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More and More Indirect Signals for Extra Dimensions at More and More Colliders *

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Abstract

It has been recently suggested by Arkani-Hamed, Dimopoulos and Dvali that gravity may become strong at energies not far above the electroweak scale and thus remove the hierarchy problem. Such a scenario can be tested at both present and future accelerators since towers of Kaluza-Klein gravitons and associated scalar fields now play an important phenomenological role. In this paper we examine several processes for their sensitivity to a low scale for quantum gravity including deep inelastic ep scattering at HERA, high precision low energy νN scattering, Bhabha and Moller scattering at linear colliders and fermion pair production at $\gamma\gamma$ colliders.

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1 Introduction

Arkani-Hamed, Dimopoulos and Dvali(ADD)[1] have recently proposed a radical solution to the hierarchy problem, *i.e.*, the problem of why the weak scale is so far removed from the Planck scale, M_{pl}, where gravity becomes as strong as the other forces. ADD hypothesize the existence of n additional large spatial dimensions in which gravity can live, called 'the bulk' whereas all of the fields of the Standard Model are constrained to lie on 'the wall', which is a 3-dimensional brane and corresponding to our conventional 4-dimensional world. It has recently been shown that a scenario of this type may emerge in string models where the effective Planck scale in the bulk is identified with the string scale[1, 2]. That the SM fields must remain on the wall without being excited into the bulk below some mass scale of order of a few TeV is argued based on the well-known behavior of QED down to rather short distances, the lack of observation of degenerate mirror copies of the SM fields and the experimental value of the width of the Z boson[1]. Thus in the ADD scenario, gravity only appears to be weak in our ordinary 4-dimensional space-time since we have up to now merely observed it's action on the wall. In such a theory the hierarchy can be simply removed by postulating that the string or effective Planck scale in the bulk, M_s, is not far above the weak scale, *e.g.*, a few TeV. Gauss' Law then provides a link between the values of M_s, M_{pl}, and the size of the compactified extra dimensions, R,

$$M_{pl}^2 \sim R^n M_s^{n+2} \,, \tag{1}$$

where the constant of proportionality depends not only on the value of n but upon the geometry of the compactified dimensions. Interestingly, if M_s is near a TeV then $R \sim 10^{30/n-19}$ meters; within Newtonian gravity and for fixed n, R can be thought of as a critical point in the power-law behavior for the force of gravity. For two masses separated by a distance greater than R one obtains the usual $1/r^2$ force law; however, for separations smaller

than R the power law changes to $1/r^{2+n}$. For $n = 1$, $R \sim 10^{11}$ meters and is thus obviously excluded, but, for $n = 2$ one obtains $R \sim 1$ mm, which is at the very edge of the range of sensitivity for existing experiments[3]. For $2 < n \leq 7$, where 7 is the maximum value of n being suggested by M-theory, the value of R is further reduced and thus we may conclude that the range $2 \leq n \leq 7$ is of phenomenological interest. While we feel the ADD scenario is quite compelling, we note that several other sets of authors have considered alternate models based on the suggestion of a low Planck or string scale within other contexts[4] through the use of extra compactified dimensions. Only the ADD scenario will concern us in what follows.

The phenomenology of the ADD model as far as the new gravitational interactions are concerned can be obtained by considering a linearized theory of gravity in the bulk, decomposing it into the more familiar 4-dimensional states and recalling the existence of Kaluza-Klein towers for each of the conventionally massless fields. The entire set of fields in the K-K tower couples in an identical fashion to those of the SM. By considering the forms of the $4 + n$ symmetric conserved stress-energy tensor for the various SM fields and by remembering that such fields live only on the wall, the relevant Feynman rules can be derived[5]. An important result of these considerations is that only the massive spin-2 K-K towers (which couple to the 4-dimensional stress-energy tensor, $T^{\mu\nu}$) and spin-0 K-K towers (which couple proportional to the trace of $T^{\mu\nu}$) are of phenomenological relevance as all the spin-1 fields can be shown to decouple from the particles of the SM. If the processes under consideration are at tree-level and involve only massless fermions and gauge fields, as will be the case below, the contributions of the spin-0 fields can also be safely ignored. There will, however, be other processes where these scalars play an important role.

Given the Feynman rules as developed in [5] it appears that the ADD scenario has two basic classes of collider tests: (i) The emission of a (kinematically cut off) tower of gravitons during a hard collision leads to missing energy final states at either lepton or hadron colliders

since the emitted gravitons essentially do not interact with the detector. The rate for such processes is quite sensitive to the value of n, falling rapidly as the number of dimensions increases beyond $n = 2$. The advantage to such processes is that their observation together with a fit to the missing energy spectrum would tell us the value of n. The clear disadvantage is due to the rapid fall off in rate with large n which makes the process difficult to observe above SM backgrounds in that case. (ii) The exchange of a K-K graviton tower between SM fields can lead to almost n-independent modifications to conventional cross sections and distributions or can possibly lead to new interactions such as $gg \to e^+e^-$ as discussed by Hewett[5]. In a simple approximation the exchange of the graviton K-K tower leads to an effective operator of dimension-eight. Here one does not produce the gravitons directly and one does not learn much about the value of n itself provided deviations attributable to gravity are indeed obtained experimentally. But this n-independence is also a strength since there is in this case no fall off in the size of the deviations with large n. For low n, both type-i and type-ii processes give comparable reach in sensitivity to the scale M_s but, due to their approximate n-independence, type-ii processes eventually win out[5] for $n > 2$.

In this paper we will extend the analyses of the ADD scenario as presented in [5] to a set of previously unconsidered reactions of type-ii in order to examine their sensitivity to values of M_s of order a few TeV or less. In section 2, we extend the previous LEP/NLC and Tevatron/LHC studies[5] to the case of neutral current interactions at HERA where K-K towers of gravitons are now exchanged in the t-channel during the $eq \to eq$ scattering process; it is important to note that such exchanges do not occur in the charged current channel since gravitons are both neutral as well as isoscalar. As is well known, the sensitivity of HERA to conventional dimension-six $eeqq$ contact interactions is both complementary and numerically comparable[6] to that obtainable from LEP and the Tevatron and hence a comparison of their M_s sensitivity in the present case is particularly interesting. Such discus-

sions naturally lead one to think about the potential sensitivity of high precision low energy νN neutral current scattering experiments, such as NuTeV, who have recently[7] obtained a very competitive measurement of the W mass (or the weak mixing angle) by employing the Paschos-Wolfenstein relation[8]. In Section 3 we will examine the sensitivity of these precise but relatively low-energy experiments to interesting values of M_s; unfortunately we find that while such processes are quite sensitive to dimension-six compositeness operators[6, 7], there is little sensitivity to the string scale in this case. In section 4, we will return to a discussion of the M_s sensitivity of various processes at lepton linear colliders by examining both Bhabha and Moller scattering, $e^{\pm}e^{-} \to e^{\pm}e^{-}$. It is often claimed that Moller scattering is the most sensitive of the purely leptonic processes accessible at lepton colliders to the existence of compositeness[9] and new neutral gauge bosons[10]. Thus it would appear natural to compare the sensitivity of these two processes to that obtained earlier by Hewett[5] who examined the reactions $e^{+}e^{-} \to f\bar{f}$, $f \neq e$. These claims will be shown to indeed be valid for the case at hand when statistical errors are dominant. In Section 5 we will consider the M_s sensitivity of the process $\gamma\gamma \to f\bar{f}$ via high energy $\gamma\gamma$ collisions obtainable at linear colliders through the backscattering of pairs of laser beams[11]. Although the M_s reach is somewhat lower here than in purely leptonic reactions, $\gamma\gamma \to f\bar{f}$ can provide complementary information. A summary of our analysis and our conclusions can be found in Section 6.

2 HERA

HERA is currently colliding 27.5 GeV electrons on 920 GeV protons, thus obtaining a center of mass energy of $\sqrt{s} = 318$ GeV. Both the H1 and ZEUS experiments are expected[12] to collect $\sim 1\ fb^{-1}$ in integrated luminosity over the next several years. After the year 2000, it is anticipated that HERA will deliver $\sim 60\%$ longitudinally polarized $e^{\pm}$ beams shared

more or less equally between the four charge and polarization assignments. These specific lumonosity and polarization parameters will be assumed in our analysis below. We recall from the discussion above that we need only to consider neutral current processes since graviton towers are not exchanged at tree level in charged current reactions. Thus potential deviations in cross sections at high Q^2 appearing in *both* channels due to, *e.g.*, leptoquarks, new gauge bosons or contact interactions *cannot* be attributed to the ADD model of low-scale quantum gravity.

The basic subprocess cross section for $e^-_{L,R}q$ elastic scattering, now including the exchange of a K-K tower of gravitons, is given by [13]

$$\begin{aligned}\frac{d\sigma_q}{dxdQ^2} &= \frac{2\pi\alpha^2}{\hat{s}^2}\Big[\mathrm{SM} - C\Big\{\left(\frac{Q_eQ_q}{t} + \frac{\sigma C'(v_e+\sigma a_e)v_q}{t-m_Z^2}\right)2(u-\hat{s})^3 \\ &\quad -\frac{C'(a_e+\sigma v_e)a_q}{t-m_Z^2}\,t\,[t^2-3(u-t)^2]\Big\} \\ &\quad +\frac{C^2}{2}\{t^4-3t^2(u-\hat{s})^2+4(u-\hat{s})^4\}\Big]\,, \end{aligned} \tag{2}$$

where 'SM' is the conventional SM contribution, $C = \lambda K/(4\pi\alpha M_s^4)$, $C' = \sqrt{2}G_F M_Z^2/4\pi\alpha$ and $\sigma = \pm 1$ for left-(right-)handed electrons. We note here that through the use of crossing symmetry, this cross section with suitable modifications can be shown to reproduce those obtained for by Hewett and by Guidice, Rattazzi and Wells[5] with the following caveat regarding the parameter K in the expressions above. For $K = 1(\pi/2)$ we recover the normalization convention employed by Hewett(Guidice, Rattazzi and Wells)[5]; we will take $K = 1$ in the numerical analysis that follows but keep the factor in our analytical expressions. We recall from the Hewett analysis that λ is a parameter of order unity whose sign is undetermined and that, given the scaling relationship between λ and M_s, experiments in the case of processes of type-*ii* actually probe only the combination $M_s/|K\lambda|^{1/4}$. For simplicity

in what follows we will numerically set $|\lambda| = 1$ and employ $K = 1$ but we caution the reader about this technicality and quote our sensitivity to M_s for $\lambda = \pm 1$.

In the case of $e^-_{L,R}\bar{q}$ scattering, we simply let $a_q \to -a_q$ in the above expression and make the replacement $q(x) \to \bar{q}(x)$ in the sum over initial state partons. Here, and in the expression above $Q^2 = -t = y\hat{s} = sxy$ and $u = -\hat{s} - t = -sx(1-y)$ with Q^2, x, y being the conventional variables of deep inelastic scattering. For positron scattering we note the relations $d\sigma^+_{R,L}(q,\bar{q}) = d\sigma^-_{L,R}(\bar{q},q)$ can be used to obtain the complementary cross sections. We note further that with the normalization employed above $a_e = -1/2$.

Of course in the ADD scenario, the $eq \to eq$ process is not the only one which contributes to deep inelastic scattering. Since both electrons and gluons have non-zero stress-energy tensors, a tower of K-K gravitons can also be exchanged in the t-channel mediating the process $eg \to eg$ where the squared matrix element is independent of the charge and helicity of the incoming lepton. The corresponding subprocess cross section for $e^\pm_{L,R}g$ scattering is thus relatively simple and is given by

$$\frac{d\sigma_g}{dxdQ^2} = \frac{-\lambda^2 K^2}{256\pi M_s^8 \hat{s}^2} u\hat{s}[(u^2 + \hat{s}^2)]\,, \tag{3}$$

there being no SM contribution in this case. Note that with K taking on the values discussed above, using crossing symmetry and rearranging color factors, we again reproduce the analogous cross section expressions given by Hewett and by Guidice, Rattazzi and Wells[5].

In order to gauge the HERA sensitivity to exchanges of a K-K tower of gravitons, we follow the current HERA analysis technique as presented by Stanco[6]. Since this new exchange only reveals itself at higher values of Q^2, we divide the Q^2 range into two regions: below $Q^2 = 1000$ GeV^2 we assume that the SM holds and use this regime to normalize the neutral current cross sections for the four charge/polarization states of the incoming lepton.

This assumption will be explicitly validated in the discussion below. Above $Q^2 = 1000$ GeV^2 we divide the range into 17 Q^2 bins up to the kinematic limit; the location and width of these bins are essentially those of the present HERA analyses with only minor modifications due to the higher anticipated integrated luminosities. We then use a toy Monte Carlo approach to generate 'data' assuming a given integrated luminosity for each of the four charge/polarization states. These data are then fit to the M_s-dependent cross section to obtain a lower bound on M_s at the 95% CL. In performing this analysis we employ the CTEQ4M parton density distributions[14] although our results are not sensitive to this particular choice. We assume that the potential of any large systematic error associated with the calorimeter energy scale can be avoided in obtaining these results.

In examining the sensitivity of the four cross sections, $d\sigma(e^{\pm}_{L,R}p)$, one finds that the process with the largest(smallest) cross section (hence the best statistics) is the one with the least(most) sensitivity to M_s. Instead of trying to choose the beam that maximizes sensitivity to M_s with the best statistics we will simply assume equal integrated luminosities are supplied for all four cases and combine the result into a single fit. One may either try to simultaneously fit to all four $e^{\pm}_{L,R}$ cross sections, *i.e.*, 4×17 bins, or simply fit to the sum of the four cross sections together in each Q^2 bin, *i.e.*, 17 bins only. Given the need to have as much statistics as possible in the highest Q^2 bins we follow the latter approach. To get an idea of the resulting sensitivity we show in Fig.1 the deviation from the bin-integrated SM cross section for M_s =800 and 1000 GeV with $\lambda = \pm 1$. Note that the deviations from the SM grows only very slowly with increasing Q^2 and are not significantly noticeable below $Q^2 = 10000 - 15000$ GeV^2.

Performing the analysis described above we arrive at the 95% CL lower bound on M_s as a function of the integrated luminosity as shown in Fig.2. With the assumption that

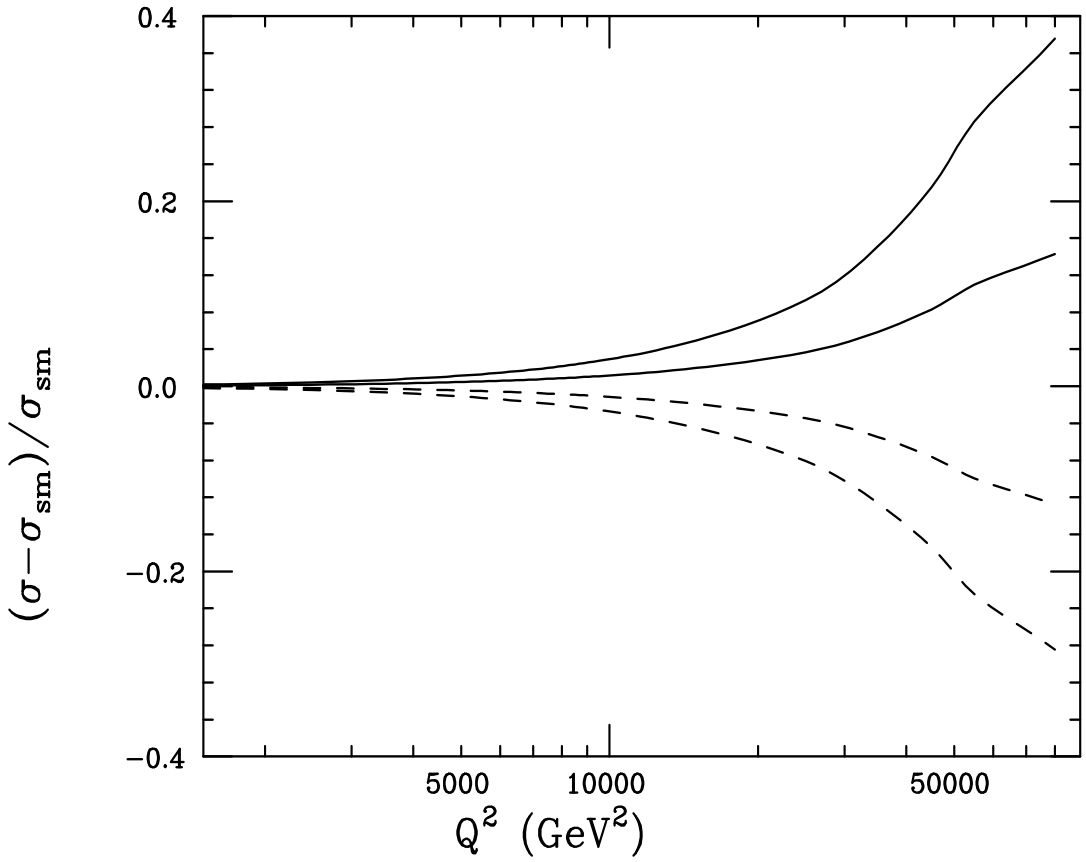

Figure 1: Deviation in equally weighted sum of the $\sigma(e^{\pm}_{L,R}p)$ deep inelastic cross sections as a function of Q^2 for λ =1(solid) and -1(dashed). The outer(inner) curve in each case corresponds to assuming $M_s = 800(1000)$ GeV.

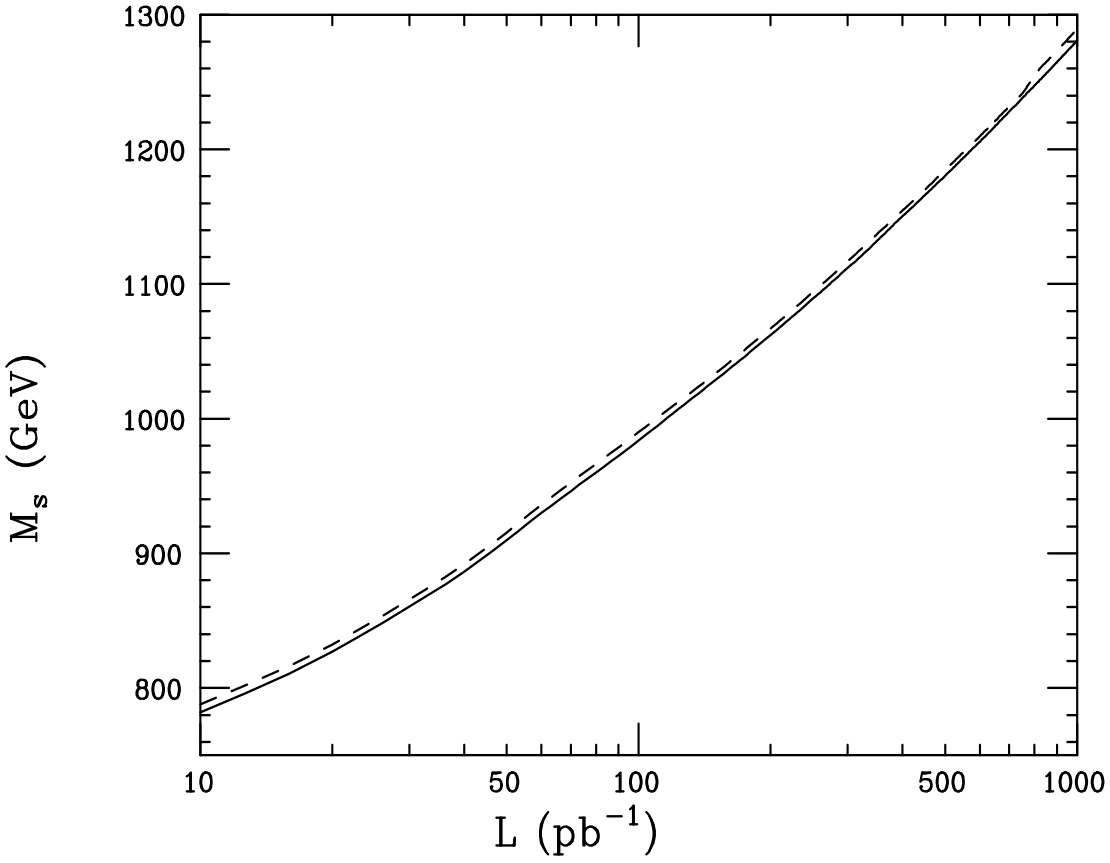

Figure 2: 95% CL lower bound on the value of M_s obtainable at HERA as a function of the integrated luminosity per charge/polarization state for $\lambda = \pm 1$.

each of the four neutral current processes, $\sigma(e^{\pm}_{L,R}p)$, obtain the same beam flux, the full 1 fb^{-1} HERA luminosity corresponds to $L = 250\ pb^{-1}$ in this figure. The limit in this case, for either sign of λ, is $\simeq 1.09$ TeV which is very comparable to the potential search reach of 1.14 TeV obtainable at LEP II from the analysis of Hewett[5]. Similarly it it comparable to, but somewhat lower than, that obtainable at Run II of the Tevatron through an analysis of the the Drell-Yan process. Clearly the bounds obtainable at HERA are complementary to those obtainable at other currently existing colliders.

What limits are obtainable from the existing HERA data, *i.e.*, approximately 40 pb^{-1} of integrated luminosity per experiment using an unpolarized e^+ beam at a center of mass energy of $\sqrt{s} \simeq 298$ GeV? The reduced center of mass energy and the use of an unpolarized e^+ beam both act to significantly suppress the reach relative to the estimate one would obtain by the use of the results shown in Fig.2 alone. We estimate that the current lower bound on M_s from HERA to be no larger than $\simeq 500 - 600$ GeV.

3 Low Energy ν Scattering

Do lower energy measurements reveal anything about M_s? Since the exchange of K-K towers of gravitons is essentially flavor independent and is a parity conserving process these new effects will not show themselves in atomic parity violation or polarized lepton nucleon scattering experiments. The only other possibility is neutrino-nucleon neutral current deep inelastic scattering.

In the case of $\nu(\bar{\nu})q$ and $\nu(\bar{\nu})\bar{q}$ scattering we can obtain the relevant cross sections from the expressions above by setting $Q_e = 0$, $v_e = a_e = 1/2$, taking $Q^2 \ll M_Z^2$, and recalling that $\nu(\bar{\nu})$'s are always left-(right-)handed. We then arrive at the following expression for the

$\nu(\bar{\nu})q$ subprocess cross section:

$$\begin{aligned}\frac{d\sigma_q^{\nu,\bar{\nu}}}{dxdy} &= \frac{G_F^2 s}{4\pi}\Big[\{(v_q \pm a_q)^2 + (v_q \mp a_q)^2(1-y)^2\} + xF\{-2(2-y)^3 v_q \\ &\pm y(y^2 - 3(2-y)^2)a_q\} + \frac{(xF)^2}{2}\{y^4 - 3y^2(2-y)^2 + 4(2-y)^4\}\Big]\,, \qquad (4)\end{aligned}$$

where $F = \lambda K s/\sqrt{2}G_F M_s^4$ and the upper(lower) sign is for the $\nu(\bar{\nu})$ scattering process. The first term is just that arising from the SM while the additional terms arise from the K-K graviton tower exchange and its interference with the SM Z exchange. The corresponding $\bar{q}$ cross section can be obtained by letting $a_q \to -a_q$ in the above expression. The corresponding $\nu(\bar{\nu})g$ subprocess cross section which has a pure graviton exchange and no SM contribution is identical in both cases and is given by

$$\frac{d\sigma_g^{\nu,\bar{\nu}}}{dxdy} = \frac{G_F^2 s}{4\pi}\frac{(xF)^2}{32}(1-y)\{1 + (1-y)^2\}\,. \qquad (5)$$

To obtain the complete scattering cross section one must weight the two expressions above with the relevant parton density functions(PDFs):

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dxdy} = \sum_q \{\frac{d\sigma_q^{\nu,\bar{\nu}}}{dxdy} xq(x) + \frac{d\sigma_{\bar{q}}^{\nu,\bar{\nu}}}{dxdy} x\bar{q}(x)\} + \frac{d\sigma_g^{\nu,\bar{\nu}}}{dxdy} xg(x)\,, \qquad (6)$$

with the sum extending over all quark flavors.

To get an idea of the sensitivity of neutrino nucleon scattering to the exchange of K-K towers of gravitons it is instructive to form the well known ratios $R^{\nu,\bar{\nu}} = \sigma_{NC}^{\nu,\bar{\nu}}/\sigma_{CC}^{\nu,\bar{\nu}}$ for an isoscalar target in the valence quark approximation which then allows us to trivially perform the integrations over the x and y variables. (We note that these quantities are not quite as well measured[15] as is the Paschos-Wolfenstein relation to be discussed later below). We

obtain the expressions

$$\begin{aligned} R^{\nu} &= g_L^2(u) + g_L^2(d) + \frac{1}{3}g_R^2(u) + \frac{1}{3}g_R^2(d) + \Delta, \\ R^{\bar{\nu}} &= g_L^2(u) + g_L^2(d) + 3g_R^2(u) + 3g_R^2(d) + 3\Delta\,, \end{aligned} \tag{7}$$

where $g_L(u) = 1/2 - 4/3\sin^2\theta_w$, *etc.*, and Δ can be expressed numerically as

$$\Delta = -3.39 \times 10^4 F' R_1 + 2.15 \times 10^{10} F'^2 R_2 + 3.83 \times 10^7 F'^2 R_3\,, \tag{8}$$

where $F' = \lambda K s/M_s^4$ (with $\sqrt{s}$ and M_s in GeV) and the R_i are ratios of integrals over the appropriate PDFs:

$$\begin{aligned} R_{1,2} &= \frac{\int x^{2,3}[u(x) + d(x)]\,dx}{\int x[u(x) + d(x)]\,dx} \\ R_3 &= \frac{\int 2x^3 g(x)\,dx}{\int x[u(x) + d(x)]\,dx}\,, \end{aligned} \tag{9}$$

which need to be evaluated at a typical value of Q^2 and over the relevant x range for a given experiment. For a typical Q^2 of 25 GeV^2 and $0.001 < x < 1$ we find, using the CTEQ4M PDFs[14], that $R_1 \simeq 0.21$, $R_2 \simeq 0.071$, and $R_3 \simeq 0.042$. Since the R_i are not too small and the numerical coefficients in Eq.(8) are large, one might anticipate a reasonable sensitivity to the string scale M_s. However, a short analysis shows this not to be the case due to the low values of the center of mass energy obtained in such collisions. Although the peak neutrino energies at NuTeV may be as high at 400 GeV, the average energies of the ν_μ and $\bar{\nu}_\mu$ from the Fermilab Tevatron Quadrupole triplet neutrino beam are roughly 165 and 135 GeV, respectively[7], implying that the typical $\sqrt{s}$ for these collisions is only $\simeq 17$ GeV. In turn, assuming $K = 1$, we arrive at $F' \simeq 3 \times 10^{-10}$ and thus $\Delta \simeq -2.15 \times 10^{-6}$ which is far too small to be observable at any forseeable level of precision. Note that this value would only

be an order of magnitude larger if all neutrinos in the beam had their maximum possible energies.

Next, from the considerations above we are able to directly construct the Paschos-Wolfenstein relationship for an isoscalar target. We anticipate that this now will take the more general form including the effects of sea quarks (since they cancel in the differences in both the numerator and denominator) but neglecting charm mass effects,

$$R_{PW} = \frac{\sigma^{\nu}_{NC} - \sigma^{\bar{\nu}}_{NC}}{\sigma^{\nu}_{CC} - \sigma^{\bar{\nu}}_{CC}} = \frac{1}{2} - \sin^2\theta_w + \Delta' \,, \tag{10}$$

where Δ' arises from graviton exchange and its interference with the SM amplitude. Note these K-K contributions only appear in the numerator of the above expression. Several things are immediately obvious. First the $\nu g \to \nu g$ and $\bar{\nu} g \to \bar{\nu} g$ contributions, being the same, cancel as do those corresponding to the pure graviton terms in the difference between $\nu q(\bar{q}) \to \nu q(\bar{q})$ and $\bar{\nu} q(\bar{q}) \to \bar{\nu} q(\bar{q})$. Secondly, the term proportional to the parity conserving vector coupling of the quarks, v_q, in the SM-graviton interference term will also cancel with the only remaining term being proportional to a_q. This leaves us, after integration over y, with the result

$$R_{PW} = \sum_V v_q a_q - \frac{15}{8} F \frac{\int \sum_V a_q x^2 q_V(x)\, dx}{\int \sum_V x q_V(x)\, dx} \,, \tag{11}$$

where the sum extends over the valence partons in the isoscalar target. The first term once expanded in terms of the conventional Z boson couplings is just that provided by the SM while the second SM-graviton interference term, Δ', can be shown to vanish! Since $u_V(x) = d_V(x)$ in an isoscalar target and $a_u = -a_d$ the sum in the numerator is identically zero. This result tells us that K-K gravitons do not influence the Paschos-Wolfenstein relation whatsoever, something we may have expected due to their isoscalar nature.

It appears that low energy neutrino measurements, however precise, will not tell us

much, if anything, about the scale M_s. One may ask why νN scattering is sensitive to traditional contact interactions but not to the exchange of a K-K tower of gravitons. The answer is directly related to the fact that traditional contact interactions are dimension-six operators while those induced by low scale quantum gravity are dimension-eight. With coefficients of order unity, a scale of order 1 TeV and an average $\sqrt{s}$ =17 GeV, the dimension-eight operators are suppressed relative to those of dimension-six by a factor of $\simeq 3500$! For these dimension-eight operators the high precision of the data cannot offset their being at rather low energies. To search for M_s in the ADD scenario we clearly need larger collision energies than those provided by νN scattering.

4 Bhabha and Moller Scattering at Linear Colliders

Linear colliders will provide the opportunity to make precision measurements of a number of elementary processes in the $\sqrt{s} = 500 - 1500$ GeV energy range. In addition to the conventional processes $e^+e^- \to f\bar{f}$, whose sensitivity to the exchange of a K-K tower of gravitons was discussed by Hewett[5], both Bhabha and Moller scattering offer complementary opportunities. In principle, Moller scattering, which takes place at a future linear collider run in the e^-e^- mode[16], may be of particular interest due to its well-known sensitivity to both contact interactions and Z' exchange[9, 10].

In analyzing both the Bhabha and Moller processes we will make an angular acceptance cut of 10^o with respect to the incoming beams, assume a 90% e^- beam polarization P, with an uncertainty of $\delta P/P = 0.3\%$[17] and a integrated luminosity uncertainty of $\delta L/L = 0.1\%$[18]. (We will ignore the possibility of polarizing the positron beam in the present analysis.) In the case of Moller scattering both e^- beams are assumed to have identical polarization so that the *effective* beam polarization will be $P_{eff} = 2P/(1+P^2) \simeq 0.9945$

with a correspondingly decreased uncertainty of $\delta P_{eff}/P_{eff} \simeq 0.032\%$. In the subsequent analysis the effects of initial state radiation will be included in all processes and we will assume a lepton identification efficiency of 100%.

In the case of Bhabha scattering the differential cross section can be written as

$$\begin{aligned}\frac{d\sigma_B}{dz} &= \frac{\pi\alpha^2}{s}\Big[\mathrm{SM} - 2C\Big\{F_1(s,t) \\ &\quad + \Big[\frac{F_2(s,t)v_e^2 + F_3(s,t)a_e^2}{(s-M_Z^2)} + (s \leftrightarrow t)\Big]\Big\} + C^2 F_4(s,t)\Big]\,, \end{aligned} \tag{12}$$

where 'SM' in the expression above now corresponds to the usual SM contribution to Bhabha scattering, $z = \cos\theta$, $C = \lambda K/(4\pi\alpha M_s^4)$ as in the expressions above and the kinematic functions F_i are given by

$$\begin{aligned} F_1(s,t) &= 9(\frac{s^3}{t} + \frac{t^3}{s}) + 23(s^2+t^2) + 30st\,, \\ F_2(s,t) &= 5s^3 + 10s^2t + 18st^2 + 9t^3\,, \\ F_3(s,t) &= 5s^3 + 15s^2t + 12st^2 + t^3\,, \\ F_4(s,t) &= 41(s^4+t^4) + 124st(s^2+t^2) + 148s^2t^2\,. \end{aligned} \tag{13}$$

Employing finite beam polarization the corresponding angular-dependent polarized Left-Right Asymmetry can be expressed as

$$A_{LR} = \frac{\Big[\mathrm{SM}' - 2Cv_e a_e\Big\{\frac{F_2(s,t)+F_3(s,t)}{(s-M_Z^2)} + (s \leftrightarrow t)\Big\}\Big]}{\Big[\mathrm{SM} - 2C\Big\{F_1(s,t) + \Big[\frac{F_2(s,t)v_e^2+F_3(s,t)a_e^2}{(s-M_Z^2)} + (s \leftrightarrow t)\Big]\Big\} + C^2F_4(s,t)\Big]}\,. \tag{14}$$

Given these expressions we can obtain the search reach for M_s for a given integrated luminosity using the assumptions discussed above by fitting to the total number of events,

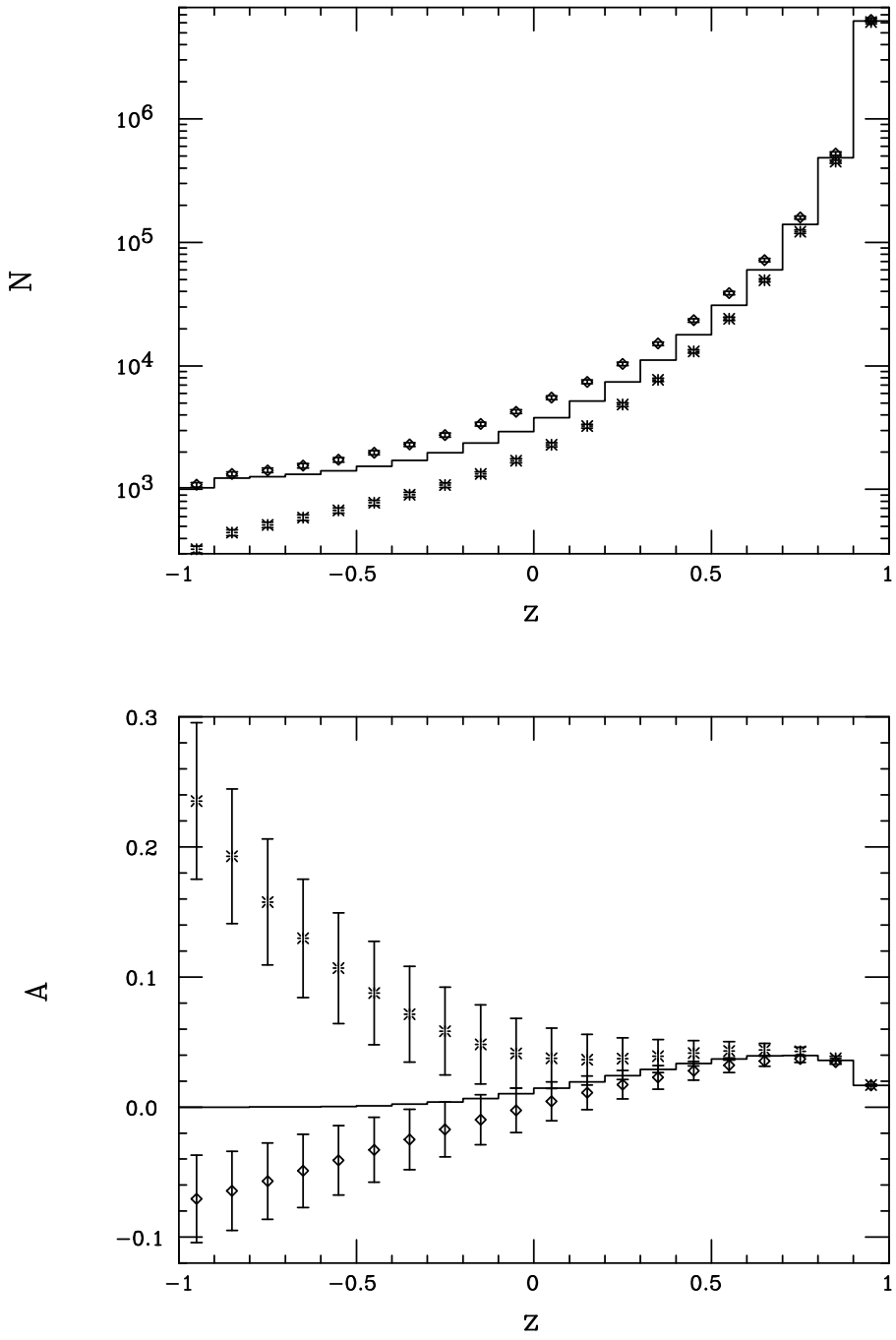

Figure 3: Deviation from the expectations of the SM(histogram) for Bhabha scattering at a 500 GeV e^+e^- collider for both the(top) number of events per angular bin, N, and the Left-Right polarization asymmetry(bottom) as a function of $z = \cos\theta$ assuming M_s=1.5 TeV. The two sets of data points correspond to the choices $\lambda = \pm 1$ and an assumed integrated luminosity of 75 fb^{-1}.

the shape of the angular distribution and the angle-dependent values of A_{LR}. We divide the angular range into 20 equal-sized $\cos\theta$ bins of width $\Delta z = 0.1$, except for those nearest the beam pipe due to the above mentioned cut. To first get an idea of the influence of finite M_s we show the distributions for Bhabha scattering in Fig. 3 for the case of a $\sqrt{s}$=500 GeV lepton collider with an integrated luminosity of 75 fb^{-1} assuming $M_s = 1.5$ TeV. In this figure the cross section in the forward direction is dominated by the photon pole but significant deviations from the SM, which is represented as the histogram, are observed away from this region in both the angular distribution and the Left-Right Asymmetry. Note the huge statistics available here. The two sets of data points show the size of the anticipated errors for both $\lambda = \pm 1$; note that they are mutually distinguishable. It is clear from this figure that for this center of mass energy and integrated luminosity the discovery reach for M_s will be significantly larger than 1.5 TeV.

In the case of Moller scattering one finds results similar to Bhabha scattering for both the cross section and Left-Right polarization asymmetry which can be obtained by crossing symmetry except for the overall factor of 2 in the normalization of the cross section:

$$\begin{aligned}\frac{d\sigma_M}{dz} &= \frac{\pi\alpha^2}{2s}\Big[\mathrm{SM} - 2C\Big\{F_1(u,t) \\ &\quad + \Big[\frac{F_2(u,t)v_e^2 + F_3(u,t)a_e^2}{(u-M_Z^2)} + (u \leftrightarrow t)\Big]\Big\} + C^2 F_4(u,t)\Big]\,. \end{aligned} \tag{15}$$

Note that the kinematic functions F_i are now functions of t and u instead of t and s as in the case of Bhabha scattering. The corresponding expression for the polarized Left-Right Asymmetry is given by

$$A_{LR} = \frac{\Big[\mathrm{SM}' - 2Cv_e a_e\{\frac{F_2(u,t)+F_3(u,t)}{(u-M_Z^2)} + (u \leftrightarrow t)\}\Big]}{\Big[\mathrm{SM} - 2C\{F_1(u,t) + \Big[\frac{F_2(u,t)v_e^2+F_3(u,t)a_e^2}{(u-M_Z^2)} + (u \leftrightarrow t)\Big]\} + C^2F_4(u,t)\Big]}\,. \tag{16}$$

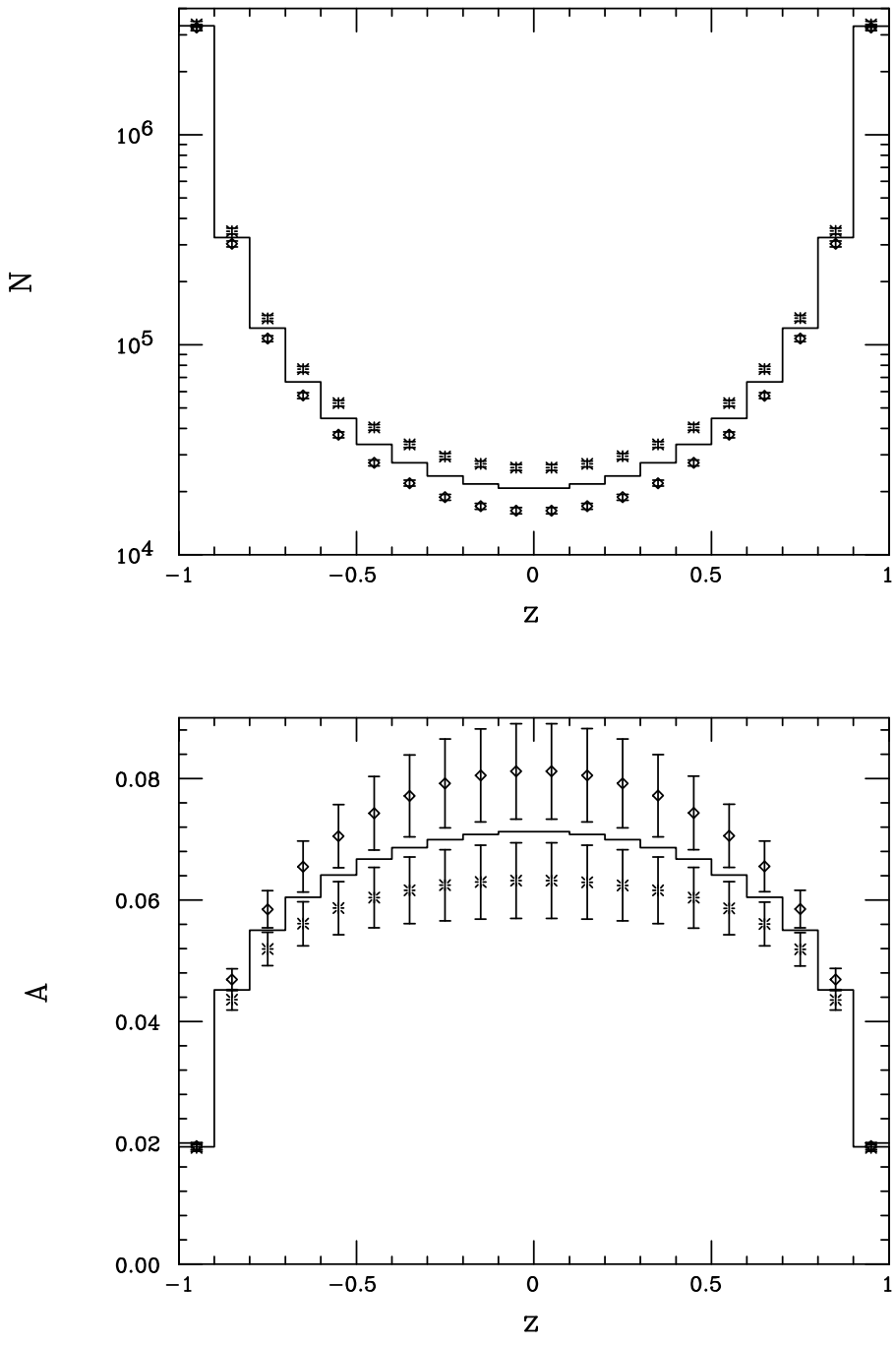

Figure 4: Same as the previous figure but now for Moller scattering.

To get an idea of the sensitivity from Moller scattering we show in Fig.4 the results of the same analysis as presented in Fig.3. While the photon poles dominate both the forward and backward directions the central regions of both the angular distribution and the Left-Right Asymmetry show clear deviations from SM expectations. We again note the huge statistics that are available. However note that the overall deviation from the SM is perhaps not as great as in the case of Bhabha scattering due to there being 2 QED poles. Of course the extra pole also leads to increased statistics. Clearly the search reach for Moller scattering exceeds 1.5 TeV for this center of mass energy and integrated luminosity.

Figs. 5, 6 and 7 show the search reaches for M_s as a function of the collider integrated luminosity for both Bhabha and Moller scattering in comparison to the 'usual' search employing $e^+e^- \to f\bar{f}$ at $\sqrt{s} = 500$ GeV, 1 TeV and 1.5 TeV colliders, respectively. (In all three cases the results for $\lambda = \pm 1$ are shown but may not be visually separable.) We note that our result for the 'usual' search confirms that of Hewett[5] but is slightly higher due a different choice of angular cuts and assumed uncertainty of the integrated luminosity. Several results are immediately obvious from these two figures. First, for reasonable integrated luminosities, the search reaches for all three modes can exceed $\simeq 6\sqrt{s}$, which is rather remarkable. At a $\sqrt{s} = 1.5$ TeV collider with a high integrated luminosity we see that string scales as high as 10 TeV can be probed. Second, since the traditional $e^+e^- \to f\bar{f}$ search with $f = \mu,\ \tau,\ b,\ c,\ t,\ etc.$ sums over many final states and employs many observables it tends to lead to the best search reach for most integrated luminosities, in particular, when large luminosity samples are available. In almost all cases the precision of this data is statistics dominated since there are only several thousands of events for each flavor. Third, the errors on the data in the cases of both Bhabha and Moller scattering are likely to be systematics dominated at typical integrated luminosities due to the huge event rates observed in Figs. 3 and 4. This explains the far shallower slopes of their luminosity dependence observed for

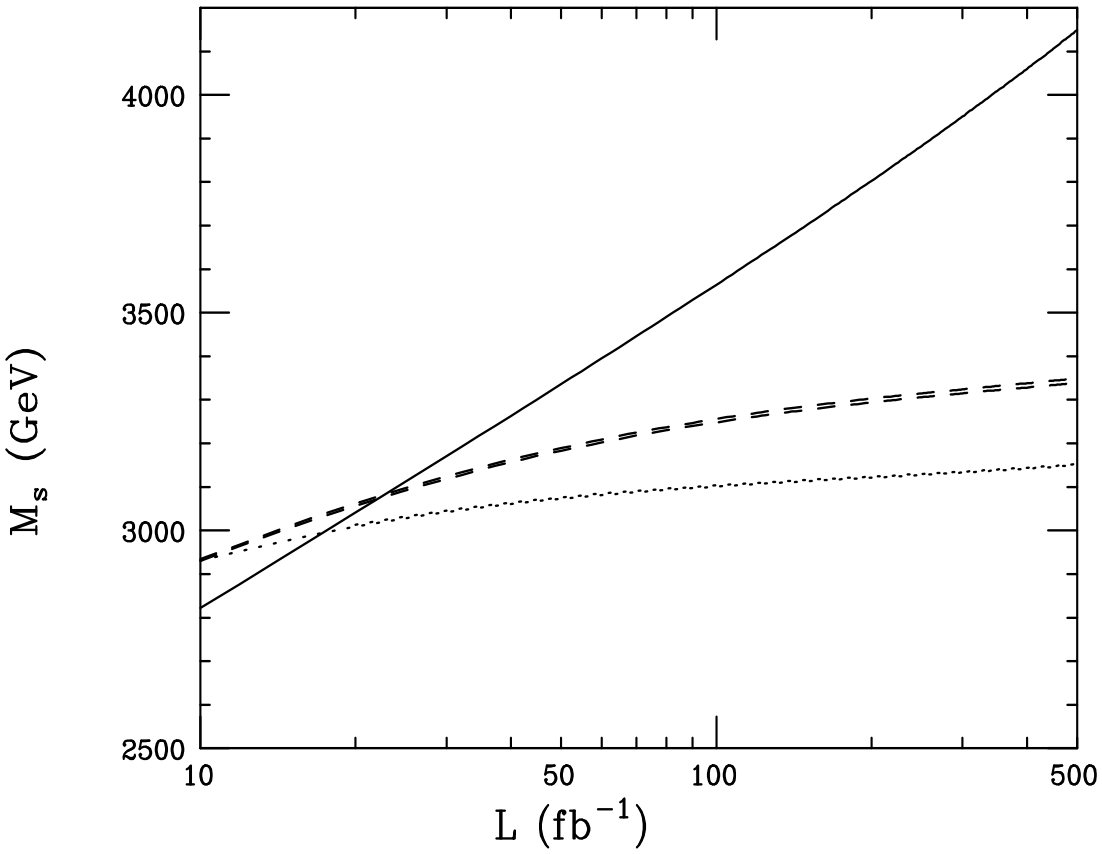

Figure 5: Search reaches for M_s at a 500 GeV e^+e^-/e^-e^- collider as a function of the integrated luminosity for Bhabha(dashed) and Moller(dotted) scattering for either sign of the parameter λ in comparison to the 'usual' search employing $e^+e^- \to f\bar{f}$(solid) as described in the text.

both the Bhabha and Moller curves in these figures. Furthermore, for a fixed integrated luminosity, we know that three event rates for all three reactions decrease with increasing values of $\sqrt{s}$ leading to different weights in the errors between statistical and systematic. Thus we note, particularly in the case of Figs. 6 and 7, that for low luminosities, where systematic errors are not as important as statistical ones, Moller scattering indeed leads to the best search reach for M_s due to the huge statistics in that data sample in comparison to either Bhabha scattering or the conventional fermion pair channel. (Thus the explanation for why Bhabha scattering is a close second to Moller scattering in the search reach for M_s for low luminosities becomes immediately obvious.) It is clear from this analysis that we again find complementarity in the search for TeV scale M_s in the ADD scenario.

5 $\gamma\gamma$ Colliders

The process $\gamma\gamma \to f\bar{f}$ is particularly clean, there being no tree level corrections from electroweak effects, and has a long tradition as a probe for higher dimensional operators. In fact, no gauge invariant operators due to contact interaction exist at dimension-six.

$\gamma\gamma$ collisions may be possible at future e^+e^- linear colliders by the use of Compton backscattering of low energy laser beams[11]. The backscattered laser photon spectrum, $f_\gamma(x = \frac{E_\gamma}{E_e})$, is far from being monoenergetic and is cut off above $x_{max} \simeq 0.83$ implying that the photons are significantly softer than their parent lepton beam energy. As we will see, this cutoff at large x, x_{max}, implies that the $\gamma\gamma$ center of mass energy never exceeds $\simeq 0.83$ of the parent collider and this will result in a significantly degraded M_s search reach. We will ignore the possibility of employing polarized photon collisions in what follows but one would anticipate that the search reach would somewhat increase beyond what we obtain below if additional polarization information were included. This possibility will be considered

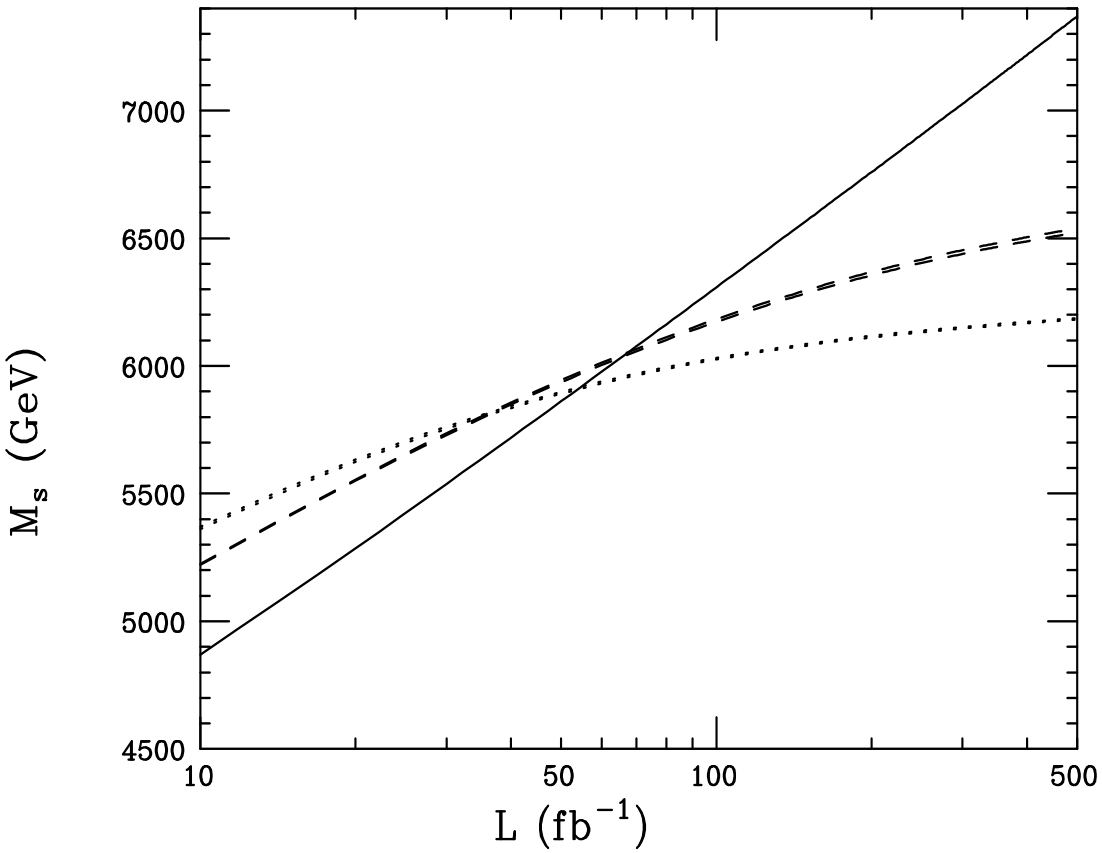

Figure 6: Same as the previous figure but now for an e^+e^-/e^-e^- collider with a center of mass energy of 1 TeV.

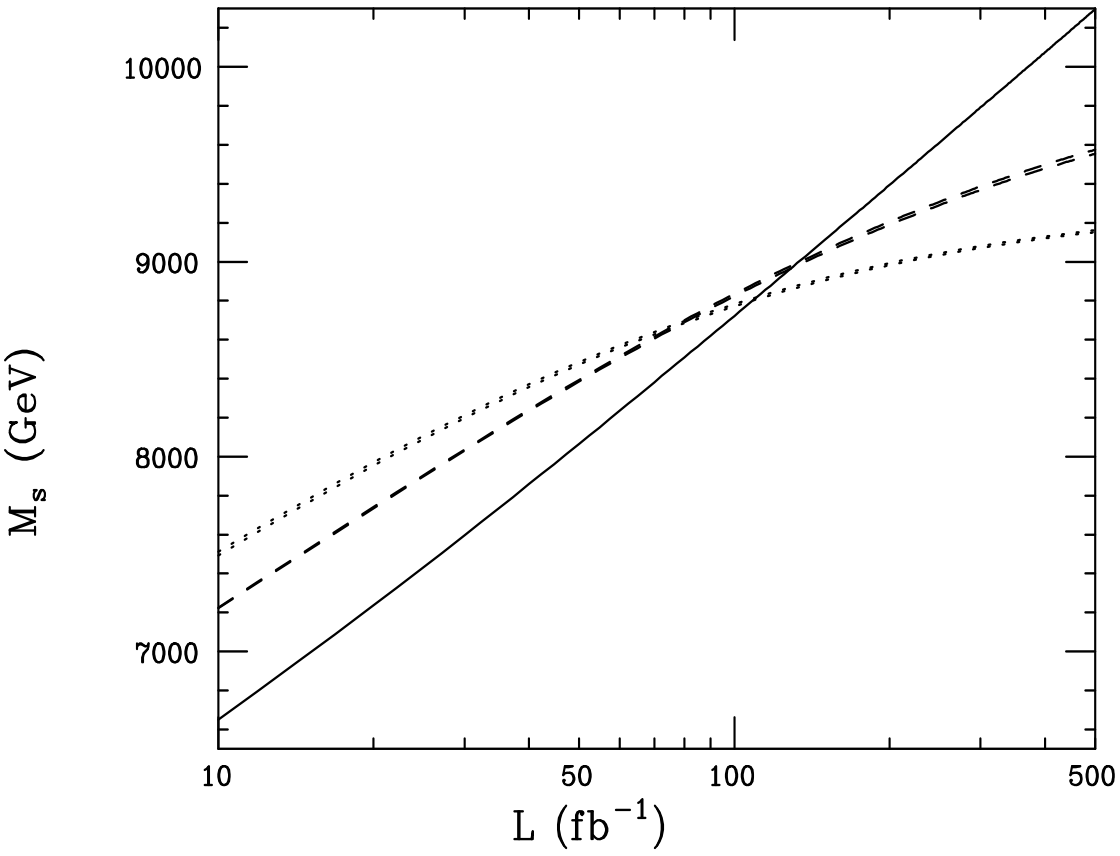

Figure 7: Same as the previous figure but now for an e^+e^-/e^-e^- collider with a center of mass energy of 1.5 TeV.

elsewhere[19].

The subprocess cross section for the unpolarized $\gamma\gamma \to f\bar{f}$ reaction including the contribution from graviton exchange can be written[5] in a rather simple form:

$$\frac{d\hat{\sigma}}{dz} = \frac{2\pi\alpha^2}{\hat{s}} N_c \frac{1+z^2}{1-z^2} \left[Q_f^2 + \lambda K \frac{\hat{s}^2(1-z^2)}{8\pi\alpha M_s^4} \right]^2 , \tag{17}$$

where as before $z = \cos\theta$ and N_c is the usual color factor for the (assumed to be massless) fermions f. To obtain the true cross section integrated over a given angular bin, assuming that the two photons have a head-on collision, we must fold in the photon fluxes and integrate over them:

$$\sigma = \int^{x_{max}} dx_1 \int^{x_{max}} dx_2 \int_{bin} dz \; f_\gamma(x_1) f_\gamma(x_2) \frac{d\hat{\sigma}}{dz} , \tag{18}$$

where we explicitly identify $\hat{s} = s_{e^+e^-} x_1 x_2$. The *lower* range of the above integrations requires some discussion. In principle, the photon fluxes persist to very low values of x; however, for very small x's we lose significant sensitivity to M_s. Hence we want to maximize as much possible the luminosity of the flux with the greatest possible value of $\hat{s}^2/M_s^4$ as is easily seen by an examination of the equation above. To this end we impose the constraint that $\hat{s}/s \geq 0.01$ and also demand $x_{1,2} \geq 0.01$ subject to this constraint. As before we will impose a 10^o angular cut in our analysis in order to obtain our search reach as a function of the total $\gamma\gamma$ integrated luminosity. Additional cuts which, for example, balance the energy of the two incoming photons, are also possible but we do not make use of them here.

To get a rough idea of the sensitivity of $\gamma\gamma$ collisions to M_s we display in Fig.8 the angular distribution for the case where f is summed over light quarks (*i.e.*, a final state of two jets without flavor tags) at a 500 GeV collider with a *diphoton* integrated luminosity of 75 fb^{-1}. Due to the SM u- and t-channel exchanges there is an enormous flux in both the forward and backward directions. However the true region of sensitivity is at large angles

where the rate is the smallest as was the case for Moller scattering. Note that the deviations are easily distinguished from both the SM and each other. It is clear from this figure that the search reach for M_s would again exceed 1.5 TeV independent of the choice of the sign of λ if $\gamma\gamma \to f\bar{f}$ were the only relevant process. However, since in this case the final state fermions are not tagged, the process $\gamma\gamma \to gg$, which occurs only through the exchange of a K-K tower, would now also contribute since the final state in both cases is just two jets as far as a detector is concerned.

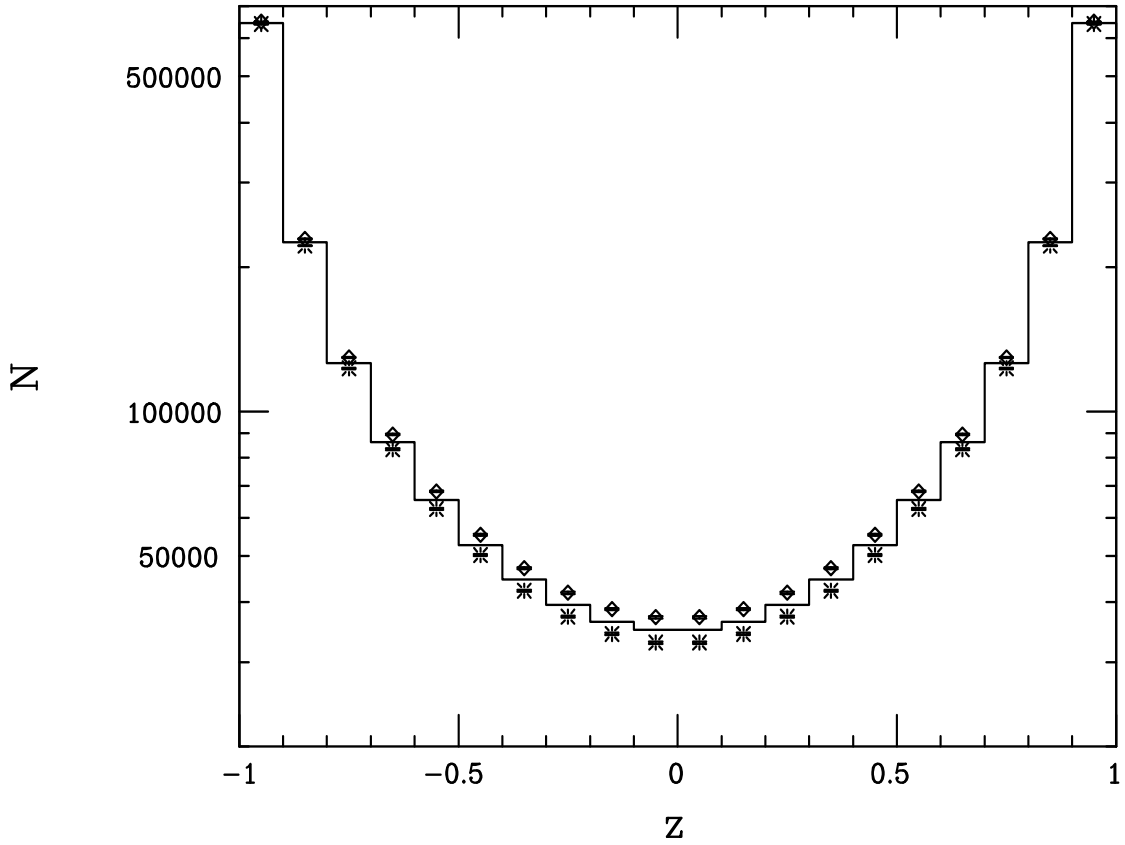

Figure 8: Angular distribution for the process $\gamma\gamma \to q\bar{q}$, with q being summed over the five light flavors of quarks at a 500 GeV e^+e^- collider with an *integrated* photon luminosity of 75 fb^{-1} assuming the cuts described in the text. The SM corresponds to the histogram while the 'data' represent the ADD scenario with M_s =1.5 TeV for $\lambda = \pm\ 1$.

To obtain the search reach there are thus two possibilities: first, one may add flavor tagging for the quarks c and b which removes the contribution from the gg final state. Second, we may drop tagging and include the gg contribution. In what follows we chose the first approach and combine the $f = c, b$ final states together with $f = e$, μ and τ and proceed as above using the efficiencies of Hewett[5]. (The corresponding analysis following the second approach will be discussed elsewhere[19].) The results of this analysis are shown in Fig.9 for $\sqrt{s_{e^+e^-}}$=500 GeV, 1 TeV and 1.5 TeV colliders. Here we see that for reasonable luminosities the search reach is $\simeq (4.5-5)\sqrt{s_{e^+e^-}}$, which is not so bad considering the minimum energy degradation of $\geq 17\%$ in going to the $\gamma\gamma$ center of mass frame. Relative to $\sqrt{s_{\gamma\gamma}^{max}}$ the search remains in the range of $\simeq 6\sqrt{s_{\gamma\gamma}}$ as found for both e^+e^- and e^-e^- collisions. Again it is quite clear that the M_s reach obtained from $\gamma\gamma$ collisions will complement those resulting from e^+e^- and e^-e^- interactions.

6 Summary and Conclusions

In this paper we have extended the phenomenological analyses of the ADD scenario presented in Ref.5 to a number of new processes involving the exchange of a Kaluza-Klein tower of gravitons at various types of colliders. The main points of our analysis are as follows:

- The collection of approximately 1 fb^{-1} of integrated luminosity at HERA balanced equally between the four intial charge and polarization states, $e^{\pm}_{L,R}$, will lead to a 95% CL bound on the values of M_s in excess of 1 TeV. This bound is comparable to that obtainable at Run II of the Tevatron employing the Drell-Yan process and that derivable by combining the results of the four LEP experiments after all data taking

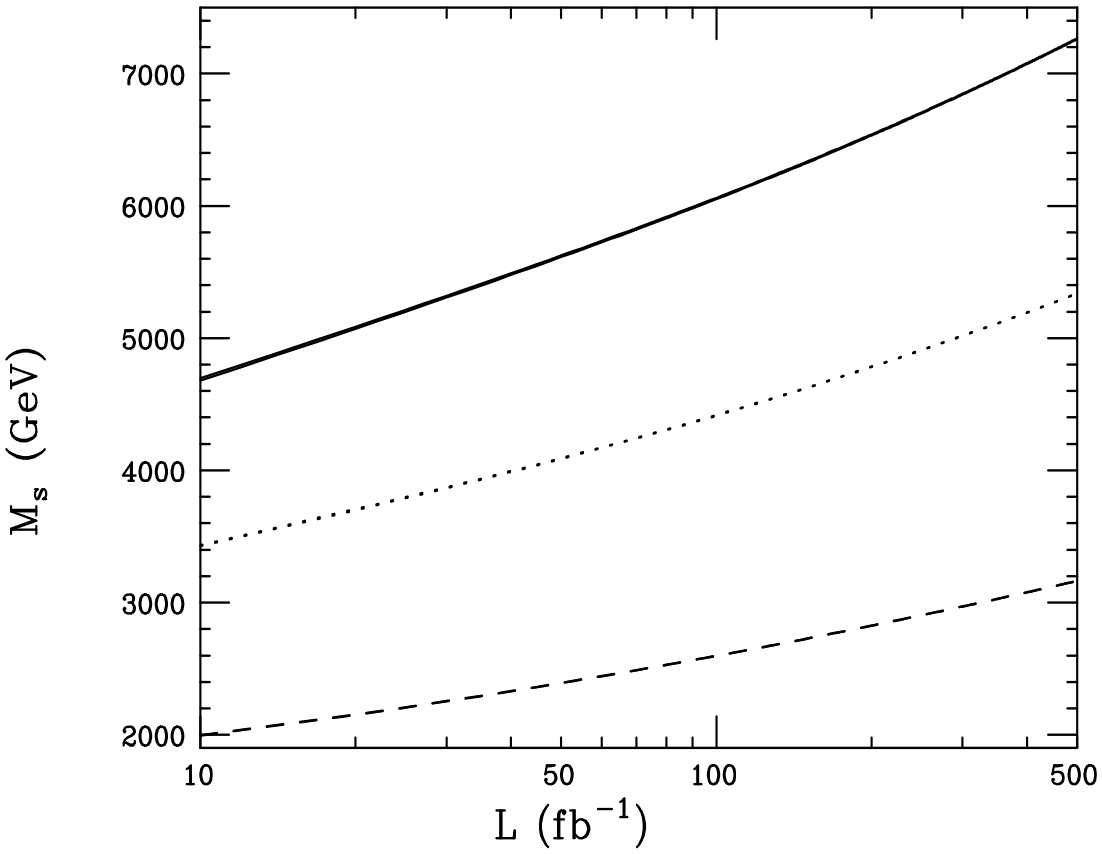

Figure 9: Search reach for the process $\gamma\gamma \to f\bar{f}$, with f being the c and b quarks together with e, μ and τ, as a function of the total $\gamma\gamma$ integrated luminosity. At a 500(1000, 1500) GeV e^+e^- collider the result is given by the dashed(dotted, solid) curve and is essentially independent of the choice $\lambda = \pm 1$. The details of the analysis are described in the text.

is completed. Clearly, the measurements at all three colliders are complementary. We estimate the current lower bound on M_s from existing HERA data using an unpolarized e^+ beam at a lower center of mass energy to be no more than $\simeq 500 - 600$ GeV.

- Low energy νN scattering data, while of high precision, are not able to significantly constraint the value of M_s although the same data is known to place respectable constraints on dimension-six operators associated with conventional contact interactions arising due to compositeness. This lack of sensitivity is directly related to the fact that the K-K tower exchange leads to dimension-eight operators which are thus suppressed by more than three orders of magnitude in comparison to contact interactions. The high precision of these measurements do not compensate in this case for the low energy at which they are made.

- Both Bhabha and Moller scattering were shown to have comparable sensitivity to the exchange of K-K towers of gravitons with search reaches of the same magnitude as those obtained by Hewett[5] for the more conventional $e^+e^- \to f\bar{f}$ process, *i.e.*, $\simeq 6\sqrt{s}$. The behavior of the search reach for these two processes with variations of integrated luminosity were, however, quite different due to the relative importance of systematic errors. This is due to the large cross sections for Bhabha and Moller scattering resulting from QED poles in the forward(and backward for Moller scattering) directions even after acceptance cuts are applied.

- The $\gamma\gamma \to f\bar{f}$ is a particularly clean channel for new physics without electroweak contributions at tree level beyond QED. In addition, there are no gauge invariant dimension-six operators arising from contact interactions in this case. As in the case of both Bhabha and Moller scattering cross sections are very large due to both t- and u-channel poles and systematic effects are important in setting limits. In comparison to $e^-e^\pm$ reactions, $\gamma\gamma$ reactions suffer in their M_s reach due to the reduced effective center

of mass energy induced by the continuous photon spectrum from the backscattered laser. However we found that by summing over all leptons as well as c, b quark flavors the search reach for M_s could be as large as $5\sqrt{s}$ which is reasonably comparable to the $e^-e^\pm$ searches and quite complementary. The use of photon beam polarization and the $t\bar{t}$ final state would almost certainly lead to an increase in this search reach. Similarly, if the untagged two jet final state were used and the process $\gamma\gamma \to gg$ included an extension of the search would be anticipated.

- Signals for an exchange of a Kaluza-Klein tower of gravitons in the ADD scenario of low energy quantum gravity appear in many complementary channels *simultaneously* at various colliders. Such signatures for new physics are rather unique and will not be easily missed.

The discovery of new dimensions may be at our doorstep and may soon make their presence known at existing and/or future colliders. Such a discovery would revolutionize the way we think of physics beyond the electroweak scale.

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Note added: After the present analysis was completed we received a paper by Mathews, Raychaudhuri and Sridhar[20] who considered the present bounds on the scale M_s from HERA data. Their resulting bound is in qualitative numerical agreement with that obtained in the discussion above.

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