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CP Violation in $\tau \to \nu_\tau + 3\pi$ *

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Abstract

We discuss the ways to find CP violation in the decay of $\tau^\pm \to \nu_\tau + 3\pi$ from unpolarized as well as polarized $\tau^\pm$.

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We discuss the ways to find CP violation in the decay of $\tau^\pm \to \nu_\tau + 3\pi$ from unpolarized as well as polarized $\tau^\pm$.

1 Introduction

In the Standard Model[1,2] no CP violation can occur in decay processes involving leptons either as a parent or a daughter because these processes involve one W exchange and thus, even if the CP violating complex coupling exists in these decays, its effect will not show up when the amplitude is squared. We need two diagrams to interfere with each other to see the CP violating effects. The CP violation is caused by the existence of a complex coupling constant whose phase changes sign as one goes from particle to antiparticle as required by the hermiticity of Lagrangian that results in the CPT theorem. Let A be the complex coupling constant for τ^- decay $A = |A|e^{i\delta_t}$, then for τ^+ decay this coupling constant must be $(CP)\,A(CP)^{-1} = A^* = |A|e^{-i\delta_t}$ in order to preserve the hermiticity of the Hamiltonian[3,4].

In quantum mechanics i is changed into $-i$ under time reversal in order to preserve the basic commutation relations such as $[x_i, p_j] = i\delta_{ij}$, $[J_i, J_j] = i\epsilon_{ijk}J_k$, etc. Thus under CPT the coupling constant A is invariant $CPT\,A(CPT)^{-1} = A$.

Let us call the particle exchanged in the new Feynman diagram the x particle. This particle could be the charged Higgs boson or an entirely new particle Nature created in order to have CP violation and the matter-dominated Universe. In this paper we do not discuss the origin of such a particle, we restrict ourselves to how the existence of such a particle with a complex coupling constant can be discovered experimentally. The x particle must have spin 0 because if it were spin 1 the x exchange diagram must be proportional to the W exchange diagram, thus even if they have a relative complex phase, the imaginary part of the relative phase can never show up[3]. Let $M_x = cM_W$, then $|M_W + M_x|^2 = |M_W|^2(1 + 2\,\mathrm{Re}\,c + |c|^2)$. Since CP violation is caused by $\mathrm{Im}\,c$, there cannot be any CP violation if x is spin 1. Higher spin particles are excluded because they are unrenormalizable.

In $\tau^\pm$ decay CP violation manifests itself in the following way:

1. Branching fraction for semileptonic decay of τ^- with final state hadronic interactions is different from the corresponding one for τ^+ decay [5].

2. The coefficient of $\vec{p}_1 \cdot \vec{q}_i$ in the rest frame of hadron decay product of τ^- is different from the corresponding one for τ^+ lepton, where $\vec{p}_1$ is the momentum of τ^- and $\vec{q}_i$ is the momentum of a decay hadron both measured in the rest frame of total hadrons cm. Since $\vec{p}_1 \cdot \vec{q}_i$ is T even, CP violation for this kind of terms can occur only if there is a strong interaction in the final state because of TCP theorem [3,4,6].

3. If τ^- is polarized with polarization vector $\vec{w}$, we have terms such as $\vec{w} \cdot \vec{q}_i$, $\vec{w} \cdot (\vec{q}_i \times \vec{q}_j)$. Under CP transformation we have [3,4]

   $$(CP)(\vec{w} \cdot \vec{q}_i)(CP)^{-1} = -\vec{w}' \cdot \vec{q}'_i$$

   and

   $$(CP)(\vec{w} \cdot (\vec{q}_i \times \vec{q}_j))(CP)^{-1} = \vec{w}' \cdot (\vec{q}'_i \times \vec{q}'_j) \ .$$

   Hence if the coefficients of these terms in τ^+ decay do not behave as above then there is CP violation. Notice that CP changes the sign of momentum but does not change the sign of $\vec{w}$.

4. For pure leptonic decay, such as $\tau^\pm \to \nu_\tau + \nu_\mu + \mu^\pm$, the only term that violates CP is $\vec{w}_\tau \cdot (\vec{P}_\mu \times \vec{w}_\mu)$. Since there is no final state interaction here we need T odd product to violate CP because of the CPT theorem. Hence we need not only τ polarization but also measurement of μ polarization. This experiment will show whether a pure leptonic system can have CP violation [7].

Since the direction of polarization can be reversed, it can be used to isolate the coefficients of different spin dependent terms such as $\vec{w} \cdot \vec{q}_i$, $\vec{w} \cdot (\vec{q}_i \times \vec{q}_j)$. Thus the polarization is essential for pure leptonic decay. For semileptonic decay it is highly desirable especially if the effect is marginal.

When dealing with CP violation it is of utmost importance to be open-minded. The reason is that experimentally[8] there is only $K_L \to 2\pi$ known to exhibit CP violation so far and the standard theory of Kobayashi and Maskawa[1] was invented to explain this. It would be a mistake to neglect testing the CP violation involving leptons just because the CKM theory says there is no CP violation for τ decay. It is also unconscionable to ignore the possibility of lighter particles participating in CP violation just because the Higgs exchange model in general favors heavy particles for this violation! Fortunately we do not have to assume any specific model for our purpose. All we need is to assume the existence of a spin 0 particle called x that is coupled to leptons and quarks with CP violating complex coupling constants. Our task is to devise a method to test whether the imaginary part of this complex coupling constant is not zero. I believe that our first priority is to discover the effect in *any* channel of decay of τ or semileptonic decay of B or D. After the effect is discovered in one particular decay mode one can worry about a systematic search for the x coupling to all quarks and leptons. Only after that can one worry about why x exists.

This paper is a sequel to my previous work[4,5,9,10,3] investigating possible CP violation involving leptons either as a parent or as a daughter. There are many issues involved in this vast subject:

(a) What is the best decay mode for the first discovery of the effect?

(b) The longitudinal polarization of $e^{\pm}$ helps, but can one do without it?

(c) Once the first discovery is made and thus confirming the existence of x boson, we have to systematically find out the coupling of the x boson to all the leptons and quarks. To find out the x coupling to the lepton we not only require τ to be polarized but also require the measurement of muon polarization in the decay $\tau \to \nu_\mu + \nu_\tau + \mu$.

(d) In order to answer the first question (a) we have to theoretically investigate many decay modes. This paper deals with one of them; namely, how to find CP violation in the decay $\tau \to 3\pi + \nu_\tau$.

We are not yet in a position to write a comprehensive summary to answer question (a). We can give only the intermediate answer to this question. For τ decay:

1. The decay mode $\tau^{\pm} \to \pi^{\pm} + \pi^0 + \nu$ has the largest branching fraction (25 %), but CP violation will be suppressed because it needs interference between s and p waves of the 2π produced by x and W bosons. The s wave is suppressed by the isospin symmetry. There is a few % breaking of isosymmetry due to mass difference of $\pi^{\pm}$ and π^0, thus the suppression is about the same order of magnitude as the Cabibbo suppression for decaying into a strange channel[3,4].

2. The decay mode $\tau^{\pm} \to e^{\pm} + \nu_e + \nu_\tau$ is next in the size of branching fraction, but here we need polarization of $\tau^{\pm}$ as well as measurement of transverse polarization of $e^{\pm}$ that is an impossibly difficult task.

3. The decay mode $\tau^\pm \to \mu^\pm + \nu_\mu + \nu_\tau$. This is similar to the above, but polarization of $\mu^\pm$ can be measured by its decay angular distribution. $\mu^\pm$ can be slow moving if it moves in the direction opposite to $\tau^\pm$ [5].

4. The decay mode $\tau^\pm \to \pi^\pm\pi^\pm\pi^\mp\nu_\tau$. This reaction was first considered by Choi *et al.*[11]. In this paper we deal with the same problem with polarization of τ taken into account. We also exhibit the phases of a_1 and π' resonances explicitly, so that it will aid the decision as to how the data should be integrated to exhibit the existence of the CP violating phase δ_t.

5. In Refs. (3) and (11) the Cabibbo suppressed mode $\tau^\pm \to K^\pm + \pi^0 + \nu_\tau$ is treated. If x is a charged Higgs boson, this one could be large.

6. More complicated cases such as $\tau \to \nu_\tau +$ more than 3π, $\tau \to \nu_\tau + K +$ more than 2π need to be considered in the future.

7. Semileptonic mode without final state interactions such as $\tau^\pm \to \pi^\pm + \nu_\tau$, $\tau^\pm \to K^\pm + \nu_\tau$ cannot have CP violation because we do not have enough kinematic variables to construct T odd triple product and the T even term is not allowed because there is no final state interaction.

For semileptonic decay of hadrons we have:

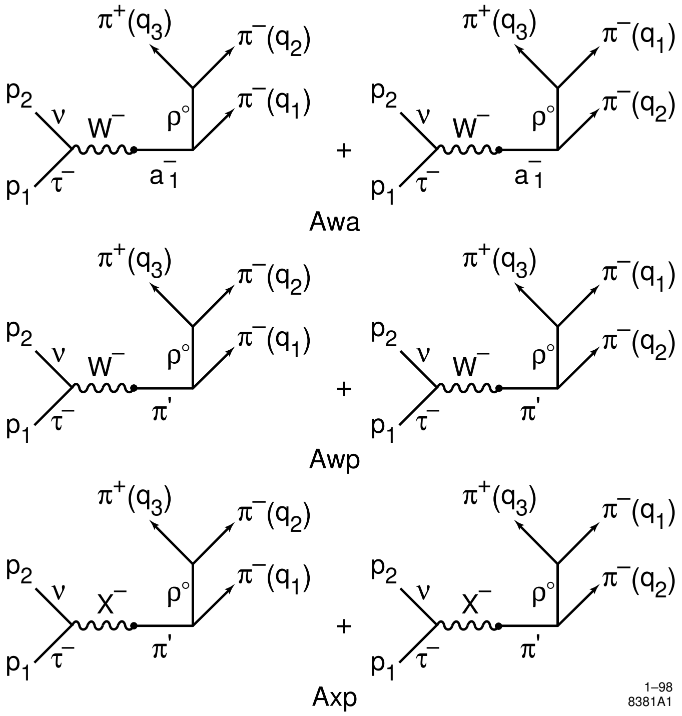
1. $K^\pm \to \pi^0 + \mu^\pm + \nu_\mu$ has been considered [12,13,14].

2. $B^\pm \to \pi^0 + \tau^\pm + \nu_\tau$, $D^\pm \to \pi^0 + \mu^\pm + \nu_\mu$, $D^\pm \to K^0 + \mu^\pm + \nu_\mu$, $B^\pm \to D^0 + \tau^\pm + \nu_\tau$ have been considered [15,3].

3. $t^\pm \to \pi^0 + \tau^\pm + \nu_\tau$, $K^0 + \tau^\pm + \nu_\tau$, $B^0 + \tau^\pm + \nu_\tau$ have been considered [16].

2 Calculations

We consider six Feynman diagrams shown in Fig. 1 for the decay $\tau^\pm \to \pi^\pm + \pi^\mp + \pi^\pm + \nu_\tau$. There are two identical particles denoted by q_1 and q_2. We may arbitrarily choose which one is called q_1 and q_2; for example, q_1 and q_2 are chosen such that $s_1 = (q_2+q_3)^2 > s_2 = (q_1+q_3)^2$. In the quark theory both a_1^- and π'^- consist of $\overline{u}d$ combination and thus they have the same CP violating weak phase $e^{i\delta_w}$ when coupled to W^- whereas the possible CP violating phase in the x coupling to π' is denoted by $e^{i\delta_x}$. Only the relative phase $\delta_t \equiv \delta_x - \delta_w$ will show up in the CP violation.


π+(q3)
π−(q2)
π−(q1)
p2
ν
W−
ρ°
p1
τ−
a1−
+
π+(q3)
π−(q1)
π−(q2)
p2
ν
W−
ρ°
p1
τ−
a1−
Awa
π+(q3)
π−(q2)
π−(q1)
p2
ν
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ρ°
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π+(q3)
π−(q1)
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p1
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π'
Awp
π+(q3)
π−(q2)
π−(q1)
p2
ν
X−
ρ°
p1
τ−
π'
+
π+(q3)
π−(q1)
π−(q2)
p2
ν
X−
ρ°
p1
τ−
π'
Axp
1–98
8381A1

Figure 1: Feynman diagrams for $\tau^- \to \pi^- + \pi^- + \pi^+ + \nu_\tau$ for CP violation.

Let

$$A_{wa} = |c_{wa}B_{a_1}|e^{i(\delta_a+\delta_w)}[e^{i\delta_1}M_a(s_1) + e^{i\delta_2}M_a(s_2)] \tag{1}$$

$$A_{wp} = |c_{wp}B_{\pi'}|e^{i(\delta_p+\delta_w)}[e^{i\delta_1}M_p(s_1) + e^{i\delta_2}M_p(s_2)] \tag{2}$$

$$A_{xp} = |c_{xp}B_{\pi'}|e^{i(\delta_p+\delta_x)}[e^{i\delta_1}M_p(s_1) + e^{i\delta_2}M_p(s_2)] \tag{3}$$

where

$$c_{wa} = \frac{G_F}{2\sqrt{2}}\ \cos\theta_c f_a f_{a\rho\pi} f_{\rho\pi\pi} = |c_{wa}|e^{i\delta_w} \tag{4}$$

$$c_{wp} = \frac{G_F}{2\sqrt{2}}\ \cos\theta_c f_{\pi'} f_{\pi'\rho\pi} f_{\rho\pi\pi} = |c_{wp}|e^{i\delta_w} \tag{5}$$

$$c_{xp} = \frac{G_x}{2\sqrt{2}}\, f_{\pi'} f_{\pi'\rho\pi} f_{\rho\pi\pi} = |c_{xp}| e^{i\delta_x} \tag{6}$$

B_{a_1}, $B_{\pi'}$ and B_r are Breit-Wigner propagators defined by

$$B_y(q^2) = \frac{1}{m_y^2 - q^2 - i m_y \Gamma_y(q^2)} \equiv \frac{e^{i\delta_y(q^2)}}{\sqrt{(m_y^2 - q^2)^2 + m_y^2 \Gamma_y^2(q^2)}} \tag{7}$$

for $y = a_1,\ \pi'$, or ρ.

Since our purpose is to investigate the effect of CP violating phase δ_w and δ_x we explicitly extract all the final state interaction phases of the problem $\delta_a, \delta_p, \delta_1$, and δ_2 for $a_1,\ \pi',\ \rho(s_1)$ and $\rho(s_2)$ respectively.

$$M_a(s_1) = \overline{u}(p_2)\gamma_\mu(1-\gamma_5)u(p_1)(2q\cdot q_2 q_{3\mu} - 2q\cdot q_3 q_{2\mu})|B_r(s_1)|\ , \tag{8}$$

$$M_a(s_2) = \overline{u}(p_2)\gamma_\mu(1-\gamma_5)u(p_1)(2q\cdot q_1\cdot q_{3\mu} - 2q\cdot q_3 q_{1\mu})|B_r(s_2)|\ , \tag{9}$$

$$M_p(s_1) = \overline{u}(p_2)(1+\gamma_5)u(p_1)(s_2 - s_3)|B_r(s_1)|\ , \tag{10}$$

$$M_p(s_2) = \overline{u}(p_1)(1+\gamma_5)u(p_1)(s_1 - s_3)|B_r(s_2)|\ . \tag{11}$$

The decay rate as well as the decay energy angle distribution of $\tau^- \to \nu_\tau + \pi^- + \pi^- + \pi^+$ from a polarized τ with a polarization vector $\overrightarrow{w}$ can be obtained from

$$\begin{aligned} d\Gamma = \frac{1}{2m_\tau} \int \frac{d^3p_2}{2E_2}\, \frac{d^3q_1}{2w_1}\, \frac{d^3q_2}{2w_2}\, \frac{d^3q_3}{2w_3}\, \delta^4(p_1 - p_2 - q_1 - q_2 - q_3) \\ \times (2\pi)^{-8} |A_{wa} + A_{wp} + A_{xp}|^2\ . \end{aligned} \tag{12}$$

The matrix elements squared can be written as

$$\begin{aligned} |A_{wa} + A_{wp} + A_{xp}|^2 = |A_{wa}|^2 + |A_{wp} + A_{xp}|^2 + (A^+_{wa} A_{wp} + A^+_{wp} A_{wa}) \\ + (A^+_{wa} A_{xp} + A^+_{xp} A_{wa})\ . \end{aligned} \tag{13}$$

Using Reduce 3.6 [17] we calculate $|A_{wa}|^2$, $|A_{wp} + A_{xp}|^2$, $(A^+_{wa} A_{wp} + A^+_{wp} A_{wa})$ and $(A^+_{wa} A_{xp} + A^+_{xp} A_{wa})$ as follows:

$$\begin{aligned} |A_{wa}|^2 = 8|c_{wa}|^2 |b_a|^2 \left\{ |br_1|^2 A_1(q_2) + |br_2|^2 A_1(q_1) \right. \\ \left. + 2|br_1 br_2| \left[\cos(\delta_1 - \delta_2) \mathrm{Re}\, A_2 - \sin(\delta_1 - \delta_2)\, \mathrm{Im}\, A_2 \right] \right\} \\ + 8|c_{wa}|^2 |b_a|^2 m_\tau \end{aligned}$$

$$
\begin{aligned}
&\{w\cdot q\left[|br_1|^2A_3(q_2)+|br_2|^2A_3(q_1)+2|br_1br_2|\cos(\delta_1-\delta_2)A_4\right]\\
&+w\cdot q_1\left[2|br_2|^2A_3(q_1)+2|br_1br_2|\cos(\delta_1-\delta_2)A_5(q_2)\right]\\
&+w\cdot q_2\left[2|br_1|^2A_5(q_2)+2|br_1br_2|\cos(\delta_1-\delta_2)A_5(q_1)\right]\\
&+w\cdot q_3\left[|br_1|^2A_6(q_2)+|br_2|^2A_6(q_1)+2|br_1br_2|\cos(\delta_1-\delta_2)A_7\right]\\
&-2|br_1br_2|q\cdot q_3\sin(\delta_1-\delta_2)A_8\}\ ,
\end{aligned}
\tag{14}
$$

where $b_a = B_{a_1}$, $b_p = B_{\pi'}$, $br_1 = B_\rho(s_1)$, $br_2 = B_\rho(s_2)$, δ_1 and δ_2 are phases of br_1 and br_2 respectively, and

$$
\begin{aligned}
A_1(q_1) &= 2[(p_1\cdot q_1)(q\cdot q_3)-(p_1\cdot q_3)(q\cdot q_1)]^2\\
&\quad+(p_1\cdot q-m_\tau^2)[m^2(q\cdot q_1)^2+m^2(q\cdot q_3)^3\\
&\quad-2(q\cdot q_1)(q\cdot q_3)(q_1\cdot q_3)]\\
\operatorname{Re}A_2 &= (p_1\cdot q)[(q\cdot q_1)(q\cdot q_2)m^2-(q\cdot q_1)(q\cdot q_3)(q_2\cdot q_3)\\
&\quad-(q\cdot q_2)(q\cdot q_3)(q_1\cdot q_3)+(q\cdot q_3)^2(q_1\cdot q_2)]\\
&\quad+2.0[(p_1\cdot q_1)(q\cdot q_3)+(p_1\cdot q_3)(q\cdot q_1)]\\
&\quad\times[(p_1\cdot q_2)(q\cdot q_3)-(p_1\cdot q_3)(q\cdot q_2)]\\
&\quad-(q\cdot q_1)m_\tau^2[(q\cdot q_2)m^2-(q\cdot q_3)(q_2\cdot q_3)]\\
&\quad+(q\cdot q_3)m_\tau^2[(q\cdot q_2)(q\cdot q_3)-(q\cdot q_3)(q_1\cdot q_2)]\\
\operatorname{Im}A_2 &= -(q\cdot q_3)q^2Eps(p_1,q_1,q_2,q_3)\\
A_3(q_1) &= 2(q\cdot q_1)(q\cdot q_3)(q_1\cdot q_3)-[(q\cdot q_1)^2+(q\cdot q_3)^2]m^2\\
A_4 &= 2.0\Big\{(q\cdot q_3)[(q\cdot q_1)(q_2\cdot q_3)+(q\cdot q_2)(q_1\cdot q_3)\\
&\quad-(q_1\cdot q_2)(q\cdot q_3)]-(q\cdot q_1)(q\cdot q_2)m^2\Big\}\\
A_5(q_1) &= (q\cdot q_3)[-(p_1\cdot q_1)(q\cdot q_3)+(p_1\cdot q_3)(q\cdot q_1)]\\
A_6(q_1) &= -2.0(q\cdot q_1)[(p_1\cdot q_1)(q\cdot q_3)-(p_1\cdot q_3)(q\cdot q_1)]\\
A_7 &= (q\cdot q_3)[(p_1\cdot q_1)(q\cdot q_2)+(p_1\cdot q_2)(q\cdot q_1)]\\
&\quad-2(p_1\cdot q_3)(q\cdot q_1)(q\cdot q_2)\\
A_8 &= (q\cdot q_1)Eps(p_2,q_2,q_3,w)-(q\cdot q_2)Eps(p_2,q_1,q_3,w)
\end{aligned}
$$

$$
\begin{aligned}
&-(q\cdot q_3)Eps(p_2,q_1,q_2,w)\\
&m=m_\pi=0.140\ GeV\ ,\quad m_\tau=1.777\ GeV\ ,\quad q=q_1+q_2+q_3\ ,\\
&s_1=(q_2+q_3)^2\ ,\quad s_2=(q_1+q_3)^2\ ,\quad s_3=(q_1+q_2)^2\ .
\end{aligned}
$$

$$
\begin{aligned}
|A_{wp}+A_{xp}|^2 &= |bp|^2[|c_{wp}|^2+|c_{xp}|^2+2|c_{wp}c_{xp}|\cos(\delta_x-\delta_w)]\\
&\quad\times\Big[M_p^+(s_1)M_p(s_1)+M_p^+(s_2)M_p(s_2)\\
&\quad+2M_p^+(s_1)M_p(s_2)\cos(\delta_2-\delta_1)\Big]\\
&=[2.0(m_\tau^2-p_1\cdot q)-2m_\tau w\cdot q]\\
&\quad\times[|c_{wp}|^2+|c_{xp}|^2+2c_{wp}c_{xp}\cos(\delta_x-\delta_w)]\,|bp|^2\\
&\quad[|br_1|^2(s_2-s_3)^2+|br_2|^2(s_1-s_3)^2\\
&\quad+2|br_1br_2|(s_1-s_3)(s_2-s_3)\cos(\delta_2-\delta_1)]
\end{aligned}
\tag{15}
$$

$A^+_{wa}A_{wp}+A^+_{wp}A_{wa}$ can be obtained from $A^+_{wa}A_{xp}+A^+_{xp}A_{wa}$ by letting $c_{wp}\leftarrow c_{xp}$ and $\delta_x-\delta_w=0$:

$$
\begin{aligned}
A^+_{wa}W_{wp}+A^+_{wp}A_{wa} = |c_{wa}c_{wp}\,ba\,bp|
&\Big\{2\cos(\delta_p-\delta_a)\,\mathrm{Re}\,[M_a^+(s_1)M_p(s_1)]\\
&-2\sin(\delta_p-\delta_a)\,\mathrm{Im}\,[M_a^+(s_1)M_p(s_1)]\\
&+2\cos(\delta_p-\delta_a+\delta_2-\delta_1)\,\mathrm{Re}\,[M_a^+(s_1)M_p(s_2)]\\
&-2\sin(\delta_p-\delta_a+\delta_2-\delta_1)\,\mathrm{Im}\,[M_a^+(s_1)M_p(s)2)]\\
&+(q_1\leftrightarrow q_1,\ s_1\leftrightarrow s_2)\Big\}\ .
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
A^+_{wa}A_{xp}+A^+_{xp}A_{wa} = |c_{wa}c_{xp}ba\,bp|
&\Big\{2\cos(\delta_p-\delta_a+\delta_x-\delta_w)\,\mathrm{Re}\,[M_a^+(s_1)M_p(s_1)]\\
&-2\sin(\delta_p-\delta_a+\delta_x-\delta_w)\,\mathrm{Im}\,[M_a^+(s_1)M_p(s_1)]\\
&+2\cos(\delta_p-\delta_a+\delta_2-\delta_1+\delta_x-\delta_w)\,\mathrm{Re}\,[M_a^+(s_1)M_p(s_2)]\\
&-2\sin(\delta_p-\delta_a+\delta_2-\delta_1+\delta_x-\delta_w)\,\mathrm{Im}\,[M_a^+(s_1)M_p(s_2)]
\end{aligned}
$$

$$+(q_1 \leftrightarrow q_2,\ s_1 \leftrightarrow s_2)\Big\} \qquad (17)$$

where

$$\mathrm{Re}\,(M_a^+(s_1)M_p(s_1)) = 4m_\tau A_9(q_2) - 4w\cdot q\,A_9(q_2) + 4w\cdot q_2(q\cdot q_3)A_{11}(s_2) - 4w\cdot q_3(q\cdot q_2)A_{11}(s_2) \qquad (18)$$

$$\mathrm{Im}\,(M_a^+(s_1)M_p(s_1)) = -4.0\,(s_1 - s_3)A_{12}(q_1) \qquad (19)$$

$$\mathrm{Re}(M_a^+(s_1)M_p(s_2)) = 4m_\tau A_9(q_2) - 4w\cdot qA_{10}(q_1) + 4w\cdot q_1(q\cdot q_3)A_{11}(s_2) + 4w\cdot q_2(q\cdot q_3)A_{11}(s_1) - 4w\cdot q_3(q\cdot q_2)A_{11}(s_1)\ . \qquad (20)$$

$$\mathrm{Im}M_a^+(s_1)M_p(s_2) = -4(s_2 - s_3)A_{12}(q_1) \qquad (21)$$

$$A_9(q_2) = (s_2 - s_3)[(p_1\cdot q_3)(q\cdot q_2) - (p_1\cdot q_2)(q\cdot q_3)] \qquad (22)$$

$$A_{10}(q_1) = (s_2 - s_3)[(p_1\cdot q_3)(q\cdot q_1) - (p_1\cdot q_1)(q\cdot q_3)] \qquad (23)$$

$$A_{11}(s_1) = (s_1 - s_3)(m_\tau^2 - p_1\cdot q) \qquad (24)$$

$$A_{12}(q_1) = q\cdot q_3\,Eps(p_1, q, q_2, w) - q\cdot q_2\,Eps(p_1, q, q_3, w)\ . \qquad (25)$$

2.1 Observations

1. A_{wa} and A_{wp} deal with W exchange, so it deals only with physics contained in the Standard Model. The a_1 decay amplitude A_{wa} is more thoroughly investigated than the π' decay amplitude A_{wp}.

2. In order to have CP violation we not only need that the gauge bosons exchanged must be different (W and x), but also that the final hadronic states must be different (a_1 and π'). For example, Eq. (15) shows that the interference between A_{wp} and A_{xp} is proportional to $\cos(\delta_x - \delta_w)$ for τ^- decay whereas for τ^+ decay it is $\cos(\delta_w - \delta_x)$ which is equal to $\cos(\delta_x - \delta_w)$ and thus we cannot have a CP violating effect even if $\delta_w - \delta_x \neq 0$. This is because A_{wp} and A_{xp} are proportional to each other and thus the imaginary part of the phase difference does not show up. Equation (16) does not contain δ_w or δ_x, so no CP violating effect. Only Eq. (17) can contain CP violating effects.

3. When τ's are not polarized only $A_9(q_1)$ and $A_9(q_2)$, shown in Eq. (22), appear in the CP violating expression. CP violation can thus be checked by comparing the coefficients of $A_9(q_1)$ and $A_9(q_2)$ for τ^- decay with those of $A_9(q_1')$ and $A_9(q_2')$ for τ^+ decay. If they are different, then CP is violated.

4. We are interested in the CP violation in the *decay* of τ. The CP violation in the *production* of τ is expected to be much less than α/π that is caused mainly by the possible existence of the electric dipole moment of tau [18]. If we ignore the CP violation in the production then $\boldsymbol{w} = \boldsymbol{w'}$; namely, the polarization vectors of τ^+ and τ^- must be equal and parallel to each other in the center-of-mass system[4] of $e^\pm$. When the energy is near threshold such as in the case of the Tau-Charm Factory we have s wave production. In the s wave production we have [4]

$$\boldsymbol{w} = \boldsymbol{w'} = \widehat{e}_z \, \frac{w_1 + w_2}{1 + w_1 w_2} \; , \tag{26}$$

   where w_1 and w_2 are longitudinal polarization of e^- and e^+ in the incident e^- direction that is chosen as the z axis. For the B factory energy $\boldsymbol{w}$ is given in Ref. (4).

5. From Eqs. (17)-(25), we see that there are much more polarization dependent terms than the polarization independent terms. Thus the polarization gives us more handles to find out whether CP is violated or how it is violated. For example, in Eq. (17) the sine terms involve only A_{12} given by Eq. (25) which vanishes when $\boldsymbol{w} = 0$. The sign of $\tau^\pm$ polarization can be reversed by reversing the polarization of initial $(e^- + e^+)$. This will help in isolating different polarization dependent terms such as the coefficients of $\vec{w} \cdot \vec{q}_1$, $\vec{w} \cdot \vec{q}_2$, $\vec{w} \cdot (\vec{q}_1 \times \vec{q}_3)$, etc.

3 Conclusions

This is a series of papers investigating new mechanisms of nonstandard CP violation using τ decay and semileptonic decay of a hadron. If CP violation is discovered in any one of these decays it will imply the existence of a spin zero boson x transmitting a CP nonconserving force. After the discovery in any one channel we can then systematically investigate how x is coupled to all particles. This will enable us to construct a correct theory of CP violation. The attractive feature of this mechanism is that it will allow leptons to participate fully in CP violation. The standard theory is unnatural in the sense that it will not allow leptons to participate in CP violation.

I am honored to be able to present this paper in memory of Professor C. S. Wu who first used angular distribution of a decay lepton from a polarized cobalt nucleus to discover the existence of $\langle\vec{\sigma}\rangle_{\text{cobalt}} \cdot \vec{p}_e$ term and thus the existence of the parity violation. The phenomenon discussed in this paper is a natural extension of her work in the sense that in the $e^\pm$ colliding machine particle and antiparticle parents ($\tau^\pm$) are produced equally abundantly. By comparing the decay angular distributions of polarized or unpolarized $\tau^{\pm}$ we hope to be able to discover a new mechanism of CP violation. I thank Professor Fang Wang for inviting me to the memorial conference. Numerical work for this paper is still in progress. The difference in detection efficiency between particle and antiparticle can be resolved by a combination of experimental and theoretical means. Experimetally the decay modes $\tau \to \pi + \nu$ and $\tau \to K + \nu$ do not have final state interactions and hence their branching fractions as well as the energy-angle distributions are not affected by the CP violations. Thus these decay modes can be used to experimentally check the difference in detection efficiency between particle and antiparticles. The difference in detection efficiency comes from difference in cross sections between particle and antiparticles on the detector materials and they can be estimated theoretically. This calculation is in progress.

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