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TEST OF CPT INVARIANCE IN B FACTORIES[a]

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Abstract

Feasibility of setting limits on CPT violating parameters for the case of both CP conjugate and semileptonic decays of neutral B_d mesons were examined. For the case of semileptonic final states of neutral B_d meson decays, bounds on CPT violating parameters at the level of few percent can be easily obtained. This sensitivity is better than the similar analysis using the CP conjugate final states of neutral B_d meson decays mainly due to the substantial increase on the statistics.

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TEST OF CPT INVARIANCE IN B FACTORIES

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Feasibility of setting limits on CPT violating parameters for the case of both CP conjugate and semileptonic decays of neutral B_d mesons were examined. For the case of semileptonic final states of neutral B_d meson decays, bounds on CPT violating parameters at the level of few percent can be easily obtained. This sensitivity is better than the similar analysis using the CP conjugate final states of neutral B_d meson decays mainly due to the substantial increase on the statistics.

1 Introduction

The CPT is known to be an exact symmetry of any local relativistic quantum field theory and all known particle interactions are consistent with the hypothesis. Thus, discovery of CPT violation in any form will signal new physics beyond the present particle paradigm. In fact, in a more general context, such as those arising from quantum gravity and superstring–inspired scenarios, CPT violating processes do occur, but are suppressed to levels consistent with presently existing experimental upper bounds.

At present, the most stringent bounds on CPT violations are obtained from the kaon sector [1,2] and are usually expressed in terms the impressively small limit on the $K^0 - \bar{K}^0$ mass difference: $|m_{K^0} - m_{\bar{K}^0}|/m_{K^0} \leq 9 \times 10^{-19}$. Note that due to the poor knowledge we have about the possible CPT violating mechanisms, there may be CPT violating reactions which would be invisible to the most sensitive experiments. Therefore, though less sensitive in an absolute sense, other manifistations of CPT violations, like the corresponding bounds achievable in the D and B sector are being investigated [3,4]. The realization of the proposed asymmetric B–factories, *i.e.* the PEP-II storage ring and *BABAR* detector at SLAC [5], and the KEK-B storage ring and *Belle* detector at KEK [6], will open a new possibility for testing CPT invariance.

After the preliminary study of Kobayashi and Sanda [7], the possibility of testing CPT in the B system has recently received a considerable attention [8,9,10]. In the present paper we will reconsider the problem, focusing on the semileptonic decays of B_d mesons [11]. Similar study with CP conjugate final states of B_d mesons was discussed elsewhere [12].

2 Mixing and decay amplitudes

The time evolution of the neutral B system is described, within quantum mechanics, by a 2×2 non–hermitian Hamiltonian and the corresponding eigenvectors are given by

$$
\begin{aligned}
|B_S\rangle &= \frac{1}{\sqrt{2}}\left[(1+\epsilon_B+\delta_B)|B^0\rangle + (1-\epsilon_B-\delta_B)|\bar{B}^0\rangle\right] \ , \\
|B_L\rangle &= \frac{1}{\sqrt{2}}\left[(1+\epsilon_B-\delta_B)|B^0\rangle - (1-\epsilon_B+\delta_B)|\bar{B}^0\rangle\right] \ . \qquad (1)
\end{aligned}
$$

In the limit of exact CPT symmetry $\delta_B = 0$, whereas $\epsilon_B = 0$ and $\delta_B = 0$ if CP is conserved. Bounds on both real and imaginary parts of δ_B at the level of 10% have been obtained [3], and recently the imaginary part of δ_B has been improved to few percent level [4]. Note that these limits are derived under the assumption that both direct CPT violating effects (*i.e.* CPT violation in the decay amplitudes) and $\Delta B = -\Delta Q$ transitions are negligible.

We conclude this section by introducing the most general parametrization of B^0 and $\bar{B}^0$ semileptonic decay amplitudes.

$$
\begin{aligned}
A^+ &= A(B^0 \to l^+\nu h^-) = F_h(1-y_h) \ , \\
\bar{A}^- &= A(\bar{B}^0 \to l^-\nu h^+) = F_h^*(1+y_h^*) \ , \\
\bar{A}^+ &= A(\bar{B}^0 \to l^+\nu h^-) = x_h F_h(1-y_h) \ , \\
A^- &= A(B^0 \to l^-\nu h^+) = \bar{x}_h^* F_h^*(1+y_h^*) \ , \qquad (2)
\end{aligned}
$$

where $h^\pm$ is a generic charged hadron. The exact CPT symmetry implies $y_h = 0$ and $x_h = \bar{x}_h$; the invariance under T requires all the amplitudes to be real; the $\Delta B = \Delta Q$ rule implies $x_h = \bar{x}_h = 0$.

3 Time dependent distributions

The decay amplitude to the final state $|a(\vec{q}\,,t_1)\rangle|b(-\vec{q}\,,t_2)\rangle$ is given by

$$
\begin{aligned}
A^{a,b}(t,\Delta t) \propto \frac{1}{\sqrt{2}}\Big\{ & A^a A^b(1+2\epsilon_B)F(\Delta t) - \bar{A}^a\bar{A}^b(1-2\epsilon_B)F(\Delta t) \\
& + \bar{A}^a A^b\left[G(\Delta t) - 2\delta_B F(\Delta t)\right] - A^a\bar{A}^b\left[G(\Delta t) + 2\delta_B F(\Delta t)\right]\Big\} \ , \qquad (3)
\end{aligned}
$$

where

$$
t = \frac{t_1+t_2}{2} \ , \qquad \Delta t = t_1 - t_2 \ , \qquad (4)
$$

and

$$F(\Delta t) = -\sinh\left(\frac{\Delta\Gamma\Delta t}{4}\right)\cos\left(\frac{\Delta m\Delta t}{2}\right) + i\,\cosh\left(\frac{\Delta\Gamma\Delta t}{4}\right)\sin\left(\frac{\Delta m\Delta t}{2}\right) \ , \quad (5)$$

$$G(\Delta t) = +\cosh\left(\frac{\Delta\Gamma\Delta t}{4}\right)\cos\left(\frac{\Delta m\Delta t}{2}\right) - i\,\sinh\left(\frac{\Delta\Gamma\Delta t}{4}\right)\sin\left(\frac{\Delta m\Delta t}{2}\right) \ . \quad (6)$$

The four terms in (3) represent the contributions due to B^0B^0, $\bar{B}^0\bar{B}^0$, $\bar{B}^0B^0$ and $B^0\bar{B}^0$ transitions to the final state. A useful observable is the so–called *time difference distribution* defined by

$$I^{ab}(\Delta t) = \int_0^\infty dt_1 dt_2 |A^{ab}(t_1,t_2)|^2 \delta(t_1 - t_2 - \Delta t) = \int_{|\Delta t|}^\infty dt |A^{ab}(t,\Delta t)|^2 \ . \quad (7)$$

This quantity is proportional to $N^{ab}(\Delta t)$, *i.e.* to the number of events where two decay $B \to a$ and $B \to b$ occur separated by a time difference Δt (in the $\Upsilon(4S)$ rest frame). The time difference distributions we are interested in are those with two semileptonic decays.

Substituting in the general expression (3) the amplitudes (2) and expanding in small quantities (*i.e.* $\Delta B = -\Delta Q$ terms, $CP-$ and CPT–violating parameters, $\Delta\Gamma\Delta t$) up to linear terms, we obtain

$$I^{+-}(\Delta t) = I^{-+}(-\Delta t) = Ne^{-\Gamma|\Delta t|}\Big[1 + \cos(\Delta m\Delta t) - 2\Delta_R(\Delta\Gamma\Delta t) - 4\Delta_I \sin(\Delta m\Delta t)\Big] \ ,$$

$$I^{\pm\pm}(\Delta t) = Ne^{-\Gamma|\Delta t|}\Big[1 \pm 4\Re(\epsilon_B - y_h)\Big]\ \Big[1 - \cos(\Delta m\Delta t)\Big] \ , \quad (8)$$

where the final states have been labeled according to the lepton sign, N is a normalization factor and $\Delta_{R,I}$ are defined by

$$\Delta_R = \Re\delta_B + \Re\ \frac{x_h - \bar{x}_h}{2} \ , \qquad \Delta_I = \Im\delta_B + \Im\ \frac{x_h + \bar{x}_h}{2} \ . \quad (9)$$

As can be deduced from the above formulae, the experimental study of the time difference distributions of semileptonic final states is to constraint the following three quantities: Δ_R, Δ_I and $\Re(\epsilon_B - y_h)$.

- Δ_R is certainly the most interesting observable, since it is a *pure* index of CPT violation. Moreover, $\Re\delta_B$ (and thus Δ_R) is the dominant CPT–violating term in the limit where direct CPT–violation is negligible. However, Δ_R is also the most difficult quantity to be measured, because it is multiplied by the small coefficient $\Delta\Gamma\Delta t$ in the time difference distribution (8).

- Δ_I is an index of CPT violation in the limit where it is possible to neglect the suppressed CP–violating and $\Delta B = -\Delta Q$ term $\Im(x_h + \bar{x}_h)$. Given the present bound on CP–violation and $\Delta B = -\Delta Q$ amplitudes in semileptonic decays, we can safely neglect the non–CPT–violating component of Δ_I at the level of 10^{-2}.

A way of extracting Δ_R and Δ_I is to measure the time–dependent asymmetries which are possible to do at asymmetric B–factories such as $BABAR$.

For example, the ratio of same–sign to opposite–sign di–lepton events

$$\begin{aligned} R(\Delta t) &= \frac{I^{++}(\Delta t) + I^{--}(\Delta t)}{I^{+-}(\Delta t)} \\ &= \frac{(1 - cos(\Delta m \Delta t))}{1 + cos(\Delta m \Delta t) - 2\Delta_R(\Delta\Gamma\Delta t) - 4\Delta_I sin(\Delta m \Delta t)} \end{aligned} \tag{10}$$

and the asymmetry on the production time of opposite–sign di–lepton events

$$\begin{aligned} R'(\Delta t) &= \frac{I^{+-}(\Delta t) - I^{-+}(\Delta t)}{I^{+-}(\Delta t) + I^{-+}(\Delta t)} \\ &= \frac{-2\Delta_R(\Delta\Gamma\Delta t) - 4\Delta_I sin(\Delta m \Delta t)}{1 + cos(\Delta m \Delta t)} \end{aligned} \tag{11}$$

serve the purpose. Whereas, the time–integrated asymmetry such as

$$R = \frac{N^{++} + N^{--}}{N^{+-}} = \frac{x_d^2}{1 + (1 + 8\delta_B^2)(1 + x_d^2)} \tag{12}$$

where $x_d = \frac{\Delta m}{\Gamma}$ and $N^{\pm\pm}$, N^{+-} denote number of equal and opposite–sign di–lepton events respectively, provides a bound on the magnitude of δ_B. We derived the equation (12) under the assumption that both direct CPT violating effects and $\Delta B = -\Delta Q$ transitions are negligible.

Note that the time–dependent asymmetry is linear in δ_B whereas the time–integrated one is quadratic on δ_B hence less sensitive than the time–dependent one when the δ_B is small.

- $\Re(\epsilon_B - y_h)$ is presumably dominated by the T–violating parameter $\Re(\epsilon_B)$. The extraction of this quantity is the simplest one since the time dependence cancels out in the following asymmetry distribution *i.e.*,

$$A_{ll} = \frac{I^{++}(\Delta t) - I^{--}(\Delta t)}{I^{++}(\Delta t) + I^{--}(\Delta t)} = 4\Re(\epsilon_B - y_h) \ . \tag{13}$$

  As noticed in reference [3], an independent measurement of the direct CPT–violating parameter $\Re(y_h)$ can be in principle achieved by looking at the time–integrated inclusive asymmetries. Denoting $N_h^{\pm}(> 0)$

the total number of events where the semileptonic decay $B \to l^{\pm}\nu h^{\mp}$ occur before any other decay (*i.e.* summing over all possible channels for the second vertex), one has

$$A_l = \frac{N_h^+(>0) - N_h^-(>0)}{N_h^+(>0) + N_h^-(>0)} = -2\Re y_h \ . \tag{14}$$

4 Experimental sensitivities

A way of extracting CPT violating parameters can be itemized as follows.

- Firstly, measure the time–integrated asymmetry R in equation (12) to extract the magnitude of δ_B in the limit of both direct CPT violating effects and $\Delta B = -\Delta Q$ transitions are negligible.

- Secondly, look at the time–dependent asymmetries such as $R(\Delta t)$ of equation (10) and extract Δ_R and Δ_I.

- Thirdly, the above extractions on CPT violating parameters are quite sensitive to the error on asymmetry R. The error on asymmetry R can be estimated assuming a binomial distribution for equal and oppisite–sign di–lepton events [10]

$$\sigma(R) = (1+R)^2 \sqrt{\frac{2N^{+-}N^{++}}{N^3}} \tag{15}$$

  where, N denotes total number of equal and oppisite–sign di–lepton events.

  For exmaple, assuming a total luminosity of $30\,fb^{-1}$, *BABAR* expects 1.5×10^7 $B^0\bar{B}^0$ pairs. With semileptonic branching fraction of 10.5% and typical lepton tagging efficiency of 50% gives about 10^4 same–sign and 5×10^4 oppsite–sign di–lepton events. The small value of $\sigma(R)$ indicates that we can tightly constrain the CPT violating parameters to few percent level.

- Lastly, we can extract the T–violating parameter $\Re(\epsilon_B)$ and the direct CPT–violating parameter $\Re(y_h)$ from the di–lepton asymmetry A_{ll} of equation (13) and the inclusive single–lepton asymmetry A_l of equation (3). If the asymmetry is small, the error on A can be written as

$$\sigma(A) \approx \sqrt{\frac{1+\frac{B}{S}}{S}} \tag{16}$$

where S and B denote number of signal and background events.

For exmaple, assuming $\frac{B}{S} = 1$, we expect to get $\sigma(A) \approx 0.01$ with a total luminosity of $30\, fb^{-1}$.

5 Conclusions

In summary, we studied the feasibility of setting limits on CPT violating parameters for the case of both CP conjugate and semileptonic decays of neutral B_d mesons. For the case of semileptonic final states of neutral B_d meson decays, bounds on CPT violating parameters at the level of few percent can be easily obtained with a total luminosity of $30\, fb^{-1}$. This sensitivity is better than the similar analysis using the CP conjugate final states of neutral B_d meson decays mainly due to the substantial increase on the statistics.

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