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Calculation of Impedance from Multibunch Synchronous Phases: Theory and Experimental Results*

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A novel beam-based method for measuring the longitudinal impedance spectrum is demonstrated using experimental data from the PEP-II High Energy Ring (HER). The method uses a digital longitudinal feedback system from which the charge and synchronous phase are measured for every bucket. Calculation of the transfer function from fill shape to synchronous phase yields the impedance seen by the beam at revolution harmonics. The experimentally-derived longitudinal impedance function and lab measurements of the impedance of parked RF cavities are compared to suggest a mechanism for the occasional instability of low-order coupled bunch modes observed in the HER during commissioning in October 1997.

1 INTRODUCTION

Single-bunch synchronous phase measurements have traditionally been used to measure integrated longitudinal beam impedance. Such measurements do not reveal much information about the frequency distribution of the impedance.

In this paper we present a novel multi-bunch method for measuring the longitudinal impedance spectrum[1]. The method involves calculation of the transfer function from fill shape (bunch current versus bunch number) to synchronous phase of a multibunch beam, which is shown to yield the longitudinal beam impedance at revolution harmonics. Derivation of the necessary equations is followed by quantitative results from measurements made during commissioning of the PEP-II High Energy Ring (HER). The HER synchronous phase measurements explain the occasional instability of low order coupled bunch modes observed at unexpectedly low total currents. Bunch currents and synchronous phases are calculated from data taken using the DSP-based PEP-II longitudinal feedback system[2, 3].

Synchronous phase transients need to be matched in the PEP-II HER and LER (Low Energy Ring) to achieve high luminosity. Matching of transients could be complicated by incomplete knowledge of the longitudinal impedance, or by additional gaps in the bunch train to combat the fast-ion instability. Multibunch synchronous phase measurements could therefore be useful during further commissioning. Synchronous phase transients are also useful as an indicator of Landau damping from bunch to bunch tune variation.

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2 TRANSFER FUNCTION

An electron bunch adjusts its synchronous phase ϕ_s so that the average kick it receives from the rf voltage cancels the average energy loss over a turn due to wakefields (V^{wk}) and synchrotron radiation (Uo):

$$V_c sin(\phi_s) = Uo + V^{wk},$$

where V_c is the peak rf cavity voltage. The synchronous phase ϕ_s^o in the absence of wakefields is given by:

$$\phi_s^o = sin^{-1}(Uo/V_c)$$

If ϕ_s^o is not very different from ϕ_s, i.e., if V^{wk} is small compared to V_c, we can write:

$$\phi = \phi_s - \phi_s^o \cong \frac{V^{wk}}{V_c cos(\phi_s^o)}$$

$$\Rightarrow \phi \cong \frac{-V^{wk}}{|V_c cos(\phi_s^o)|},$$

since the PEP-II beams are above transition. If a beam is filled with N bunches at an even spacing, we have:

$$\phi_k \cong \frac{-V_k^{wk}}{|V_c cos(\phi_s^o)|}; \qquad k = 0, 1, ...N\text{–}1 \tag{1}$$

Since the bunch currents i_k and synchronous phase offsets ϕ_k are periodic discrete-time signals, it is useful to define their discrete-time Fourier transforms:

$$\mathbf{I}_n = \sum_{k=0}^{N-1} i_k \mathrm{e}^{-j2\pi kn/N},$$

$$\mathbf{\Phi}_n = \sum_{k=0}^{N-1} \phi_k e^{-j2\pi kn/N}; \qquad n = 0, 1, ...N\text{–}1$$

It can be shown[1] that these equations lead to the following relation between $\mathbf{\Phi}$ and $\mathbf{I}$:

$$\frac{\mathbf{\Phi}_n}{\mathbf{I}_n} = \frac{-NZ_n}{|V_c cos(\phi_s^o)|} = \frac{-N}{|V_c cos(\phi_s^o)|} \sum_{m=-\infty}^{\infty} Z[j(mN+n)\omega_o], \tag{2}$$

where $Z(j\omega)$ is the longitudinal impedance and ω_o is the revolution frequency in rad/s. The above equation contains the assumption that the bunches are vanishingly small in length. To take the finiteness of the bunch length into account we must replace $Z(j\omega)$ with $F(\omega)Z(j\omega)$, where the form factor $F(\omega)$ is the squared magnitude of the Fourier

transform of the normalized bunch line density. Of course, this need not be done if the bunch length is much smaller than the wavelength of the relevant resonances in $Z(j\omega)$.

If the bunch currents i_k and synchronous phase offsets ϕ_k are known, the aliased longitudinal impedance Z_n can be calculated using the above equation.

3 MEASUREMENTS FROM PEP-II HER COMMISSIONING

Bunch signals recorded by the feedback system are proportional to the product of charge and longitudinal phase error[4]. Since line harmonics from the klystron impose the same low frequency motion on all bunches, bunch currents can be measured by calculating the signal amplitudes at selected noise-driven frequencies for each bunch. These amplitudes are proportional to the individual bunch currents - the total current is normalized to the DCCT reading. This method of bunch current monitoring is feasible at most machines, since power supply ripple is ubiquitous.

Figure 1 shows a typical averaged low frequency bunch signal spectrum. In this case we calculate bunch currents by projecting individual bunch signals onto the 720 Hz line in the averaged spectrum and then scaling the result to agree with the total current reading of the DCCT.

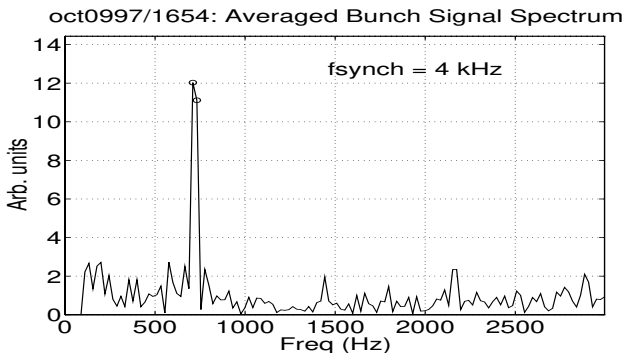

Figure 1: 720 Hz line from klystron is demodulated to extract bunch currents from lfb system data.

Synchronous phases of individual bunches are calculated by averaging their digitized current-phase products and dividing the averages by the bunch currents.

3.1 Time Domain

In the time domain we would like to observe the response of the synchronous phase to impulses in the fill shape. Figure 2(a) shows a 96mA 291-bucket fill with a discontinuity that is impulsive at low frequencies. The resultant synchronous phase ringing is shown in Figure 2(b). We can see from the figure that the "impulse response" goes through about three oscillations and dies out in one revolution period. This indicates that $Z(j\omega)$ has a strong resonance three revolution harmonics away from some multiple of the bunch frequency, which is a twelfth of the rf rfequency in this case.

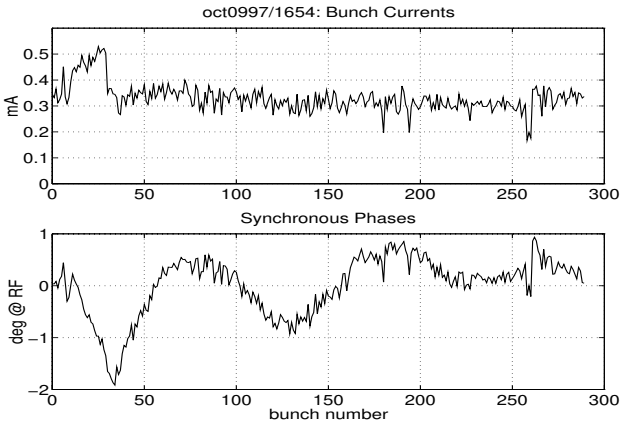

Figure 2: (a) Bunch current measurement, 291-bunch fill, 96mA total current. (b) Synchronous phase variation around the ring (three oscillations in one revolution period).

3.2 Impedance Extraction

As shown in Equation 2, the scaled transfer function from i_k to ϕ_k is the aliased longitudinal impedance Z_n. Of course, we should only calculate Z_n at revolution harmonics n that have a reasonably good signal to noise ratio (SNR). By looking at the shapes of the fill and the synchronous phase transient in Figure 2, we can tell that an estimate of Z_n made from this piece of data would be most reliable for small values of n.

It must be pointed out here that the absolute DC synchronous phase is not known, since it is canceled in the feedback front end by a DC offset designed to prevent the phase signal from saturating the digitizer. This precludes the calculation of Z_0 from the data presently available. Z_0 can be measured by keeping the offset fixed at a nominal value and varying the total beam current.

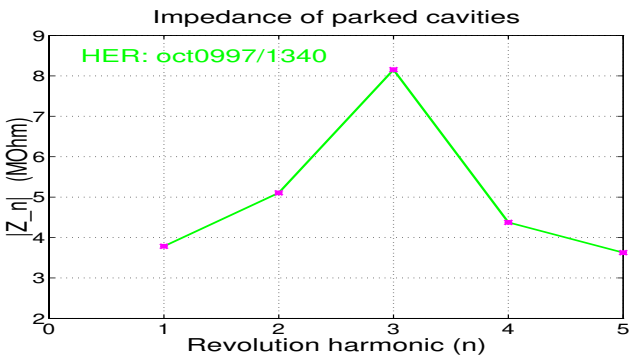

Figure 3: Estimate of Z_n at first few revolution harmonics, extracted from bunch currents and synchronous phases. Large value at n = 3 is due to the fundamental resonances of parked cavities.

Transfer functions have been calculated from 15 different sets of data. The resulting impedance estimates are consistent to within 20% of each other. Figure 3 shows the estimate of Z_n obtained by averaging transfer functions

from four consecutive data sets with similar fill shapes. The aliased impedance is calculated only for the first five values of n, since the SNR is low for $n > 5$. As expected from Figure 2, there is a strong resonance in Z_n at $n = 3$, with $Z_3 = 8.2M\Omega$. The HER has 20 installed rf cavities, each with a loaded shunt impedance of $761k\Omega$ [5]. The experiment was performed with eight active cavities, tuned about 5kHz away from the rf frequency. This detuning is small compared to the revolution frequency, which is 136.3kHz. Six idle cavities were nominally parked exactly halfway between the second and third revolution harmonics above f_{rf}, while the other six were thought to occupy a symmetric location below f_{rf}. If, however, they were all parked exactly three revolution harmonics away from f_{rf}, their impedances would add up to $9.2M\Omega$ at $n = 3$. The asymmetry between Z_2 and Z_3 in Figure 3, together with the fact that $Z_3 = 8.2M\Omega$, indicates that the 12 idle cavities were indeed parked closer to the third revolution harmonic than the second.

3.3 Coupled Bunch Instability

Ideally, idle cavities should be parked symmetrically around f_{rf} so that they do not drive coupled bunch instabilities. The impedance estimates shown in Figure 3 suggest that they were not parked accurately. This conclusion is supported by the fact that coupled bunch modes 2 and 3 were sometimes seen to be unstable. It was also seen that the low mode instabilities disappeared when the idle cavity tuners were shifted from their nominal positions[6].

Figure 4 shows the beam pseudospectrum[7] (beam spectrum without revolution harmonics, calculated from digitized data) for a 291-bunch 84mA fill, taken a few days before the data displayed in the previous figures. The pseudospectrum shows that mode 3 is unstable, with a steady state amplitude of 2° at the rf frequency.

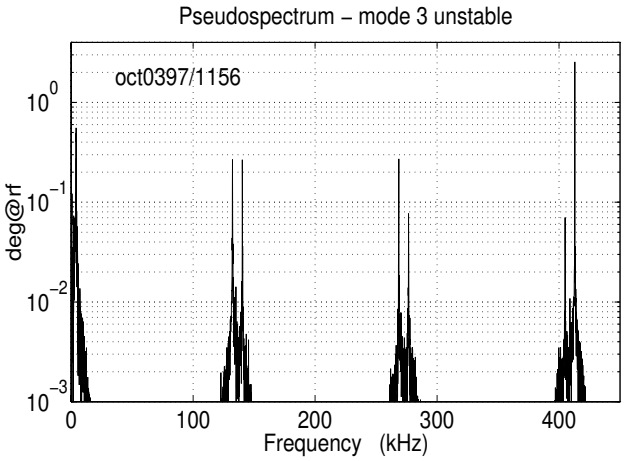

Figure 4: Beam pseudospectrum of 84 mA 291-bunch beam, showing an unstable upper sideband at the third revolution harmonic (410 kHz).

4 SUMMARY

We have demonstrated the use of a novel beam-based technique to measure the longitudinal impedance spectrum at the PEP-II HER. The technique involves calculation of the transfer function from fill shape to multibunch synchronous phase. Bunch currents and synchronous phases have been extracted from data taken using the longitudinal feedback system during PEP-II HER commissioning.

Impulsive discontinuities in the fill have been seen to cause the multi-bunch synchronous phase to ring at thrice the revolution frequency. The calculated impedance agrees well with that of the parked cavities, if we assume that they were tuned closer to $f_{rf} + 3f_o$ than to $f_{rf} + 2f_o$.

Our ability to measure the longitudinal impedance has been limited mainly by noise in the bunch current measurements. Cleaner current monitoring by injecting low frequency signals into the feedback front end has already been demonstrated, and should improve the situation significantly. Specific regions of the impedance spectrum can be explored by adjusting the fill shape to excite the targeted revolution harmonics. For example, we could investigate the impedance around $100f_o$ with a 582 bucket fill by creating a periodicity of approximately 5.82 buckets in the fill. We could get a good measurement by squirting a little extra charge into every sixth bucket (every 18th bucket at 238MHz).

Multibunch synchronous phase measurements take on added importance at PEP-II because of the need to match gap transients in the two rings.

5 ACKNOWLEDGEMENTS

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