Gluon Virtuality and Heavy Sea Quark Contributions to the Spin-Dependent g_1 Structure Function

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Abstract

We analyze the quark mass dependence of photon gluon fusion in polarized deep inelastic scattering for both the intrinsic and extrinsic gluon distributions of the nucleon. We calculate the effective number of flavors for each of the heavy and light quark photon gluon fusion contributions to the first moment of the spin-dependent structure function $q_1(x)$.

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1 Introduction

One of the most interesting aspects of deep inelastic lepton-proton scattering is the contribution to the g_1^p spin-dependent structure function from photon-gluon fusion subprocesses $\gamma^*(q)g(p) \to q\bar{q}$. Naively, one would expect zero contributions from light mass $q\bar{q}$ pairs to the first moment $\int_0^1 dx g_1^p(x, Q^2)$ since the q and \bar{q} have opposite helicities. In fact, this is not the case if the quark mass m_q is small compared to a scale set by the spacelike gluon virtuality p^2 . This is the origin of the so-called anomalous correction $-3\frac{\alpha_s}{2\pi}\Delta g$ [1]-[4] to the Ellis-Jaffe sum rule [5] for isospin-zero targets assuming three light flavors. Here Δg is the helicity carried by gluons in the hadron target, $\Delta g(Q) = \int_0^1 dx [g_{\uparrow}(x,Q) - g_{\downarrow}(x,Q)]$, at the factorization scale Q. In the language of the operator product expansion, the photon-gluon subprocess contributions to the first moment of $g_1(x,Q)$ correspond to the anomalous VVA triangle graph [6, 7] contribution to the hadronic matrix element of the local axial current.

For fixed gluon virtuality $P^2 = -p^2$ the photon-gluon fusion process induces two distinct contributions to the first moment of g_1 in polarized deep inelastic scattering. Let m_q denote the mass of the struck sea quark. When Q^2 is much greater than both m_q^2 and P^2 the box graph contribution to the first moment of g_1 for a gluon target is [8]:

$$\int_{0}^{1} dx g_{1}^{\gamma^{*}g} = -\frac{\alpha_{s}}{2\pi} \left[1 + \frac{2m_{q}^{2}}{P^{2}} \frac{1}{\sqrt{1 + 4m_{q}^{2}/P^{2}}} \ln \left(\frac{\sqrt{1 + 4m_{q}^{2}/P^{2}} - 1}{\sqrt{1 + 4m_{q}^{2}/P^{2}} + 1} \right) \right] .$$
(1)

The first, mass-independent term $\left(-\frac{\alpha_s}{2\pi}\right)$ in Eq. (1) comes from the region of phase space where the struck quark carries large transverse momentum squared $k_T^2 \sim Q^2$ relative to the photon-gluon direction. It measures a contact photon-gluon interaction and is associated [3] with the axial anomaly [6, 7]. The second mass-dependent term comes from the region of phase space where the struck quark carries transverse momentum $k_T^2 \sim P^2$, m_q^2 . This mass dependent term vanishes in the limit $P^2 \gg m_q^2$ and tends to $+\frac{\alpha_s}{2\pi}$ when $P^2 \ll m_q^2$. The "soft" mass dependent term in Eq. (1) is associated with the quark parton distribution of the gluon $\Delta q^{(\text{gluon})}$; it can safely be neglected for the light (up and down) quarks.

On the other hand, the magnitude of the gluon virtuality is important for gauging the contribution of the massive sea quarks. If the sea quark mass is heavy compared to the gluon virtuality $4m_q^2 \gg P^2 = -p^2$, the photon-gluon fusion contribution to $\int_0^1 dx g_1(x, Q^2)$ vanishes to leading order in $\alpha_s(Q^2)$. This result follows from a general theorem based on the Drell-Hearn-Gerasimov sum rule [9] which states that the logarithmic integral over the photoabsorption cross section

$$\int_{\nu_{\pi}}^{\infty} \frac{d\nu}{\nu} \,\Delta\sigma_{\gamma a \to bc}(\nu) = 0(\alpha^3) \;; \tag{2}$$

it vanishes at order α^2 for any $2 \to 2$ Standard Model process [10, 11]. Here $\Delta \sigma$ is the cross section difference for parallel versus anti-parallel incident helicities. In the present application, the gluon (for $p^2 = 0$) takes the role of the on-shell photon γ and the particle *a* can be taken as a real or virtual photon. As the photon virtuality Q^2 becomes large, the DHG integral evolves to the first moment of the helicitydependent structure function $g_1(x, Q^2)$. Thus the fusion $\gamma^*g \to q\bar{q}$ Born contribution to $\int_0^1 dx \, g_1(x, Q^2)$ vanishes for small gluon virtuality $P^2 \ll 4m_q^2$, $P^2 \ll Q^2$. Notice that the Born photon-gluon fusion contribution to the Ellis-Jaffe moment is zero even for very light quarks as long as the gluon virtuality can be neglected.

The above application of the DHG theorem holds for any photon virtuality $q^2 = -Q^2$, and is thus more general than leading twist [12]. In fact, the leading-order fusion contribution to the $d\nu/\nu$ moment of the difference of helicity-dependent photoabsorption cross sections vanishes even if $Q^2 < 4m_q^2$, as long as the gluon virtuality can be neglected. The result also holds for the weak as well as electromagnetic current probes [11, 13].

It is clearly important to ascertain the actual numerical contribution of heavy quarks $s\bar{s}$, $c\bar{c}$, $b\bar{b}$, $t\bar{t}$ to the first moment of g_1 ; i.e., what is the effective number of sea quark contributions to the Ellis-Jaffe moment? From the above discussion, the specific contribution of a given sea quark pair $q\bar{q}$ depends not only on Q^2 , but more critically on the ratio of scales $p^2/4m_q^2$. In a full QCD calculation of photon-gluon fusion contributions to the first moment of g_1 one needs to integrate over the distributions of extrinsic and intrinsic gluon virtualities in the target nucleon. For small gluon virtualities ($P^2 \ll m_q^2$) the "hard" anomaly contribution to the first moment of $g_1^{\gamma^*g}$ cancels with the "soft" mass dependent contribution. For deeply-virtual gluons the mass-independent anomaly contribution dominates over the mass-dependent term which tends to zero. Therefore, we shall investigate the effect of retaining the finite quark masses and performing a more exact analysis, in which we integrate over P^2 . Our aim is to understand the role of heavy quarks (e.g. strange and charm) in the photon-gluon fusion process.

As we shall show in the next section, the exact form of the spectrum $N(p^2)$ of gluon virtuality in the target nucleon depends in detail on the physics of the nucleon wavefunction. "Extrinsic" gluon contributions, which arise from gluon bremmstrahlung $q_V \rightarrow q_V g$ of a valence quark, have a relatively hard virtuality $dN_{ext}(p^2)/dp^2 \sim \alpha_s(p^2)/p^2$, above a minimum virtuality p_{\min}^2 . The mean virtuality of the extrinsic gluons depends on the upper limit of integration, which in turn depends on the kinematic phase space. On the other hand, intrinsic gluons, which are associated with the physics of the nucleon wavefunction (for example, gluons emitted by one valence quark and absorbed by another quark), have a relatively soft spectrum. We will characterize the shape of the intrinsic gluon virtuality by the convergent form $dN_{int}(p^2)/dp^2 \sim dN_{ext}(p^2)/dp^2/[1 + p^2/\mathcal{M}^2]$, where \mathcal{M} is a typical hadronic mass scale. We shall use such model forms for the extrinsic and intrinsic gluon distributions to predict specific contributions of the heavy sea quarks to the first moment of $g_1(x, Q^2)$.

In addition to the photon-gluon fusion contributions, additional contributions to the first moment of $g_1(x, Q^2)$ arise from intrinsic heavy sea quarks associated with higher Fock states in the target hadron. For example, meson-baryon fluctuations such as $p \to K\Lambda$ imply a negative intrinsic strange quark contribution to $\int_0^1 dx g_1(x, Q^2)$ [14]. In the case of charm, the small probability 0(1%) of intrinsic charm present in the proton implies a small intrinsic charm contribution to $\int_0^1 dx g_1(x, Q^2)$.

The charm contribution to the nucleon helicity-dependent structure functions and sum rules will be addressed by several new experiments. The COMPASS [15] and HERMES [16] experiments will measure charm production [17]-[25] in polarized deep inelastic scattering. Experiments have also been proposed at SLAC [26]. The aim of these experiments is to learn about the gluon polarization in a nucleon through the photon-gluon fusion process.

2 Polarized gluons and g_1

In order to analyze the sensitivity of the anomaly to the sea quark mass in the photon-gluon fusion subprocesses, let us start by expressing the contributing gluon distributions in terms of the corresponding bound-state wavefunctions. In general the Q^2 dependence of the parton distributions comes from the integral of the boundstate wavefunction over the virtuality of the corresponding parton up to the scale Q^2 . Schematically, for the polarized gluon distribution, we have

$$\Delta G(x,Q^2) = \int^{Q^2} dP^2[|\Psi_{g\uparrow/p\uparrow}(P^2,x)|^2 - |\Psi_{g\downarrow/p\uparrow}(P^2,x)|^2], \qquad (3)$$

which means that

$$\frac{\partial}{\partial P^2} \Delta G(x, P^2) = |\Psi_{g\uparrow/p\uparrow}(P^2, x)|^2 - |\Psi_{g\downarrow/p\uparrow}(P^2, x)|^2 \equiv \frac{d^2 \Delta N_{g/p}}{dP^2 dx}.$$
 (4)

Here $\Psi_{g\uparrow/p\uparrow}$ and $\Psi_{g\downarrow/p\uparrow}$ are the gluon wavefunctions for positive and negative helicities relative to the proton helicity as functions of the gluon virtuality $P^2 = -p^2$ and the fraction x of the plus component of the target nucleon's momentum.

In perturbative QCD the total photon-gluon fusion contribution to g_1 for a nucleon target is given by

$$g_1^{(G)}(x,Q^2) = \frac{1}{2} \sum_q e_q^2 g_1^{(Gq)}(x,Q^2) , \qquad (5)$$

where $g_1^{(Gq)}$ is the contribution where the struck quark carries flavor q

$$g_1^{(Gq)}(x,Q^2) = \int_{P_{\min}^2}^{Q^2} dP^2 \ \frac{\partial(\Delta G(x,P^2))}{\partial P^2} \otimes \mathcal{A}_q(x,Q^2,P^2) \ . \tag{6}$$

Here \otimes denotes the convolution over x and \mathcal{A}_q denotes the contribution to the spin structure function g_1 of a "gluon target" with virtuality P^2 , where the struck quark carries flavor q. The infra-red cut-off P_{\min}^2 is the minimum gluon virtuality at which we can apply perturbative QCD—that is, where the current-quark and gluon degrees of freedom in perturbative QCD give way to dynamical chiral symmetry breaking and confinement. The GRV [27] and Bag model [28] analyses of deep inelastic structure functions involve taking a QCD-inspired model-input for the leading twist parton distributions at some low scale μ_0^2 , evolving the distributions to deep inelastic Q^2 and comparing with data. The optimal GRV and Bag model fits to deep inelastic data are found with $\mu_0^2 \simeq 0.2 - 0.3 \text{ GeV}^2$. Motivated by this phenomenological observation, we shall set $P_{\min}^2 = 0.3 \text{GeV}^2$.

In the Born approximation \mathcal{A}_q is calculated from the box graph contribution to photon-gluon fusion. We can define the "hard" part of \mathcal{A}_q by imposing a cutoff on the transverse momentum squared of the struck quark $k_T^2 > \lambda^2$ [8]:

$$\mathcal{A}_{q}|_{\text{hard}}(x,Q^{2},P^{2},\lambda^{2}) = -\frac{\alpha_{s}}{2\pi} \frac{\sqrt{1 - \frac{4(m_{q}^{2} + \lambda^{2})}{W^{2}}}}{1 - \frac{4x^{2}P^{2}}{Q^{2}}} \left[(2x-1)(1 - \frac{2xP^{2}}{Q^{2}}) \right]$$
(7)
$$\left(1 - \frac{1}{\sqrt{1 - \frac{4(m_{q}^{2} + \lambda^{2})}{W^{2}}} \sqrt{1 - \frac{4x^{2}P^{2}}{Q^{2}}} \ln\left(\frac{1 + \sqrt{1 - \frac{4x^{2}P^{2}}{Q^{2}}} \sqrt{1 - \frac{4(m_{q}^{2} + \lambda^{2})}{W^{2}}}}{1 - \sqrt{1 - \frac{4x^{2}P^{2}}{Q^{2}}} \sqrt{1 - \frac{4(m_{q}^{2} + \lambda^{2})}{W^{2}}}}\right) + (x-1 + \frac{xP^{2}}{Q^{2}}) \frac{\left(2m_{q}^{2}(1 - \frac{4x^{2}P^{2}}{Q^{2}}) - P^{2}x(2x-1)(1 - \frac{2xP^{2}}{Q^{2}})\right)}{(m_{q}^{2} + \lambda^{2})(1 - \frac{4x^{2}P^{2}}{Q^{2}}) - P^{2}x(x-1 + \frac{xP^{2}}{Q^{2}})} \right].$$

Here m_q is the fermion mass, x is the Bjorken variable and $W^2 = Q^2(\frac{1-x}{x}) - P^2$ is the center of mass energy for the photon-gluon collision. The running coupling, α_s , in Eq. (7) is evaluated at the scale P^2 . Following Parisi and Petronzio [29] we shall use a modified running $\alpha_s(P^2)$ —see Eq. (15) below—which freezes in the infrared, to describe \mathcal{A}_q when P^2 becomes small.

Keeping contributions where the struck quark carries transverse momentum squared $k_T^2 \ge \lambda^2$, the photon-gluon fusion contribution to the first moment of $g_1^{(Gq)}$ is obtained from (6):

$$\Gamma_q(Q^2,\lambda^2) = \int_{P_{\min}^2}^{Q^2} dP^2 \ \mathcal{I}_q(P^2,\lambda^2) \ \frac{d\Delta N_{g/p}}{dP^2}(P^2) \ . \tag{8}$$

Here

$$\mathcal{I}_q(P^2,\lambda^2) = \int_0^{x_{\text{max}}} dx \ \mathcal{A}_q(x,Q^2,P^2,\lambda^2) \tag{9}$$

and

$$\frac{d\Delta N_{g/p}}{dP^2}(P^2) = \int_0^{z_{\max}(P^2)} dz \, \frac{d^2 \Delta N_{g/p}}{dP^2 dz}(z, P^2) \,. \tag{10}$$

The cutoffs x_{max} and z_{max} in Eqs. (9,10) come from the kinematics. For the box graph term \mathcal{A}_q , the cutoff $x_{\text{max}} = Q^2/(Q^2 + P^2 + 4(m_q^2 + \lambda^2))$ in Eq. (9) is obtained from the phase space factor $\sqrt{1 - \frac{4(m_q^2 + \lambda^2)}{W^2}}$ in Eq. (8). The cutoff z_{max} in Eq. (10) is derived from the explicit form of the polarized gluon distribution—see below.

In the rest of this Section we discuss the contribution of $q\bar{q}$ pairs with small transverse momentum (when we relax the λ^2 cut-off), the size of higher-twist contributions to the first moment of $g_1^{(Gq)}$, and the jet signature of the different contributions to the first moment.

When we integrate over the full range of possible impact parameters we need to include small values of k_T^2 in Eqs. (8–10). This necessarily involves extrapolating

the calculation into the domain of non-perturbative QCD. Shore and Veneziano [30] have considered the analogous process of the spin structure function of the polarized photon for a virtual photon target. They argue that the target photon virtuality where $\int_0^1 dx g_1^{\gamma}$ grows from zero (at $P^2 = 0$) to $-\frac{\alpha}{\pi}N_c$ depends on the realization of chiral symmetry breaking in QCD. In perturbative QCD the individual quark flavor contributions to $\int_0^1 dx g_1^{\gamma}(x, Q^2)$ grow rapidly from zero when $P^2 \sim 4m_q^2$. In full QCD, spontaneous chiral symmetry breaking means that the scale of the transition virtuality is set by the constituent quark mass rather than by the current quark mass — that is, we expect $\int_0^1 dx g_1^{\gamma}(x, Q^2)$ to grow rapidly from zero when $P^2 \sim m_{\rho}^2$. Motivated by this result, one might expect the gluon-virtuality where \mathcal{I}_q grows rapidly to depend on any possible diquark structure of the gluon at low k_T^2 . Spontaneous chiral symmetry breaking is a considerably more dramatic effect in the light quark masses than it is in the heavy quark masses. Thus we can expect that perturbative QCD will provide a reasonable model-independent estimate of the heavy-quark \mathcal{A}_q when λ^2 becomes small.

When $Q^2 \to \infty$ the expression for \mathcal{A}_q simplifies to the leading twist (=2) contribution:

$$\mathcal{A}_{q}(x,Q^{2},P^{2},\lambda^{2}) = \frac{\alpha_{s}}{2\pi} \left[(2x-1) \left(\ln \frac{Q^{2}}{\lambda^{2}} + \ln \frac{1-x}{x} - 1 \right) + (2x-1) \ln \frac{\lambda^{2}}{x(1-x)P^{2} + (m_{q}^{2}+\lambda^{2})} + (1-x) \frac{2m_{q}^{2} - P^{2}x(2x-1)}{x(1-x)P^{2} + m_{q}^{2} + \lambda^{2}} \right]$$
(11)

which has the first moment

$$\mathcal{I}_{q}(P^{2},\lambda^{2}) = -\frac{\alpha_{s}}{2\pi} \left[1 + \frac{2m_{q}^{2}}{P^{2}} \frac{1}{\sqrt{1 + 4(m_{q}^{2} + \lambda^{2})/P^{2}}} \ln \left(\frac{\sqrt{1 + 4(m_{q}^{2} + \lambda^{2})/P^{2}} - 1}{\sqrt{1 + 4(m_{q}^{2} + \lambda^{2})/P^{2}} + 1} \right) \right].$$
(12)

For finite quark masses, the cutoff x_{max} protects \mathcal{A}_q from reaching the $\ln(1-x)$ singularity in Eq. (8). To quantify this effect, in Table 1 we list the values of \mathcal{I}_q for different values of P^2 and λ^2 . The " $Q^2 = \infty$ " values are obtained by keeping only the leading twist contribution, Eq. (13). For the "hard" cut-off $\lambda^2 = 1 \text{ GeV}^2$ the cut-off itself acts as a major source of higher-twist. When we relax the cut-off by setting λ^2 to zero we find a large $\simeq 63\%$ higher twist suppression of \mathcal{I}_c at $Q^2 = 10$

P^2	λ^2	Q^2	light	$\operatorname{strange}$	charm
0.5	1.0	10.0	0.77	0.74	0.16
0.5	1.0	100.0	0.98	0.94	0.30
0.5	1.0	∞	1.00	0.96	0.32
0.5	0.0	10.0	0.99	0.61	0.013
0.5	0.0	100.0	1.00	0.63	0.033
0.5	0.0	∞	1.00	0.63	0.035

Table 1: Heavy quark effects in \mathcal{I}_q (in units of $-\frac{\alpha_s(P^2)}{2\pi}$)

GeV². Steffens and Thomas [24] have observed that \mathcal{I}_c is, to good approximation, independent of P^2 for values of P^2 between zero and 1 GeV². The rise in \mathcal{I}_c with increasing λ^2 corresponds to removing a greater amount of the mass dependent term in Eq. (1) which cancels against the mass-independent (anomaly) term which is associated with $k_T^2 \sim Q^2$ in the limit $Q^2 \to \infty$. When we increase the cut-off λ^2 on the transverse momentum squared of the struck quark we increase the infrared cut-off on the invariant mass of the $q\bar{q}$ pairs produced by the photon-gluon fusion. Quark mass dependent contributions to \mathcal{I}_q go to zero when we increase the invariant mass $\mathcal{M}_{q\bar{q}}$ much greater than $4m_q^2$. Note that the light-quark \mathcal{I}_l in Table 1 is significantly suppressed below unity with the $\lambda^2 = 1$ GeV² cut-off at $Q^2 = 10$ GeV². If we decrease the cut-off this \mathcal{I}_l grows to 0.87 ($\lambda^2 = 0.5$ GeV²), 0.91 ($\lambda^2 = 0.3$ GeV²) and 0.96 ($\lambda^2 = 0.1$ GeV²) when we use our modified α_s (—see Eq. (15) below) together with the current light-quark mass through-out.

Consider the large Q^2 limit $(Q^2 \gg 4m_q^2)$. When we set $\lambda^2 = 0$ in Eq. (12) to obtain Eq. (1), the "hard" anomaly contribution to \mathcal{I}_q (the first moment of $g_1^{\gamma^* g}$) cancels with the "soft" mass dependent contribution for small gluon virtualities $(P^2 \ll m_q^2)$. For deeply virtual gluons the mass-independent anomaly contribution dominates over the mass-dependent term which tends to zero in the limit $P^2 \gg 4m_q^2$. It is interesting to observe that in a semi-inclusive experiment one can in principle identify events which correspond specifically to the contributions to the first moment of $g_1(x)$. These are events with three jets recoiling, taking up the large momentum q_T transferred by the lepton. The final state consists of the $q\bar{q}$ pair with perpendicular momentum $\vec{q}_T - \vec{p}_T$ plus the quark that emitted the gluon, with mass M and transverse momentum \vec{p}_T . These events have gluon virtuality $-p^2 \ge m_q^2$. As shown in [31], the events where only the $q\bar{q}$ pair recoils produce no contribution to the first moment of $g_1(x)$. This corresponds to events with small transverse momentum of the quark that emitted the gluon $p_T^2 \ll m_q^2$, or with gluon virtuality $-p^2 \ll m_q^2$. The vanishing of the first moment of $g_1(x)$ for heavy quark production implies that there must be a polarization asymmetry zero, which in principle can be measured experimentally [31].

3 Extrinsic and intrinsic glue

Having established the theoretical framework, we now investigate photon-gluon fusion using simple models for the exclusive and inclusive gluon distributions of the nucleon.

The "extrinsic" glue consists of gluons which are radiatively generated from individual valence quarks in the target whereas the "intrinsic" glue is associated with gluon exchange between valence quarks. For example, consider a gluon which is exchanged between two valence quarks in the proton. In constituent quark models these "gluon exchange currents" contribute to the proton-Delta mass difference [32]. They also renormalize the valence contributions to the nucleon's axial charges, which are measured in β -decays and in the first moment of $g_1(x, Q^2)$. When cut, the exchanged gluon gives an intrinsic gluon. A gluon which contributes to the quark self energy when cut gives an extrinsic gluon.

We now estimate the size of the extrinsic and intrinsic gluon contributions to the first moment of g_1 . We calculate the ratio $n_{\text{eff}} = \Gamma_h / \Gamma_l$ of the heavy to light quark contributions to photon gluon fusion for both the extrinsic and intrinsic glue.

The extrinsic and intrinsic gluon distributions are dominated by gluons with small virtuality. The virtuality distribution $\frac{dN_{g/p}}{dP^2}$ for the extrinsic glue contains a logarithmic tale extending to the kinematic limits. Phenomenologically [33], the momentum distribution of the intrinsic glue is found to be weighted by the factor

$$\mathcal{W}(P^2) = N_g \left(\frac{1}{1 + \frac{P^2}{\mathcal{M}^2}}\right) \tag{13}$$

relative to the extrinsic glue. The mass parameter \mathcal{M}^2 can be estimated as 0.71 GeV², the mass scale of the dipole fit to the proton form factor [33]; N_g is a model dependent normalization constant.

We start with a simple model for the gluon distributions taking into account their correct P^2 distribution. Model dependent normalization uncertainties cancel in the ratios Γ_h/Γ_l . We treat the nucleon target as a three-quark system where the target quarks are treated as "elementary" with constituent quark mass M equal to 300 MeV. The polarized extrinsic gluon distribution is given by

$$\frac{d^2 \Delta N_{g/p}}{dP^2 dz}(z, P^2) = \mathcal{N}C_F \frac{\alpha_s}{2\pi} \left(\frac{1}{z}\right) \frac{(1 - (1 - z)^2)p_T^2 + M^2 z^4}{(p_T^2 + M^2 z^2)^2} (1 - z)$$
(14)

where the (1-z) factor is a Jacobian factor for the change of variables from p_T^2 to P^2 in Eq. (8). The QCD factor $C_F = \frac{4}{3}$; $\mathcal{N} \simeq 0.6$ [34, 35, 36] is the spin depolarization factor found in relativistic quark models, which parametrizes the transfer of the proton's angular momentum from intrinsic spin of the quarks to orbital angular momentum through relativistic effects and quark-pion coupling. Since the gluon transverse momentum squared $p_T^2 = P^2(1-z) - M^2 z^2$ is non-negative, we obtain the z_{max} cutoff in Eq. (10): $z_{\text{max}}(P^2) = (-1 + \sqrt{1+4C})/2C$, where $C = M^2/P^2$.

Our simple model, Eq. (14), for the gluon distributions exhibits the $x \to 0$ behavior predicted by color coherence [37]. By construction, it also exhibits the large x behavior associated with an elementary quark target. In a more sophisticated model one should also include gluon exchange between the valence quarks in addition to the gluon involved in the $\gamma^*g \to q\bar{q}$ process. However, this is beyond the scope of the present paper.

We work in the analytically extended [38] α_V scheme [39]. This means that we use the running coupling

$$\alpha_s(P^2) = \frac{4\pi}{\beta_0 \ln(\frac{P^2 + 4m_g^2}{\Lambda_V^2})},$$
(15)

in Eqs. (7,14). Here $m_g^2 = 0.2 \text{GeV}^2$, $\Lambda_V = 0.16 \text{GeV}$ and the number of flavors which contribute to β_0 is taken as a continuous variable which depends on P^2 [38]: $\beta_0 = (11 - \frac{2}{3} \sum_{i=1}^{4} N_i)$ where

$$N_i \simeq \left(1 + \frac{5}{\rho_i}\right)^{-1}, \qquad (\rho_i = P^2/m_i^2) .$$
 (16)



Figure 1: The effective number of flavors $n_{\rm eff}$ for heavy sea quarks $s\bar{s}, c\bar{c}$, and $b\bar{b}$ contributing to the first moment of $g_1(x, Q^2)$, arising from γ^* -(extrinsic gluon) fusion, as a function of momentum transfers $Q^2 < 100 \,{\rm GeV^2}$. In Fig. 1a, the cutoff on quark transverse momentum $k_T^2 > \lambda^2$ is set equal to zero. In Figs. 1b and 1c, $\lambda^2 = 1 \,{\rm GeV^2}$ and $\lambda^2 = 10 \,{\rm GeV^2}$, respectively.

Q^2	λ^2	$n_{\rm s}$	$n_{ m c}$	$n_{ m b}$
Extrinsic glue				
10.0	0.0	0.78	0.12	0.007
10.0	1.0	0.97	0.35	0.037
10.0	10.0	1.00	0.78	0.23
100.0	0.0	0.82	0.21	0.023
100.0	1.0	0.97	0.41	0.052
100.0	10.0	1.00	0.79	0.24
Intrinsic glue				
10.0	0.0	0.71	0.07	0.004
10.0	1.0	0.97	0.33	0.034
10.0	10.0	1.00	0.78	0.23
100.0	0.0	0.71	0.08	0.005
100.0	1.0	0.97	0.33	0.035
100.0	10.0	1.00	0.78	0.23

Table 2: The effective number of heavy-flavors $n_{\rm eff} = \Gamma_h / \Gamma_l$.

In Figs. 1a–1c we show the effective number of flavors, $n_{\text{eff}} = \Gamma_h/\Gamma_l$, for the heavy flavor $s\bar{s}$, $c\bar{c}$ and $b\bar{b}$ production contributions to the first moment of $g_1(x, Q^2)$ from γ^* -(extrinsic gluon) fusion up to $Q^2 = 100 \text{GeV}^2$. Figure 1a is obtained by setting λ^2 equal to zero. (In this calculation we have used "modified" α_s , Eq. (15), together with the current light-quark mass through-out.) In Figs. 1b and 1c we set the cut-off λ^2 equal to 1 GeV² and 10 GeV² respectively. We repeat these calculations for γ^* -(intrinsic gluon) fusion in Figs.2a–2c. The results in Figs. 1 and 2 are summarized in Table 2.

The effective number of flavors $n_{\text{eff}} = \Gamma_h / \Gamma_l$ increases for the heavy quarks when we increase the cutoff λ^2 on the transverse momentum squared of the struck quark.



Figure 2: The effective number of flavors n_{eff} for heavy sea quarks $s\bar{s}, c\bar{c}$, and $b\bar{b}$ contributing to the first moment of $g_1(x, Q^2)$, arising from γ^* -(intrinsic gluon) fusion. In Fig. 2a, the cutoff on quark tranverse momentum $k_T^2 > \lambda^2$ is set equal to zero. In Figs. 2b and 2c, $\lambda^2 = 1$ GeV² and $\lambda^2 = 10$ GeV², respectively.



Figure 3: The integrand in Eq. (8) as a function of P^2 for the light and charm quark contributions at $Q^2 = 100 \text{ GeV}^2$ for extrinsic glue. Figure 3b shows the corresponding integrand for intrinsic glue.

This corresponds to removing a greater amount of the mass dependent term in Eq. (1) which cancels against the mass-independent (anomaly) term from $k_T^2 \sim Q^2$ in the limit $Q^2 \to \infty$. By increasing λ^2 we are increasing the cut on the invariant mass of the $q\bar{q}$ pairs produced by the photon-gluon fusion. Quark-mass dependent terms become less important when the invariant mass $\mathcal{M}_{q\bar{q}}$ becomes much greater than $4m_q^2$.

In Fig. 3a we show the integrand in Eq. (8) as a function of P^2 for the light and charm quark contributions at $Q^2 = 100 \text{ GeV}^2$ with the extrinsic glue. Figure 3b shows the corresponding integrand for the intrinsic glue. Figures 3a and 3b both involve $\lambda^2 = 0$. Note that the heavy and light quark curves come closer together with increasing P^2 . This result corresponds to the fact that the mass-dependent term in Eq. (1) tends to zero in the limit $P^2 \gg 4m_q^2$.

4 Phenomenology and discussion

Gluon polarization offers a possible explanation for the small value of $g_A^{(0)}$ (the three-flavor, singlet axial charge) extracted from polarized deep inelastic scattering [40, 41, 42, 43]:

$$g_A^{(0)} \simeq 0.2 - 0.35.$$
 (17)

Relativistic binding [36] and constituent-quark pion coupling [34, 35] models predict $g_A^{(0)} \simeq 0.6$ — a factor of two larger than the measured $g_A^{(0)}$. In these semi-classical models $g_A^{(0)}$ is interpreted as the fraction of the nucleon's helicity which is carried by its quark constituents. In QCD the axial anomaly [6, 7] induces various gluonic contributions to $g_A^{(0)}$. One finds [1, 2, 3, 44]

$$g_A^{(0)} = \left(\sum_q \Delta q - 3\frac{\alpha_s}{2\pi}\Delta g\right)_{\text{partons}} + \mathcal{C} .$$
 (18)

Here $\frac{1}{2}\Delta q$ and Δg are the amount of spin carried by quark and gluon partons in the polarized proton. The $-3\frac{\alpha_s}{2\pi}\Delta g_{\text{partons}}$ term is associated with the mass-independent, local $\gamma^* g$ interaction in Eq. (1) assuming three light flavors. The soft mass-dependent contributions to photon-gluon fusion are included in $\Delta q_{\text{partons}}$. The last term, C, is associated with non-trivial gluon topology [44] and a possible $\delta(x)$ term in g_1 . It is missed by polarized deep inelastic scattering experiments which measure the combination $(g_A^{(0)} - C)$.

How large are the photon-gluon fusion sea-quark contributions Γ_q to $g_A^{(0)}$ if we allow for finite sea-quark masses and a spectrum of gluon virtuality?

Since extrinsic glue is radiatively generated from single quark lines in the target, we believe that our model should provide a good order-of-magnitude estimate for the normalization of the extrinsic Γ_q . We find $\Gamma_c^{\text{ext}} = -0.0024$ and $(\Gamma_u + \Gamma_d + \Gamma_s)^{\text{ext}} = -0.033$ at $Q^2 = 100 \text{ GeV}^2$ for the extrinsic glue.

The magnitude of the intrinsic gluon contribution to $g_A^{(0)}$ depends on the normalization of the gluon polarization. Taking the estimate $\Delta g_{\text{intrinsic}} = +0.5$ at 1 GeV² [37], we obtain $\Gamma_c^{\text{int}} = -0.0020$ and $(\Gamma_u + \Gamma_d + \Gamma_s)^{\text{int}} = -0.07$ when $Q^2 = 100 \text{ GeV}^2$.

It is interesting to compare these estimates of Γ_c with the results of heavy-quark effective theory [21] and heavy-quark operator product expansion [45] calculations. These calculations express the total heavy-quark contribution to the first moment of g_1 in terms of the three-light-flavor singlet axial-charge $g_A^{(0)}$. Using the most recent value, Eq. (17), of $g_A^{(0)}$, Manohar's effective theory calculation of the heavy-charmquark axial-charge becomes

$$g_A^{(\text{charm})}(Q^2 >> m_c^2) = -0.0055 \pm 0.0018 + O(1/m_c)$$
 (19)

Manohar gives an estimate $\simeq 0.003$ (magnitude) for the $O(1/m_c)$ corrections.

The sum of our extrinsic and intrinsic charm contributions ($\Gamma_c^{\text{ext}} + \Gamma_c^{\text{int}} = -0.0044$) is in good agreement with Eq. (19). However, with the same gluonic input, photongluon fusion can account for only about one-third of the difference between the value of $g_A^{(0)}$ extracted from polarized deep inelastic scattering and the quark model prediction. Next-to-leading order QCD fits to the present world data for g_1 are consistent with a value of Δg between zero and +2 at 1 GeV² [46]. The value $\Delta g = 2$ would increase our estimate of the intrinsic gluon contribution by a factor of 4 and would bring theory into agreement with the empirical determinations of $g_A^{(0)}$. We look forward to a more precise measurement of Δg from forthcoming experiments on open charm production.

Finally, it is interesting to note that since the contributions due to heavy sea quarks come from highly virtual gluons, one expects minimal nuclear shadowing for their contribution to the first moment of g_1^N .

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