi^0) = i \frac{G_F}{\sqrt{2}} m_B^2 f_+(0) f_\pi |\lambda_c| \cdot \quad (19)$$

$$\cdot R_b / \sqrt{2} \cdot e^{-i\gamma} (a_1^u(\pi\pi) + a_2^u(\pi\pi)).$$

Here  $R_b = (1 - \lambda^2/2)|V_{ub}/V_{cb}|/\lambda$ , where  $\lambda = 0.22$  is the sine of the Cabibbo angle,  $\gamma$  is the phase of  $V_{ub}^*$ , and we will use  $|V_{cb}| = 0.039 \pm 0.002$ ,  $|V_{ub}/V_{cb}| = 0.085 \pm 0.020$ . We find the branching fractions

$$Br(\bar{B}_d \rightarrow \pi^+\pi^-) = 6.5 [6.1] \cdot 10^{-6} \left| e^{-i\gamma} + 0.09 [0.18] e^{i \cdot 12.7 [6.7]^\circ} \right|^2, \quad (20)$$

$$Br(\bar{B}_d \rightarrow \pi^0\pi^0) = 5.2 [7.7] \cdot 10^{-8} \left| e^{-i\gamma} + 0.73 [1.11] e^{-i \cdot 137 [149]^\circ} \right|^2, \quad (21)$$

$$Br(B^- \rightarrow \pi^-\pi^0) = 4.3 [4.3] \cdot 10^{-6}, \quad (22)$$

where the default values correspond to neglecting  $a_6^p(\pi\pi)$  and the values in brackets use  $a_6^p(\pi\pi)$  at leading order. While the predictions for the  $\pi^+\pi^-$  and  $\pi^-\pi^0$  final states are relatively robust, with errors on the order of  $\pm 30\%$

due to the input parameters, the decay into  $\pi^0\pi^0$  depends very sensitively on the parameter  $\lambda_B$  that controls the hard spectator scattering. If it is significantly smaller than 0.3 GeV, a branching fraction of order  $10^{-6}$  cannot be excluded. Eq. (20) can be converted into a result for the time-dependent CP asymmetry as a function of the CKM angle  $\alpha$ . The direct CP asymmetry in the  $\pi^+\pi^-$  mode is approximately  $4\% \cdot \sin \gamma$ .

The approach discussed here allows us to formulate, for the first time, rigorous predictions of QCD for exclusive non-leptonic  $B$  decays in the heavy quark limit. On the other hand, as the dependence on the formally power-suppressed coefficient  $a_6^p(\pi\pi)$  demonstrates, the asymptotic limit may be problematic at  $m_b \approx 5$  GeV and the applicability of the theory has to be decided on a case-by-case basis. The most important power corrections are those that depend on the chirally enhanced combination (16). The  $\alpha_s$  corrections to all such terms can in fact be identified. However, the factorization formula breaks down in this case, because the relevant twist-3 wave functions do not fall off fast enough at the endpoints. A detailed discussion of this point, and of its phenomenological consequences, will be given elsewhere.

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- [1] G.P. Lepage and S.J. Brodsky, Phys. Rev. **D22**, 2157 (1980).
  - [2] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
  - [3] J.D. Bjorken, Nucl. Phys. (Proc. Suppl.) **B11**, 325 (1989).
  - [4] H.D. Politzer and M.B. Wise, Phys. Lett. **B257**, 399 (1991).
  - [5] M. Neubert and B. Stech, hep-ph/9705292.
  - [6] A.J. Buras et al., Nucl. Phys. **B400**, 37 (1993).
  - [7] M. Bander, D. Silverman and A. Soni, Phys. Rev. Lett. **43**, 242 (1979).

TABLE I. The QCD coefficients  $a_i^p(\pi\pi)$  at NLO for three different renormalization scales  $\mu$ . Leading order values are shown in parenthesis for comparison.

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2m_b$
$a_1^u(\pi\pi)$	$1.047 + 0.033i$ (1.038)	$1.038 + 0.018i$ (1.020)	$1.027 + 0.010i$ (1.010)
$a_2^u(\pi\pi)$	$0.061 - 0.106i$ (0.066)	$0.082 - 0.080i$ (0.140)	$0.108 - 0.064i$ (0.200)
$a_3(\pi\pi)$	$0.005 + 0.003i$ (0.004)	$0.004 + 0.002i$ (0.002)	$0.003 + 0.001i$ (0.001)
$a_4^u(\pi\pi)$	$-0.030 - 0.019i$ (-0.027)	$-0.029 - 0.015i$ (-0.020)	$-0.026 - 0.013i$ (-0.014)
$a_4^c(\pi\pi)$	$-0.038 - 0.009i$ (-0.027)	$-0.034 - 0.008i$ (-0.020)	$-0.031 - 0.007i$ (-0.014)
$a_5(\pi\pi)$	$-0.006 - 0.004i$ (-0.005)	$-0.005 - 0.002i$ (-0.002)	$-0.003 - 0.001i$ (-0.001)
$a_6^p(\pi\pi) r_\chi$	- (-0.036)	- (-0.030)	- (-0.024)