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Using Scalars to Probe Theories of Low Scale Quantum Gravity *

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Abstract

Arkani-Hamed, Dimopoulos and Dvali have recently suggested that gravity may become strong at energies near 1 TeV which would remove the hierarchy problem. Such a scenario can be tested at present and future colliders since the exchange of towers of Kaluza-Klein gravitons leads to a set of new dimension-8 operators that can play important phenomenological roles. In this paper we examine how the production of pairs of scalars at e^+e^-, $\gamma\gamma$ and hadron colliders can be used to further probe the effects of graviton tower exchange. In particular we examine the tree-level production of pairs of identical Higgs fields which occurs only at the loop level in both the Standard Model and its extension to the Minimal Supersymmetric Standard Model. Cross sections for such processes are found to be potentially large at the LHC and the next generation of linear colliders. For the $\gamma\gamma$ case the role of polarization in improving sensitivity to graviton exchange is emphasized.

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1 Introduction

Arkani-Hamed, Dimopoulos and Dvali(ADD) [1] have recently proposed a radically interesting solution to the hierarchy problem. ADD hypothesize the existence of n additional large spatial dimensions in which gravity can live, called 'the bulk', whereas all of the fields of the Standard Model are constrained to lie on 'a wall', which is our conventional 4-dimensional world. Gravity only appears to be weak in our ordinary 4-dimensional space-time since we merely observe it's action on the wall. It has recently been shown [1] that a scenario of this type may emerge in string models where the effective Planck scale in the bulk is identified with the string scale. In such a theory the hierarchy can be removed by postulating that the string or effective Planck scale in the bulk, M_s, is not far above the weak scale, *e.g.*, a few TeV. Gauss' Law then provides a link between the values of M_s, the conventional Planck scale M_{pl}, and the size of the compactified extra dimensions, R,

$$M_{pl}^2 \sim R^n M_s^{n+2}\,, \tag{1}$$

where the constant of proportionality depends not only on the value of n but upon the geometry of the compactified dimensions. Interestingly, if M_s is near a TeV then $R \sim 10^{30/n-19}$ meters; for separations between two masses less than R the gravitational force law becomes $1/r^{2+n}$. For $n=1$, $R \sim 10^{11}$ meters and is thus obviously excluded, but, for $n=2$ one obtains $R \sim 1$ mm, which is at the edge of the sensitivity for existing experiments[2]. For $2 < n \leq 7$, where 7 is the maximum value of n being suggested by M-theory, the value of R is further reduced and thus we may conclude that the range $2 \leq n \leq 7$ is of phenomenological interest. Astrophysical arguments[3] suggest that $M_s > 50$ TeV for $n=2$, but allow $M_s \sim 1$ TeV for $n>2$.

The Feynman rules for this theory are obtained by considering a linearized theory of gravity in the bulk, decomposing it into the more familiar 4-dimensional states and recalling the existence of Kaluza-Klein towers for each of the conventionally massless fields. The entire set of fields in the K-K tower couples in an identical fashion to the particles of the SM. By considering the forms of the $4+n$ symmetric conserved stress-energy tensor for the various SM fields and by remembering that such fields live only on the wall one may derive all of the necessary couplings. An important result of these considerations is that only the massive spin-2 K-K towers (which couple to the 4-dimensional stress-energy tensor, $T^{\mu\nu}$) and spin-0 K-K towers (which couple proportional to the trace of $T^{\mu\nu}$) are of phenomenological relevance as all the spin-1 fields can be shown to decouple from the particles of the SM. For processes that involve massless fields at at least one vertex the contributions of the spin-0 fields can also be ignored.

The details of the phenomenology of the ADD model has begun to be explored in a series of recent papers [4]. Given the Feynman rules as developed by Guidice, Rattazzi and Wells and by Han, Lykken and Zhang[4], it appears that the ADD scenario has two basic classes of collider tests. In the first class, type-i, a K-K tower of gravitons can be emitted during a decay or scattering process leading to a final state with missing energy. The rate for such processes is strongly dependent on the number of extra dimensions as well as the exact value of M_s. In the second class, type-ii, which we consider here, the exchange of a K-K graviton tower between SM or MSSM fields can lead to almost n-independent modifications to conventional cross sections and distributions or they can possibly lead to new interactions. The exchange of the graviton K-K tower leads to a set of effective color and flavor singlet contact interaction operator of dimension-eight with the scale set by the parameter M_s. The universal overall order one coefficient of these operators, λ, is unknown but its value is conventionally set to ± 1. Given the kinematic structure of these operators the modifications in the relevant cross sections and distributions can be directly calculated.

Until now the phenomenological analyses have concentrated[4] on the collider processes $f_1\bar{f}_2 \to f_3\bar{f}_4, VV$ and $\gamma\gamma, gg \to f\bar{f}$, where the f_i's are fermions and V's are vector bosons, since these are the traditional components of the SM. However in SUSY models, in particular in the case of the Minimal Supersymmetric Standard Model(MSSM), scalar fields exist on the same footing as do fermions and vectors. Therefore in this paper we turn to a complementary examination of the influence of the exchange of a tower of K-K gravitons on the processes $e^+e^-, \gamma\gamma \to S\bar{S}$ where S is a real or complex scalar. From the nature of these new operators the application to the processes $q\bar{q}, gg \to S\bar{S}$ becomes immediately apparent. While we do not expect the existence of such K-K exchanges to be discovered in these channels the processes we consider do offer new ways to probe this phenomena and possibly confirming information through the universality expected in the couplings of low scale quantum gravity. As we will see, many of the modifications of cross sections and asymmetries due to K-K tower exchange observed earlier in the case of fermion final states will have their parallel in the case of scalar final states. Of particular interest, as we will see below, is the tree level production of identical pairs of neutral Higgs bosons via K-K tower exchange in e^+e^-, $\gamma\gamma$ as well as hadronic collisions with rather unique kinematic properties. In the SM or MSSM these processes can only occur at the loop level with rather small cross sections.

2 $e^+e^- \to$ Scalar Pairs

The production of pairs of scalars is a basic elementary e^+e^- process in the MSSM. For the reaction $e^-(k_1)e^+(k_2) \to S(p_1)\bar{S}(p_2)$ [with the $k_i(p_i)$ incoming(outgoing)], a K-K tower of gravitons contributes an additional amplitude of the form

$$\mathcal{M} = \frac{2\lambda K}{M_s^4}(t-u)(p_1-p_2)_\mu \bar{e}(k_2)\gamma^\mu e(k_1)\,, \tag{2}$$

with u, t being the usual Mandelstam kinematic variables. Thus, including the contributions from γ, Z and K-K graviton tower exchanges, the full differential cross section for $e^+e^- \to S\bar{S}$ is given compactly by

$$
\begin{aligned}
\frac{d\sigma}{dz} &= N_c \frac{\pi\alpha^2}{4s}\beta^3(1-z^2)\Big[\sum_{ij} c_i c_j (v_i v_j + a_i a_j) P_i P_j \\
&- 2C\beta z \sum_i c_i v_i P_i + C^2(\beta z)^2\Big], \qquad (3)
\end{aligned}
$$

where $z = \cos\theta$, s is the square of the collider center of mass energy, N_c is the color factor for the scalar in the final state, v_i, a_i are the electron's vector and axial vector couplings to the gauge boson $i(=\gamma, Z)$, c_i are the corresponding couplings for the scalar S, $P_i = s/(s-M_i^2)$ are propagator factors for the gauge bosons of mass M_i, $C = \lambda K s^2/2\pi\alpha M_s^4$ and $\beta^2 = 1-4m_S^2/s$, with m_S being the scalar mass. Our couplings are normalized such that $v_\gamma = -1$ and $c_\gamma = Q$, the scalar's electric charge. For $K = 1(\pi/2)$ we recover the normalization convention employed by Hewett(Guidice, Rattazzi and Wells)[4]; we will take $K = 1$ in the numerical analysis that follows but keep the factor in our analytical expressions. We recall from the Hewett analysis that λ is a parameter of order unity whose sign is undetermined and that, given the scaling relationship between λ and M_s, experiments in the case of processes of type-*ii* actually probe only the combination $M_s/|K\lambda|^{1/4}$. For simplicity in what follows we will numerically set $|\lambda| = 1$ and employ $K = 1$ but we caution the reader about this technicality and quote our sensitivity to M_s for $\lambda = \pm 1$.

Note that in the SM or MSSM the well-known scalar pair production cross section angular distribution behaves as $\sim \sin^2\theta$ as expected. However, the exchange of a K-K-tower of gravitons can alter this distribution in several statistically significant ways provided s/M_s^2 is not too small. First, both the interference and pure graviton terms lead to higher powers of z in the angular distribution as was first observed in the case of fermion pair final states by Hewett[4]. Second, the interference term leads to a forward-backward asymmetry, A_{FB}, in the angular distribution–something not generally expected for scalar pair production. Note that this interference term, being odd in z, vanishes upon integration and makes no contribution to the total cross section, as was also the case for fermion pair production. This implies that the total cross section can only increase with respect to the expectations of the SM when graviton exchanges are present.

With at least single e^- beam polarization one can define a z-dependent Left-Right asymmetry, $A_{LR}(z)$, associated with scalar pair production given by

$$A_{LR}(z) = \frac{(1-z^2)\Big[\sum_{ij} c_i c_j (v_i a_j + a_i v_j) P_i P_j - 2C\beta z \sum_i c_i a_i P_i\Big]}{(1-z^2)\Big[\sum_{ij} c_i c_j (v_i v_j + a_i a_j) P_i P_j - 2C\beta z \sum_i c_i v_i P_i + C^2(\beta z)^2\Big]}\,, \tag{4}$$

where the expression in the denominator is essentially that for the differential cross section above. Note that in the case of the SM, MSSM or in any model with an additional Z' exchange, the z dependence completely cancels and A_{LR} becomes a constant for a fixed value of $\sqrt{s}$. A third effect of the exchange of K-K-towers is thus to give A_{LR} a finite (approximately odd) z dependence which may be observable provided the ratio s/M_s^2 is not too small. Combined with the first two effects described above we see that the exchange of a K-K-tower of gravitons leads to rather unique signatures in the case of the production of scalar pairs. The angular-averaged value of $A_{LR}(z)$ can also be obtained from this expression by separately integrating the numerator and denominator of Eq.4 over z. In a similar fashion we note that the odd terms in z contained in $A_{LR}(z)$ can be directly probed by forming an integrated Left-Right Forward-Backward asymmetry, A_{FB}^{LR}, by analogy with the more conventional integrated Forward-Backward asymmetry, A_{FB}. We remind the reader that both of these quantities are expected to be zero in the MSSM but will now differ from zero

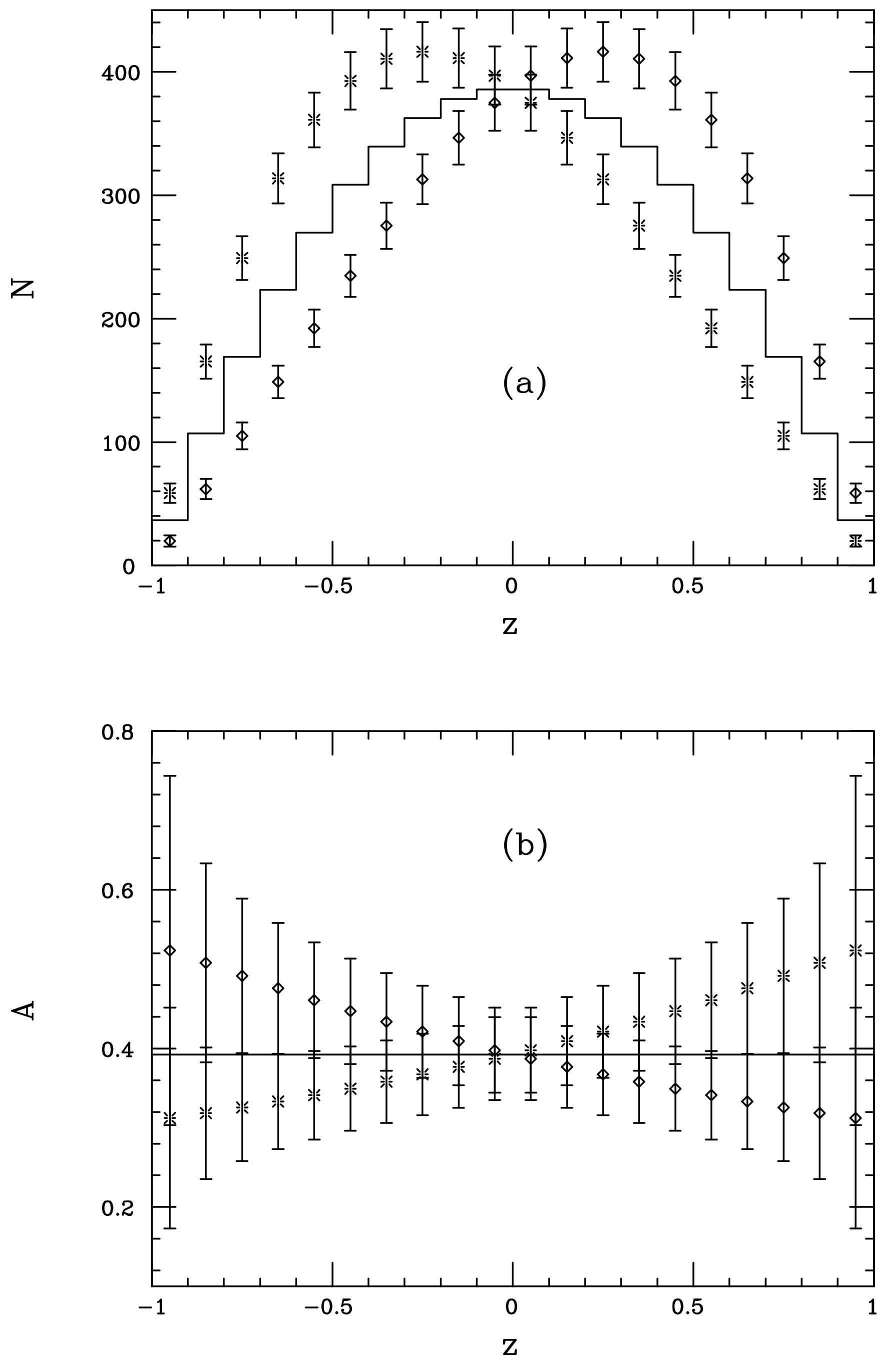
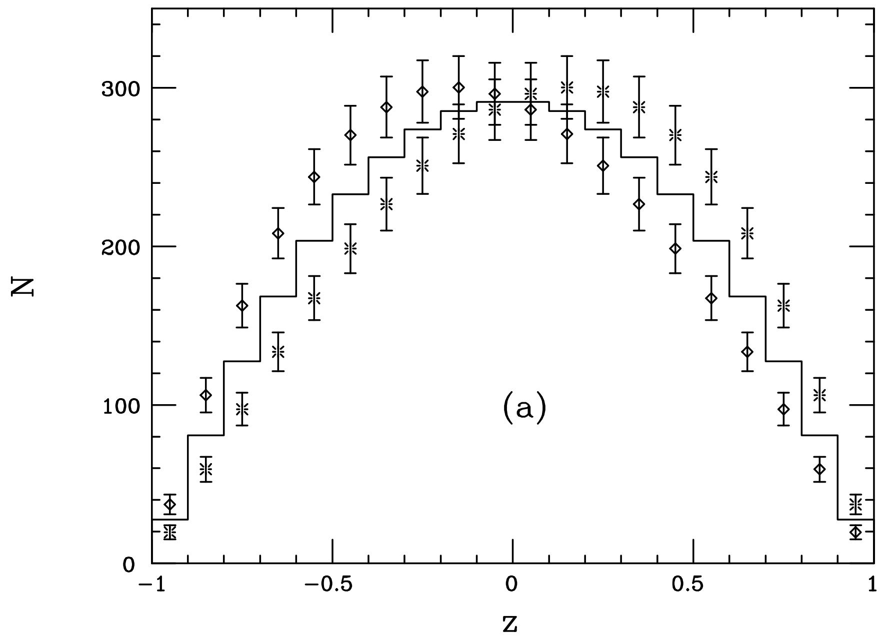


Figure 1: (a)Cross section and (b)A_{LR} for $\tilde{t}$ production at a 500 GeV e^+e^- collider assuming $m_{\tilde{t}}$ = 150 GeV and an integrated luminosity of 100 fb^{-1}. The stop is assumed to be maximally mixed, *i.e.*, $\theta_{\tilde{t}} = 45^o$. The histogram(solid line) shows the MSSM expectations while the two sets of data points include the contributions from a K-K tower of gravitons with $M_s = 1.5$ TeV and reflect the anticipated measurement errors.

due to K-K tower exchange by terms of order s^2/M_s^4.

At this point it is informative to go through a few typical examples. In the MSSM the scalars we have available to study are the SUSY partners of the quarks and leptons as well as the five physical Higgs fields. For purposes of demonstration, we consider the pair production of these fields at a 500 GeV e^+e^- collider. In this case, except for the possibility of highly mixed stop squark($\tilde{t}$), we expect that the squarks will most likely be too massive to be pair produced. Thus we will consider the pair production of scalars for the following four cases: (i) A maximally mixed light $\tilde{t}_1$, *i.e.*, where the stop mixing angle between $\tilde{t}_L$ and $\tilde{t}_R$ which determines the $Z\tilde{t}_1\tilde{t}_1^*$ is given by $\theta_{\tilde{t}} = 45^o$, (ii) a charged Higgs or $\tilde{\mu}_L$ (they have identical electroweak couplings), (iii) a $\tilde{\mu}_R$ and (iv) the production of a pair of identical neutral Higgs bosons. This last possibility is particularly interesting since it cannot occur at the tree level in the MSSM. Instead of $\tilde{\mu}_{L,R}$, one could just as well examine selectron pair production but in that case the additional t-channel graphs make the analysis and the influence of the K-K tower contributions less transparent.

Fig.1 shows the angular distribution for the production of 150 GeV maximally mixed stop squark pairs at a 500 GeV e^+e^- collider assuming an integrated luminosity of 100 fb^{-1}. In the absence of contributions from low scale quantum gravity the distribution is forward-backward symmetric, $\sim \sin^2\theta$. When the K-K tower exchange contribution is turned on it leads to a skewing of the distribution to either the forward or backward direction depending upon the sign of λ. Overall there is not a large change in the total cross section, $\sim 2\%$ in the example shown in the figure. However, in this case the value of A_{FB} differs from zero by many σ as one might expect from a simple visual inspection of the angular distribution; for this integrated luminosity we obtain $A_{FB} = \pm 0.210 \pm 0.013$, where the overall sign is directly correlated with the sign of λ. Also one sees that A_{LR} picks up a z dependence



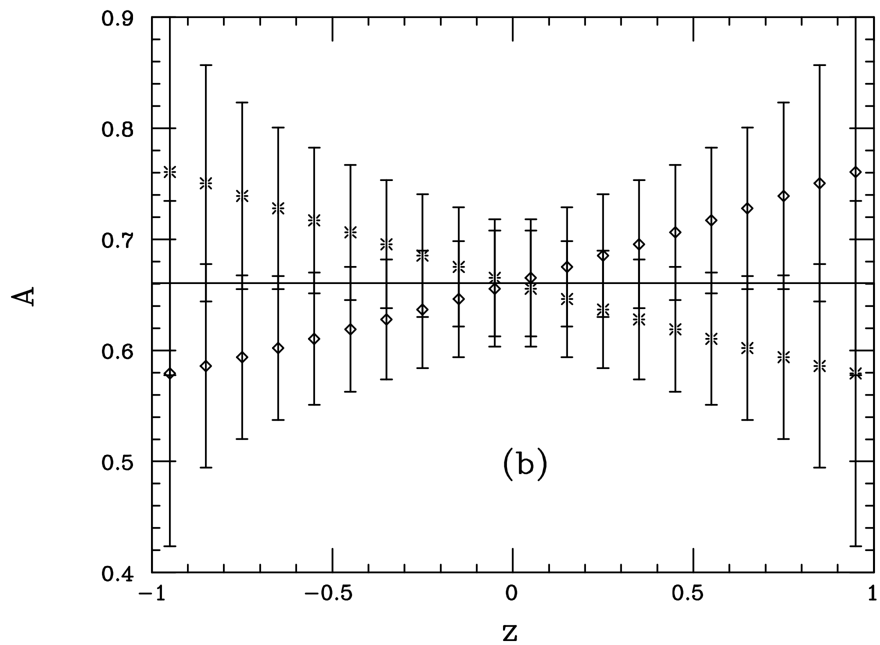
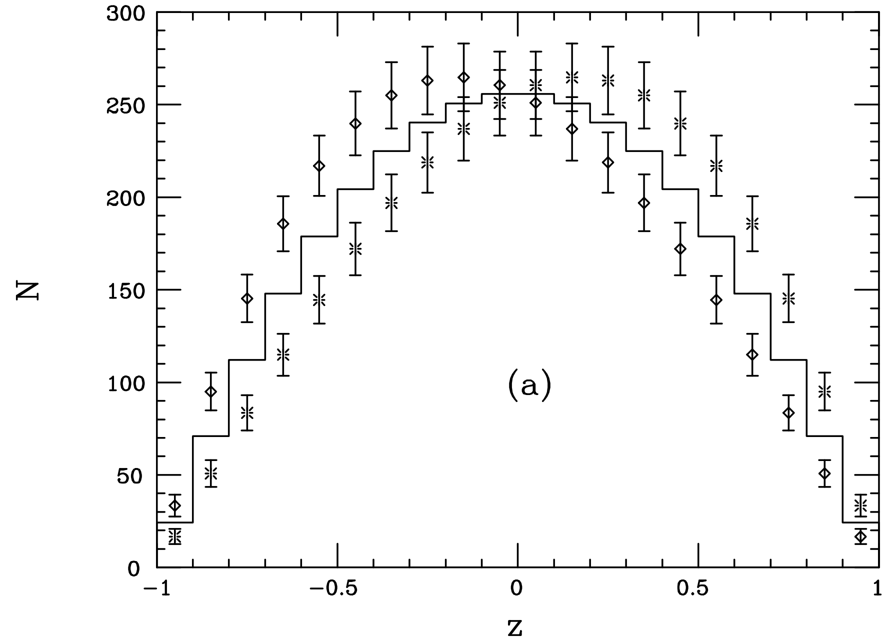


Figure 2: Same as the previous figure but now for either a $\tilde{\mu}_L$ or a charged Higgs boson, $H^{\pm}$. Note that the slope of A_{LR} in this case is opposite to that for $\tilde{\mu}_L$ or $H^{\pm}$.



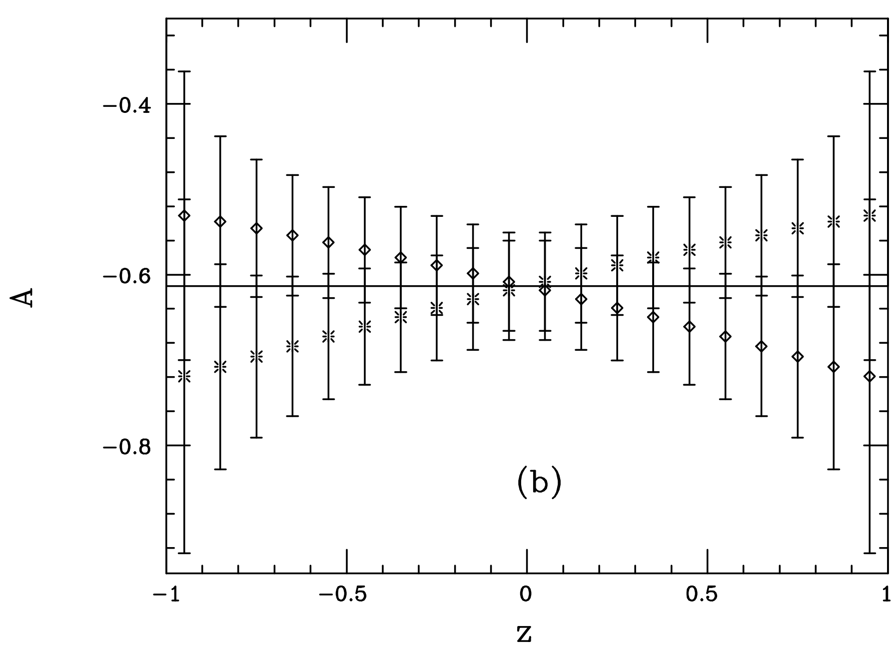


Figure 3: Same as the previous figure but now for a $\tilde{\mu}_R$.

as promised, tilting the distribution into either the forward or backwards direction. While the angular averaged value of A_{LR} differs from the MSSM expectation by less than 1% since it is almost purely odd in z, the integrated Left-Right Forward-Backward asymmetry, A_{FB}^{LR}, differs from zero by approximately $\sim 3\sigma$ with this assumed integrated luminosity; we obtain $A_{FB}^{LR} = \pm 0.043 \pm 0.015$ with the sign the same as that of A_{FB}. Combining all of the observables and assuming 100% efficiencies, the lack of observation of such effects would place a 95% CL lower bound of $\simeq 4.65$ TeV on M_s, which is comparable to the *discovery* reaches obtained using the more conventional fermionic channels[4] at this center of mass energy and assumed integrated luminosity.

Figs.2 and 3 show that the case of stop pair production is not in any way special. Although the overall effect of the exchange of the K-K tower of gravitons is now somewhat softer in the case of sleptons and charged Higgs and less statistics is available due to the smaller cross sections, it is clearly present in both of these figures. In both these cases, the graviton tower exchange increases the total cross section by less than 1% and shifts the integrated value of A_{LR} by a comparable amount. In the $\tilde{\mu}_L(H^-)$ case we find, however, that $A_{FB} = \pm 0.127 \pm 0.016$ and $A_{FB}^{LR} = \pm 0.048 \pm 0.018$ with common signs. In the $\tilde{\mu}_R$ example, we find instead that $A_{FB} = \pm 0.137 \pm 0.017$ and $A_{FB}^{LR} = \mp 0.047 \pm 0.019$ with the signs *anti-correlated*. The bounds that one could potentially obtain on M_s in either one of these two examples would be somewhat smaller than that obtainable from $\tilde{t}$ pair production; for both cases we obtain at 95% CL the bound of 3.40 TeV assuming 100 fb^{-1} of integrated luminosity and perfect efficiencies.

One interesting consequence of a K-K graviton tower exchange is the existence of new operators that can lead to unusual tree-level processes such as reaction $e^+e^- \rightarrow 2h^0$, where h^0 can be the SM Higgs, the light or heavy CP-even Higgs of the MSSM, h, H, or

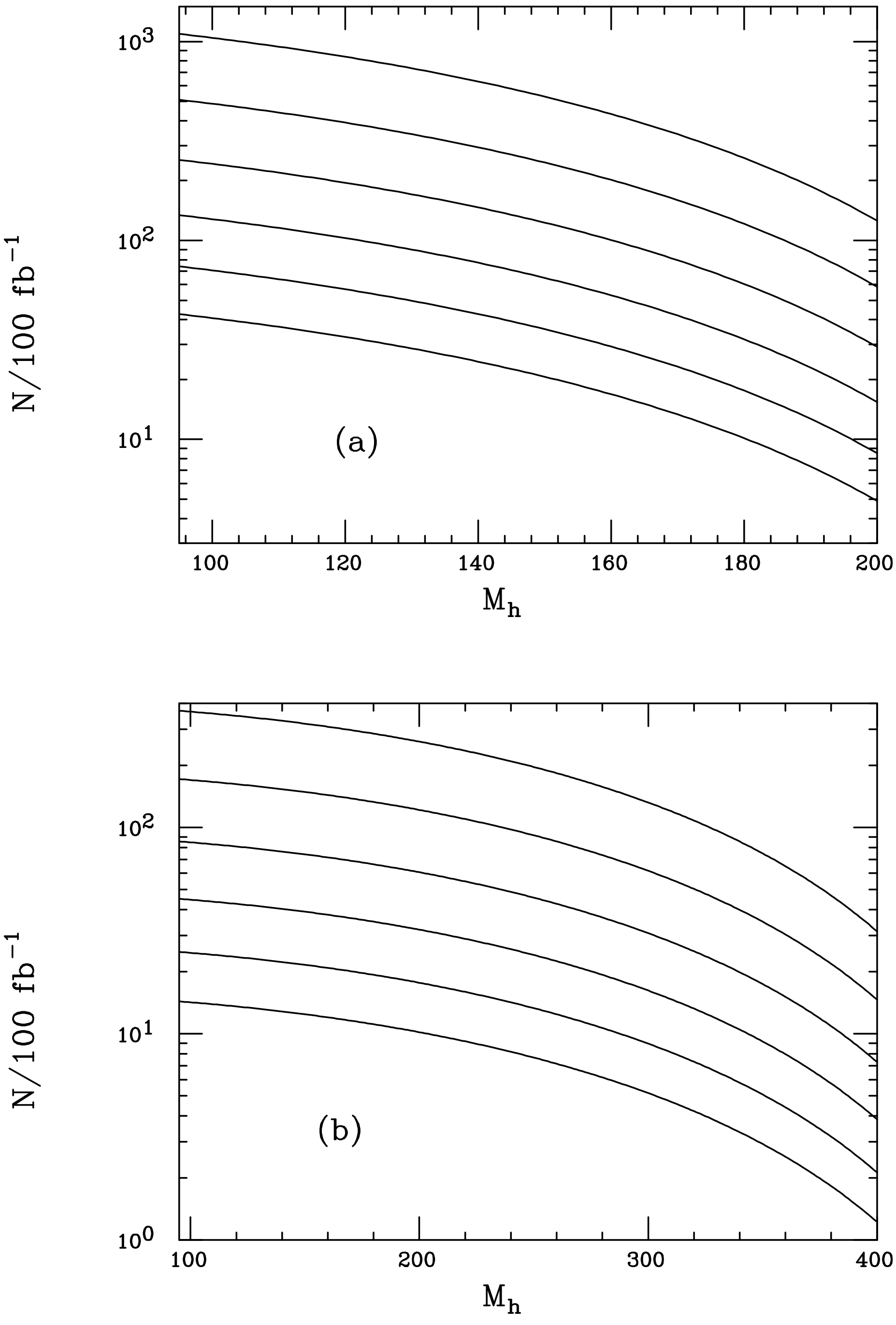


Figure 4: Tree level production rate for Higgs boson pairs due to graviton tower exchange at a (a)500 GeV or a (b)1 TeV e^+e^- collider as a function of the Higgs mass scaled to an integrated luminosity of 100 fb^{-1}. From top to bottom in (a)[(b)] the curves correspond to the choice $M_s = 1[2]$ TeV increasing in steps of 0.1[0.2] TeV.

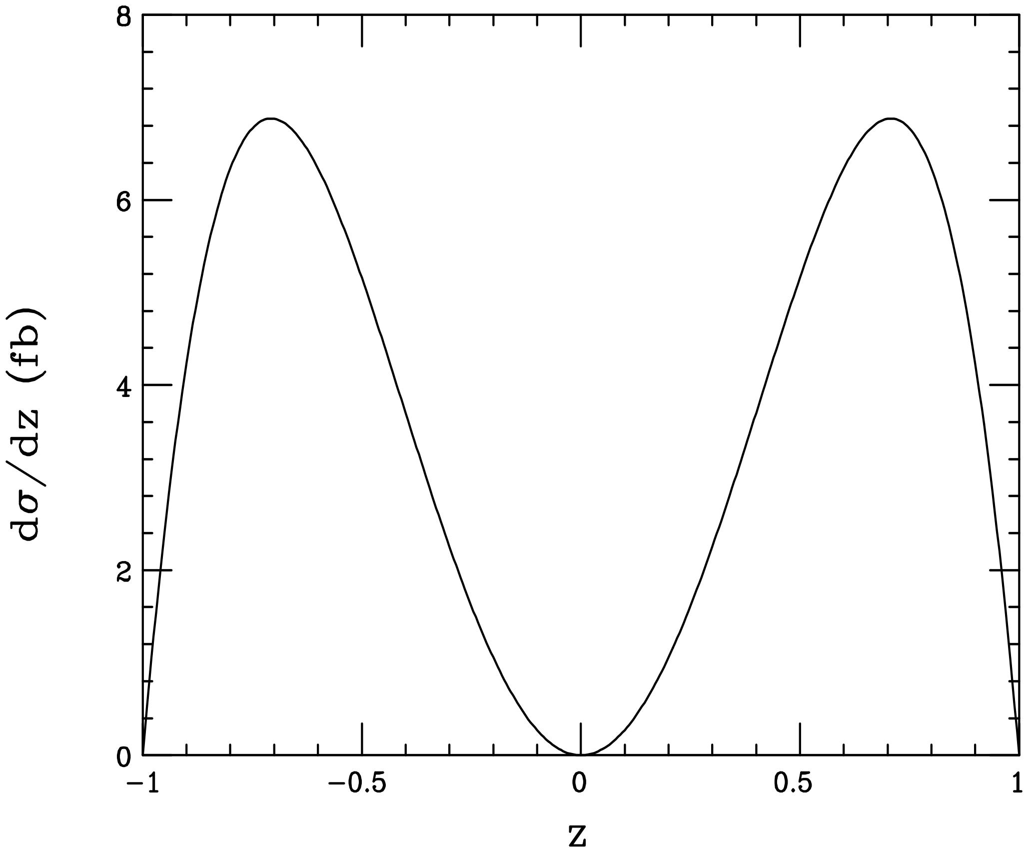


Figure 5: The tree level unpolarized differential cross section for $e^+e^- \to 2h^0$ at a 500 GeV e^+e^- linear collider due to the exchange of a Kaluza-Klein tower of gravitons assuming $m_h = 130$ GeV and $M_s = 1$ TeV. Note the canonical shape arising from the nature of the spin-2 exchange.

the corresponding CP-odd field A. As we will comment upon in more detail below, such processes can indeed occur in the SM or MSSM but only at loop level[5]. Unlike the MSSM case, however, both fields in the final state *must* be identical, *e.g.*, the hH or hA final states are not accessible. The cross section can easily be obtained from the expressions above by turning off the photon and Z contributions and dividing by a factor of 2 since identical particles are being produced in the final state. We find, allowing for polarization of either beam,

$$\frac{d\sigma}{dz} = \frac{\lambda^2 K^2 s^3}{32\pi M_s^8}\beta^5 z^2(1-z^2)(1-\lambda_{e^+}\lambda_{e^-})\,, \tag{5}$$

where the $\lambda_{e^\pm}$ are the positron and electron helicities, respectively. The unpolarized cross section is obtained by averaging over all four helicity combinations; note that both the $e^-_L e^+_R \to 2h^0$ and the $e^-_R e^+_L \to 2h^0$ cross sections due to K-K graviton exchange are predicted to be identical.

In a number of ways the K-K graviton-induced cross section for Higgs pair production can be easily distinguished from the loop-induced SM or MSSM contribution[5]. First, for light Higgs, both the SM and MSSM cross sections are quite small, of order 0.1-0.2 fb for $\sqrt{s}=500$ GeV, and have a rough $\sim \sin^2\theta$ angular distribution. In the K-K case, as shown in Fig.4, this small of a cross section is only obtained when $\sqrt{s}/M_s < 1/4$. The shape of the angular distribution in the case of graviton exchange is also quite distinctive, as shown in Fig.5, owing to the nature of the spin-2 exchange. Secondly, while graviton exchange leads to identical cross sections for both $e^-_L e^+_R \to 2h^0$ and $e^-_R e^+_L \to 2h^0$ processes, these are found to differ by a factor of 2 in both the SM and MSSM cases. Lastly, as stated above, the graviton induced cross section is the same for hh, HH and AA final states with the same mass which will not necessarily be the case in the MSSM.

3 $\gamma\gamma \to$ Scalar Pairs

$\gamma\gamma$ collisions offer a unique and distinct window on the possibility of new physics in a particularly clean environment. At tree level the cross section for particle pair production depends only upon QED-like couplings and is thus independent of many other factors such as weak isospin and various mixing parameters. Unlike e^+e^- collisions, however, Bose symmetry forbids the existence of non-zero values for either A_{FB} or A_{FB}^{LR}, which were powerful weapons in probing for K-K graviton tower exchanges. In the case of $\gamma\gamma$ collisions our remaining tools are the angular distributions of the produced scalar pairs and their sensitivity to the polarization of the initial state photons.

Polarized $\gamma\gamma$ collisions may be possible at future e^+e^- colliders through the use of Compton backscattering of polarized low energy laser beams off of polarized high energy electrons[6]. The backscattered photon distribution, $f_\gamma(x = E_\gamma/E_e)$, is far from being monoenergetic and is cut off above $x_{max} \simeq 0.83$ implying that the colliding photons are significantly softer than the parent lepton beam energy. As we will see, this cutoff at large x, x_{max}, implies that the $\gamma\gamma$ center of mass energy never exceeds $\simeq 0.83$ of the parent collider and this will result in a significantly degraded M_s sensitivity. In addition, the shape of the function f_γ is somewhat sensitive to the polarization state of both the initial laser (P_l) and electron (P_e) whose values fix the specific distribution. While it is anticipated that the initial laser polarization will be near 100%, *i.e.*, $|P_l| = 1$, the electron beam polarization is expected to be be near 90%, *i.e.*, $|P_e| = 0.9$. We will assume these values in the analysis that follows. With two photon 'beams' and the choices $P_l = \pm 1$ and $P_e = \pm 0.9$ to be made for each beam it would appear that 16 distinct polarization-dependent cross sections need to be examined. However, due to the exchange symmetry between the two photons and the fact that a simultaneous flip in the signs of all the polarizations leaves the product of the fluxes and the cross sections invariant, we find that there only six physically distinct polarization combinations. In what follows we will label these possibilities by the corresponding signs of the electron and laser polarizations as $(P_{e1}, P_{l1}, P_{e2}, P_{l2})$, For example, the configuration $(-++-)$ corresponds to $P_{e1} = -0.9$, $P_{l1} = +1$, $P_{e2} = 0.9$ and $P_{l2} = -1$.

Clearly some of these polarization combinations will be more sensitive to the effects of K-K towers of gravitons than will others so our analysis can be used to pick out those particular cases.

For the reaction $\gamma(k_1)\gamma(k_2) \to S(p_1)\bar{S}(p_2)$ [with the $k_i(p_i)$ incoming(outgoing)], the K-K tower of gravitons contributes an additional amplitude of the form

$$\begin{aligned}\mathcal{M} &= \frac{8\lambda K}{M_s^4}\Big[2k_1 \cdot k_2 \left\{p_1 \cdot \epsilon_1 p_2 \cdot \epsilon_2 + p_1 \cdot \epsilon_2 p_2 \cdot \epsilon_1\right\} + 2\epsilon_1 \cdot \epsilon_2 \\ &\quad \left\{k_1 \cdot p_1 k_2 \cdot p_2 + k_1 \cdot p_2 k_2 \cdot p_1 - k_1 \cdot k_2 p_1 \cdot p_2\right\}\Big], \qquad (6)\end{aligned}$$

following our earlier notation and with the ϵ_i being the polarization vectors of the incoming photons. The full subprocess cross section for polarized $\gamma\gamma \to S\bar{S}$ reaction can now be written as

$$\begin{aligned}\frac{d\hat{\sigma}}{dz} &= N_c \frac{\pi\alpha^2}{2\hat{s}}\beta\Big[\{(Y - Q^2)^2 + [Q^2(1-2X) + Y]^2\} \\ &- 2\xi_1\xi_2(Y - Q^2)[Q^2(1-2X) + Y]\Big], \qquad (7)\end{aligned}$$

where the ξ_i are the photon helicities[7] and

$$\begin{aligned}X &= \frac{\beta^2(1-z^2)}{1-\beta^2 z^2} \\ Y &= \frac{\lambda K \hat{s}^2 \beta^2 (1-z^2)}{4\pi\alpha M_s^4}, \qquad (8)\end{aligned}$$

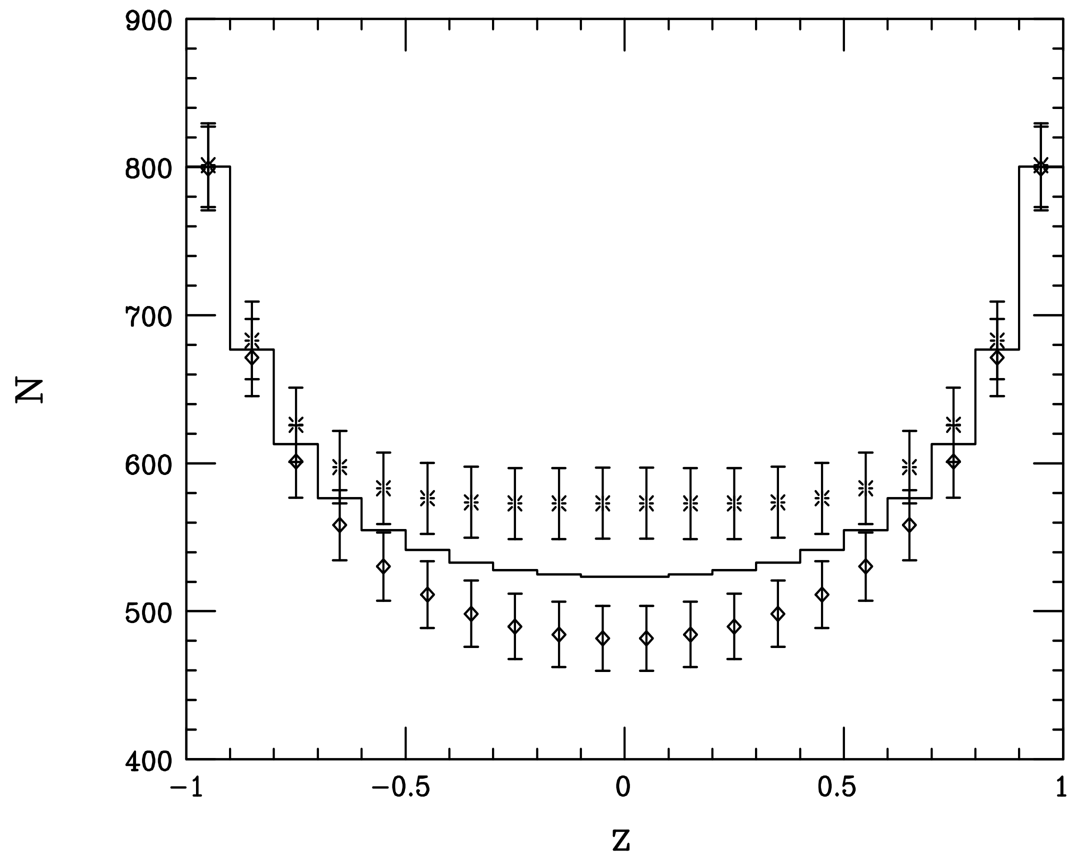


Figure 6: Binned angular distribution for stop pairs produced in unpolarized $\gamma\gamma$ collisions assuming an e^+e^- center of mass energy of 1 TeV for a stop mass of 250 GeV and an integrated luminosity of 200 fb^{-1}. The histogram shows the result in the MSSM while the two sets of 'data' are the predictions including the exchange of a tower of K-K gravitons with $\lambda = \pm 1$ and $M_s = 2.5$ TeV. The errors shown are statistical only.

with $z = \cos\theta$, where θ being the partonic center of mass scattering angle as above, $\hat{s}$ is the square of the sub-process center of mass energy, β is defined as above but now with the replacement, $s \to \hat{s}$, and Q is the scalar's electric charge. Note that the contribution due to graviton tower exchange is largest when $z = 0$, *i.e.*, at scattering angles of 90^o. To obtain the corresponding unpolarized cross section we simply average over the values of the photon helicities $\xi_{1,2}$. To derive the experimentally accessible cross sections, we must fold in the polarized photon fluxes and integrate over the associated energy fractions:

$$\sigma = \int^{x_{max}} dx_1 \int^{x_{max}} dx_2 \; f_\gamma(x_1, \xi_1(x_1), P_{e_1}, P_{\gamma_1}) f_\gamma(x_2, \xi_2(x_2), P_{e_2}, P_{\gamma_2}) \frac{d\hat{\sigma}}{dz} \,, \tag{9}$$

where we explicitly note the dependence of the fluxes on the laser and electron beam polarization and the average helicities on the beam energy. We remind the reader that the ξ's are also dependent on the initial laser and electron beam polarizations. (We also must identify $\hat{s} = s_{e^+e^-} x_1 x_2$.) In the present case the kinematics require the photon energies to satisfy the constraint $\tau = \hat{s}/s = x_1 x_2 \geq 4m_S^2/s = \tau_{min}$ which, together with the value of x_{max}, then determines the lower bounds on both $x_{1,2}$: $x_1^{min} = \tau_{min}/x_{max}$ and $x_2^{min} = \tau_{min}/x_1$.

We begin as before with the example of stop pair production. We now no longer need to specify the $\tilde{t}_L - \tilde{t}_R$ mixing angle since the cross section depends only on $Q_{\tilde{t}}$ and $m_{\tilde{t}}$, though our ability to detect and reconstruct this final state may remain dependent on this mixing parameter. For simplicity, we first consider the case of *unpolarized* photons in which case the resulting binned angular distribution is shown in Fig.6 for both the standard MSSM scenario and that including graviton tower exchange. For the chosen set of parameters it is quite clear that new physics is present by the fact that the dip in the angular distribution near 90^o has been made deeper or more shallow in a statistically significant manner. Assuming no deviations from the MSSM expectations are observed with this assumed integrated luminosity we estimate that the 95% CL lower bound placed on M_s would be $\simeq 3.2$ TeV. As

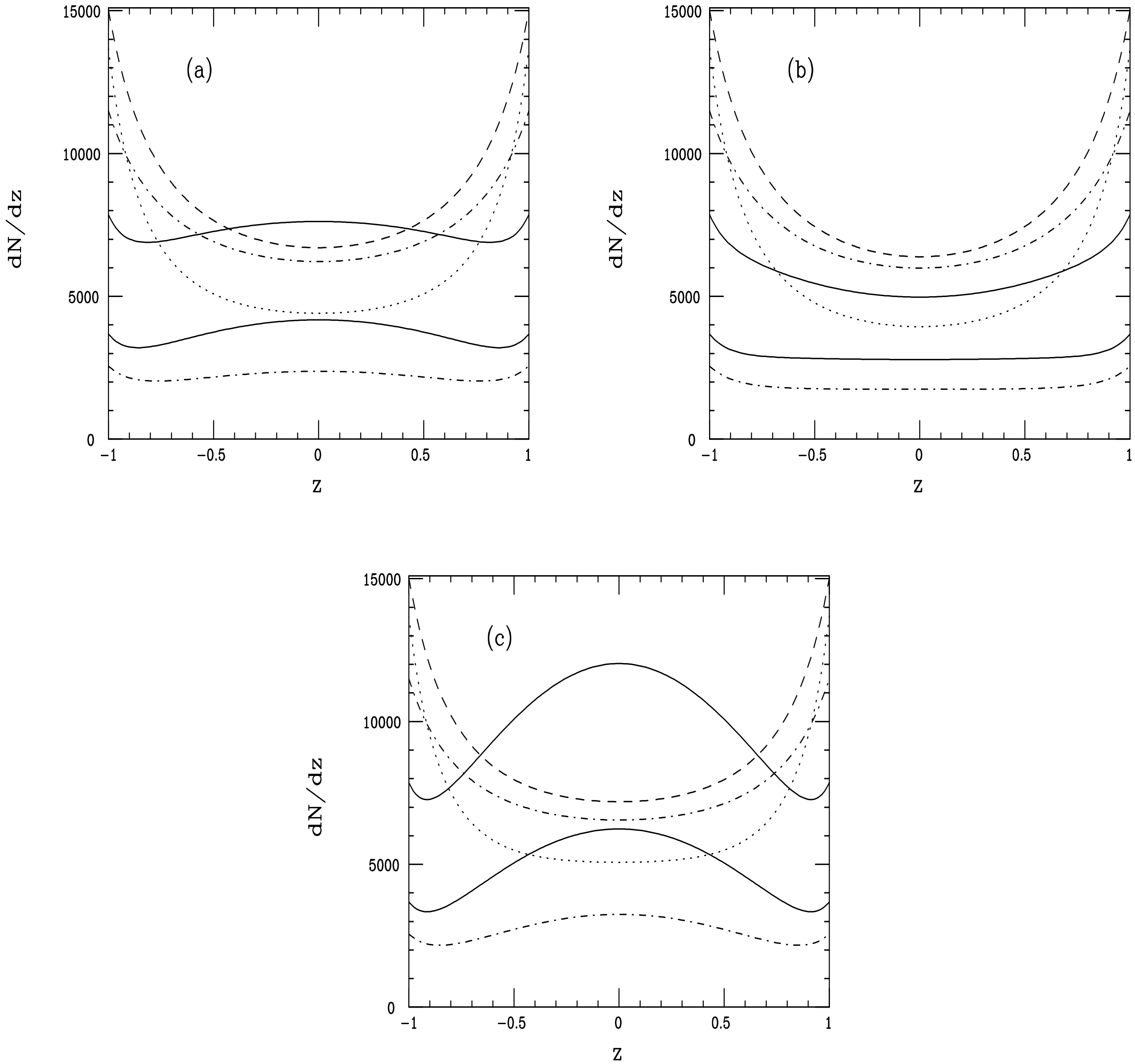


Figure 7: Un-binned angular distribution as in the previous figure but now broken down into the various helicity contributions. A stop mass of 250 GeV has been assumed along with an integrated luminosity of 200 fb^{-1} and a collider center of mass energy of 1 TeV. (a) is the MSSM while (b) and (c) are the results including graviton exchange with $\lambda = \pm 1$ with $M_s = 2$ TeV. In all panels, the helicities are as follows: $(++++)$ is the upper dash-dotted curve, $(+++-)$ is the dashed curve, $(++--)$ is the lower dash-dotted curve, $(+-+-)$ is the dotted curve, $(-++-)$ is the upper solid curve and $(+---)$ is the lower solid curve.

we will see below, beam polarization will significantly improve this bound.

What happens when we polarize the colliding photons? In this case the results are shown in the 3 panels of Fig.7 for the six independent polarization choices. We see immediately that two of the cases which yield the largest cross sections, $(++++)$ and $(+++-)$, are hardly effected by the contributions due to graviton tower exchange whereas those corresponding to both $(-++-)$ and $(+---)$ are quite significantly modified. $(++--)$ and $(+-+-)$ are seen to be somewhat less affected by K-K exchange but the cross sections are visibly shifted by a substantial amount. The reasons why these particular choices are the most sensitive to K-K exchange is easily seen by expanding Eq.7 and examining the new pieces due to gravitons:

$$\delta\frac{d\hat{\sigma}}{dz} \sim [-4Q^2XY + 2Y^2](1-\xi_1\xi_2)\,. \tag{10}$$

This means that polarization choices that lead to large negative values of the product $\xi_1\xi_2$ in the kinematic region of interest will display the greatest sensitivity to K-K tower exchange. Indeed both the polarization choices $(-++-)$ and $(+---)$ fulfill this expectation as can be seen by an explicit calculation. Employing these polarizations and assuming no signal for K-K graviton tower exchange is observed, we can place a 95% CL bound on M_s assuming an integrated luminosity of 200 fb^{-1} of $\simeq$ 3.8 TeV–a value significantly above that obtained in the unpolarized case which reflects the added sensitivity.

In Figs. 8 and 9 we show the corresponding cross sections for the production of charged Higgs or slepton pairs using both unpolarized as well as polarized photons. These cross sections are all somewhat larger than the corresponding ones for stop pairs since, in the exact MSSM limit, the total cross section is directly proportional to the product N_cQ^4. (Thus the results in Fig.8 and Fig.6 for the case of the MSSM differ only by an overall constant.) This also implies that for fixed M_s the $H^\pm$ and $\tilde{\ell}$ pair cross sections are overall less sensitive to graviton tower exchange due to the larger value of the scalar's electric charge, though this is partially compensated for by the increase in the available statistics in the case of unpolarized beams and nearly completely so when beam polarization is available. (This would further imply that $\tilde{b}$ pair production would be the most sensitive to graviton exchange due to the small size of $Q^2 = 1/9$ in this case. Indeed this is true but the reduced size of the MSSM cross section, suppressed by Q^4, leads to very small statistical samples.) These observations and expectations are indeed verified by examining Fig.9. We again see from these figures that the best sensitivity to K-K exchanges arises from the particular polarization choices $(-++-)$ and $(+---)$. In the case of unpolarized(polarized) beams the absence of any deviation from MSSM expectations would lead to a 95% CL lower bound on M_s of $\simeq 2.8(3.8)$ TeV for this value of the integrated luminosity.

As was the case of e^+e^- collisions, the existence of K-K graviton exchange now allows for the production of neutral scalars, such as sneutrinos or Higgs bosons, in $\gamma\gamma$ collisions. The cross section for the production of identical Higgs boson pairs is trivially obtainable from the above by setting $Q = 0$ and dividing by a factor of 2 since identical particles now reside in the final state. We obtain

$$\frac{d\hat{\sigma}}{dz} = \frac{\lambda^2 K^2 \hat{s}^3}{32\pi M_s^8}\beta^5(1-z^2)^2(1-\xi_1\xi_2)\,, \tag{11}$$

following the above notation. As in the case of e^+e^- annihilation, the Higgs pairs appearing in the final state for the case of the MSSM can be either hh, AA, or HH but no 'off-diagonal' couplings are present. For particles of the same mass all of these cross sections are predicted to be identical. Such processes cannot occur at the tree level in the SM or MSSM but they can appear at one loop[8] to which we will later compare our results. Note that the above expression for the differential cross section leads to a very distinctive angular distribution $\sim \sin^4\theta$ for the production of neutral Higgs pairs which only populates the amplitude where

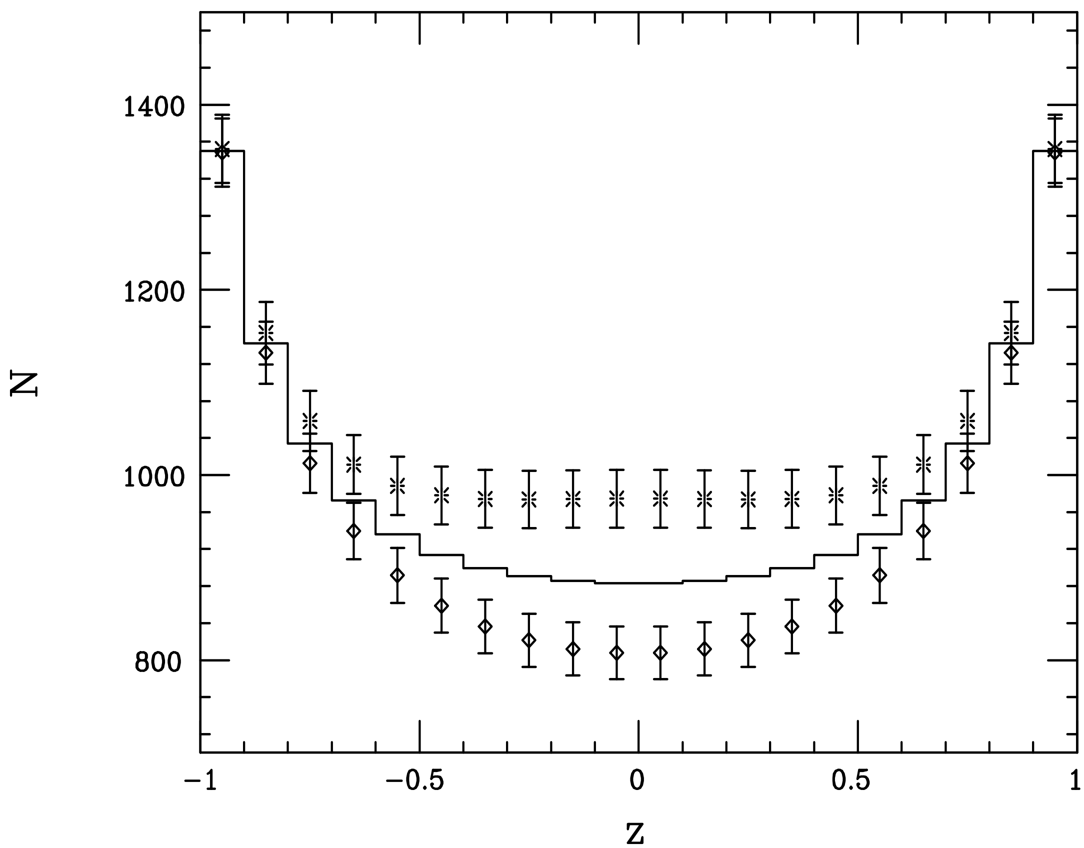


Figure 8: Same as Fig.6 but now for either H^- or $\tilde{\ell}$ pair production by unpolarized photons.

the two photon helicities are opposite. This rather unique angular distribution, resulting from spin-2 exchange, is shown for the case of unpolarized photons in Fig.10 for the very simple choice $M_s = \sqrt{s}$=1 TeV and $m_h = 130$ GeV. The total integrated cross section for these parameters is found to be quite large, $\simeq 410\ fb$ implying the availability of very large rates; results for other values of M_s and $\sqrt{s}$ can be obtained by very simple scalings using Eq.11.

For the case of polarized photons the angular distribution for this process is found to be independent of the choice of electron or laser polarizations unlike the case of charged scalar pair production and is shown in Fig.11. Here the large rates obtained by the polarization choices $(-++-)$ and $(+---)$ are easily explained. The argument essentially parallels the explanation for the sensitivities of these polarization choices to K-K exchanges. First, both these distributions are somewhat larger than the others at values of $\sqrt{\tau} \geq 0.6$ where most of the cross section originates. Second, the dominant enhancement arises since both of these polarization choices lead to large negative values of the product $\xi_1\xi_2$ in the same invariant mass range. From the $1-\xi_1\xi_2$ dependence of the cross section this leads to a large enhancement in the production rate. The choice $(++--)$ also yields a somewhat large negative value of $\xi_1\xi_2$ and it too is seen to be somewhat enhanced.

How large are these cross sections in comparison to SM and MSSM expectations and do their angular distributions and polarization dependencies differ? In the SM, the loop induced cross section[8] obtains contributions from both the same and opposite sign photon helicity combinations unlike the case of graviton exchange. With $\sqrt{s}$=1 TeV and a Higgs mass of 130 GeV a cross section of order 0.2 fb is obtained and found to be reasonably insensitive (at the $\pm 50\%$ level) to the choice of laser and electron polarizations. In the unpolarized case such a cross section is significantly below that obtained from K-K graviton tower exchange unless $M_s > 3.5$ TeV. As the Higgs boson mass increases the tree-level K-K

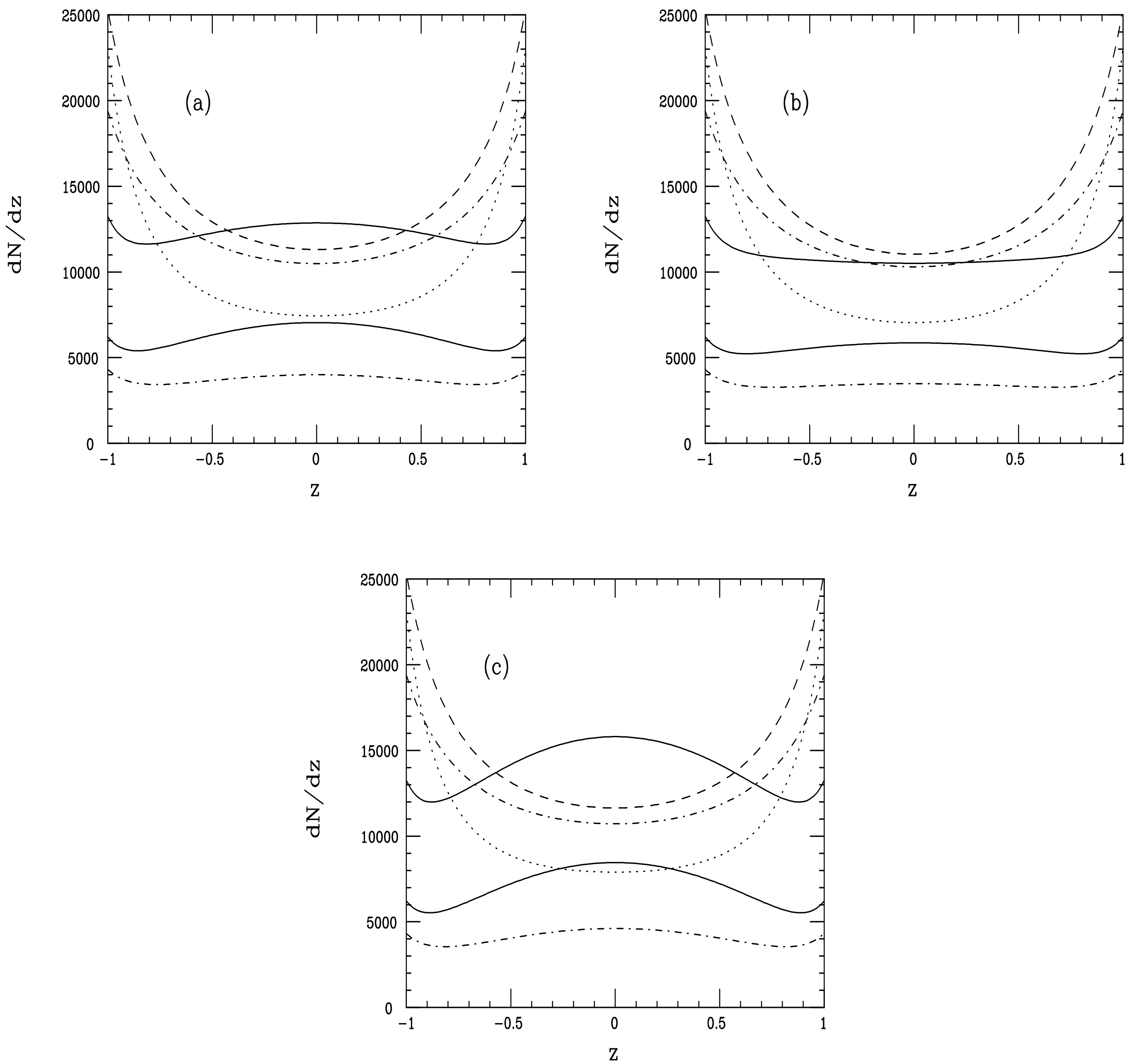


Figure 9: Same as Fig.7 but now for either H^- or $\tilde{\ell}$ pair production by polarized photons.

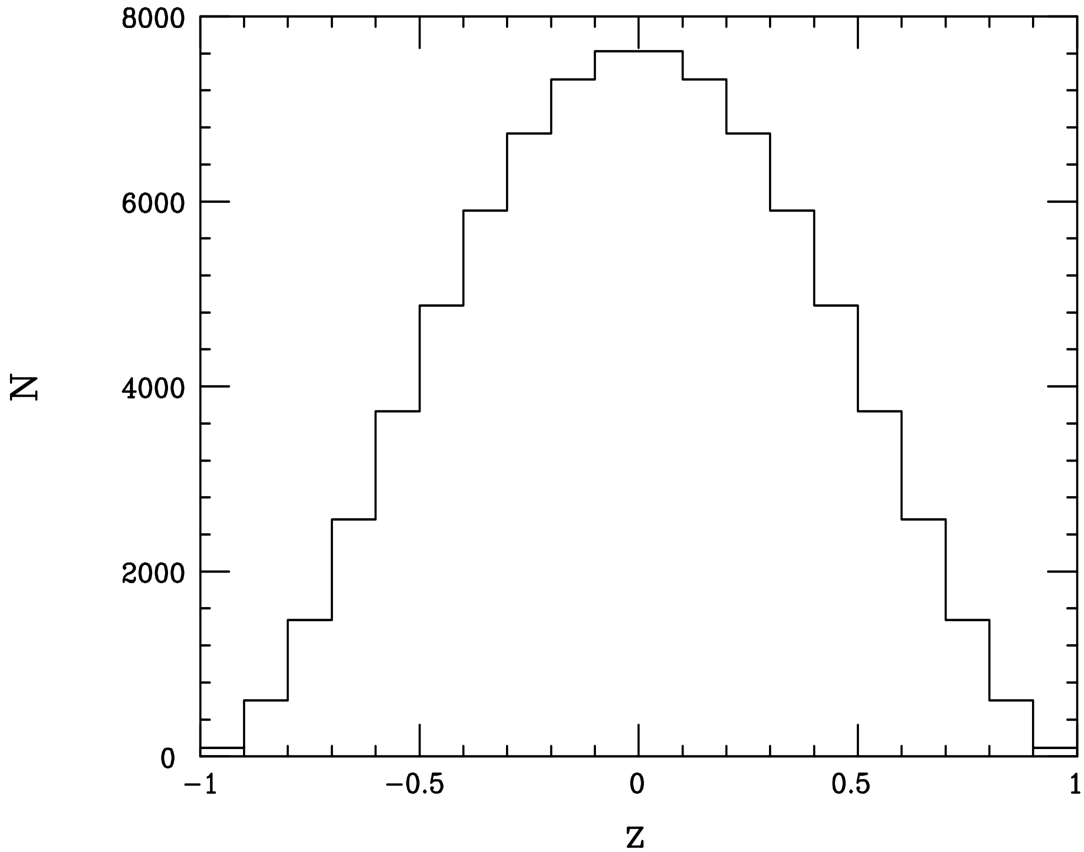


Figure 10: Binned angular distribution for Higgs boson pair production in unpolarized $\gamma\gamma$ collisions at a 1 TeV e^+e^- assuming $m_h = 130$ GeV, $M_s = 1$ TeV and an integrated luminosity of 200 fb^{-1}.

induced rate falls slower than does that induced by SM or MSSM loops. The apparent size of this cross section for the CP-even Higgs in the ordinary Two-Higgs Doublet extension is quite comparable to that found in the SM whereas in the MSSM with large stop mixing the rate can be dramatically enhanced[8]. If stop mixing is reasonably small then the MSSM and SM predictions are again comparable.

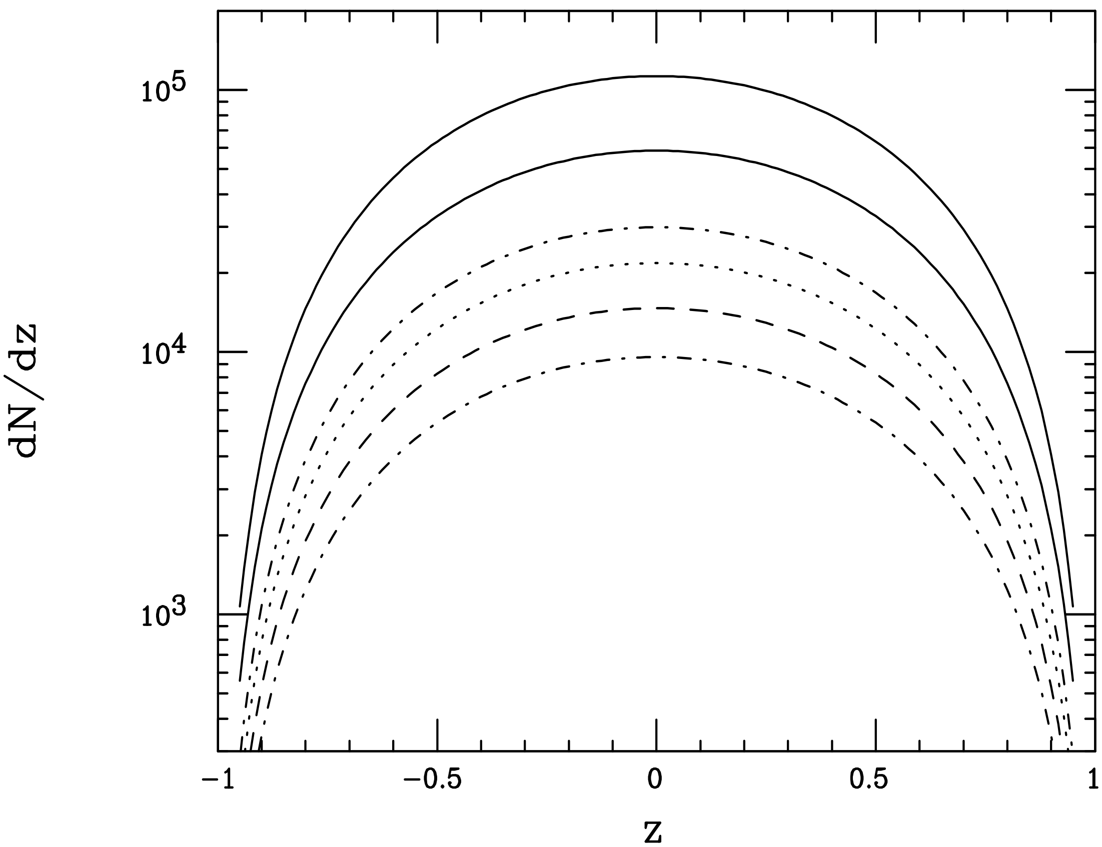


Figure 11: Un-binned angular distribution for the process $\gamma\gamma \to 2h^0$ using polarized beams for the same parameter choice as in the previous figure. From top to bottom the polarization parameter choices are given by $(-++-)$, $(+---)$, $(++--)$, $(+-+-)$, $(+++-)$ and $(++++)$, respectively.

4 Higgs Pair Production at Hadron Colliders

By analogy with the processes $e^+e^-, \gamma\gamma \to 2h^0$ discussed above, it will be possible to pair produce Higgs bosons at hadron colliders by very similar mechanisms: $q\bar{q}, gg \to 2h^0$. Since the K-K tower of gravitons represents a color singlet operator the cross sections for these reactions are very easily obtained from the expressions above by dividing by color factors of 3 or 8, respectively, and then weighting them with the appropriate parton densities. The result of this straightforward analysis yields the results shown in Fig.12 at leading order for both the Run II Tevatron and the LHC. (We note that NLO QCD corrections may significantly increase these rates by as much as a factor of two as they do for both the SM and MSSM loop induced processes; we will ignore such effects in our discussion below.) Recall that in either case the cross section scales as $\sim M_s^{-8}$ thus falling rapidly as the string scale is increased. Also recall that the cross sections for HH or AA production due to graviton exchange are numerically identical. The final state may, in principle, be observed in either the $4b$ or $2b\tau^+\tau^-$ mode.

The first thing to notice is that at the Tevatron the cross section leads to only a very small handful of events for this process (for a representative value of M_s=1 TeV) even before any cuts are imposed. This implies that if M_s is only slightly larger than the assumed value there will be essentially no statistics available to discover or observe Higgs boson pairs in this channel assuming an integrated luminosity of 2 fb^{-1}. This situation could change if significantly higher integrated luminosities were to be achieved. We note that the current lower bound on M_s from the analysis of Hewett[4] is already in excess of 1.0-1.1 TeV or so based on data from LEPII and the Run I of the Tevatron. For the LHC, taking $M_s = 3$ TeV and an integrated luminosity of 100 fb^{-1} a sizeable number of events can be obtained over a reasonable range of Higgs boson masses. For $M_s = 3$ TeV, these cross sections range from somewhat larger to quite substantially larger than those obtainable in the SM or in the MSSM[9] for CP-even scalars, expect in the case where the $H \to hh$ channel opens up with the H produced directly on-shell. The cross section at the LHC for the CP-odd pairs in the MSSM is always quite small in comparison to the CP-even case implying that the K-K contributions can be very large and possibly dominant.

It thus appears that if the string scale is not too large then Higgs boson pair production via K-K tower graviton exchange will become an exciting possibility at the LHC and, if we are very lucky, the Tevatron.

5 Summary and Conclusions

In this paper we have extended the phenomenological analyses of the ADD scenario to a number of new processes involving scalar final states and the exchange of a Kaluza-Klein tower of gravitons at various types of colliders. The main points of our analysis are as follows:

- As in the case of $e^+e^- \to f\bar{f}$ processes, the exchange of a K-K tower of gravitons in the s-channel can significantly modify the angular distribution and left-right polarization asymmetry in the case of scalar pair production. In particular, graviton exchange leads to a qualitatively flavor independent and statistically significant forward-backward asymmetry provided the scale M_s is not too far above $\sqrt{s}$. Such an asymmetry for squarks, sleptons and charged Higgs produced in e^+e^- annihilation is essentially impossible to mimic by other forms of new physics such as a Z', R-parity violation or leptoquark exchange. Perhaps more exciting, we observed that K-K tower exchange leads to the tree-level production of identical Higgs boson pairs with a reasonable cross section. In contrast, within the SM or MSSM, Higgs pair production in e^+e^- collisions can only occur through loops.

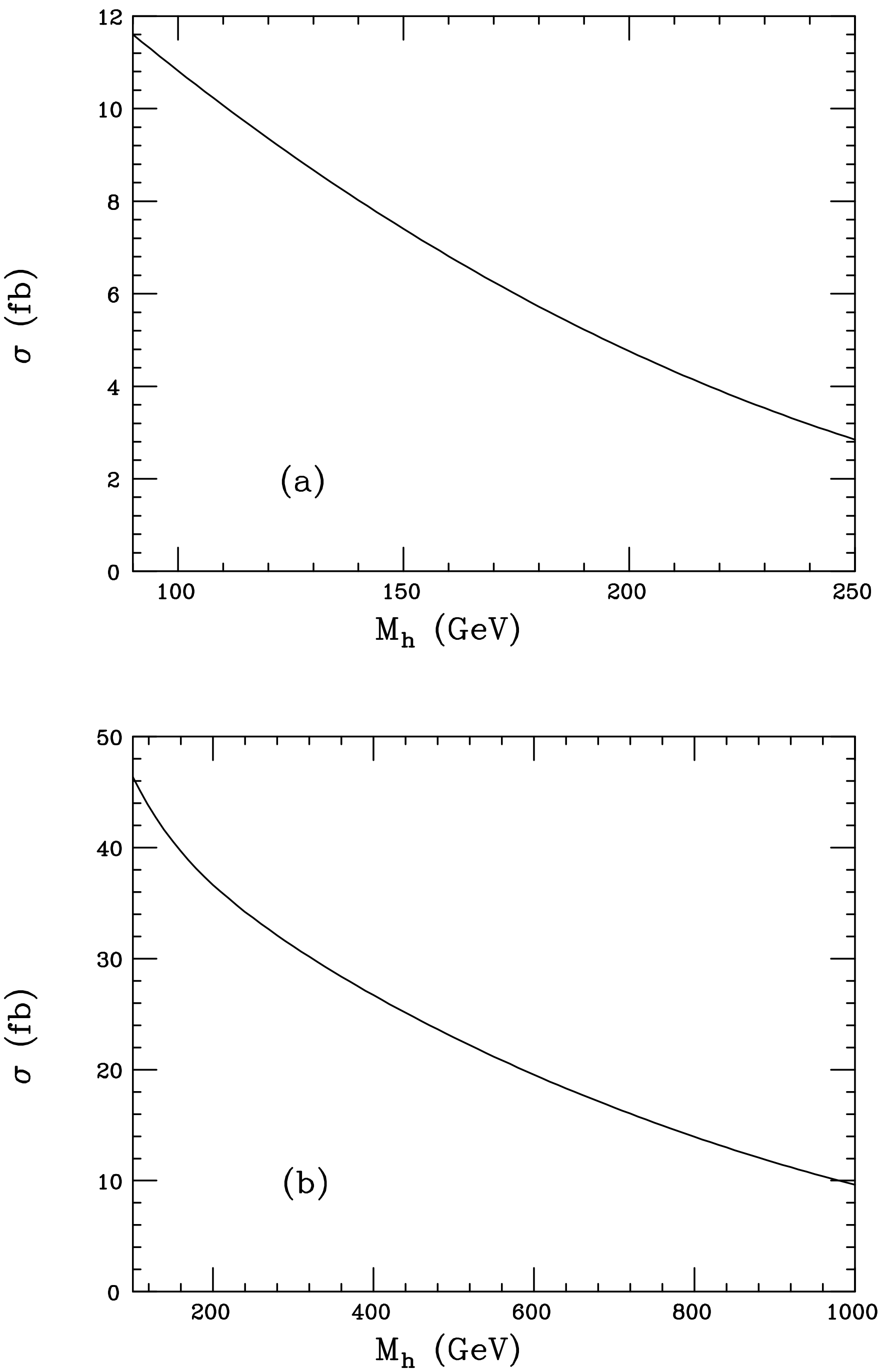


Figure 12: Leading order production cross sections for Higgs boson pairs at (a) TeV II or (b) the LHC as a function of the Higgs boson mass at the tree level due to the exchange of a Kaluza-Klein tower of gravitons. We assume that $M_s = 1(3)$ TeV for the case of the Tevatron(LHC).

- $\gamma\gamma \to S\bar{S}$ using polarized Compton backscattered laser photons offers another window on graviton exchange. As in the case of e^+e^- annihilation, K-K towers introduce distortions in the scalar pair angular distributions which were shown to be particularly sensitive to the polarizations choices made for the lasers and electrons in the initial state. Since the SM cross section is here proportional to Q^4 the flavor of the scalar plays an intricate role in determining the sensitivity to M_s: scalars with large(small) values of Q have larger SM cross sections hence more(less) statistical power. On the otherhand, the *fractional* shift in the total amplitude due to graviton exchange is clearly smaller(larger) in that case. For $\tilde{t}$ and $\tilde{l}$ pairs these two effects approximately cancel with the help of the additional color factor for $\tilde{t}$'s.

- Graviton tower exchange leads to new operators which can lead to tree level processes which cannot occur in either the SM or the MSSM that involve scalar pairs in the final state. Here we have considered as particularly interesting examples of this phenomena the processes $e^+e^-, \gamma\gamma \to 2h^0$ and $\gamma\gamma \to \tilde{\nu}\tilde{\nu}^*$ at lepton colliders and $q\bar{q}, gg \to 2h^0$ at hadron colliders. These reactions were shown to have potentially significant cross sections and are likely to be easily separable from the corresponding loop induced processes in the SM and MSSM.

- Scalar pair production at colliders, though not the likely channel for the discovery of new operators associated with extra dimensions, provides an additional channel with which to explore the implications of theories of low scale quantum gravity.

New dimensions may soon make their presence known at existing and/or future colliders. Such a discovery would revolutionize the way we think of physics beyond the electroweak scale.

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