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SPECIAL CONFORMAL SYMMETRY OF WORLDVOLUME ACTIONS

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Abstract

Kappa-symmetric worldvolume actions of the D3-, M5- and M2-branes can be coupled consistently to their near horizon bosonic geometry background. We study the gauge-fixed action in the approximation in which only the transverse radial direction of the brane is allowed to fluctuate. The generalized special conformal symmetry of these self-interacting actions is established. This opens up a possibility to find out if the full superconformal symmetry of the free actions of these branes survives in the presence of coupling defined by the size of the anti-deSitter throat.

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Recently the relation between superconformal symmetry of the worldvolume actions and near horizon geometry of branes was studied [1] [2].

In [1] the kappa-symmetric M5-brane action [3] was gauge-fixed in the Killing gauge [4] in the flat 11-dimensional target space. This resulted in a self-interacting non-linear action of (0,2) tensor multiplet in d=6. The full non-linear action has 32 worldvolume supersymmetries, half of which are of Volkov-Akulov type. The quadratic part of this action was shown to have full superconformal symmetry. However the interaction terms were shown to break the superconformal symmetry. It was suggested that the quantization of the M5-brane action in the curved background may change the situation and some generalization of the rigid superconformal symmetry may exist for the interacting theory. The indication of this was found via the study of special conformal transformations using the properties of the *adS* geometry. It was found that one must use a particular field dependent modification of the standard special conformal symmetry (see eq. (6.13) in [1]):

$$\Lambda^\alpha_K = \Lambda^\alpha_- + \frac{(6\mu)^2}{\phi^2(x)}\Lambda^\alpha_+ \tag{1}$$

Here Λ^α_- is a global parameter of special conformal symmetry, $\phi(x)$ is related to a radial coordinate of the brane, which fluctuates on the worldvolume** and μ defines the size of the *adS* throat.

On the other hand, an important observation was made in [2]. If one has a scaling invariant action of the type

$$S = b\int d^4xU^4\left[\sqrt{(1-\tilde{R}^4\partial_\alpha U\partial^\alpha U/U^4)}-1\right]\ , \tag{2}$$

one can verify the invariance of this action under the generalized special conformal symmetry with additional field dependent terms of the form shown in eq. (1)

$$\delta x^\alpha = \epsilon^\beta x_\beta x^\alpha - \epsilon^\alpha(x^2 - \frac{\tilde{R}^4}{U^2})/2 \tag{3}$$

$$\delta U \equiv U'(x') - U(x) = -\epsilon^\alpha x_\alpha U\ , \tag{4}$$

where ϵ^α is an infinitesimal parameter.

But the relation of the action (2) to the gauge-fixed kappa symmetric action of the D3 brane, which has a unique supersymmetric generalization, remained unclear. Additionally, one might wish to check if the actions of other branes which have a near-horizon $AdS_{p+2}\times S^{d-p-2}$ geometry, such as the M-5 brane or M-2 brane, are also conformal.

One can establish generically that when only radial excitations are allowed, the bosonic part of such an action [5] in the background of the near horizon geometry is

$$S = -C\int d^{p+1}x\left(\frac{U}{\tilde{R}}\right)^{\frac{p+1}{w}}\left[\sqrt{-Det\left(\eta_{\alpha\beta}+\tilde{R}^{\frac{p+1}{w}}\frac{\partial_\alpha U\partial_\beta U}{U^{\frac{p+1}{w}}}\right)}-1\right] \qquad w=\frac{p-1}{2} \tag{5}$$

**We renamed here the field $w(x)$ of [1] into $\phi(x)$ to avoid confusion with conformal weight w.

with $\alpha, \beta = 0, 1, ..., p$. The conformal weight of U is w. The radial direction on the brane is the only non-Killing direction since the geometry is spherically symmetric and depends only on radial coordinate.

We can simplify the expression under the square root:

$$-Det\left(\eta_{\alpha\beta} + X_{\alpha\beta}\right) = Det\left(\delta_\alpha{}^\gamma + X_\alpha{}^\gamma\right)\left(-Det(\eta_{\gamma\beta})\right) = Det\left(\delta_\alpha{}^\gamma + X_\alpha{}^\gamma\right) \tag{6}$$

where

$$X_{\alpha\beta} \equiv \tilde{R}^{\frac{p+1}{w}} \frac{\partial_\alpha U \partial_\beta U}{U^{\frac{p+1}{w}}} \tag{7}$$

We then use the identity

$$Det(\eta_\alpha{}^\beta + X_\alpha{}^\beta) = exp(Trlog(\eta_\alpha{}^\beta + X_\alpha{}^\beta)) = expTr(\sum_{n=1} \frac{(-1)^{n+1}}{n} X^n) \tag{8}$$

and we note that

$$Tr(X^n) = \left(\frac{\tilde{R}^{\frac{p+1}{w}}}{U^{\frac{p+1}{w}}}\right)^n (\partial_{\alpha_1} U)(\partial^{\alpha_2} U)(\partial_{\alpha 2} U)(\partial^{\alpha_3} U)...(\partial_{\alpha_n} U)(\partial^{\alpha_1} U) = (TrX)^n \tag{9}$$

Thus we find that the logarithm and trace operations commute, and

$$Det(\eta_\alpha{}^\beta + X_\alpha{}^\beta) = exp(log(1 + TrX_\alpha{}^\beta))) = 1 + TrX_\alpha{}^\beta \tag{10}$$

We thus conclude that the truncated action may be written as

$$S = -C \int d^{p+1}x \left(\frac{U}{\tilde{R}}\right)^{\frac{p+1}{w}} \left[\sqrt{1 + \tilde{R}^{\frac{p+1}{w}} \frac{(\partial_\alpha U)(\partial^\alpha U)}{U^{\frac{p+1}{w}}}} - 1\right] \tag{11}$$

This action is invariant under the following special conformal transformation:

$$\begin{aligned} \delta x^\alpha &= \epsilon^\beta x_\beta x^\alpha - \epsilon^\alpha (x^2 + \frac{w^2 \tilde{R}^{\frac{p+1}{w}}}{U^{\frac{2}{w}}})/2 \\ \delta U &\equiv U'(x') - U(x) = -w\epsilon^\alpha x_\alpha U \ , \end{aligned} \tag{12}$$

One can see this simply by using the substitution

$$U = \phi^w \tag{13}$$

where ϕ is now a weight 1 field. We then see that

$$\frac{(\partial_\alpha U)(\partial^\alpha U)}{U^{2+\frac{2}{w}}} = w^2 \phi^{2w-2} \frac{(\partial_\alpha \phi)(\partial^\alpha \phi)}{\phi^{2w+2}} = w^2 \frac{(\partial_\alpha \phi)(\partial^\alpha \phi)}{\phi^4} \tag{14}$$

$$S = -C' \int d^{p+1}x \phi^{p+1} \left[\sqrt{1 + \mu^4 \frac{(\partial_\alpha \phi)(\partial^\alpha \phi)}{\phi^4}} - 1\right] \ , \qquad \mu^4 = w^2 \tilde{R}^{\frac{p+1}{w}} \tag{15}$$

The proof of conformal invariance of this truncated action under the above transformation (up to a different choice of a sign [††]) was shown in [2].

Thus we have found that part of the bosonic actions of the gauge-fixed kappa-symmetric D3- and M5- and M2-branes has a generalized special conformal symmetry of the type shown in eq. (1) as predicted by the study of the anti-deSitter geometry and as found in [2] for a closely related theory (2). This opens up an exciting possibility that there exist non-linear interacting superconformal worlvolume field theories, waiting to be discovered. There are two possibilities to proceed. The first one, indicated in [2] is to generalize the action (5) to the full superconformal symmetry. The second one is to use the available kappa-symmetric actions of the branes and to work out the quantization of kappa-symmetry in $AdS_{p+2} \times S^{d-p-2}$ backgrounds following the simpler case developed in [4] for the flat background. This procedure will automatically supply us with the supersymmetric version of the action (5) which either will be superconformally symmetric or not. An indication that it might be superconformally symmetric is given by the fact that the supersymmetric actions in quadratic approximation have a full superconformal symmetry. This corresponds to the approximation where the size of the anti-deSitter throat can be sent to zero. In our case here, at $\tilde{R} \to 0$ the truncated actions (11) of the D3- and M2- and M5-brane tend to

$$S = C \int d^{p+1}x \left(-\frac{1}{2}(\partial_\alpha U)^2 + \frac{1}{8}\tilde{R}^{\frac{p+1}{w}} \frac{[(\partial_\alpha U)^2]^2}{U^{\frac{p+1}{w}}} - \frac{1}{16}(\tilde{R}^{\frac{(p+1)}{w}})^2 \frac{[(\partial_\alpha U)^2]^3}{U^{\frac{2(p+1)}{w}}} + \dots \right) \qquad (16)$$

which at $\tilde{R} = 0$ has special conformal symmetry under the transformation (12) with $\tilde{R} = 0$. In this approximation we know that all 3 type of free supersymmetric actions do have a superconformal symmetry. For the M2-brane there is a superconformal scalar multiplet action in d=3, for the D3-brane there is a superconformal vector multiplet action in d=4. For the M5-brane (0,2) tensor multiplet the superconformal symmetry of the free action was established recently [1]. The size of the anti-deSitter throat $\tilde{R}$ plays the role of the coupling constant for these non-gravitational theories. It remains to be seen if the superconformal symmetry of free actions can be generalized in the presence of coupling as it already happened for the special conformal part of the symmetry, presented in eq. (12) for the approximation of the theory with only radial excitations.

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[††]Our choice of a sign between two terms in square root is defined by the original action (5) with the square root of the determinant.

REFERENCES

[1] P. Claus, R. Kallosh and A. Van Proeyen, "M 5-brane and superconformal (0,2) tensor multiplet in 6 dimensions", hep-th/9711161.

[2] J. Maldacena, " The large N limit of superconformal field theories and supergravity", hep-th/9711200.

[3] I. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin, and M. Tonin, Phys. Rev. Lett. **78** (1997) 4332.

[4] R. Kallosh, "Worldvolume supersymmetry," hep-th/9709069, and "Quantization of P-branes, D-P-branes and M-branes,", hep-th/9709202.

[5] M. Cederwall, A. von Gussich, B.E.W. Nilsson, and A. Westerberg, Nucl. Phys. **B490**, 163 (1997); M. Aganagic, C. Popescu, and J.H. Schwarz, Phys. Lett. **B393**, 311 (1997), Nucl. Phys. **B495**, 99 (1997); E. Bergshoeff and P.K. Townsend, Nucl. Phys. **B490**, 145 (1997).