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VMD, the WZW Lagrangian and ChPT : The Third Mixing Angle

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Abstract

A recently proposed vector meson dominance (VMD) model with SU(3) and nonet symmetry breaking has provided a successful description of all radiative decays of light mesons ($VP\gamma$ and $P\gamma\gamma$). It is shown here that it also accounts for parameter values, definitions and leading order expressions from Chiral Perturbation Theory (ChPT). We compare the determination of symmetry breaking parameters from both ChPT and the Wess–Zumino–Witten (WZW) anomalous Lagrangian, and discuss the relation between the two methods. We find that the traditional one–angle mixing scheme of Current Algebra, used for $P\gamma\gamma$ amplitudes, arises naturally from VMD/WZW and provides a satisfactory description of data, without any contradiction of ChPT. In particular, following the recent work of Kaiser and Leutwyler, this model, using fit results to radiative decays reported elsewhere, yields $\theta_8 \simeq -21^\circ$, $\theta_0 \simeq 0^\circ$. The single angle occurring in η/η' two–photon decay amplitudes is $\theta_P \simeq -11^\circ$, where $\theta_P \simeq \theta_8/2$ is the relevant angle for the η/η' mixing. The quark mass ratio is found $\simeq 21.2 \pm 2.4$. The Gell–Mann–Okubo mass relation is found fulfilled at the percent level. The result $\theta_0 = 0$, is supported by all data on radiative decays and gives a novel relation between mixing angles and the violation of nonet symmetry in the pseudoscalar sector. Nonet symmetry breaking is found to remove any need for glue coupling in the η/η' system.

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I. INTRODUCTION.

Recently, a model [1,2] has been proposed which describes quite successfully all radiative decays of light mesons, that is, those of the kind $V \to P\gamma$ and $P \to \gamma\gamma$. This vector meson dominance (VMD) based model relies on the hidden local symmetry (HLS) approach developed in Ref. [3] which introduces the vector mesons as gauge bosons of a spontaneously broken hidden local symmetry. Its anomalous sector [3,4] (referred to hereafter as FKTUY), describes the radiative decays of light flavor mesons. In its orginal form this Lagrangian possesses both nonet symmetry for the pseudoscalars and SU(3) flavor symmetry.

In order to successfully describe the full pattern of light mesons radiative decays, this original scheme needs to be supplemented with symmetry breaking mechanisms. As recalled in Ref. [1], nonet symmetry in the sector of vector mesons is well fulfilled; however, it is also shown there that it is not true for the pseudoscalar sector. Nonet symmetry has thus to be broken, but only for pseudoscalar mesons; it is done in the manner of Ref. [5]. Additionally, it is also shown in Ref. [1] that breaking the SU(3) flavor symmetry cannot be avoided. This is mainly performed following the mechanism proposed by Bando, Kugo and Yamawaki (BKY) [6,7] and does not depend on a free parameter. This quite simple breaking pattern[1] is able to account for all radiative decays of light mesons, except for $K^{*\pm} \to K^{\pm}\gamma$. The treatment of this last mode necessitates a slight complication of this simple picture; we will not discuss this further, though, as it is not relevant for the purpose of the present paper and we refer the reader to Refs. [1,2] for further details.[2]

The problem of η/η' mixing [8,9] is tightly connected with the breaking of nonet symmetry. This can be performed at the level of the coupling constants as stated above [5], but this breaking can also be connected with a possible glue component inside the light mesons [10,11]. This was investigated in Ref. [1] where it was clearly shown that the effects of such a glue component can only affect the η' meson, but cannot be disentangled from genuine nonet symmetry breaking effects without a *a priori* knowledge of one of these twin phenomena.

In connection with this particular problem, but even more closely connected with the effects of broken symmetries in Chiral Perturbation Theory (ChPT), Kaiser and Leutwyler [12,13] advocate an η/η' mixing scheme, more complicated than the usual one, depending on two decay constants and two mixing angles. Some phenomenological analyses [14,15] have been done investigating this new scheme. However, the successful analysis of radiative decays of Ref. [1] does not find any need for such a complicated picture, if one limits oneself to only radiative decays of light mesons (that is, however, 14 independent modes).

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[1] Concerning radiative decays, it depends on only four free parameters and describes 13 decay modes ; one more parameter is needed in order to describe completely the K^* sector with 14 modes at total. This seems by far the simplest successful scheme proposed so far. Nevertheless, the present work will allow to reduce the number of free parameters by one unit, without any loss in fit quality. We will also emphasize that, in principle, this number of free parameters can be reduced by two units.

[2] Though these two references deal essentially with the same physics their perspectives are somewhat complementary.

More appealing, the anomalous WZW Lagrangian [16,17], with SU(3) symmetry broken as explained in Refs. [1,2], leads to definitions of the mixing angle and decay constants as per Current Algebra expressions. The mixing angle was found in Ref. [1] to be $\simeq -11^\circ$, and moreover, the value for the octet decay constant is $f_8 = 0.82 f_\pi$.

All this seems in glaring disagreement with the expectations of ChPT [18,8,9,19,12,13]. Taking into account the special role of ChPT in low energy phenomenology, a possible contradiction between ChPT (and hence QCD) and VMD is a worrying question which must be addressed. This is the purpose of the present paper, which will show that the contradiction is illusory and only due to different definitions of the same parameters in the WZW approach (with encompasses the Current Algebra definitions) and in ChPT. It will even be shown that the VMD approach, relying on the HLS model broken as in Refs. [1,2], is in accord with ChPT expectations. This will be illustrated by deriving from VMD the leading order expressions for ChPT mixing angles and decay constants, and by giving their numerical values.

The outline of the paper is as follows. In Section II we first describe the full breaking scheme of the VMD model of Ref. [1]. Section III gives the VMD description of the $\eta/\eta' \to \gamma\gamma$ decay which depends[3] on one mixing angle and nonet symmetry breaking (as parametrized by x). In Section IV the corresponding description is derived, starting from the BKY broken WZW Lagrangian, and it is shown that WZW and VMD coincide. In Section V, we derive the set of relations which allows one to define mixing angles and decay constants in accord with the standard (or extended) ChPT approach. Here we show, first that the definitions of mixing angles and decay constants from VMD/WZW and ChPT do not coincide once symmetry breaking is included and, second, that VMD provides expressions and values for the ChPT parameters of relevance in accord with all current expectations. This is illustrated by several examples, including the relation between the VMD mixing angle θ_P and the ChPT angle θ_8. In Section VII, we show that nonet symmetry breaking and pseudoscalar mixing angle(s) are functionally related, which is a completely new result. This allows us to perform a fit of radiative decays with only 3 free parameters. The level of nonet symmetry breaking correlated with the fit value of the pseudoscalar mixing angle is shown to remove any need for glue in the η' meson. A few other points of interest are also examined (quark mass ratio, isoscalar mass matrix). Finally, Section VIII is devoted to conclusions.

II. A VMD BASED MODEL FOR RADIATIVE DECAYS

The model developed in Refs. [1,2] in order to describe all light meson radiative decays relies on breaking nonet symmetry and flavor SU(3) in the HLS Lagrangian [3], and especially in its anomalous (FKTUY) sector [4]. We will briefly sketch these breaking schemes here, referring the reader to Refs. [1,2] for all details and previous references.

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[3] SU(3) flavor breaking arises through a dependence upon the ratio f_K/f_π and non–zero mixing angles.

A. Basic Ingredients

The basic ingredients of the Effective Lagrangian approach to the interaction of vector and pseudoscalar mesons are the matrices V and P of the corresponding fields expressed in the flavor (u, d, s) basis.

The vector meson field matrix V is usually written[4]

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} (\rho^0 + \omega^I)/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & (-\rho^0 + \omega^I)/\sqrt{2} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\phi^I \end{pmatrix}. \tag{1}$$

in terms of ideally mixed states (ω^I, ϕ^I). Correspondingly the pseudoscalar field matrix is usually defined as

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\pi_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\pi_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\pi_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}, \tag{2}$$

using the conventional octet and singlet components (π_8, η_0) for the isoscalar mesons.

The physical states (ω, ϕ, η, η') are generated from the ideally mixed states by means of standard rotations of angles δ_V or δ_P for vector and pseudoscalar mesons. Correspondingly, the rotation angles from singlet and octet states to the physically observed mesons are traditionally named θ_V and θ_P. These well known relations can be found in Refs. [1,5,7,20].

B. Nonet Symmetry Breaking

These expressions for V and P can also be written

$$V = V_8 + V_0, \quad P = P_8 + P_0 \tag{3}$$

in order to exhibit their octet and singlet component combinations, and show that nonet (U(3)) symmetry is implicitly assumed. Breaking of nonet symmetry is allowed by making the change $P_8 + P_0 \to P_8 + xP_0$. In this way, we accept changing the relative weight of the octet and singlet parts in only the pseudoscalar field matrix. This allows to recover the radiative decay model of Ref. [5] in the particular case when nonet symmetry is broken in only the pseudoscalar sector. It happens [1,2] that data favor a value $x \simeq 0.9$ instead of 1, if there is no coupling to glue. Departures from nonet symmetry in the sector of vector mesons are not supported by the data [1,21] and thus need not be accounted for.

Another way to allow for nonet symmetry breaking is to assume that the singlet sector contains another component than the standard SU(3)/U(3) singlet; we will name it glue only for convenience. A possible coupling of the η/η' doublet to glue can be accounted for

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[4]The sign in front of ϕ^I means that we define $\phi^I = -|s\overline{s}\rangle$.

[1] by means of an additional angle γ, which is zero if one chooses to decouple this doublet from glue :

$$\begin{bmatrix} \eta \\ \eta' \\ \eta'' \end{bmatrix} = \begin{bmatrix} \cos\theta_P & -\sin\theta_P & 0 \\ \sin\theta_P\cos\gamma & \cos\theta_P\cos\gamma & \sin\gamma \\ -\sin\theta_P\sin\gamma & -\cos\theta_P\sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} \pi_8 \\ \eta_0 \\ gg \end{bmatrix} \tag{4}$$

Indeed, following the analysis of Ref. [1], we do not have to introduce any coupling of the η meson to glue, which would turn out to introduce an additional angle (β in Ref. [1]). The angle γ produces a coupling of (only) the η' meson to glue. We have named η'' the possible triplet companion of the η/η' mesons, and we will not attempt to identify it. If $\gamma = 0$, one clearly recovers the usual mixing pattern for the η/η' system, by decoupling it from glue.

When fitting the data on radiative decays of light mesons, the level of correlation between x and γ is found such [1,2] that assuming glue and exact nonet symmetry ($x = 1$), or assuming no glue ($\gamma = 0$) and some nonet symmetry breaking, provide the same description of the data. Therefore, whether glue is requested in order to describe the η' properties is still a pending question which will be addressed in the present paper (see Section VII), when a reliable guess about the value of x will have been done.

However, whatever the kind of deviation from nonet symmetry, condition $x \neq 1$ – and/or $\gamma \neq 0$ – is absolutely required in order to get a reasonable fit to the data. Indeed, Ref. [1] shows, for instance, that $x = 0.92$ instead of 1, allows to lessen the minimum χ^2 from about 30 to about 10, a quite significant effect for one parameter.

C. SU(3) Breaking Mechanism of the HLS Model

SU(3) symmetry breaking of the HLS Lagrangian originates from Refs. [3,6]. A comprehensive account of the HLS model is given in Ref. [3]. Brief accounts and some new developments can be found in Refs. [7,22], connected more precisely with the anomalous sector [4]. Basically, the SU(3) breaking scheme we use has been introduced by Bando, Kugo and Yamawaki [6] (already referred to as BKY) and has given rise to a few variants [7,22]. We refer the reader to Refs. [1,2,6,7,22] for detailed analyses of the properties of known variants of the BKY breaking scheme.

In the following, we use basic consequences common to the original BKY mechanism [6], its Hermitized variant and the so-called new scheme, both discussed in Ref. [7]. In these cases, SU(3) symmetry breaking results in defining a renormalized pseudoscalar field matrix P' in terms of the bare one P given above by :

$$P' = X_A^{1/2} P X_A^{1/2}, \tag{5}$$

where the breaking matrix X_A writes diag$(1,\ 1,\ 1 + c_A)$ and we have [6,7]

$$\ell_A \equiv 1 + c_A = \left(\frac{f_K}{f_\pi}\right)^2 = 1.495 \pm 0.030 \quad , \tag{6}$$

It should be noted [7], that the field renormalization (Eqs. (5) and (6)) is required in order to recover the charge normalization condition, $F_{K^+}(0) = 1$. The numerical value just given is deduced from the experimental information quoted in Ref. [20] ; fitting this parameter [1] in radiative decays, gives a value quite consistent with this number. Self–consistency of the broken HLS model implies that this parameter, which has a well defined value, is not free.

It should be mentioned that, if the BKY breaking mechanism results in a renormalization of the pseudoscalar field matrix, it does not result likewise in a renormalization of the vector field matrix [6,7]. Nevertheless, this might have to be introduced in connection with the description of the $K^{*\pm}$ radiative decay [1,2]. The final word, however, might have to wait for a confirmation of the decay rate, which could be measured at B–factories from τ lepton decays.

III. THE VMD DESCRIPTION OF $\eta/\eta' \to \gamma\gamma$ DECAYS

Following FKTUY [4], the anomalous U(3) symmetric Lagrangian describing PVV interactions is

$$\mathcal{L} = -\frac{3g^2}{4\pi^2 f_\pi}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[\partial_\mu V_\nu \partial_\rho V_\sigma P]. \tag{7}$$

$PV\gamma$ and $P\gamma\gamma$ transitions amplitudes are obtained by means of this Lagrangian and of the non–anomalous HLS Lagrangian, needed in order to describe the direct transition from vector mesons to photons. This non–anomalous HLS Lagrangian is given in its expanded form in Ref. [7].

Postulating that the same formulae apply when breaking nonet symmetry ($x \neq 1$), is confirmed by its formal agreement with the (quasi–axiomatic) O'Donnell derivation of the coupling constant formulae [1,5,21].

The HLS model contains the Vector Meson Dominance (VMD) assumption [23]; it thus gives a way to relate the radiative decay modes $VP\gamma$ to each other and to the $P\gamma\gamma$ decays for light mesons, by giving a precise meaning to the equations sketched in Fig. 1.

From known information [20] on single photon radiative decays and on leptonic decay widths of vector mesons only, this equation can be used in order to derive the two–photon widths of the π^0, η, η' mesons. The accuracy of such a crude calculation is quite good, as can be seen from Table 1 in Ref. [2]. Additionally, this equation implies an algebraic relationship among both kind of radiative decays which can be worked out analytically [1].

FIGURES

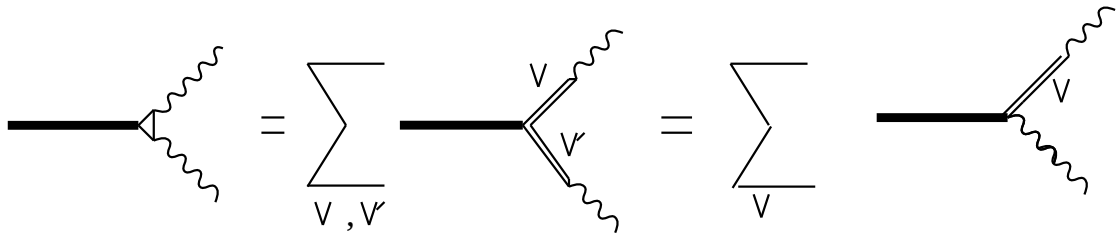

FIG. 1. .

Graphical representation of the relation among various kind of coupling constants. V and V' stand for the lowest lying vector mesons (ρ^0, ω, ϕ) ; the internal vector meson lines are propagators at $s = 0$ and are approximated by the corresponding tabulated [20] masses squared.

When breaking SU(3) symmetry à la BKY [6], we have to reexpress the FKTUY Lagrangian in terms of the renormalized matrix P', instead of the bare one P; this is done using Eq. (5), with the (fixed) parameter given in Eq. (6). We remind the reader that P' is also modified by the replacement $\eta_0 \longrightarrow x\eta_0$.

Propagating this field renormalization down to the FKTUY Lagrangian we have

$$\mathcal{L} = -\frac{3g^2}{4\pi^2 f_\pi}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}[\partial_\mu V_\nu \partial_\rho V_\sigma X_A^{-1/2} P' X_A^{-1/2}]. \tag{8}$$

Then, the VVP Lagrangian is changed in a definite way by the symmetry breaking parameter ℓ_A defined above (see Eq. (6)) and supposed to have a well understood numerical value (practically 1.5). The parameter g here is the universal vector meson coupling [3,4].

It is interesting to note that the VMD broken model we discuss expresses all radiative coupling constants in terms of g , x and of the two mixing angles θ_P and θ_V. Even if f_π, f_K serve to define the SU(3) breaking, they can hardly be considered as (free) parameters ; it is thus legitimate to consider this model as a 4–parameter one.

The expanded form of the Lagrangian (8) is given in the Appendix of Ref. [1]. The expressions for the two–photon decays widths[5] of the η and η' mesons can be derived herefrom. They are

$$\begin{aligned}
G_{\eta\gamma\gamma} &= -\frac{\alpha_{em}}{\pi\sqrt{3}f_\pi}\left[\frac{5-2Z}{3}\cos\theta_P - \sqrt{2}\frac{5+Z}{3}x\sin\theta_P\right] \\
G_{\eta'\gamma\gamma} &= -\frac{\alpha_{em}}{\pi\sqrt{3}f_\pi}\left[\frac{5-2Z}{3}\sin\theta_P + \sqrt{2}\frac{5+Z}{3}x\cos\theta_P\right] \\
G_{\pi^0\gamma\gamma} &= -\frac{\alpha_{em}}{\pi f_\pi}
\end{aligned} \tag{9}$$

where $Z = 1/\ell_A = [f_\pi/f_K]^2$. Actually, the last expression in Eq. (9) is a normalization condition which allows to fix the numerical coefficient in Eq. (8). It is clear that Eqs. (9)

[5]The corresponding expressions when glue is coupled ($\gamma \neq 0$) can be found in Ref. [1]. We will only comment here about the results they provide.

give the two–photon decay widths in terms of f_π, f_K, x and only one mixing angle θ_P. This will be frequently referred to as the wave–function mixing angle (see Eq. (4) with $\gamma = 0$).

Then, using standard Feynman rules, the HLS model provides definite expressions for the two–photon couplings of the pseudoscalar mesons, through its anomalous (FKTUY) sector. It should be noted that they exhibit the traditional form [8,9,24] originally obtained through Current Algebra. These couplings relate to partial widths by

$$\Gamma(X \to \gamma\gamma) = \frac{M_X^3}{64\pi}|G_{X\gamma\gamma}|^2 \quad , \quad X = \pi^0,\ \eta,\ \eta' \quad . \tag{10}$$

As a test, one can fit the parameters x and θ_P on only the radiative decays of kind $VP\gamma$ and use these values and errors in order to predict the values for the two–photon decay widths of the η and η' mesons. The fit values used for these computations [1] are

$$x = 0.917 \pm 0.017 \quad , \quad \theta_P = -11.59^\circ \pm 0.76^\circ \tag{11}$$

and will remain our reference values throughout this paper. The results are given in Table 1 and clearly illustrate that the expressions in Eq. (9) are valid and that the $VP\gamma$ processes predict the two–photon decay widths quite nicely. Stated otherwise, one does not need more than one angle θ_P in order to describe the η and η' radiative decays and this receives an especially strong support from all $VP\gamma$ modes considered together. Additionally, despite textbook claims [19,8], this angle is found at a value ($\simeq -11^\circ$) clearly inconsistent with ChPT expectations [19,8,18] ($\simeq -20^\circ$).

Mode	VMD Fit	PDG	Comment
$\eta \to \gamma\gamma$ [keV]	0.464 ± 0.026	0.514 ± 0.026	$\gamma\gamma$
		0.46 ± 0.04	PDG mean
		0.324 ± 0.046	Primakoff
$\eta' \to \gamma\gamma$[keV]	4.407 ± 0.233	4.27 ± 0.19	PDG mean

Table 1 : Partial decay widths of the η/η' mesons, as reconstructed from fit to solely the radiative decays $VP\gamma$ (leftmost data column) and their direct measurements [20] (rightmost data column).

IV. THE WZW DESCRIPTION OF $\eta/\eta' \to \gamma\gamma$ DECAYS

Therefore, starting from broken HLS and FKTUY, the VMD model of Ref. [1] recovers the traditional form for the two–photon decay amplitudes, (*i.e.* the one mixing angle expressions of Current Algebra [8,9,24]). Using these standard expressions, one indeed gets through identification with our Eq. (9)

$$\frac{f_\pi}{f_8} = \frac{5 - 2Z}{3} \quad , \quad \frac{f_\pi}{f_0} = \frac{5 + Z}{6}x \tag{12}$$

where $Z = [f_\pi/f_K]^2$, and $\overline{f}_{0,8}$ denote the (Current Algebra) singlet and octet decay constants ; we have already defined θ_P the (single) mixing angle occurring in this approach.

Practically, these formulae show that, instead of going through the whole machinery of VMD by starting from the broken FKTUY Lagrangian, one could get these decay constants directly from the WZW Lagrangian [16,17]. Indeed, this can be written

$$\mathcal{L}_{WZW} = -\frac{e^2}{4\pi^2 f_\pi}\epsilon^{\mu\nu\rho\sigma}\partial_\mu A_\nu \partial_\rho A_\sigma \mathrm{Tr}[Q^2 P] \tag{13}$$

where $Q = \mathrm{diag}(2/3,\text{–}1/3,\text{–}1/3)$ is the quark charge matrix, A is the electromagnetic field and P is the bare pseudoscalar field matrix. Changing to the renormalized field P' through Eq. (5) allows us to recover the couplings in Eqs. (9) quite easily.[6]

This illustrates clearly that, what is named f_8 in the Current Algebra [24] expressions for η/η' decays to two photons, can be expressed solely in terms of f_π and f_K, in a way which fixes its value to $\overline{f_8} = 0.82 f_\pi$. Correspondingly, we have $\overline{f_0} = 1.15 f_\pi$ which includes a correction of approximately 8% due to nonet symmetry breaking. The fact that the WZW Lagrangian leads to the same results as the FKTUY Lagrangian simply states their expected equivalence when deriving two–photon decays amplitudes, as soon as the same breaking scheme is consistently implemented within both Lagrangians.

However, the SU(3) sector of Chiral Perturbation Theory (ChPT) [19,8,18] is well known to predict $f_8/f_\pi \simeq 1.25$ and a mixing angle of $\simeq -21^\circ$. Then, the question is whether there is an inconsistency with respect to ChPT, or if there is a mismatch among definitions in ChPT ($f_{0,8}$) and in the VMD/WZW approach ($\overline{f}_{0,8}$), after symmetry breaking.

V. THE CHPT DESCRIPTION OF $\eta/\eta' \to \gamma\gamma$ DECAYS

We have seen above that in the VMD procedure developed in Refs. [1,2,7], the expressions for f_8, f_0 and the mixing angle are the same as are obtained from the matrix elements for $\langle\gamma\gamma|\mathcal{L}_{WZW}|\eta\rangle$ and $\langle\gamma\gamma|\mathcal{L}_{WZW}|\eta'\rangle$. We have denoted the parameters, obtained in this manner, $\overline{f}_{0,8}$ and θ_P. In ChPT, however, the corresponding quantities ($f_{0,8}, \theta_8, \theta_0$) are defined through other matrix elements, namely $\langle 0|\partial^\mu \mathcal{A}^{8,0}_\mu|\eta\rangle$ and $\langle 0|\partial^\mu \mathcal{A}^{8,0}_\mu|\eta'\rangle$, where the $\mathcal{A}$'s are axial-vector fields of the appropriate flavor. It seems, however, traditionally admitted [19,8,25] that both sets of definitions necessarily coincide. It is this last property which we shall examine now.

A. Usual ChPT Parameters from Broken VMD

The main ingredient for the derivation of mixing parameter expressions in agreement with the ChPT current definitions is already given in our Ref. [7]; it involves the coupling of axial fields with the pseudoscalar meson fields. The Lagrangian piece of relevance is Eq. (33) in this reference and is expressed in term of the bare field matrix P and of an axial field matrix named $\mathcal{A}$. In term of the renormalized P' field matrix, it can be rewritten

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[6] Actually, these formulae have been already been written in Ref. [7], but they were expressed in terms of c_A, which leads to non–transparent formulae.

$$\mathcal{L}_{\mathcal{AP}} = -2f_\pi \mathrm{Tr}[\mathcal{A} X_A^{1/2} \partial P' X_A^{1/2}] \tag{14}$$

using obvious notations. We have explicitly treated in Ref. [7] the question of the kaon decay constant. The isoscalar part of Eq. (14) is

$$\mathcal{L}_{\mathcal{AP}} = -f_\pi \left[\frac{(1+2z)}{3}\ \mathcal{A}_\mu^8 \partial^\mu \pi_R^8 + \frac{(2+z)}{3} x\ \mathcal{A}_\mu^0 \partial^\mu \eta_R^0 \right.$$
$$\left. + \frac{\sqrt{2}}{3}(1-z)\mathcal{A}_\mu^0 \partial^\mu \pi_R^8 + \frac{\sqrt{2}}{3}(1-z)x\mathcal{A}_\mu^8 \partial^\mu \eta_R^0 \right] \tag{15}$$

with obvious notations, and where we have defined $z = [f_K/f_\pi]^2 = 1/Z$ which was also named ℓ_A above and in Refs. [1,2]. One should note the occurrence in Eq. (15) of terms coupling the octet (singlet) axial field to the singlet (octet) component of the isoscalar pseudoscalar mesons. These terms vanish in the limit of unbroken SU(3) flavor symmetry ($z = 1$) as expected.

These terms allow us to define the following matrix elements

$$\langle 0|\mathcal{A}_\mu^8|\pi^8(q)\rangle = i f_8 q_\mu\,, \quad \langle 0|\mathcal{A}_\mu^0|\eta^0(q)\rangle = i f_0 q_\mu$$
$$\langle 0|\mathcal{A}_\mu^8|\eta_0(q)\rangle = i b_8 q_\mu\,, \quad \langle 0|\mathcal{A}_\mu^0|\pi^8(q)\rangle = i b_0 q_\mu \tag{16}$$

where

$$f_8 = \frac{(1+2z)}{3} f_\pi = (1.33 \pm 0.02) f_\pi\,, \quad b_8 = \frac{\sqrt{2}}{3}(1-z) x f_\pi = (-0.22 \pm 0.02) f_\pi$$
$$f_0 = \frac{(2+z)}{3} x f_\pi = (1.07 \pm 0.02) f_\pi\,, \quad b_0 = \frac{\sqrt{2}}{3}(1-z) f_\pi = (-0.24 \pm 0.01) f_\pi \tag{17}$$

One readily observes a mismatch between the VMD/WZW definition for f_8 and f_0 (see Eqs. (12)) and the ChPT definitions explicited just above ; this mismatch is both algebraic and numerical. Otherwise, *this* f_8 corresponds to its standard ChPT expected value [19,8,18,25].

In order to switch to the matrix elements for $\langle 0|\mathcal{A}_\mu^{0,8}|\eta/\eta'\rangle$, we use Eq. (4) and we specialize[7] to $\gamma = 0$. In this case, using the notations of Kaiser and Leutwyler [12,13]

$$\langle 0|\mathcal{A}_\mu^{0,8}|\eta/\eta'(q)\rangle = i F_{\eta/\eta'}^{0,8} q_\mu \tag{18}$$

we get[8]

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[7] We have checked that [1] using the fit value $\gamma = 26.38^\circ \pm 2.64^\circ$ with $x = 1$ leads also to the numbers given below, within errors.

[8] If we introduce coupling η' to glue by means of the angle γ defined in Refs. [1,2], Eq. (19) for $F_\eta^{0,8}$ in term of z, x, and θ_P are unchanged, while the corresponding expressions for $F_{\eta'}^{0,8}$ are formally multiplied by $\cos\gamma$.

$$
\begin{array}{rcccccl}
F^8_\eta & = & F^8\cos\theta_8 & = & f_8\cos\theta_P - b_8\sin\theta_P & = & (1.263\pm 0.007)f_\pi \\
F^8_{\eta'} & = & F^8\sin\theta_8 & = & f_8\sin\theta_P + b_8\cos\theta_P & = & -(0.480\pm 0.017)f_\pi \\
F^0_\eta & = & -F^0\sin\theta_0 & = & b_0\cos\theta_P - f_0\sin\theta_P & = & -(0.016\pm 0.015)f_\pi \\
F^0_{\eta'} & = & F^0\cos\theta_0 & = & b_0\sin\theta_P + f_0\cos\theta_P & = & (1.095\pm 0.019)f_\pi
\end{array} \tag{19}
$$

using the parameter values for x and θ_P given in Eq. (11). These relations lead to

$$
\begin{array}{cc}
F^8 = (1.351\pm 0.007)f_\pi & F^0 = (1.095\pm 0.020)f_\pi \\
\theta_8 = -20.80^\circ \pm 0.78^\circ & \theta_0 = 0.84^\circ \pm 0.79^\circ
\end{array} \tag{20}
$$

In the exact SU(3) limit ($z = 1$), the expressions in Eq. (19) imply $\theta_0 = \theta_8 = \theta_P$. Thus, the difference between them is, indeed, an effect of SU(3) flavor symmetry breaking, with only a marginal influence of the nonet symmetry breaking parameter x. If one simply replaces, as suggested by our Refs. [1,2], nonet symmetry breaking by coupling to glue, the numbers just given are practically unchanged, showing a relative independence upon the detailed parametrization of the nonet symmetry breaking phenomenon.

The values given in Eqs. (20) can be compared with ChPT expectations, as they now correspond to the standard ChPT definition of mixing parameters. The value for F^8 compares impressively to the parameter free prediction of Ref. [13] ($1.34 f_\pi$). The value for F^0 is harder to estimate theoretically because of its scale dependence ; however, from the information given in Refs. [12,13], $F^0 \simeq 1.10 f_\pi$ seems a quite reasonable value. The VMD value $\theta_8 = -20.80^\circ \pm 0.78^\circ$ is quite consistent with all reported ChPT expectations (for example, $-20^\circ \pm 4^\circ$ from Ref. [18], -20.5° from Ref. [12]).

For θ_0, the situation is unclear because the reported expectation [12] $\theta_0 = -4^\circ$ does not quote the accuracy of this estimate. However the estimate of Kaiser and Leutwyler for $\theta_0 - \theta_8 \simeq 14^\circ$, is seemingly much smaller than what comes out from VMD phenomenology ($21.64^\circ \pm 1.14^\circ$). It is clear that the disagreement (if actual) refers to the estimate for θ_0. Answering this question needs an improved ChPT prediction for $\theta_0 - \theta_8$ and/or θ_0, together with their accuracies, as will be seen shortly.

B. Further Comparison of VMD with ChPT

One can ask about the correspondence of the expressions for the ChPT parameters coming from VMD with their usual expressions in terms of f_π and f_K. For some of them, irrespective of the quality of the comparison reported above, it is easy to check that they actually coincide at leading order. Let us illustrate with a few examples referred to in Ref. [12].

From Eqs. (19), one can easily derive

$$
[F^8]^2 = [F^8_\eta]^2 + [F^8_{\eta'}]^2 = \left[\frac{(1+2z)}{3}\right]^2 f_\pi^2 + \frac{2x^2}{9}(1-z)^2 f_\pi^2 \quad . \tag{21}
$$

Let us define

$$
z = 1 + 2\varepsilon \ , \quad x = 1 + \delta \ . \tag{22}
$$

Neglecting terms of order ε^2, we have $\varepsilon = f_K/f_\pi - 1 \simeq 0.22$ and $\delta = x - 1 \simeq -0.08$ as values for these perturbations to exact SU(3) flavor and nonet symmetries. Using Eq. (22), Eq. (21) gives

$$[F^8]^2 = \left[1 + \frac{8\varepsilon}{3} + \frac{8\varepsilon^2}{3} + \mathcal{O}(\varepsilon^2\delta)\right] f_\pi^2 \quad . \tag{23}$$

At leading order, this also gives

$$3[F^8]^2 = 4f_K^2 - f_\pi^2 \tag{24}$$

which is indeed the leading term in Ref. [12] (see Eq. (11) there); the leading neglected term is numerically about 0.04. For F^0, we cannot really compare with a known expression, however, assuming the approximate relation [13]

$$F^0 \simeq \sqrt{1 + \Lambda_1}\ f_\pi \quad , \tag{25}$$

we may conclude that the coupling constant [12] $\Lambda_1 \simeq 0.20 \pm 0.04$, which might serve to estimate the other coupling constants [12] Λ_2 and Λ_3 from phenomenology. On the other hand, Eq. (19) also gives

$$F^0 F^8 \sin(\theta_8 - \theta_0) = f_8 b_0 + f_0 b_8 . \tag{26}$$

Using the expressions in Eq. (17) and in Eq. (22), this is

$$3F^0F^8 \sin(\theta_0 - \theta_8) = 2\sqrt{2}(f_K^2 - f_\pi^2)\,[1 + \varepsilon + \delta] . \tag{27}$$

which coincides at leading order with the corresponding quantity given in Ref. [12] (see Eq. (13) there). One should note, however, that the leading correction is reduced with respect to neglecting deviations from nonet symmetry, from 22% to 14%. On the other hand, one can check, at first non–leading order, that

$$F^0 = 1 + \frac{2}{3}\varepsilon \quad , \quad \text{and} \quad F^8 = 1 + \frac{4}{3}\varepsilon \tag{28}$$

in units of f_π, which also corresponds to the expectation that F^0 and F^8 differ only at first non–leading order [12]. Therefore, the difference between our VMD result for $\theta_0 - \theta_8$ ($21.64^\circ \pm 1.14^\circ$) and the estimate of Refs. [12,13] (14° to 16°) can only be due to non–leading contributions, if they were confirmed.

Under this condition, we can consider our numerical results given in Eq. (20) as a quite accurate input, valid for standard and extended ChPT.

C. The Relation between θ_8 and θ_P

Eqs. (19) and (17) together give

$$\tan\theta_8 = \frac{f_8 \tan\theta_P + b_8}{f_8 - b_8 \tan\theta_P} = \tan(\theta_P + \varphi_P) \quad , \text{with} \quad \tan\varphi_P = \sqrt{2}\frac{(1-z)}{(1+2z)}x \tag{29}$$

The difference between $\varphi|_{(x=1)} = -10.02^\circ$ and $\varphi|_{(x=0.917)} = -9.21^\circ$ is small enough that it is legitimate to consider the main effect to be SU(3) breaking.

Let us also consider the condition $\theta_0 = 0$ as exact, which is clearly close to real life (see Eq. (20)). One can easily check that it can be cast under the form

$$\tan\varphi_P = x^2 \tan\theta_P. \tag{30}$$

Therefore, Eq. (29) can be written

$$\tan\theta_8 = \tan\left[\theta_P + \arctan\left(x^2 \tan\theta_P\right)\right] \quad . \tag{31}$$

If nonet symmetry were unbroken, this equation gives $\theta_8 = 2\theta_P$, which clearly implies that the ChPT (decay constant) mixing angle is twice the (wave–function) mixing angle. Allowing for nonet symmetry breaking, we have at first non leading order in $\delta = x - 1$

$$\theta_8 = 2\theta_P + \delta \sin 2\theta_P \simeq 2\theta_P[1+\delta] \quad , \tag{32}$$

the last equation taking account of the smallness of θ_P. Therefore, up to deviations from nonet symmetry (*i.e.* less than 10%), the (wave–function) mixing angle fulfills $\theta_P \simeq \theta_8/2$, which clearly exhibits the connection between VMD/WZW and ChPT angles. This connection actually relies on the smallness of θ_0, which is confirmed within ChPT directly [12].

D. Estimate of the Quark Mass Ratio

Since we have some reason to trust the reliability of our numerical results in Eq. (19), we can use the following equation [12]

$$3\left\{[F_\eta^8 M_\eta]^2 + [F_{\eta'}^8 M_{\eta'}]^2\right\} = 4[f_K M_K]^2 \frac{2S}{(S+1)} - [f_\pi M_\pi]^2(2S-1) \tag{33}$$

in order to extract the ratio S of the strange quark mass to the mean value of the non–strange quark masses ($S = m_s/\hat{m}$). This equation is actually second degree and thus surely admits a spurious solution. The M_i terms are the corresponding meson masses.

Using the mass values for neutral mesons and the information given in Ref. [20], we get

$$\frac{m_s}{\hat{m}} = 21.23 \pm 2.42 \quad \text{or} \quad \frac{m_s}{\hat{m}} = 2.5^{+1.3}_{-0.7} \tag{34}$$

The first solution compares well to the expectation of Current Algebra (25.9) and to the estimate (26.6) of Ref. [12]. The magnitude of the uncertainties is dominated by the errors on decay constants; errors due to choosing the neutral masses for the relevant mesons are not accounted for. Thus, Eq. (33) is probably not the most accurate way to determine the ratio S, but our result is a quite interesting estimate. Anyway, this does not signal that the parameter values computed from the VMD basic fit parameters lead to some drawback.

VI. VMD, THE WZW LAGRANGIAN AND CHPT

The broken VMD model of Ref. [1] which relies on the HLS approach [3,4] to the interactions of vector and pseudoscalar mesons among each other and with the electromagnetic field has been shown to provide an accurate description of all radiative decays [1,2]. Because of the VMD assumption itself, it provides a unified and simultaneous description of the decays processes of kinds $VP\gamma$ and $P\gamma\gamma$, and both data sets are successfully accounted for.

We have shown here, as in Refs. [1,2], that the broken FKTUY Lagrangian provides the same description as breaking directly the WZW Lagrangian à la BKY [6], supplemented with nonet symmetry breaking. As far as only the basic ChPT parameters are concerned, effects of having $x \neq 1$ are clearly marginal.

Comparing the expressions coming out from there to usual Current Algebra expressions, some definition of the decay constants is exhibited, and only one mixing angle shows up in order to account for the decay modes $\eta/\eta' \to \gamma\gamma$ and for the $VP\gamma$ processes. This angle is $\theta_P \simeq \theta_8/2 \simeq -11^\circ$.

On the other hand, using results on the same modelling already derived in our Ref. [7], we have succeeded in reconstructing the standard expressions currently used in Chiral Perturbation Theory in order to describe the physics of isoscalar pseudoscalar mesons. We have also shown that the leading contribution to the decay constants and mixing angle coincide with the predictions of ChPT. The numerical values have been given and most have been shown to correspond to ChPT expectations.

Comparing the two sets of parameters, both derived from within a (VMD) common framework, the single clear conclusion is that there is a mismatch between the VMD/WZW definitions of decay constants and mixing angles and that currently stated within ChPT.

The consequences are rather practical. Usually, the experimental determination of the pseudoscalar parameters (f_0, f_8, θ) is derived using the (quite standard) VMD/WZW Eq. (9), which are nothing but the former equations of Current Algebra [8,9,24].

However, as generally, the model reconstruction quality [21] is defined by comparing fit results for $\overline{f_8}$, $\overline{f_0}$ or θ_P with ChPT numerical expectations (F^8, F^0, θ_8). Sometimes, the numerical ChPT expectations are even used to constraint these two–photon equations [8,9,14,15,25]. What has been illustrated above is that such a phenomenological approach and such a theoretical treatment, are probably intrinsically inconsistent. The fact that this inconsistency has been derived within a quite specific framework (VMD), using a quite specific breaking scheme (BKY and nonet symmetry breaking) cannot dismiss the conclusion, except if one were able to derive, directly from QCD, THE unique acceptable breaking scheme. Moreover, VMD and the breaking scheme just referred to, succeed in providing a quite consistent and successful picture of *all* data it should deal with [1]. From this point of view, it is quite important that $P\gamma\gamma$ modes are so easily constrained by the $PV\gamma$ modes of light mesons, as shown in Table 1.

To be more specific, we have also shown that $\overline{f_8} = (0.82 \pm 0.01) f_\pi$, $\overline{f_0} = (1.15 \pm 0.02) f_\pi$ and a (single) $\theta_P \simeq -11^\circ$ which describes the η/η' mixing at the wave function level, are perfectly consistent with the ChPT traditional expectations, mainly $F^8 \simeq (1.25 - 1.35) f_\pi$ and $\theta_8 \simeq -21^\circ$. VMD has been able to provide new information ($\theta_0 \simeq 0$ and $F^0 \simeq 1.10 f_\pi$) of relevance for ChPT.

The results above show that the mixing angle which occurs in two–photon decay amplitudes is not θ_8, but about half of it, which is nothing but the fit value [1] of θ_P. Stated otherwise : There is no contradiction between the one angle mixing scheme of VMD/WZW and the two–angle mixing scheme of ChPT, and the numbers in Eqs. (20) are all a reliable phenomenological input for the parameters [12,13] of ChPT.

The analysis we have presented clearly shows that the decay constants occurring in the Current Algebra formulae are *not* necessarily the decay constants defined in a standard way in ChPT. Related to this point, one should also recall, from Ref. [7], that SU(3) breaking does not affect the box anomalies for $\gamma\pi^+\pi^-\eta/\eta'$. It is clear however from Eq. (39) in this reference, that nonet symmetry breaking can play some (numerically) minor role. Thus, the existing analyses [21,25] of the anomaly equations [24] might have to be redone from scratch, at least for consistency, knowing that the expected parameter values are not (directly) the ChPT ones.

VII. FEEDBACK FROM THE CHPT PARAMETRIZATION

The ChPT parameter values we have obtained (see Eq. (20)) allow for several remarks of importance which are to be discussed in this Section.

A. A Hidden Relation between x and θ_P/θ_8

At the level of accuracy permitted by the whole set of radiative decays of light mesons, the results gathered in Eqs. (20) indicate that $\theta_0 = 0$ is well fulfilled experimentally. At its level of accuracy ($\theta_0 = 0.8^\circ \pm 0.8^\circ$), one can even ask oneself whether this relation is only approximate; this means that θ_0 does not undergo effects of SU(3) breaking, instead of θ_8 and θ_P. As remarked in Ref. [12], this also means that, in the sense that $|\eta\rangle$ is orthogonal to $\mathcal{A}^0|0\rangle$, the η meson is practically pure octet. As shown above, and at the meantime, the same $|\eta\rangle$ happens also to be a mixture of $|\pi^8\rangle$ and $|\eta^0\rangle$ with an angle $\theta_P \simeq -11^\circ$. This illustrates the duality of definitions from another point of view.

The numerical result $\theta_0 = 0$ indicates that the state mixing angle θ_P fulfills

$$\tan\theta_P = \frac{\sqrt{2}}{x}\frac{(1-z)}{(2+z)} \tag{35}$$

to good accuracy. As a glue component inside the η meson is clearly not requested by the data on radiative decays of light mesons, we know that the corresponding coupling angle (β in Refs. [1,2]) can be safely chosen to be zero. Under this condition, Eq. (35) happens to be independent of a possible glue coupling to the η' meson. So, even if possibly somewhat approximate, this equation is not influenced by glue at all.

Eq. (35) indicates a surprising correlation (x, θ_P) completely missed in the study of Ref. [1]. Looking at the correlation table of the corresponding fits, we indeed find a correlation coefficient of –33.5%, small but visible, and with the right sign. The small magnitude of this number could be due to the fact that the effective range of variation of these parameters is too small that correlations fully develop.

Therefore, even if somewhat approximate, Eqs. (35) and (32) reveal an unexpected algebraic relation between the mixing angles θ_P/θ_8 and the nonet symmetry breaking level x. To our knowledge, it is the first time such a relation is reported. It teaches that nonet symmetry breaking is determined by the value of either of the mixing angles (θ_P or θ_8). If $z = 1$ (then, all θ's vanish), the value of x becomes unconstrained.

As Chiral Perturbation Theory is able to provide an expression of the mixing angle θ_8 in terms of pseudoscalar meson masses, this also means that x can be defined in terms of the pseudoscalar meson masses, f_K and f_π in an reliable way. This has several consequences that have to be examined now.

B. A New Fit to Radiative Decays

From Eq. (35), we can conclude that x and θ_P are algebraically related, at least to a very good approximation. This leads us to redo the fits given in Ref. [1], requiring moreover this functional relation. This turns out to describe all radiative decays in terms of only 3 independent parameters (g, θ_P, θ_V), which is, by far, the most constrained fit of the 14 radiative decay modes ever attempted[9].

The fit quality obtained is $\chi^2/\text{dof}= 12.02/11$, and does not exhibit any degradation compared to the fit quality $\chi^2/\text{dof}= 11.07/10$, reached in Ref. [1] using the so–called K^* model[10] which involved additionally x as free parameter.

Practically, the fit returns all parameters at the values obtained when leaving x and θ_P unrelated, except for θ_P which changes from $\theta_P = -11.59^\circ \pm 0.76^\circ$ to $\theta_P = -12.31^\circ \pm 0.18^\circ$. The sharp reduction of the statistical error is an effect of removing the correlations by accounting explicitly for the functional relation in Eq. (35). The corresponding value for x is 0.926, which is of important consequence, as will be seen in the next subsection.

The results are given in Table 2, together with the previous (best) fit result of Ref. [1]. We refer the reader to this reference for the comparison with the data recommanded by the PDG [20]. We simply add a few remarks.

The first one is about the η meson decay to two–photon ; the value reported in Ref. [20], is determined by the 6 e^+e^- experiments and the experiment based on the Primakoff effect. If one retains the mean value of the e^+e^- experiments, generally considered as more reliable, the corresponding reference value becomes 43.56% (instead of 39.21%) and compares somehow better with (each) of the fit values.

---

[9] As, now, we know that θ_P and θ_8 are functionally related, and that $\theta_8 \simeq -21^\circ$ is equivalent to the favored [1] $\theta_P \simeq -11^\circ$, we could even fix θ_8 (and then θ_P) to its ChPT expectation, and consequently x too. We will not perform this exercise, as the accuracy on the ChPT estimate of θ_8 prevents [18] to really fix it. Nevertheless, it indicates that, from first principles, one can perform a fit to the 14 radiative decays, and parameters to be fit refer to vector mesons only.

[10] The specific purpose of this model, is to account for the decay mode $K^{*\pm} \to K^\pm\gamma$, which calls for a specific modification of the K^* sector in an otherwise "natural" and simple treatment. The discussion about this modified model is discussed from complementary points of view in Refs. [1] and [2]. We refer for this purpose the reader to both papers.

We do not observe any degradation in the quality of the description of the various branching fractions. Comparing the two sets of predictions in Table 2, one should note the sharp reduction of the statistical errors produced by having switched on the $x-\theta_{8/P}$ relation on all decay modes invloving the η meson. This also affects the modes involving η', however to a lesser extent.

One should also note that recent measurements [26] of $\phi \to \eta'\gamma$ by the CMD–2 Collaboration at VEPP–2M, using new η' decay modes, confirm the central value of Ref. [27] rather than that of Ref. [28] reported in the PDG [20]; the agreement with the fit values we always get for this mode [1] is thus improved. Indeed, the new measurement ($[5.8 \pm 1.8]10^{-4}$) reported by Ref. [26] (or by Ref. [27]) would provide a minimum χ^2 smaller than reported above by one unit ; it has not been used in the fit in order to keep consistency with a previous work [1] and with the 1998 accepted values [20].

C. The x–θ_P Kills Glue in the η' Meson

In the previous study of Refs. [1,2] it was shown that the correlation between glue and nonet symmetry breaking was huge. As said several times above, accounting generally for glue coupling to the η/η' system implies that two angles have to be introduced in addition to θ_P. One (β) is such that $\beta = 0$ implies that the η meson does not couple to glue, the other (γ) is such that $\gamma = 0$ implies a decoupling of the η' from glue.

However Table IV in Ref. [1] or Fig. 2 in Ref. [2], clearly show that : **i/** whatever is the value of the nonet symmetry breaking parameter x, the angle β is not observed to deviate sensitively from zero ; **ii/** additionally, for $x \simeq 0.92$ the angle γ is also consistent with zero.

We have mentioned in the previous subsection that the value for x which comes correlatively from the fit value of θ_P is 0.926 and the equation which produces this correlation ($\theta_0 = 0$) is free of any influence from the precise value of γ. So, we can conclude that within the picture presented in this paper, there is no signal for glue inside the η and the η' meson, but instead there is a significant signal for deviation from exact nonet symmetry : $x = 0.926$ relative to 1 reveals a 4σ significance level.

This is confirmed by performing the fit with γ and x, now decorrelated because of the functional relation in Eq. (35). In this case, the fit quality is strictly unchanged χ^2/dof= 12.02/10 and the minimum is reached for $\gamma = 0.17^\circ \pm 40^\circ$; this shows that no glue is requested. The magnitude of the error tells that the (x, γ) correlation is somewhat replaced by a (θ_P, γ) correlation. However, the fact that the minimum χ^2 is unchanged also signals that we do not get improvement by letting γ deviate from zero.

D. θ_P and the Isoscalar Mass Matrix

Then, the ChPT picture happens to be consistent with the standard one angle state mixing scheme of fields (or wave–functions) :

$$\begin{bmatrix} \eta \\ \eta' \end{bmatrix} = \begin{bmatrix} \cos\theta_P & -\sin\theta_P \\ \sin\theta_P & \cos\theta_P \end{bmatrix} \begin{bmatrix} \pi_8 \\ \eta_0 \end{bmatrix} \simeq \begin{bmatrix} \cos\dfrac{\theta_8}{2} & -\sin\dfrac{\theta_8}{2} \\ \sin\dfrac{\theta_8}{2} & \cos\dfrac{\theta_8}{2} \end{bmatrix} \begin{bmatrix} \pi_8 \\ \eta_0 \end{bmatrix} \tag{36}$$

where the last relation becomes a strict equality if $x = 1$ (no breaking of nonet symmetry). The most accurate value for θ_P comes out from fit to all radiative decays with the $x - \theta_P$ correlation set up. Indeed, from the subsection just above, we know that it is legitimate to neglect coupling to glue. Actually, one cannot assert that glue (or any other singlet state) is not present inside the η/η' system, but what is shown in Refs. [1,2] for η, and just above for η', is that no glue contribution is required. A kind of minimum complexity argument, then leads to state $\beta = \gamma = 0$ and to decouple glue from the η/η' system. The angle value is $\theta_P = -12.31^\circ \pm 0.18^\circ$, a hardly constrained value.

With this, it is possible to revisit the determination of the isoscalar mass matrix [8,19,25]. Indeed we know that the mass matrix $\mathcal{M}$:

$$\mathcal{M} \;\; = \left[\begin{array}{cc} m_{88}^2 & m_{08}^2 \\ \\ m_{08}^2 & m_{00}^2 \end{array} \right] \tag{37}$$

admits the following eigenvectors :

$$\left\{ \begin{array}{l} v_\eta = \;\; (\cos\theta_P \;, -\sin\theta_P) \\ \\ v_{\eta'} = \; (\sin\theta_P \;, \quad \cos\theta_P) \end{array} \right. \tag{38}$$

with eigenvalues M_η^2 and $M_{\eta'}^2$. This gives $m_{88}^2 = 0.326 \pm 0.001$, $m_{00}^2 = 0.889 \pm 0.001$ and $m_{08}^2 = -0.129 \pm 0.002$ in units of GeV^2. This corresponds to $m_{00} = 0.943 \pm \pm 0.001$ GeV, and $m_{88} = 0.571 \pm 0.003$ GeV, the former close to the η' mass and the latter close to the η mass as one might expect from the value of θ_P. The off–diagonal term can be written $m_{08}^2 = -0.522 M_K^2$. The solution favored by VMD phenomenology is a very small deviation from the classical Gell–Mann–Okubo formula [8,19,25]. More precisely, from :

$$M_{\pi^8}^2 = \frac{(4M_K^2 - M_\pi^2)}{3}\,[1 + \Delta] \tag{39}$$

(with $M_{\pi^8} \equiv m_{88} \simeq M_\eta$) one extracts $\Delta \simeq 0.01$.

E. The Differential Effect of Nonet Symmetry Breaking

Ref. [1], and Ref. [21] before, clearly proved that nonet symmetry breaking plays a major role in accounting for radiative decays of light mesons within a relatively simple and constrained framework. The effect is small ($\delta \simeq -0.08$) but statistically significant ($\simeq 4\sigma$). In this Section, and in the previous one, it was instead shown that the main ChPT parameters have values which are not sharply sensitive to having $x \neq 1$. To be more specific, effects of the small value of δ are competing with SU(3) breaking, always by modifying non–leading corrections ; for instance, the magnitude of the correction terms to $\theta_0 - \theta_8$ (see Eq. (22), for instance) are reduced by about 30% if one takes into account both ε and δ corrections, instead of only ε corrections.

VIII. CONCLUSION

From a previous work, we were faced to a paradoxical problem. Using the HLS model and its anomalous FKTUY sector, together with a definite breaking scheme, it is possible to achieve a quite satisfactory description of all radiative decays, including $\eta/\eta' \to \gamma\gamma$. Within this framework, it was moreover possible to predict accurately these last rates, using only numerical information obtained by fitting the $VP\gamma$ processes in isolation.

This quite satisfactory pattern was obscured by some strange results : the (single) pseudoscalar mixing angle was found at $\theta_P \simeq -11^\circ$ and the octet decay constant was $f_8 = 0.82 f_\pi$, both in obvious disagreement with unavoidable ChPT expectations.

The origin of this disagreement has been investigated. Starting from VMD, we have shown how to deduce the (WZW) η/η' two–photon amplitudes on the one hand, and the expectation values $\langle 0|\mathcal{A}^{0,8}|\eta/\eta'\rangle$ on the other hand ($\mathcal{A}$ is an axial field), which gives the customary (ChPT) definition of decay constants. Consistency of both information is guaranteed by their having been derived from within the same VMD framework. The origin of the disagreement reported above has been traced back to inconsistent definitions for the same parameters provided by ChPT and the WZW Lagrangian (through the former definitions of Current Algebra). For instance, it was shown that none of the two angles of extended ChPT can appear *as such* in the two–photon decay amplitudes. In some way, beside the angle θ_8 and θ_0 recently introduced by Kaiser and Leutwyler, the standard angle θ_P, which still describes the η/η' wave–function mixing, goes on playing an important role, the main one in radiative decays. It has been derived that $\theta_P = \theta_8/2$, up to higher order corrections due to our breaking of nonet symmetry.

Whether all this is a pure effect of VMD and/or deeper, is hard to evaluate. However, it does not seem cautious to avoid the help of all radiative decays as source of physics information. We thus have clearly illustrated that, besides the successful description of all radiative decays, broken VMD was also able to meet all the requirements of extended ChPT (the two mixing angles and the two decay constants) with a good accuracy. It was then shown that the relevant (and self–consistent) angle pattern is $\theta_0 \simeq 0^\circ$, $\theta_8 \simeq -20^\circ$ and $\theta_P \simeq -12^\circ$. Correspondingly, we have simultaneously $\overline{f_8} = 0.82 f_\pi$ when using the WZW (or Current Algebra) definition and $f_8 = 1.33 f_\pi$, when using standard ChPT definition. Therefore, the VMD phenomenology is indeed able to provide ChPT with quite reliable inputs. One may, for instance, guess that the consequences of $\theta_0 = 0^\circ$ within ChPT have still to be explored.

This study has led us to several additional conclusions :

i/ Because $\theta_0 = 0$ is observed with a quite impressive precision, it has been possible to relate functionally nonet symmetry breaking (the parameter x) and the mixing angle θ_P. To our knowledge such a relation had never been reported.

ii/ As consequence of the value obtained for x (0.926), it has been shown, that no glue component was requested inside the η' meson.

iii/ A fit of all radiative decay modes can be performed with practically all pseudoscalar meson parameters fixed from outside (f_K, and the ChPT mixing angle) ; awaiting that an accurate estimate of θ_8 becomes available, we have illustrated the fit quality reached already by imposing the x–θ correlation. Indeed, it follows from the inconsistency shown above that previous values for θ_8, reported from phenomenological analyses of η/η' decays, are artefacts.

iv/ We observe that departure from the Gell–Mann–Okubo quadratic mass relation is at the level of the percent.

v/ The quark mass ratio deduced from VMD information is $m_s/\hat{m} = 21.2 \pm 2.4$.

All this clearly illustrates that VMD is not only a reasonable approximate guess, but also a powerful tool in analyzing experimental data in its field of relevance.

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Process	x and θ_P Related	x and θ_P Unrelated	PDG
$\rho \to \pi^0\gamma$ $(\times 10^4)$	5.16 ± 0.03	5.16 ± 0.03	6.8 ± 1.7
$\rho \to \pi^\pm\gamma$ $(\times 10^4)$	5.13 ± 0.03	5.12 ± 0.03	4.5 ± 0.5
$\rho \to \eta\gamma$ $(\times 10^4)$	3.40 ± 0.02	3.31 ± 0.09	$2.4^{+0.8}_{-0.9}$
$\eta' \to \rho\gamma$ $(\times 10^2)$	32.71 ± 1.45	33.0 ± 1.8	30.2 ± 1.3
$K^{*\pm} \to K^\pm\gamma$ $(\times 10^4)$	9.83 ± 0.95	9.80 ± 0.93	9.9 ± 0.9
$K^{*0} \to K^0\gamma$ $(\times 10^3)$	2.32 ± 0.02	2.32 ± 0.02	2.3 ± 0.2
$\omega \to \pi^0\gamma$ $(\times 10^2)$	8.50 ± 0.05	8.50 ± 0.05	8.5 ± 0.5
$\omega \to \eta\gamma$ $(\times 10^4)$	8.30 ± 0.08	8.12 ± 0.19	6.5 ± 1.0
$\eta' \to \omega\gamma$ $(\times 10^2)$	2.74 ± 0.13	2.80 ± 0.20	3.01 ± 0.30
$\phi \to \pi^0\gamma$ $(\times 10^3)$	1.25 ± 0.12	1.26 ± 0.13	1.31 ± 0.13
$\phi \to \eta\gamma$ $(\times 10^2)$	1.19 ± 0.01	1.22 ± 0.04	1.26 ± 0.06
$\phi \to \eta'\gamma$ $(\times 10^4)$	0.65 ± 0.01	0.63 ± 0.02	$1.2^{+0.7}_{-0.5}$
$\eta \to \gamma\gamma$ $(\times 10^2)$	43.15 ± 0.06	41.5 ± 1.4	39.21 ± 0.34
$\eta' \to \gamma\gamma$ $(\times 10^2)$	2.10 ± 0.08	2.10 ± 0.10	2.11 ± 0.13

Table 2 : Radiative decay branching fractions. The first two data columns display the fit results using the K^* model of Refs. [1,2]; in the first data column (present work) the x–θ_P of Eq. (35) is switched on, while in the second one it is not (numbers taken from Ref. [1]). The last data column gives the accepted values from the 1998 Review of Particle Properties [20].

REFERENCES

[1] M. Benayoun *et al.*, hep-ph/9902326, (to appear in Phys. Rev. **D**).

[2] M. Benayoun *et al.*, Proceeding of the international Workshop "e^+e^- Collisions from ϕ to J/ψ", Novosibirsk, March 1–5, 1999, to be published.

[3] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985); M. Bando, T. Kugo and K. Yamawaki, Phys. Rep. **164**, 217 (1985).

[4] T. Fujiwara, T. Kugo, H. Terao, S. Uehara and K. Yamawaki, Prog. Th. Phys. **73**, 926 (1985).

[5] P. O'Donnell, Rev. of Mod. Phys. **53** (1981) 673.

[6] M. Bando, T. Kugo and K. Yamawaki, Nucl. Phys. **B259**, 493 (1985).

[7] M. Benayoun and H.B. O'Connell, Phys. Rev. **D58** (1998) 074006.

[8] J. F. Donoghue, B. R. Holstein and Y. R. Lin, Phys. Rev. Lett. **55** (1985) 2766.

[9] F. J. Gilman and R. Kauffman, Phys. Rev. **D36** (1987) 2761.

[10] P. Ball, J.–M. Frère and M. Tytgat, Phys. Lett. **B365** (1996) 367.

[11] G.M. Shore and G. Veneziano, Nucl. Phys. **B381** (1992) 3.

[12] H. Leutwyler, Nucl. Phys. Proc. Suppl. **64**, 223 (1998) [hep-ph/9709408].

[13] R. Kaiser and H. Leutwyler, hep-ph/9806336.

[14] T. Feldman and P. Kroll, Eur. Phys. J. **C5** (1998) 327 ; Phys. Rev. **D58** (1998) 057501.

[15] T. Feldman, P. Kroll and B.Stech, hep-ph/9812269.

[16] J. Wess and B. Zumino, Phys. Lett. **37B** (1971) 95.

[17] E. Witten, Nucl. Phys. **B223** (1983) 422.

[18] J. Gasser and H. Leutwyler, Nucl. Phys. **B250** (1985) 465; Ann. Phys. (NY) **158** (1984) 142.

[19] J.F. Donoghue, E. Golowich, B.R. Holstein, "Dynamics of the Standard Model", Cambridge University Press, Cambridge UK, 1996, P. 204.

[20] C. Caso *et al.* Eur. Phys. Journ. **C3** (1998) 1.

[21] M. Benayoun, Ph. Leruste, L. Montanet and J.–L. Narjoux, Zeit. Phys. **C65** (1995) 399.

[22] A. Bramon, A. Grau and G. Pancheri, Phys. Lett. **B344**, 240 (1995).

[23] H.B. O'Connell, B.C. Pearce, A.W. Thomas and A.G. Williams, Prog. Part. Nucl. Phys. **39** (1996) 201.

[24] M.S. Chanowitz, Phys. Rev. Lett. **35** (1975) 977 ; Phys. Rev. Lett. **44** (1980) 59.

[25] E.P. Venugopal and B.R. Holstein, Phys. Rev. **D 57** (1998) 4397.

[26] T. Purlatz *al.*, Proceeding of the international Workshop "e^+e^- Collisions from ϕ to J/ψ", Novosibirsk, March 1–5, 1999, to be published.

[27] V. M. Aulchenko *et al.*, Phys. Lett. **B436** (1998) 199 ; M.N. Achasov *et al.*, Preprint Budker INP 98–65, Novosibirsk, 1998.

[28] R.R. Akhmetshin *et al.*, Phys. Lett. **B415** (1997) 445.