complex would occupy 100 km. These numbers could be relaxed, with non-interleaved chromatic correction, or harmonic rf energy spread compensation, or with a breakthrough in the understanding of breakdown. Regardless, the disruption parameter is sufficiently large that the beams would be violently unstable in collision.

One way out of this dilemma is to relax the constraint on Y, and accept and deal with the copious pair production that would result. In this case, one could contemplate a machine at wavelengths of cm, gradients as high as 200 MeV/m, and a linac 25 km long. There is a second alternative, and this is the subject of the present work.

2 NEUTRAL BEAM COLLISIONS

We consider luminosity production by collision of two neutral beams, each consisting of two co-propagating e^+ and e^- beams. We put aside the issue of initial state tagging. Parameters we have in mind are $\sigma_x=\sigma_y \equiv \sigma_r \sim 2$ nm, $\beta_y \sim 220$ µm, $\varepsilon_{nx} = \varepsilon_{ny} \sim 1\text{x}10^{-7}$ m-rad, $f \sim 550$Hz, $eN_b \sim 2$nC. Corresponding two-beam parameters are $\delta \sim 1$, Y ~ $3\text{x}10^4$, $\eta \sim 1\%$. As a check of consistency, we take note of the Oide limit [7], arising from synchrotron radiation in the final focus,

$$\frac{\sigma_y}{r_e} \approx 3.6\left(\frac{\varepsilon_{ny}}{r_e}\right)^{5/7} F^{1/7}\ , \quad \frac{\beta_y}{r_e} \approx 9.3\gamma\left(\frac{\varepsilon_{ny}}{r_e}\right)^{3/7} F^{2/7}\ ,$$

where the function F depends on the optics, and is of order unity. The bunch length is subject to $\sigma_z \leq \beta_y$, and the uncompensated disruption parameter $D_y \sim 1.4\sigma_z$ (µm).

We consider next, what collective limits arise in this system. As noted by Balakin and Solyak[8] and Rosenzweig *et al* [9] neutralized beams in collision suffer from a charge separation instability. A minute deviation from neutrality is amplified as the like-charge beams repel each other. This effect needs to be evaluated quantitatively, and a strong-strong, multi-particle simulation has been written for that purpose. The following section describes that simulation and the work that has been done to date to verify it.

3 INSTABILITY OF NEUTRAL BEAMS

We employ a particle-in-cell simulation based on a three dimensional grid with charge allocation by area weighting in the transverse plane, and solution of Poisson's equation in the transverse plane. The transverse algorithm is described in references [10, 11], and is best suited for approximately round beams which is the case of interest. The modification of previous work for this paper is that the simulation has been made strong-strong by splitting the beams into a number of slices longitudinally. For the results reported in this paper the azimuthal bin size was π/8, the radial bin size was $0.01\sigma_r$ and there were 16 longitudinal slices, with 10^5 particles/slice/beam.

The charge separation instability can be solved analytically (see next paragraphs) for uniform charge density beams with the assumption that the beam radius is constant through the collision, and the simulation has been tested by comparing results with this analytical solution.

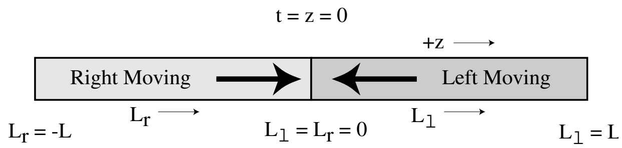

Figure 1: Configuration and coordinates for the calculation. L_r and L_l are right-handed coordinates that move with the beams. The figure is drawn at the start of the interaction.

Let R_r and R_l denote the transverse separations of the beams moving to the right and left, respectively. The equations of motion, derived from Gauss' Law for uniform beams, are

$$\frac{d^2\vec{R}_l}{dt^2} = \frac{8c^2D}{L^2}\vec{R}_r;\quad \frac{d^2\vec{R}_r}{dt^2} = \frac{8c^2D}{L^2}\vec{R}_l$$

where L is the full length of the beams, D is the disruption given by

$$D = \frac{r_e N_b L}{\gamma\sigma_r^2}\ ,$$

and σ_r is the beam radius. Let L_r and L_l denote coordinates that move with the bunch as shown in Fig. 1. The equations of motion can be rewritten in terms of L_r and L_l as

$$\frac{\partial^2}{\partial L_r^2}\vec{R}_l(L_l, L_r) = \frac{2D}{L^2}\vec{R}_r(L_l, L_r)\ ,$$

$$\frac{\partial^2}{\partial L_l^2}\vec{R}_r(L_l, L_r) = \frac{2D}{L^2}\vec{R}_l(L_l, L_r)\ .$$

These equations can be solved by Laplace transformation. For example, the equation for the Laplace transform of R_r with respect to L_r is

$$\tilde{r}_r(L_l, p) = \tilde{r}_{r0}\cosh\left(\frac{2DL_l}{L^2p}\right) + \frac{L^2p}{2D}\tilde{r}'_{r0}\sinh\left(\frac{2DL_l}{L^2p}\right)$$
$$+\int_0^{L_l} dx_l \sinh\left(\frac{2D(L_l - x_l)}{L^2p}\right)\left\{\vec{R}_{l0} + \frac{1}{p}\vec{R}'_{l0}\right\}\ ,$$

where p is the Laplace transform variable and the subscripts "0" denote initial values.

For the specific case where the centroids of the right moving beams are offset from each other by 2Δ, $\vec{R}_{l0} = \vec{R}'_{l0} = \vec{R}'_{r0} = 0;\ \vec{R}_{r0} = 2\Delta$. Substituting into the above equation

$$\tilde{r}_r(L_1, p) = \frac{2}{p}\Delta \cosh\left(\frac{2DL_1}{L^2 p}\right).$$

which can be inverse transformed after expanding the cosh in a Taylor series to give

$$\frac{1}{2\Delta} R_r(L_1, L_r) = 1 + \sum_{n=1}^{\infty} \left(\frac{1}{(2n)!}\right)^2 \left(2D\frac{L_1}{L}\frac{L_r}{L}\right)^{2n} .$$

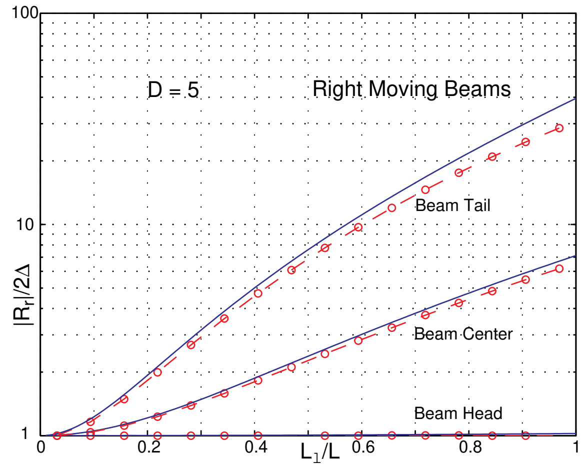

Figure 2: Centroid separation normalized to the initial offset for the right moving beams that were initially offset. The circles are from the simulation, and the solid curve is the analytical result.

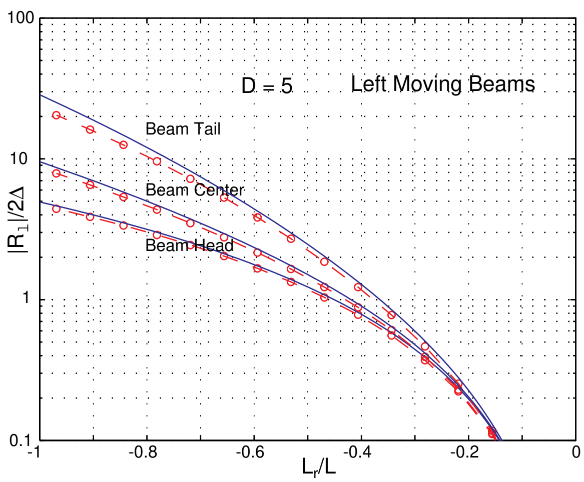

Figure 3: Centroid separation normalized to the initial offset for the left moving beams that were not offset initially. The circles are from the simulation, and the solid curve is the analytical result.

The separation of the left moving beams for this case is

$$\frac{1}{2\Delta} R_1(L_1, L_r) = \frac{1}{2D}\left(\frac{L}{L_1}\right)^2 \sum_{n=1}^{\infty} \frac{1}{(2n)!}\frac{1}{(2n-2)!}\left(2D\frac{L_1}{L}\frac{L_r}{L}\right)^{2n}.$$

Note that the last two equations are exact solutions.

These two equations can be compared with simulation results. Figures 2 and 3 show the results for the case of for $D = 5$ and $2\Delta/\sigma_r = 0.05$. The simulation and calculation are in good agreement. We consider this to indicate that the simulation is correct and can be used to study tolerances for realistic situations including Gaussian profiles and unequal charges.

4 CONCLUSIONS

Collision of neutral beams permits operation in a region of IP parameter space that, for simple e+e- collisions would correspond to Y, $\delta >> 1$. With neutral beams much larger emittances, higher bunch charges, and shorter wavelength linacs may be contemplated. Control of 2.5 TeV neutral beams in collision are likely to require an uncompensated disruption parameter $D < 10$, and a correspondingly short bunch length.

We have developed and tested a simulation that can be used to make quantitative estimates of tolerances in neutral beam collisions.

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