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Neutrino propagation in matter with general interactions

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Abstract

We present a general analysis of the effective potential for neutrinos propagation in matter, assuming a generic set of Lorentz invariant non-derivative interactions. We find that besides the known vector and axial vector terms, in a polarized medium also tensor interactions can play an important role. We compute the effective potential arising from a tensor interaction. We show that the components of the tensor potential transverse to the direction of the neutrino propagation can induce a neutrino spin-flip, similar to the one induced by a transverse magnetic field.

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I. INTRODUCTION

Neutrino physics currently provides the strongest experimental evidence for physics beyond the Standard Model (SM). The atmospheric neutrino anomaly [1] and the solar neutrino [2] problem are best explained by neutrino oscillations.

Neutrino oscillations occur when the produced neutrinos are not eigenstates of the Hamiltonian that describes their propagation. In vacuum, this is the case if the flavor eigenstates are non-trivial linear combinations of different mass eigenstates. This requires massive neutrinos that mix. It is well known that the neutrino propagation in matter can be very different from that in vacuum. The crucial fact is that coherent interactions with the background give to the neutrino an "index of refraction" which depends on its flavor. This is because normal matter, that contains only first generation fermions, is flavor asymmetric. For example, for standard weak interactions, only electron neutrinos can have charged current interactions with the background electrons. Thus, in matter the effective electron neutrino mass depends on the electron density and is enhanced with respect to the other flavors. This allows for the possibility of level crossing between different neutrino eigenstates in matter. If the electron neutrinos are produced with an effective mass above the level crossing (the "resonance") an adiabatic transition through the resonance can induce a significant amplification of neutrino oscillations. This is known as the Mikheyev-Smirnov-Wolfenstein (MSW) effect [3]. If light sterile neutrinos exist, then also neutral current interactions are important since only the active neutrinos are subject to it [4]. In a polarized medium the neutrino effective mass also depends on the average polarization of the background, and on the angle between the neutrino momentum and the polarization vector [5,6].

Many extensions of the SM imply massive neutrinos. It is important to stress that these new physics models often predict also new neutrino interactions. In this case the SM picture can be significantly changed [7], since the neutrino effective mass will depend on both the SM and the new interactions. For example, non-universal interactions may give rise to matter effects that distinguish between muon and tau neutrinos. Lepton flavor violating interactions can induce an effective mixing in matter, allowing for a resonant conversion even in the absence of vacuum mixing. The two effects combined together could induce neutrino flavor transitions even for massless neutrinos.

Most of the discussions of neutrino oscillations in matter are based on the effective Hamiltonian

$$\mathcal{H}_{(V,A)} = \frac{G_F}{\sqrt{2}} \sum_{a=V,A} \left(\bar{\nu}\,\Gamma_a\,\nu\right) \left[\bar{\psi}_f\,\Gamma^a\,\left(g_a + g'_a\gamma^5\right)\psi_f\right] + \text{h.c.}\,, \tag{1.1}$$

where $\Gamma^V = \gamma^\mu$, $\Gamma^A = \gamma^\mu\gamma^5$, ψ_f are the field operators for the background fermions $f = e, p, n, \nu$ and g_a, g'_a are suitable coupling constants parametrizing the strength of the interactions.

Clearly, the standard neutral current and the Fierz rearranged charged current $(V-A)(V-A)$ structures are included in (1.1). However, $\mathcal{H}_{(V,A)}$ describes in fact a larger set of interactions. For example, several models where neutrinos couple to new heavy scalars (like supersymmetric models without R-parity and left-right symmetric models) imply low energy effective interactions of the form $\bar{\nu}(S \pm P)\psi_f \, \bar{\psi}_f(S \mp P)\nu$ that, after Fierz rearrangment, are also accounted for by (1.1). The interactions in (1.1) only induce transitions between neutrinos of the same chirality. Therefore the couplings between different helicity states that would flip the neutrino spin are suppressed by the ratio between the neutrino mass and its energy, m/E, and can be safely neglected. Thus the matter effects induced by (1.1) only allow for flavor transitions that conserve the neutrino spin. (Note that transitions into sterile neutrinos [4] are no exception. The sterile neutrino is a SM singlet, but the state that is produced via oscillations has negative helicity.)

In contrast, neutrino transitions induced by a magnetic field result in a spin-flip [8]: a left-handed neutrino is rotated into a right-handed one. The rate of this transition depends on the neutrino magnetic moment and on the strength of the component of the magnetic field orthogonal to the direction of the neutrino propagation. If the SM is extended just by introducing right-handed neutrinos, the resulting neutrino magnetic moment is vanishingly small, and spin-flipping transitions are negligible even for the largest conceivable magnetic fields. Therefore, spin-flipping transitions can be relevant for solar or supernova neutrinos only in the presence of new physics that induce a very large neutrino magnetic moment.

While the couplings in (1.1) account for the SM weak interactions as well as for some new physics interactions they are clearly not the most general ones. In this paper we systematically study the effects of all Lorentz invariant non-derivative interactions of neutrinos with the background fermions. Namely, we add scalar (S), pseudoscalar (P) and tensor (T) interactions, to the vector (V) and axial-vector (A) interactions in (1.1). In our analysis we reproduce the known results for V and A interactions [5]. The S and P interactions that couple states with opposite chirality but the same helicity are suppressed by m/E and therefore are negligible. Our main result is that transverse tensor interactions induce effects which are not helicity suppressed, because they couple states of both opposite chirality and opposite helicity. We find that in a polarized medium these interactions can flip the neutrino spin. The overall effect depends on the strength of the interaction, on the density of the background and on the average polarization of the medium. The physics is similar to the electromagnetic spin-flip, however in this case spin-flipping transitions can be effective even for a vanishing neutrino magnetic moment. We note that an effective tensor potential does not need to arise from a fundamental tensor interaction. It can also result after Fierz reordering from some specific scalar and pseudoscalar couplings of the neutrinos to the background fermions.

II. NEUTRINO PROPAGATION IN MATTER WITH GENERAL INTERACTIONS

In this section we derive the neutrino propagation equation in matter in the presence of the most general pointlike and Lorentz invariant four-fermion interaction with the background fermions ($f = e, p, n, \nu$). That is, we generalize (1.1) to

$$\mathcal{H}_{\rm int} = \frac{G_F}{\sqrt{2}} \sum_a (\bar{\nu}\,\Gamma^a\,\nu)\,\left[\bar{\psi}_f\,\Gamma_a\,(g_a + g'_a\gamma^5)\,\psi_f\right] + \text{h.c.}\,, \tag{2.1}$$

where $\Gamma^a = \{I, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}\}$, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ and $a = \{S, P, V, A, T\}$. Here the neutrino ν is assumed to be of the Dirac type (we will comment on the Majorana case later). In general, ν is a vector of the different neutrino types, and g_a, g'_a are 10 matrices in the space of neutrino flavors that describe the coupling strengths. In (2.1) the Fermi constant G_F has been factored out so that all the couplings are dimensionless. From the hermiticity of $\mathcal{H}_{\rm int}$ in (2.1) it follows that all g_a as well as g'_V, g'_A are hermitian while g'_S, g'_P and g'_T are antihermitian. In particular, the diagonal elements in g_a and g'_V, g'_A are real while those of g'_S, g'_P and g'_T are imaginary. We stress that new interactions in general include both flavor diagonal and off-diagonal couplings. The SM charged current interactions of a ν_e with background electrons correspond to $g_V = -g'_V = g_A = -g'_A = 1$ for the $\nu_e - \nu_e$ entries, while all the other couplings vanish.

The derivation of the equation of motion describing the neutrino propagation in a medium proceeds as follows. First we average the effective interactions over the background fermions. We are not interested in incoherent effects that become negligible after averaging. Therefore, while we do allow for neutrino spin-flipping interactions, we require that the background fermions do not undergo spin-flip. That is, we select coherent transitions that leave the many-fermion background system in the same state. Next we add the effective neutrino interaction to the free Lagrangian, and we derive the equation of motion for neutrino propagation in matter. Finally, we study the neutrino dynamics described by the equation of motion, under the assumption that the masses and potential terms are much smaller than the neutrino energy.

A. Computing the effective neutrino potential

The effect of the medium on the neutrino propagation in the presence of the general interactions (2.1) can be described by the Lagrangian

$$-\,\mathcal{L}_{\rm int} = \sum_{a,f} (\bar{\nu}\,\Gamma^a\,\nu)\,V_a^f\,, \tag{2.2}$$

where

$$V_a^f = \frac{G_F}{\sqrt{2}} \sum_{\boldsymbol{\lambda}} \int \frac{d^3p}{(2\pi)^3} \rho_f(\boldsymbol{p},\boldsymbol{\lambda}) \mathcal{M}_a^f \,, \tag{2.3}$$

is given by the expectation value of the background fermion current $\mathcal{M}_a^f$, averaged over the fermion distribution $\rho_f(\boldsymbol{p},\boldsymbol{\lambda})$. Here $\boldsymbol{p}$ and $\boldsymbol{\lambda}$ denote, respectively, the momentum and polarization vectors of the background fermion f. According to the requirement of leaving the many-fermion background system unmodified, the matrix element

$$\mathcal{M}_a^f \equiv \langle f,\boldsymbol{p},\boldsymbol{\lambda}|\bar{\psi}_f\,\Gamma_a\,(g_a + g'_a\gamma^5)\,\psi_f|f,\boldsymbol{p},\boldsymbol{\lambda}\rangle \tag{2.4}$$

is taken between initial and final states with the same quantum numbers. The computation of the various $\mathcal{M}_a^f$ is straightforward and is given in the Appendix. We find

$$V^S = \frac{G_F}{\sqrt{2}}\, n_f\, g_S \left\langle \frac{m_f}{E_f} \right\rangle \,, \tag{2.5}$$

$$V^P = \frac{G_F}{\sqrt{2}}\, n_f\, g'_P \left\langle \frac{m_f}{E_f} \right\rangle \,, \tag{2.6}$$

$$V_\mu^V = \frac{G_F}{\sqrt{2}}\, n_f \left[g_V \left\langle \frac{p_\mu}{E_f} \right\rangle + g'_V\, m_f \left\langle \frac{s_\mu}{E_f} \right\rangle \right] \,, \tag{2.7}$$

$$V_\mu^A = \frac{G_F}{\sqrt{2}}\, n_f \left[g'_A \left\langle \frac{p_\mu}{E_f} \right\rangle + g_A\, m_f \left\langle \frac{s_\mu}{E_f} \right\rangle \right] \,, \tag{2.8}$$

$$V_{\mu\nu}^T = \frac{G_F}{\sqrt{2}}\, n_f \left[-g_T\, \epsilon_{\mu\nu\rho\sigma} \left\langle \frac{p^\rho s^\sigma}{E_f} \right\rangle + i g'_T \left\langle \frac{p_\mu s_\nu - p_\nu s_\mu}{E_f} \right\rangle \right] \,, \tag{2.9}$$

where the spin-vector s, which satisfies $s^2 = -1$ and $s_\mu\, p^\mu = 0$, is given explicitly in (A5), and

$$n_f = \sum_{\boldsymbol{\lambda}} \int \frac{d^3p}{(2\pi)^3} \rho_f(\boldsymbol{p},\boldsymbol{\lambda}) \,, \qquad \langle x \rangle = \frac{1}{n_f} \sum_{\boldsymbol{\lambda}} \int \frac{d^3p}{(2\pi)^3} \rho_f(\boldsymbol{p},\boldsymbol{\lambda})\, x(\boldsymbol{p},\boldsymbol{\lambda}) \tag{2.10}$$

denote, respectively, the number density of the fermion f and the average of some function $x(\boldsymbol{p},\boldsymbol{\lambda})$ over the fermion distribution.

We can now perform the contractions $\Gamma^a\, V_a^f$ in (2.2), which yield

$$\Sigma^{SP} \equiv \Sigma^0 \left[V^S + V^P\,\gamma^5 \right] \;= \frac{G_F}{\sqrt{2}}\, n_f \left\langle \frac{m_f}{E_f} \right\rangle \left(g_S + g'_P\,\gamma^5 \right) \tag{2.11}$$

$$\Sigma^{VA} \equiv \gamma^\mu \left[V_\mu^V + V_\mu^A\,\gamma^5 \right] \;= \frac{G_F}{\sqrt{2}}\, n_f \left[\left\langle \frac{\not{p}}{E_f} \right\rangle \left(g_V + g'_A\,\gamma^5 \right) + m_f \left\langle \frac{\not{s}}{E_f} \right\rangle \left(g'_V + g_A\,\gamma^5 \right) \right] \tag{2.12}$$

$$\Sigma^T \equiv \Sigma^i \left[V_i^B + i V_i^E\,\gamma^5 \right] = \frac{G_F}{\sqrt{2}}\, n_f \left\langle \frac{[\not{s},\not{p}]}{E_f} \right\rangle \left(g'_T + g_T\,\gamma^5 \right) \tag{2.13}$$

where $\Sigma^\mu \equiv \mathrm{diag}\,(\sigma^\mu,\sigma^\mu)$ with $\sigma^\mu = (\sigma^0\,,\sigma^i)$ and $\sigma^0 = I$, and we have used $\sigma^{ij} = \epsilon^{ijk}\,\Sigma^k$ and $\sigma^{0i} = i\,\Sigma^i\gamma^5$. In (2.13) we have decomposed the tensor term $V_{\mu\nu}^T$, in analogy to the electro-magnetic field tensor $F_{\mu\nu}$, as $V_i^B = \epsilon_{ijk}\,V_{jk}^T$ and $V_i^E = 2\,V_{0i}^T$. Note that the second equality in (2.13) makes apparent that the tensor interaction can contribute only in the presence of a polarized background.

B. Equations of Motion

We turn now to study the effects of the potential on the neutrino propagation. The equation of motion can be deduced from the neutrino Lagrangian

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int} = \bar{\nu}(i\partial\!\!\!/ - m - \Sigma)\nu \tag{2.14}$$

where the matrix of the potentials

$$\Sigma \equiv \Sigma^{SP} + \Sigma^{VA} + \Sigma^{T} \tag{2.15}$$

depends on the background density and polarization, and in general will vary along the neutrino propagation path. In the general case both Σ and m are matrices in the space of neutrino types. It is instructive to write the interaction part in (2.14) explicitly in the chiral basis, see (A12)

$$-\mathcal{L}_{int} = \bar{\nu}\,\Sigma\,\nu = \begin{pmatrix} \nu_L^\dagger \\ \nu_R^\dagger \end{pmatrix}^T \begin{pmatrix} V_\mu^{LL}\bar{\sigma}^\mu & V_\mu^{LR}\sigma^\mu \\ V_\mu^{RL}\sigma^\mu & V_\mu^{RR}\sigma^\mu \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}, \tag{2.16}$$

where $\bar{\sigma}^\mu = (\sigma^0\,, -\sigma^i)$ and

$$V_\mu^{LL} \equiv V_\mu^V - V_\mu^A\,, \qquad V_\mu^{RR} \equiv V_\mu^V + V_\mu^A\,, \tag{2.17}$$

$$V_0^{RL} \equiv V^S - V^P\,, \qquad V_0^{LR} \equiv V^S + V^P\,, \tag{2.18}$$

$$V_i^{RL} \equiv V_i^B - i\,V_i^E\,, \qquad V_i^{LR} \equiv V_i^B + i\,V_i^E\,. \tag{2.19}$$

The explicit form (2.16) makes apparent that the (axial)vector potentials (contained in V^{LL} and V^{RR}) couple neutrinos of the same chirality, while the (pseudo)scalar and tensor potentials (in V^{RL} and V^{LR}) couple neutrinos of opposite chirality.

From (2.14) it follows that the equations of motion for neutrinos and antineutrinos are, respectively,

$$\gamma_0(k\!\!\!/ - m - \Sigma)u = 0\,, \qquad \gamma_0(k\!\!\!/ + m + \Sigma)v = 0\,. \tag{2.20}$$

We note that the signs of m and Σ are opposite for the antineutrinos. The dispersion relations for the neutrino propagation are given by the solutions of

$$\det\left[\mathcal{O}\right] = \det\left[\gamma_0(k\!\!\!/ - m - \Sigma)\right] = 0. \tag{2.21}$$

Solving (2.21) is simplified by working in the following approximation. Let us chose the neutrino momentum along the z-axis ($\boldsymbol{k} = k\hat{\boldsymbol{z}}$). Then $\sigma_{0,3}$ couple between states of the same helicity while $\sigma_{1,2}$ couple neutrinos of opposite helicity. Hence, for ultra-relativistic neutrinos, $V_{1,2}^{LL}$ and $V_{1,2}^{RR}$ in the chirality conserving diagonal blocks in (2.16) and $V_{0,3}^{LR}$ and $V_{0,3}^{RL}$ in the chirality flipping off-diagonal blocks are suppressed as $m/E \ll 1$, and can be

neglected. Thus, the relevant potential terms in (2.16) are $V^{LL}_{0,3}$, $V^{RR}_{0,3}$ and the tensor potential components $V^{LR}_{1,2}$ and $V^{RL}_{1,2}$ that are transverse with respect to the neutrino propagation direction. In this approximation we get

$$\mathcal{O} = \begin{pmatrix} E+k-V^{LL}_{0+3} & 0 & -m & -V^{LR}_{-} \\ 0 & E-k-V^{LL}_{0-3} & -V^{LR}_{+} & -m \\ -m & -V^{RL}_{-} & E-k-V^{RR}_{0-3} & 0 \\ -V^{RL}_{+} & -m & 0 & E+k-V^{RR}_{0+3} \end{pmatrix}, \tag{2.22}$$

where $V_{0\pm3} \equiv V_0 \pm V_3$ and $V_{\pm} \equiv V_1 \pm iV_2$. Note that since $V^{V,A,T}$ are hermitian, which implies that $(V^{RL}_{\pm})^{\dagger} = V^{LR}_{\mp}$, the matrix (2.22) is manifestly hermitian. Solving the determinant equation for (2.22) in the limit of ultra-relativistic neutrinos, assuming $V^{V,A,T}, m \ll E$. we obtain the general Hamiltonian that governs the neutrino propagation

$$i\frac{d}{dt}\begin{pmatrix}\nu_L \\ \nu_R\end{pmatrix} = \mathcal{H}_{\nu}\begin{pmatrix}\nu_L \\ \nu_R\end{pmatrix} \quad \text{with} \quad \mathcal{H}_{\nu} \equiv k + \frac{m^2}{2k} + \begin{pmatrix} V^{LL}_{0+3} & V^{LR}_{+} \\ V^{RL}_{-} & V^{RR}_{0+3} \end{pmatrix}. \tag{2.23}$$

The two energy eigenvalues are

$$E = k + \frac{m^2}{2k} + \tfrac{1}{2}\left[V^{LL}_{0-3} + V^{RR}_{0-3} \pm \sqrt{(V^{LL}_{0-3} - V^{RR}_{0-3})^2 + 4\,V^{LR}_{+}V^{RL}_{-}}\right], \tag{2.24}$$

where the plus (minus) sign refers to neutrinos that are mainly left(right)-handed states. The solutions for antineutrinos can be obtained from (2.24) by changing the sign of the potentials ($V \to -V$). Note that the contribution to the energy levels from the tensor term, which is quadratic in V^T, does not change sign. In the case of more than one neutrino flavor (2.24) is a matrix equation in the space of the neutrino types. It is interesting to note that in general we should not expect that the various interactions in (2.24) will be diagonal in the same basis. In this case even in the massless limit (or for degenerate neutrinos) flavor oscillations can occur in matter. In the one flavor case, the energy gap between the two states is

$$\Delta E_{\nu} = \sqrt{(V^{LL}_{0-3} - V^{RR}_{0-3})^2 + 4\,V^{LR}_{+}V^{RL}_{-}}. \tag{2.25}$$

In the limit of vanishing tensor interaction ($V_T = 0$) ν_L decouples from ν_R, and we obtain

$$E_L = k + \frac{m^2}{2k} + V^{LL}_{0-3}, \qquad E_R = k + \frac{m^2}{2k} + V^{RR}_{0-3}. \tag{2.26}$$

Clearly in this case we can have oscillations only between different neutrino flavors. Moreover, if there is a basis where the full V^{LL} (or V^{RR}, for the SM sterile states) is flavor diagonal, then oscillations can occur only in the presence of non-trivial mixings in the mass matrix. Setting $V^{RR} = 0$ and V^{LL} equal to the SM charged current and neutral current interactions, we recover the SM case, with non-interacting right-handed states.

So far we only discussed the case of neutrinos propagating in a background of particles. If also antiparticles (e.g. positrons) are present in the background, one has to take into

account the corresponding interactions. Assuming CP conservation we find that neutrino scattering off antifermions leads to the Hamiltonian in (2.23), but with opposite sign for the potential matrix.

III. IMPLICATIONS AND DISCUSSION

The general interactions that we have studied in the previous section can give rise to several effects for neutrino oscillations in matter. It is well-known that the vector and axial-vector interactions can be very important for neutrino propagation in dense matter. These interactions do not change the neutrino spin, but they can enhance flavor transitions when the neutrino moves through a resonance [3].

To recover the SM result for the potential felt by an electron neutrino propagating in an electron background, we set $g_V = -g'_A = g_A = -g'_V = 1$ in (2.12) and obtain

$$\Sigma^{SM} = \sqrt{2}\, G_F\, n_e \left(\left\langle \frac{\not{p}}{E_e} \right\rangle - m_e \left\langle \frac{\not{s}}{E_e} \right\rangle \right) P_L \,, \tag{3.1}$$

with $P_L = \frac{1}{2}(1-\gamma_5)$. Defining $\hat{\boldsymbol{k}} = \boldsymbol{k}/|\boldsymbol{k}|$ as a unit vector in the direction of the neutrino momentum $\boldsymbol{k}$ and using the explicit expression (A5) for the spin vector s we obtain

$$V^{SM}_{\nu,\bar{\nu}} = \pm\sqrt{2}\, G_F\, n_e \left[1 - \left\langle \frac{\hat{\boldsymbol{k}}\cdot\boldsymbol{p}}{E_e} \right\rangle - \left\langle \frac{\boldsymbol{p}\cdot\boldsymbol{\lambda}}{E_e} \right\rangle + m_e \left\langle \frac{\hat{\boldsymbol{k}}\cdot\boldsymbol{\lambda}}{E_e} \right\rangle + \left\langle \frac{(\hat{\boldsymbol{k}}\cdot\boldsymbol{p})\,(\boldsymbol{p}\cdot\boldsymbol{\lambda})}{E_e(m_e+E_e)} \right\rangle \right] , \tag{3.2}$$

which is valid for an arbitrary neutrino direction. The plus-sign in (3.2) refers to neutrinos and the minus-sign to antineutrinos. We note that (3.2) is in agreement with the results given in [5].

Our main result is, however, that in the presence of a neutrino tensor interaction with the background fermions, the neutrino can undergo spin-flip. This effect is similar to the spin-precession induced by a transverse magnetic field $B_\perp$ that couples to the neutrino magnetic dipole moment μ_ν. In fact, if we substitute in (2.23) the off-diagonal term $V^{LR}_{\pm}$ by $\mu_\nu B_\perp$ we obtain the equation of motion for a neutrino that propagates in a magnetic field [8,9]. Thus, while these two scenarios originate from different physics, formally they can be treated in the same way.

To illustrate the effects of neutrino oscillations due to the presence of a non-zero transverse tensor potential, we consider the simplest case of one neutrino generation. A left-handed neutrino that was produced at $t = 0$ and propagates for a time t in a constant medium will be converted into a right-handed neutrino with a probability

$$P^{LR}_{\nu}(t) = \sin^2 2\theta \, \sin^2 \left(\frac{\Delta E_\nu \, t}{2} \right) . \tag{3.3}$$

The effective mixing angle θ is given by

$$\sin^2 2\theta = \frac{|2V_+^{LR}|^2}{(\Delta E_\nu)^2}\,, \tag{3.4}$$

where the energy splitting ΔE_ν is defined in (2.25). (Note that for one neutrino flavor we have $V_+^{LR}\,V_-^{RL} = |V_+^{LR}|^2$). In the case of more than one neutrino flavor, propagation in a medium with changing density can lead to resonance effects in complete analogy to the magnetic field induced resonant spin-flip. We will not discuss the details of the resonant case here (which can be found in the existing literature [9]), but we want to discuss shortly the results for different types of background matter.

First consider the case where the background fermions are assumed to have an isotropic momentum distribution. Then, the relevant (transverse) component of the tensor potential which determines the effective mixing in (3.4) is given by

$$|V_+^{LR}| = \sqrt{2}\,G_F\,n_f\sqrt{|g_T|^2+|g_T'|^2}\,\left\langle \lambda_\perp\,\left(\sin^2\vartheta + \frac{m_f}{E_f}\cos^2\vartheta\right)\right\rangle\,, \tag{3.5}$$

where ϑ is the angle between the momentum and the transverse polarization of the of the background fermion and $\lambda_\perp = \sqrt{\lambda_1^2+\lambda_2^2}$. Note that $|V_+^{LR}|$ vanishes if the neutrino propagates along the direction of the average background polarization ($\lambda_\perp = 0$). For a non-relativistic background only p_0 contributes and we obtain from (3.5) that the effective mixing angle is determined by

$$|V_+^{LR}| = \sqrt{2}\,G_F\,n_f\sqrt{|g_T|^2+|g_T'|^2}\,\langle\lambda_\perp\rangle\,. \tag{3.6}$$

In the ultra-relativistic limit the effective mixing depends on $\langle\lambda_\perp\,\sin^2\vartheta\rangle$ which equals to $\langle\lambda_\perp/2\rangle$ if $\lambda_\perp$ is uncorrelated to the momentum of the background fermion. Finally, for a degenerate background in the presence of a magnetic field, only the fermions in the lowest Landau level contribute to the polarization, with the spin oriented antiparallel to the momentum. In this case the background is not isotropic, and eq. (3.5) is not applicable. One obtains for this case

$$|V_+^{LR}| = \sqrt{2}\,G_F\,n_f\sqrt{|g_T|^2+|g_T'|^2}\,\left\langle \lambda_\perp\frac{m_f}{E_f}\right\rangle\,, \tag{3.7}$$

which vanishes in the ultra-relativistic limit.

Let us now comment on the possible source of the tensor interaction. Of course, one cannot rule elementary tensor interactions. However, it is interesting to note that also certain neutrino scalar interactions can generate, after Fierz rearrangement, effective tensor couplings. For example, consider the tree level Lagrangian

$$-\,\mathcal{L}_{\rm tree} = \lambda_\phi\phi\,(\overline{L_L}\,e_R) + \lambda_\phi'\tilde{\phi}\,(\overline{L_L}\,\nu_R)\;+\text{h.c.}\,, \tag{3.8}$$

where L_L is the left-handed lepton $SU(2)_L$ doublet, e_R (ν_R) is the right-handed electron (neutrino) singlet, ϕ is a doublet scalar field, of mass m_ϕ, $\tilde{\phi} = i\sigma_2\phi^*$ and $\lambda_\phi, \lambda_\phi'$ are real

elementary couplings. At low energy $E \ll m_\phi$, the interaction in (3.8) induces a set of four-fermion effective interactions, which also contains the following coupling

$$\mathcal{H}^{\phi}_{\rm int} = \frac{\lambda'_\phi \lambda_\phi}{m^2_\phi} (\overline{e_R}\,\nu_L)\,(\overline{\nu_R}\,e_L) = -\frac{\lambda'_\phi \lambda_\phi}{m^2_\phi} \left[\tfrac{1}{2} (\overline{\nu_R}\,\nu_L)\,(\overline{e_R}\,e_L) + \tfrac{1}{8} (\overline{\nu_R}\,\sigma_{\mu\nu}\,\nu_L)\,(\overline{e_R}\,\sigma^{\mu\nu}\,e_L)\right] . \quad (3.9)$$

From (3.9) it follows that $g_T \sim \lambda'_\phi \lambda_\phi / m^2_\phi$. Finally, we mention that the above four-fermion operator can also be generated when different scalar fields mix. This possibility exist, for example, in supersymmetric models without R-parity.

Throughout this paper we assumed the neutrinos to be of the Dirac type. For the case of Majorana neutrinos there are additional constraints on some of the couplings. Namely, one can show [10] that the flavor diagonal elements of the vector couplings g_V, g'_V as well as the tensor couplings g_T, g'_T vanish identically, while the axial-vector couplings are twice the value corresponding to the Dirac case. As a consequence the standard MSW effect does not distinguish between Dirac and Majorana neutrinos, but a tensor-induced spin-flip requires at least two neutrino flavors in the Majorana case.

To conclude, in this paper we have studied the effects on neutrino propagation in matter due to the most general Lorentz-invariant interactions with the background fermions. Scalar, pseudo-scalar and longitudinal tensor interactions couple states of opposite chirality but do not flip the helicity, and hence are suppressed by ratio between the neutrino mass and its energy. Our crucial observation is that transverse tensor interactions are not suppressed, since they couple states of both opposite chirality and opposite helicity. In the presence of a non-vanishing background polarization, such interactions can result in a neutrino spin-flip during propagation, much alike the magnetic moment spin-precession [8,9]. The requirement that the background is not changed by the neutrino spin-flip singles out the coherent part of the interactions, implying that in a macroscopic system the effect can be coherently enhanced. While we have analyzed in some detail the basic neutrino dynamics, we did not study its relevance for real physical systems (like the sun or a supernova) nor did we perform a phenomenological analysis of the couplings which are relevant for producing this new effect. We just mention that the presence of right-handed neutrino components implies in general a non-vanishing magnetic moment, so that the effect of the tensor interaction will be accompanied by the similar effect of the magnetic field that polarizes the medium. For the constraints on the couplings, we expect that in some cases one can derive severe constraints from $SU(2)_L$ related interactions [11] and from the bounds on neutrino masses. This and related issues will be discussed elsewhere [12].

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APPENDIX A:

We present here the details on the computation of the matrix elements $\mathcal{M}_a$ [c.f. (2.4)] that determine the potentials V_a [c.f. (2.3)]. We have

$$\mathcal{M}_a^f \equiv \langle f, \boldsymbol{p}, \boldsymbol{\lambda} | \overline{\psi}_f \, \Gamma_a \, (g_a + g_a' \gamma^5) \, \psi_f | f, \boldsymbol{p}, \boldsymbol{\lambda} \rangle \tag{A1}$$

$$= \tfrac{1}{2E_f} \overline{u}_f(\boldsymbol{p}, \boldsymbol{\lambda}) \, \Gamma^a \, (g_a + g_a' \gamma^5) \, u_f(\boldsymbol{p}, \boldsymbol{\lambda}) \tag{A2}$$

$$= \tfrac{1}{4E_f} \mathrm{Tr} \left[\Gamma^a \, (g_a + g_a{}' \gamma^5) \, (\not{p} + m_f) \, (1 + \gamma^5 \not{s}) \right] , \tag{A3}$$

where E_f and m_f denote respectively the energy and the mass of the background fermion f. In (A2) we have assumed the background fermions to be free, so that a plane wave expansion for the field operators can be used. In obtaining (A3) we have used the identity

$$u_f(\boldsymbol{p}, \boldsymbol{\lambda}) \, \overline{u}_f(\boldsymbol{p}, \boldsymbol{\lambda}) = \tfrac{1}{2} (\not{p} + m_f) \, (1 + \gamma^5 \not{s}) \, , \tag{A4}$$

where the spin vector s is defined as

$$s \equiv \left(\frac{\boldsymbol{p} \cdot \boldsymbol{\lambda}}{m_f}, \boldsymbol{\lambda} + \frac{\boldsymbol{p} \, (\boldsymbol{p} \cdot \boldsymbol{\lambda})}{m_f \, (m_f + E_f)} \right) , \tag{A5}$$

and satisfies $s^2 = -1$ and $s_\mu \, p^\mu = 0$. Using $\gamma^5 \sigma^{\mu\nu} = \frac{i}{2} \epsilon^{\mu\nu}{}_{\rho\sigma} \, \sigma^{\rho\sigma}$ and the elementary traces $\frac{1}{4} \mathrm{Tr} \left[\Gamma^{S,P,V,A,T} (\not{p} + m_f)(1 + \gamma^5 \not{s}) \right] = m_f \, , 0 \, , p^\mu \, , s^\mu \, , -\epsilon^{\mu\nu\rho\sigma} \, p_\rho s_\sigma$ we obtain

$$\mathcal{M}^S = g_S \, \frac{m_f}{E_f} \, , \tag{A6}$$

$$\mathcal{M}^P = g_P' \, \frac{m_f}{E_f} \, , \tag{A7}$$

$$\mathcal{M}^V = g_V \, \frac{p^\mu}{E_f} + g_V' \, \frac{m_f}{E_f} \, s^\mu \, , \tag{A8}$$

$$\mathcal{M}^A = g_A' \, \frac{p^\mu}{E_f} + g_A \, \frac{m_f}{E_f} \, s^\mu \, , \tag{A9}$$

$$\mathcal{M}^T = -g_T \, \epsilon^{\mu\nu\rho\sigma} \, \frac{p_\rho s_\sigma}{E_f} + i g_T' \, \frac{p^\mu s^\nu - p^\nu s^\mu}{E_f} \, . \tag{A10}$$

While the identity (A4) provides a simple way to calculate $\mathcal{M}_a$ by means of standard "trace technology", we find it useful to present also an alternative calculation which is based on the spinorial expression (A2) for $\mathcal{M}_a$. In this derivation the details of the fermion polarization

$$\boldsymbol{\lambda} = \xi_f^\dagger \, \boldsymbol{\sigma} \, \xi_f \quad (\xi_f^\dagger \, \xi_f = 1) \, , \tag{A11}$$

are more transparent (ξ_f denotes the two-component spinor of the fermion f).

To compute $\overline{u}(\boldsymbol{p}, \boldsymbol{\lambda}) \, \Gamma_a \, (g_a + g_a' \gamma^5) \, u(\boldsymbol{p}, \boldsymbol{\lambda})$ we choose the chiral representation for Γ_a, where

$$1 = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} , \qquad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} , \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} . \tag{A12}$$

with $\sigma^\mu = (I, \sigma^i)$ and $\bar{\sigma}^\mu = (I, -\sigma^i)$. Since

$$u(\boldsymbol{p}, \boldsymbol{\lambda}) \equiv \begin{pmatrix} u_L(\boldsymbol{p}, \boldsymbol{\lambda}) \\ u_R(\boldsymbol{p}, \boldsymbol{\lambda}) \end{pmatrix} = \begin{pmatrix} \sqrt{p\,\sigma}\,\xi_f \\ \sqrt{p\,\bar{\sigma}}\,\xi_f \end{pmatrix} = \sqrt{\frac{E_f + m_f}{2}} \begin{pmatrix} \left(I - \frac{\boldsymbol{p}\cdot\boldsymbol{\sigma}}{E_f + m_f}\right)\,\xi_f \\ \left(I + \frac{\boldsymbol{p}\cdot\boldsymbol{\sigma}}{E_f + m_f}\right)\,\xi_f \end{pmatrix} \tag{A13}$$

it is sufficient to calculate $u_C^\dagger(\boldsymbol{p}, \boldsymbol{\lambda})\,\sigma^\mu\,u_{C'}(\boldsymbol{p}, \boldsymbol{\lambda})$ for $C, C' \in \{L, R\}$. Using the identities

$$(\boldsymbol{p}\cdot\boldsymbol{\sigma})\sigma_i + \sigma_i(\boldsymbol{p}\cdot\boldsymbol{\sigma}) = 2p_i \tag{A14}$$

$$(\boldsymbol{p}\cdot\boldsymbol{\sigma})\sigma_i - \sigma_i(\boldsymbol{p}\cdot\boldsymbol{\sigma}) = 2i\epsilon_{kji}\sigma_k p_j \tag{A15}$$

$$(\boldsymbol{p}\cdot\boldsymbol{\sigma})\sigma_i(\boldsymbol{p}\cdot\boldsymbol{\sigma}) = 2p_i(\boldsymbol{p}\cdot\boldsymbol{\sigma}) - |\boldsymbol{p}|^2\sigma_i \tag{A16}$$

we obtain

$$u_{L,R}^\dagger(\boldsymbol{p}, \boldsymbol{\lambda})\,I\,u_{L,R}(\boldsymbol{p}, \boldsymbol{\lambda}) = (E_f \mp \boldsymbol{p}\cdot\boldsymbol{\sigma}) \tag{A17}$$

$$u_{L,R}^\dagger(\boldsymbol{p}, \boldsymbol{\lambda})\,I\,u_{R,L}(\boldsymbol{p}, \boldsymbol{\lambda}) = m_f \tag{A18}$$

$$u_{L,R}^\dagger(\boldsymbol{p}, \boldsymbol{\lambda})\,\boldsymbol{\sigma}\,u_{L,R}(\boldsymbol{p}, \boldsymbol{\lambda}) = m_f\boldsymbol{\lambda} + \left(\frac{\boldsymbol{p}(\boldsymbol{p}\cdot\boldsymbol{\sigma})}{E_f + m_f} \mp \boldsymbol{p}\right) \tag{A19}$$

$$u_{L,R}^\dagger(\boldsymbol{p}, \boldsymbol{\lambda})\,\boldsymbol{\sigma}\,u_{R,L}(\boldsymbol{p}, \boldsymbol{\lambda}) = E_f\boldsymbol{\lambda} \pm i(\boldsymbol{p}\times\boldsymbol{\lambda}) - \frac{\boldsymbol{p}(\boldsymbol{p}\cdot\boldsymbol{\sigma})}{E_f + m_f}\,. \tag{A20}$$

This allows us to compute

$$J_a \equiv \overline{u}(\boldsymbol{p}, \boldsymbol{\lambda})\,\Gamma_a\,u(\boldsymbol{p}, \boldsymbol{\lambda}) \tag{A21}$$

for $a = S, P, V, A$:

$$J_S = u_L^\dagger u_R + u_R^\dagger u_L = 2m_f \tag{A22}$$

$$J_P = u_L^\dagger u_R - u_R^\dagger u_L = 0 \tag{A23}$$

$$J_V = u_R^\dagger \sigma_\mu u_R + u_L^\dagger \bar{\sigma}_\mu u_L = 2p_\mu \tag{A24}$$

$$J_A = u_R^\dagger \sigma_\mu u_R - u_L^\dagger \bar{\sigma}_\mu u_L = 2m_f s_\mu\,, \tag{A25}$$

where s_μ is defined in (A5). From the above one immediately obtains $\mathcal{M}_a$ for $a = S, P, V, A$ as in (A6-A9). To compute the tensor terms we define

$$\Sigma_i \equiv \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}, \qquad \Sigma_i' \equiv \Sigma_i\,\gamma^5 = \begin{pmatrix} -\sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}. \tag{A26}$$

The respective currents are

$$J_\Sigma = u_L^\dagger \boldsymbol{\sigma} u_R + u_R^\dagger \boldsymbol{\sigma} u_L = 2E_f\boldsymbol{\lambda} - \frac{2\boldsymbol{p}(\boldsymbol{p}\cdot\boldsymbol{\lambda})}{E_f + m_f} = 2E_f\boldsymbol{s} - 2\boldsymbol{p}s_0 \tag{A27}$$

$$J_{\Sigma'} = u_L^\dagger \boldsymbol{\sigma} u_R - u_R^\dagger \boldsymbol{\sigma} u_L = 2i(\boldsymbol{p}\times\boldsymbol{\lambda}) = 2i(\boldsymbol{p}\times\boldsymbol{s})\,. \tag{A28}$$

Noting that $\sigma_{ij} = \epsilon_{ijk}\Sigma_k$, $\sigma_{ij}\gamma_5 = \epsilon_{ijk}\Sigma_k'$, $\sigma_{0i} = i\Sigma_i'$ and $\sigma_{0i}\gamma_5 = i\Sigma_i$ one can easily verify the expression for $\mathcal{M}_T$ in eq. (A10).

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