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Search For New Mechanism of CP Violation through Decay of Leptons and Semileptonic Decay of Hadrons *

Yung Su Tsai

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309
e-mail:ystth@slac.stanford.edu

Abstract

If CP is violated in any decay process involving leptons it will signify the existense of a new scalar boson (called the X boson) responsible for CP violation that may be the key to understanding matter-antimatter asymmetry in the universe. We discuss the signatures of CP violation in (1) the decay of tau and mu leptons, and (2) the semileptonic decay of π, K, D, B and t particles by measuring the polarization of the charged lepton in the decay. We discuss how the coupling constants and their phases of the coupling of the X boson to 9 quark vertices and 3 lepton vertices can be obtained through 12 decay processes.

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1 INTRODUCTION

The only particle that exhibits CP violation so far is the neutral K [1] and in a few years we would know whether B and D mesons will have CP violation from the B factories. The standard theory of CP violation due to Kobayashi and Maskawa predicts [2, 3] that there is no CP violation whenever lepton is involved in the decay either as a parent or as a daughter. This prediction applies for example to the decay of a muon and a tau lepton or semileptonic decay of a hadron such as π, K, D, B or t as shown in Table 1. Since leptons constitute a sizable fraction of the total number of particles in the universe this is a statement of utmost importance and thus must be tested experimentally. In order to have CP violation involving leptons we must go beyond the standard model of Maskawa and Kobayashi. The reasons why in the standard model leptons can not participate in CP violation are as follows:

1. Unlike K_L decay into two pions, all the decays listed in Table 1 involve only a tree diagram of one W exchange where W is coupled to each quark and leptonic vertex with one coupling constant in the standard model.

2. T violation or CP violation occurs in quantum mechanics via existence of a complex coupling constant in the vertex. The phase of this complex coupling constant can not manifest itself if we have only W exchange diagram, we need another diagram to interfere with it to obtain its phase. Hence T or CP violation in the standard model can not occur in the lowest order weak interaction.

Thus if we see CP or T violation in any of the decay processes shown in Table 1 we can infer immediately the existence of another charged boson mediating the weak interaction. We shall call this new boson the X boson and the diagram involving the X boson exchange A_X and the standard model diagram A_W as shown in the figures. In Appendix B we show that the X particle must have spin zero. Since only the relative phase of the two diagrams matters we shall assume that the coupling constants appearing in A_W are real and those appearing in A_X are allowed to be complex.

The only theoretical candidate for the X particle is the charged Higgs bosons proposed by Lee and Weinberg [4, 5]. The most striking feature of its prediction is that the heavier the particle the larger the CP violation. This should be tested experimentally. We shall thus not assume that charged Higgs is the X particle. This is to avoid having a prejudice against testing CP violation involving lighter particles. The Kobayashi-Maskawa model is based on the assumption that CP violation is caused by a complex coupling constant between the lepton and the vacuum expectation value of the Higgs fields and the Lee-Weinberg Model is based on the assumption that CP violation occurs because the vacuum expectation values of multi-Higgs field could have complex relative phases among them. It is possible that neither of these models is true and we must devise a model independent way of testing the CP violation experimentally. Tests of the Standard Model involves testing the unitarity of the CKM

matrix. If any of the 6 unitarity triangles is found to be not closed then CKM model of CP violation is wrong. If the sum of the absolute squares of three elements in any row or column is not unity it will also show that CKM matrix is not unitary. However to prove that all 12 conditions of unitarity of the CKM matrix experimentally is an impossible task.

The vertex function for hadrons for the X exchange diagram is related to that for W boson exchange diagram in the quark model, hence no new form factor for X coupling to hadron is needed. The purpose of this paper is as follows:

1. We give 12 examples of test of CP violation involving leptons in one place so that experimentalists can choose a most suitable one to carry out the experiment. After one experiment is found to have T or CP violation, there will be an onrush of effort to finish all the experiments in the table. We will then know the coupling constants between X boson and all the quarks and leptons. We will also know approximately the mass of the X boson.

2. We have avoided purposely to assume that the X boson is the charged Higgs boson of Lee and Weinberg [4, 5] so that experimenters will not have prejudice to avoid testing CP violation for light particles. By avoiding a discussion of the origin of complex phases in the coupling constants for X particles, we can concentrate on model independent features of CP violation such as the role of CPT theorem, the role of complex phases due to final state interactions, the role played by the complex part of the W propagator when it is on the mass shell such as in the top decay, and the use of polarization in the initial state and the final states to obtain CP violating effects. We also discuss test of CP using partially integrated cross section without using polarizations.

3. The X boson could be the particle we need for causing the matter-antimatter asymmetry in our universe [6, 7].

In Section 2 we treat the semileptonic decay of Tau using $\tau^{\pm} \to \nu_{\tau} + K^{\pm} + \pi^0$ as an example. This is the only case in Table 1 that involves final state interaction. We show that only the interference term between s wave from X exchange and p wave from W exchange diagrams can produce CP violation. In Section 3 we treat a simple semileptonic decay of a hadron. We chose spin zero hadrons for initial and final state because they are simplest to analyze. We also chose final hadrons to be neutral so that we do not have to worry about corrections due to electromagnetic final state interactions. Since there is no final state interaction, we can have only the T odd term to look for CP violation. The only T odd term in the problem is $(\mathbf{p_3} \times \mathbf{p_4}) \cdot \mathbf{W}$, where $\mathbf{p_3}$, $\mathbf{p_4}$ and $\mathbf{W}$ are the incident hadron momentum, the outgoing hadron momentum and the lepton polarization, all measured in the rest frame of the lepton.

Nelson et al. [8] were the first ones to consider the possibility of CP violation in the semileptonic decay of tau lepton. However their treatments of the problem

Table 1: Test of CP and T Violations Involving Leptons. (Test of Existence of a New Boson Responsible for CP Violation.) Determination of Coupling of X Boson to 12 Vertices.

Vertex	Experiment(where)	Signature of CP, T or CPT violation	Obtains	Ref.
X_{ud}	1. $\pi^+ \to \pi^0 + e^+ + \nu$ (LA)	$C_1(p_{\pi^+} \times p_{\pi^0}) \cdot W_{e^+}$ $C_1 \neq 0$ (T)	$\mathrm{Im}(X_{ud}X_{e\nu})$	[9]
X_{us}	2. $\tau^\pm \to K^\pm + \pi^0 + \nu$ (TCF)	$C_2^\pm(p_{K^\pm} \cdot W_{\tau^\pm})$ $C_2^+ \neq -C_2^-$ (CP)	$\mathrm{Im}(X_{us}X_{\tau\nu})$	[10]
		$C_3^\pm(p_{\pi^0} \cdot W_{\tau^\pm})$ $C_3^+ \neq -C_3^-$ (CP)	$\mathrm{Im}(X_{us}X_{\tau\nu})$	
		$C_4^\pm(p_{\pi^0} \times p_{K^\pm}) \cdot W_{\tau^\pm}$ $C_4^+ \neq C_4^-(CP)$	$\mathrm{Im}(X_{us}X_{\tau\nu})$	
	3. $K^\pm \to \pi^0 + \mu^\pm + \nu$ (BNL)	$C_5^\pm(p_{\pi^0} \times p_{\mu^\pm}) \cdot W_{\mu^\pm}$ $C_5^\pm \neq 0$ (T)	$\mathrm{Im}(X_{us}X_{\mu\nu})$	[11-13]
X_{ub}	4. $B^\pm \to \pi^0 + \tau^\pm + \nu$ (BF)	$C_5^\pm(p_{\pi^0} \times p_{\tau^\pm}) \cdot W_{\tau^\pm}$ $C_5^+ \neq -C_5^-(CPT)$	$\mathrm{Im}(X_{ub}X_{\tau\nu})$	[14]
X_{cd}	5. $D^\pm \to \pi^0 + \mu^\pm + \nu$ (TCF)	$C_5^\pm(p_{\pi^0} \times p_{\mu^\pm}) \cdot W_{\mu^\pm}$	$\mathrm{Im}(X_{cd}X_{\mu\nu})$	[14]
X_{cs}	6. $D^\pm \to K^0 + \mu^\pm + \nu$ (TCF)	$C_5^\pm(p_{K^0} \times p_{\mu^\pm}) \cdot W_{\mu^\pm}$	$\mathrm{Im}(X_{cs}X_{\mu\nu})$	[14]
X_{cb}	7. $B^\pm \to D^0 + \tau^\pm + \nu$ (BF)	$C_5^\pm(p_{D^0} \times p_{\tau^\pm}) \cdot W_{\tau^\pm}$	$\mathrm{Im}(X_{cb}X_{\tau\nu})$	[14]
X_{td}	8. $t^\pm \to \pi^0 + \tau^\pm + \nu$ (FNL)	$C_5^\pm(p_{\pi^0} \times p_{\tau^\pm}) \cdot W_{\tau^\pm}$	$\mathrm{Im}(X_{td}X_{\tau\nu})$	[15]
X_{ts}	9. $t^\pm \to K^0 + \tau^\pm + \nu$ (FNL)	$C_5^\pm(p_{K^0} \times p_{\tau^\pm}) \cdot W_{\tau^\pm}$	$\mathrm{Im}(X_{ts}X_{\tau\nu})$	[15]
X_{tb}	10. $t^\pm \to B^0 + \tau^\pm + \nu$ (FNL)	$C_5^\pm(p_{B^0} \times p_{\tau^\pm}) \cdot W_{\tau^\pm}$	$\mathrm{Im}(X_{tb}X_{\tau\nu})$	[15]
$X_{e\nu}$	11. $\mu^\pm \to e^\pm + \nu_\mu + \nu_e$ (LA)	$C_6^\pm(W_{e^\pm} \times p_{e^\pm}) \cdot W_{\mu^\pm}$ $C_6^\pm \neq 0$ (T)	$\mathrm{Im}(X_{\mu\nu}X_{e\nu})$	[16]
$X_{\mu\nu}$	12. $\tau^\pm \to \mu^\pm + \nu_\mu + \nu_\tau$ (TCF)	$C_6^\pm(W_{\mu^\pm} \times p_{\mu^\pm}) \cdot W_{\tau^\pm}$ $C_6^+ \neq C_6^-(CPT)$	$\mathrm{Im}(X_{\mu\nu}X_{\tau\nu})$	[16]
$X_{\tau\nu}$	Obtainable from Experiment 1, 2, 4, 7, 8, 9, 10 or 12.			

violates some sacred principles such as TCP invariance and rotational invariance (see Appendix A).

2 SEMILEPTONIC DECAY OF TAU

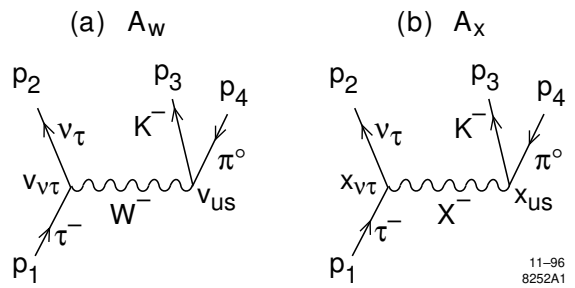

Figure 1: Feynman diagrams for the reaction $\tau^- \to \nu_\tau + K^- + \pi^0$. A_W is the Standard Model $W^\pm$ exchange diagram. A_X is the CP violating scalar exchange, $X^\pm$ may or may not be the charged Higgs boson.

Let us consider the decay $\tau^- \to K^- + \pi^0 + \nu$ (see Fig. 1). The CP conserving W exchange diagram of the Standard Model can be written as

$$A_W = V_{us}[(p_3 - p_4)_\mu f_-(s) + (p_3 + p_4)_\mu f_+(s)]\overline{u}(p_2)\gamma_\mu(1 - \gamma_5)u(p_1) \qquad (1)$$

where V_{us} is the CKM matrix which can be chosen to be real. p_3, p_4, p_2, and p_1 are four momenta of K^-, π^0, ν_τ and τ respectively, and $s = (p_3 + p_4)^2$.

The hadronic current in Eq. (1) is a pure vector (not an axial vector) because $K^-\pi^0$ can only be $J^P = 0^+$ and 1^-. Hence

$$J_\mu \equiv \left\langle K^-\pi^- \,|\, \overline{\psi}_u(0)\gamma_\mu\psi_s(0)\,|\,0\right\rangle = (p_3-p_4)_\mu f_-(s) + (p_3+p_4)_\mu f_+(s) \tag{2}$$

In *the rest frame* of p_3+p_4 the zeroth component of J_μ, denoted by J_{ro} which is rotationally invariant, represents the s wave and the vector part $\vec{J}_r$ represents the p wave. The s wave part is thus

$$J_{ro} = \left[\frac{m_3^2-m_4^2}{\sqrt{s}}\, f_-(s) + \sqrt{s}\, f_+(s)\right] \tag{3}$$

and the p wave part is

$$\vec{J}_r = \left[2\vec{p}_{3r}\, f_-(s)\right] \ , \tag{4}$$

where $\vec{p}_{3r} = -\vec{p}_{4r}$ is the momentum of K^- in the rest frame of the $K^- + \pi^0$ system. The amplitude for the CP nonconserving X scalar boson exchange is

$$A_X = C X_{us} X_{\tau\nu}\, J_{ro}\, \overline{u}(p_2)(1+\gamma_5)\, u(p_1) \tag{5}$$

where X_{us} and $X_{\tau\nu}$ are the complex coupling constants. The proportionality constant C can be obtained from

$$\left\langle K^-\pi^0 \,|\, \overline{\psi}_d(0)\psi_s(0)\,|\,0\right\rangle = C J_{ro} = \frac{(p_3+p_4)_\mu}{m_s-m_u}\, J_\mu = \frac{\sqrt{s}}{m_s-m_u}\, J_{ro} \tag{6}$$

and hence

$$C = \frac{\sqrt{s}}{m_s-m_u} \ . \tag{7}$$

From Eq. 5, 6 and 7, we have

$$A_X = X_{us} X_{\tau\nu}\, \frac{s}{m_s-m_u}\, f_0 \overline{u}(p_2)(1+\gamma_5)u(p_1) \ , \tag{8}$$

where

$$f_0(s) = \frac{{M_3}^2 - {M_4}^2}{s}\, f_-(s) + f_+(s) \tag{9}$$

is the s wave form factor and its phase is defined by

$$f_0(s) = |f_0(s)| e^{i\delta_0(s)} \ . \tag{10}$$

Only the p wave part of A_W interfering with A_X can produce CP violation [17, 18, 19]. Therefore instead of $f_-(s)$ and $f_+(s)$ in Eq. (1) we write

$$\begin{aligned} A_W \;=\; & V_{us}\Big[f_1(s)\left\{(p_3-p_4)_\mu - \frac{{M_3}^2-{M_4}^2}{s}\,(p_3+p_4)_\mu\right\} \\ & + f_0(s)(p_3+p_4)_\mu\Big]\,\overline{u}(p_2)\gamma_\mu(1-\gamma_5)u(p_2) \end{aligned} \tag{11}$$

where $f_1(s) = f_-(s)$ is the p wave form factor, and its phase if defined by

$$f_1(s) = |f_1(s)|e^{i\delta_1(s)} \ . \tag{12}$$

The decay probability of $\tau^- \to \nu_\tau + K^- + \pi^0$ can be written as

$$\Gamma = \frac{1}{2M_\tau}\frac{1}{(2\pi)^5}\int\frac{d^3p_2}{2E_2}\int\frac{d^3p_3}{2E_3}\int\frac{d^3p_4}{2E_4}\delta^4(p_1-p_2-p_3-p_4)\left|\frac{g^2}{{m_W}^2}A_W + \frac{1}{{m_X}^2}A_X\right|^2 \tag{13}$$

where $g^2/m_W^2 = G/\sqrt{2}$ with $G = 1.116\times 10^{-5}\ GeV^{-2}$, hence $g = 0.2303$. Let

$$\begin{aligned} V &= f_0(\not{p}_2 + \not{p}_4) + f_1\left\{\not{p}_3 - \not{p}_4 - D(\not{p}_3 + \not{p}_4)\right\}, \\ V^* &= f_0^*(\not{p}_3 + \not{p}_4) + f_1^*\left\{\not{p}_3 - \not{p}_4 - D(\not{p}_3 + \not{p}_4)\right\}. \end{aligned}$$

We have

$$\begin{aligned} \frac{A_W^+ A_W}{2|V_{us}|^2} &= \frac{\mathrm{Tr}}{4}\left[(1+\gamma_5\not{W})(\not{p}_1 + M_1)V(1-\gamma_5)\not{p}_2(1+\gamma_5)V^*\right] \\ &= |f_0|^2\Big\{W_3(-2M_1^3) + W_4(-2M_1^3) + M_1^2(-s+M_1^2)\Big\} \\ &\quad +|f_1|^2\Big\{W_3 2M_1\Big[2Q\cdot p_1(D-1) - D^2M_1^2 + D(s+M_1^2) - s - 2(M_3^2+M_4^2)\Big] \\ &\quad +W_4 2M_1\Big[2Q\cdot p_1(D+1) - D^2M_1^2 - D(s+M_1^2) + s - 2(M_3^2+M_4^2)\Big] \\ &\quad +\Big[2Q\cdot p_1 - D(s+M_1^2)\Big]^2 + (s-M_1^2)(2M_3^2+2M_4^2-s-D^2s)\Big\} \\ &\quad +2\mathrm{Re}(f_1 f_1^*)\Big\{W_3M_1(-2Q\cdot p_1 + 2DM_1^2 + s - M_1^2) \\ &\quad +W_4M_1(-2Q\cdot p_1 + 2DM_1^2 - s + M_1^2) + M_1^2[2Q\cdot p_1 - D(s+M_1^2)]\Big\} \\ &\quad -8M_1\mathrm{Im}(f_0 f_1^*)Eps(W,p_3,p_4,p_1)\ , \end{aligned} \tag{14}$$

where $W_3 = W\cdot p_3,\ W_4 = W\cdot p_4,\ D = (M_3^2 - M_4^2)/s,\ s = (p_3+p_4)^2, Q = p_3 - p_4$;

$$\begin{aligned} &\frac{A_W^+A_X + A_X^+A_W}{2|V_{us}X_{us}X_{\tau\nu}|} = \frac{\mathrm{Tr}}{4}(1+\gamma_5\not{W})(\not{p}_1+M_1)\Big[(1-\gamma_5)\not{p}_2(1+\gamma_5) \\ &V^*f_0 + V(1-\gamma_5)\not{p}_2(1+\gamma_5)f_0^*\Big] \\ =\ &\cos(\delta_t)|f_0|\left\{-4W_3M_1^2 - 4W_4M_1^2 + 2M_1(-s+M_1^2)\right\} + 2\cos(\delta_0-\delta_1+\delta_t)|f_1| \\ &\Big\{W_3(-2Q\cdot p_1 + 2DM_1^2 - s - M_1^2) + W_4(-2Q\cdot p_1 + 2DM_1^2 - s + M_1^2) \\ &+M_1[2Q\cdot p_1 - D(s+M_1^2]\Big\} - 8|f_1|\sin(\delta_0-\delta_1+\delta_t)Eps(W,p_3,p_4,p_1)\ , \end{aligned} \tag{15}$$

where δ_t is the CP violating phase in A_X , i.e. $X_{us}X_{\tau\nu} = |X_{us}X_{\tau\nu}|\exp(i\delta_t)$.

The decay angular distribution of the charge conjugate decay $\tau^+ \to \overline{\nu}_\tau + K^+ + \pi^0$ can be obtained from above by reversing the sign of the four momenta, i.e. $p_{1\mu} \to -p'_{1\mu}$, etc., but keeping the polarization vector unchanged. The strong interaction

is invariant under charge conjugation so f_0, f_1, δ_0 and δ_1 remain unchanged. The complex phase δ_t for causing T violation changes sign because of hermiticity of the Hamiltonian and this sign change also insures TCP invariance [17, 18, 19].

Observations:

1. Since $\cos(-\delta_t) = \cos(\delta_t)$, the interference term between the s wave part of A_W, represented by terms associated with $|f_0|$ in Eq. (15), and the A_X is not CP violating. Only the s-p interference represented by the term with $|f_1|$ in Eq. (15) can have CP violation. [See also Appendix A, and [17, 18, 19].]

2. The coefficients of terms proportional to $\cos(\delta_0 - \delta_1 + \delta_t)$ is T even because these terms cannot violate CP in the limit $\delta_0 - \delta_1 = 0$ according to the TCP theorem. Similarly the coefficient of term proportional to $\sin(\delta_0 - \delta_1 + \delta_t)$ is T odd for similar reasons.

3. The purpose of the experiment is to find out whether CP is violated by comparing the decay $\tau^- \to \nu_\tau + K^- + \pi^0$ with that of CP conjugate decay $\tau^+ \to \nu_\tau + K^+ + \pi^0$. $\delta_t \neq 0$ or π means CP violation. From the T even part of Eq. (15) and that for τ^+ decay, the CP violation is proportional to

$$\cos(\delta_0 - \delta_1 + \delta_t) - \cos(\delta_0 - \delta_1 - \delta_t) = 2\sin(\delta_1 - \delta_0)\sin\delta_t \tag{16}$$

   and for the T odd term we have

$$\sin(\delta_0 - \delta_1 + \delta_t) - \sin(\delta_0 - \delta_1 - \delta_t) = 2\cos(\delta_1 - \delta_0)\sin\delta_t \ . \tag{17}$$

   These two equations tell us three important things:

   - CP violation comes from the imaginary part of the coupling constants, i.e. $\sin\delta_t$, as expected.
   - If there is no final state interactions, i.e. $\delta_1 = \delta_0 = 0$, only the T odd part can have CP violation in agreement with the TCP theorem.
   - Near the K^* resonance, we have $\delta_1 \sim \frac{1}{2}\pi$ and $\delta_0 \sim 0$, hence the T odd part hardly contributes to CP violation whereas the T even part contributes maximally near the resonance.

4. We notice that other than the phase functions, the square of the s wave part and sp interference part of Eqs. (14) and (15) have identical expressions. Only the square of the p wave in Eq. (14) is unique there. These functions are something like the relativistic version of Legendre polynomials and thus universal in all similar calculations.

5. For the Tau-Charm Factory, the production of τ is more than 99% s wave [17], hence the τ polarization is either along the initial electron beam direction or opposite to it depending upon the beam polarization. The τ angular distribution

is almost isotropic and the decay length is negligible. This means the production angle must be integrated. In the B factories, about 25% of the cross section is d wave so polarization of the τ is only about 75% even if the incident beam is 100% polarized. On the other hand, in the B factories the energy of the τ is high enough to have observable trajectory, so the production angle for $\tau^{\pm}$ need not be integrated.

6. The spin independent term $[2Q \cdot p_1 - D(s + M_1^2)]$ in Eq. (15) averages to zero when integrated with respect to the direction of p_1. Thus it was ignored in Ref. [17]. However, Mirkes and Kühn have pointed out [10] that if we do not integrate completely but only partially the phase space of p_1 we can obtain a test of CP violation without using the polarized beam. In the following we summarize their discovery.

2.1 Test of CP violation in the semileptonic decay of τ without using polarized τ

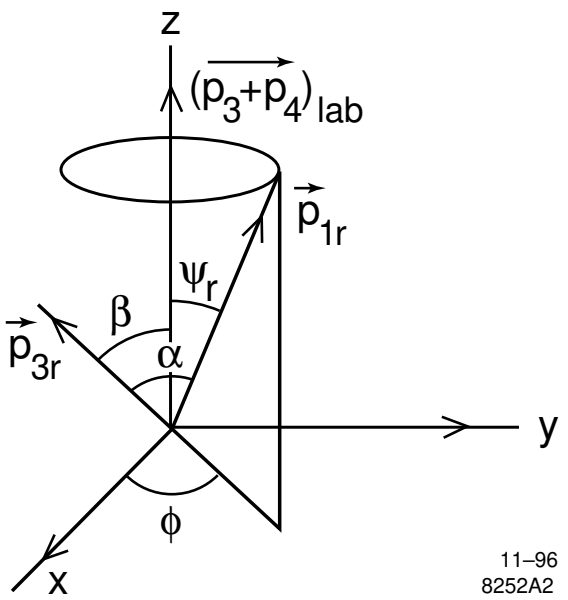

Figure 2: Definition of angles α, β, and ψ_r used in Fig. 3.

In the rest frame of $p_3 + p_4$ the term we are interested in (Eq. (15)) can be written as [see Fig. 2]

$$\frac{D(s + M_1^2)}{2} - Q \cdot p_1 = \vec{Q}_r \cdot \vec{p}_{1r} = |\vec{Q}_r||\vec{p}_{1r}| \cos\alpha \tag{18}$$

where $\vec{Q}_r$ and $\vec{p}_{1r}$ are the 3 vectors $\vec{Q}$ and $\vec{p}_1$ respectively in the rest frame of $p_3 + p_4$. We notice that $\delta_0 - \delta_1 + \delta_t$ is a function of s alone. So we hold s fixed and plot

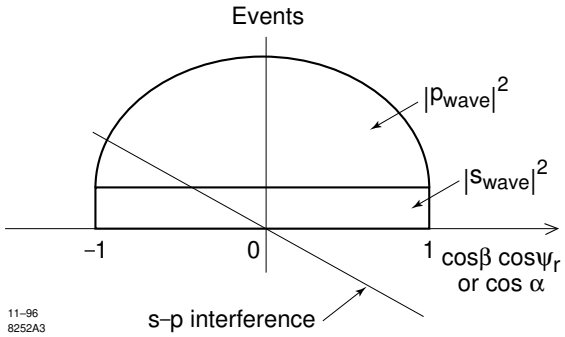

Figure 3: Schematic illustration of the test of CP violation in tau decay without using polarized τ. CP violation is manifested by the difference in slopes of the straight line labeled s-p interference between τ^- and τ^+ decays.

the decay distribution as a function of $\cos\alpha$ if the direction of τ is known. This distribution is different for τ^+ and τ^- decay if CP is violated. Thus one can obtain δ_t by comparing the $\cos\alpha$ distributions of τ^+ and τ^- decay (see Fig. 3).

In the Tau-Charm Factory, the direction of $\tau^\pm$ is not measurable. Mirkes and Kühn pointed out that even in this case one can still measure δ_t by plotting the decay distribution as a function of $\cos\beta\,\cos\psi_r$ where β and ψ_r are the azimuthal angles of $\vec{p}_{3r}$ and $\vec{p}_{1r}$, respectively, with the z axis that is in the direction of $(\vec{p}_3+\vec{p}_4)_{\rm lab}$ as shown in Fig. 2. Even though $\vec{p}_1$ is not measured experimentally, its component along the z direction in the laboratory is confined along the surface of a cone around the z axis, because

$$p_2^2 = 0 = (p_1 - p_3 - p_4)^2 = M_1^2 + s - 2(E_3+E_4)E_1 + 2p_{1z}|\vec{p}_3+\vec{p}_4| \ . \qquad (19)$$

The z component of p_1 in the rest frame of p_3+p_4 can be obtained by boosting along the direction of $\vec{z}$ as shown in Fig. 2.

$$p_{1rz} = |\vec{p}_{1r}|\cos\psi_r = -E_1\gamma\beta + \gamma p_{1z} \qquad (20)$$

with

$$\gamma = \frac{E_3+E_4}{\sqrt{s}} \ , \qquad \beta\gamma = \frac{|\vec{p}_3+\vec{p}_4|}{\sqrt{s}} \ . \qquad (21)$$

Now $\cos\alpha$ in Eq. (18) is (see Fig. 2)

$$\cos\alpha = \cos\beta\cos\psi_r + \sin\beta\sin\psi_r\cos\phi \ . \qquad (22)$$

After integrating with respect to ϕ, the second term drops out and thus by comparing the distributions of decay events as a function of $\cos\beta\,\cos\psi_r$ for τ^+ and τ^- we can find out if CP is violated (see Fig. 3).

Physically, the CP violation in this case can be understood in the following way:

$$CP\,\vec{Q}_r(CP)^{-1} = -\vec{Q}',$$

and

$$CP\,\vec{p}_{1r}(CP)^{-1} = -\vec{p}'_{1r},$$

where prime means the corresponding quantity for the charge conjugate particle. Thus $CP\,\vec{Q}_r \cdot \vec{p}_{1r}(CP)^{-1} = \vec{Q}'_r \cdot \vec{p}'_{1r}$.

This means that the coefficient of $\vec{Q}_r \cdot \vec{p}_1$ must be equal to that of $\vec{Q}'_r \cdot \vec{p}'_{1r}$ if CP is conserved. When p_1 is not observed we simply replace it with the ϕ averaged direction of p_1 as the z axis, and instead of plotting the events as a function of $\cos\alpha$, we plot the events as a function of $\cos\beta\,\cos\psi_r$ in order to find out whether CP is violated (see Fig. 3).

Since $\vec{Q}_r \cdot \vec{p}_{1r}$ is T even, we must have final state interaction phases to have CP violation for this kind of test.

As this stage we might ask whether polarized $\tau^{\pm}$'s are still needed for testing CP violation in τ decay. Let me give my opinion on this subject:

1. For a test of T violation in $\tau^{\pm} \to \nu^{\tau} + \mu^{\pm} + \nu_{\mu}$ the polarized τ is absolutely necessary because $(\vec{W}_{\tau} \times \vec{p}_{\mu}) \cdot \vec{W}_{\mu}$ is the only T violating quantity one can construct. This reaction involves only leptons, thus if one finds CP violation in this reactions we can conclude that a pure leptonic system violates CP.

2. If τ is polarized we can check CP violation using W_3, W_4 and $\vec{W} \cdot (\vec{p}_3 \times \vec{p}_4)$ terms. This is equivalent to quadrupling the number of events compared with using angular asymmetry discussed above.

3. Since the polarization vector can be reversed, we can check whether the CP violating effect is real or not by switching the sign of the polarization.

4. The overall production rate can be increased by a factor $(1 + w_1 w_2)$ where w_1 and w_2 are longitudinal polarization of e^- and e^+ respectively (see Ref. [17]).

3 SEMILEPTONIC DECAY OF HADRONS

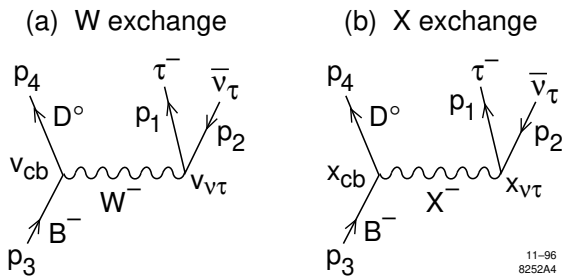

Figure 4: Feynman diagrams for the reaction $B^- \to D^0 + \tau^- + \overline{\nu}_{\tau}$. A_W and A_X are explained in the caption of Fig. 1.

This subject has been treated fully by many people in conjunction with Weinberg's three Higgs doublet model [12, 13, 14]. For tau decay we have final state interactions but here we selected final states so that they have neither strong nor electromagnetic final state interactions. Since there is no final state interaction we have $\delta_1 = \delta_0 = 0$ and thus $\cos(\delta_1 - \delta_0 + \delta_t) = \cos(\delta_t) = \cos(-\delta_t)$ and hence we cannot have CP or T violation coming from T even terms such as $S \cdot p_3$ and $S \cdot p_4$ or polarization independent angular distributions. The only possible T or CP violating term is $\sin\delta_t Eps(p_1, S, p_3, p_4)$. In the rest frame of the lepton, $p_1 = M_1$, we have

$$Eps(p_1, S, p_3, p_4) = M_1(\vec{p}_3 \times \vec{p}_4) \cdot \vec{S},$$

where $\vec{S}$ is the direction of the spin of p_1. When the lepton is an unstable particle such as τ or μ, the polarization can be analyzed by measuring the energy angle distribution of a decay particle and hence S is replaced by the momentum of any of the decay particles of τ.

Let us consider the decay $B^{\pm} \to D^0 + \tau^{\pm} + \nu_\tau$ as an example. The Feynman diagrams for W exchange and X exchange are shown in Fig. 4. We assume that the X exchange contribution to be much smaller than the W exchange one, and thus we ignore $|A_X|^2$ compared with $|A_W|^2$ and the interference between the two. The treatment here is similar to the previous section dealing with the tau decay, except here we do not have to separate out the p wave and s wave parts in A_W because we do not have to deal with the phase shift due to the final state interactions.

$$A_W = V_{cb} J_\mu \overline{u}(p_1)\gamma_\mu(1-\gamma_5)v(p_2) \tag{23}$$

where

$$J_\mu = \left\langle D^0 \,|\, \overline{\psi}_c(0)\gamma_\mu\psi_b(0) \,|\, B^- \right\rangle = (p_3 - p_4)_\mu f_-(t) + (p_3 + p_4)_\mu f_+(t) \tag{24}$$

where p_1, p_2, p_3, and p_4 are defined in Fig. 4 and $t = (p_3 - p_4)^2$.

$$A_X = X_{cb}X_{\tau\nu} \left\langle D^0 \,|\, \overline{\psi}_c(0)\psi_b(0) \,|, B^- \right\rangle \overline{u}(p_1)(1-\gamma_5)v(p_2) \ . \tag{25}$$

Now

$$\begin{aligned} \left\langle D^0 \,|\, \overline{\psi}_c(0)\psi_b(0) \,|, B^- \right\rangle &= \frac{(p_3 - p_4)_\mu J_\mu}{m_c - m_b} = \frac{t}{m_c - m_b}\, f_- + \frac{M_3^2 - M_4^2}{m_c - m_b}\, f_+ \\ &\equiv f_0\, \frac{t}{m_c - m_b} \ . \end{aligned} \tag{26}$$

The decay rate and decay distribution can be calculated by:

$$\Gamma(B^- \to D^0 + \tau^- + \overline{\nu}_\tau) = \tag{27}$$

$$= \frac{1}{2M_B}\frac{1}{(2\pi)^5}\int\frac{d^3p_4}{2E_4}\int\frac{d^2p_1}{2E_1}\int\frac{d^3p_2}{2E_2}\ \delta^4(p_3 - p_4 - p_1 - p_2)\left|\frac{g^2}{m_W^2}\,A_W + \frac{1}{m_X^2}\,A_X\right|^2 \ .$$

Let

$$V = f_-(\not{p}_3 - \not{p}_4) + f_+(\not{p}_3 + \not{p}_4) \ ,$$

$$V^* = f_-^*(\not{p}_3 - \not{p}_4) + f_+^*(\not{p}_3 + \not{p}_4) \ .$$

We have

$$\begin{aligned}
&\frac{A_W^+ A_W}{2|V_{cb}|^2} = \frac{\mathrm{Tr}}{4}\Big[(1+\gamma_5\ \not{S})(\not{p}_1 + M_1)V(1-\gamma_5)\ \not{p}_2(1+\gamma_5)V^*\Big] \\
= \ & |f_-|^2\Big\{(S\cdot p_3)2M_1^3 - (S\cdot p_4)2M_1^3 + M_1^2(t-M_1^2)\Big\} \\
& + |f_+|^2\Big\{(S\cdot p_3)2M_1\left[2p\cdot p_1 - t + 4M_4^2\right] + (S\cdot p_4)2M_1\left[2p\cdot p_1 + t - 4M_3^2\right] \\
& -4(p\cdot p_1)^2 + 4(p\cdot p_1)(M_3^2 - M_4^2) + (M_1^2 - t)(2M_3^2 + 2M_4^2 - t)\Big\} \\
& + \mathrm{Re}(f_- f_+^*)\Big\{(S\cdot p_3)2M_1\left[2p\cdot p_1 - t + M_1^2\right] + (S\cdot p_4)2M_1\left[-2p\cdot p_1 - t + M_1^2\right] \\
& + 4M_1^2(-p\cdot p_1 + M_3^2 - M_4^2)\Big\} - 8\mathrm{Im}(f_- f_+^*)M_1 Eps(S, p_3, p_4, p_1) \qquad (28)
\end{aligned}$$

where S is the direction of spin of τ and $p = p_3 + p_4$.

$$\begin{aligned}
&\frac{A_W^+ A_X + A_X^+ A_W}{\left\langle 2\,|\,V_{cb}X_{cb}X_{\tau\nu}\,|\,\frac{t}{m_c - m_b}\right\rangle} = \\
= \ & \frac{\mathrm{Tr}}{4}(1+\gamma_5\ \not{S})(\not{p}_1 + M_1)\Big[(1-\gamma_5)\ \not{p}_2(1+\gamma_5)V^* f_0 + V(1-\gamma_5)\ \not{p}_2(1+\gamma_5)f_0^*\Big] \\
= \ & \cos(\delta_t)(f_0 f_-^*)\Big\{(S\cdot p_3)4M_1^2 - (S\cdot p_4)4M_1^2 + 2M_1(t-M_1^2)\Big\} \\
& + \cos(\delta_t)(f_0 f_+^*)\Big\{2(S\cdot p_3)\left[2p\cdot p_1 - t + M_1^2\right] + 2(S\cdot p_4)\left[-2p\cdot p_1 - t + M_1^2\right] \\
& + 4M_1(-p\cdot p_1 + M_3^2 - M_4^2)\Big\} - 8\sin(\delta_t)(f_0 f_+^*)Eps(S, p_3, p_4, p_1) \ , \qquad (29)
\end{aligned}$$

where $p = p_3 + p_4$, δ_t is the CP violating phase in A_X , i.e. $X_{cb}X_{\tau\nu} = |X_{cb}X_{\tau\nu}|\exp(i\delta_t)$. Since there is no final state interaction we assume all form factors to be real. Since only the relative phase between A_W and A_X is observable we assume the KM matrix element to be real.

APPENDIX A—Comments on Nelson et al.'s paper.

In this appendix I would like to resolve two conflicting results in the literature. Nelson et al. [8] discussed in a series of papers the possibility of CP violation in the decay $\tau^\pm \to \nu_\tau + \rho^\pm$ and my papers [17, 18, 19] which showed that in the decay $\tau^\pm \to \nu_\tau + \pi^\pm + \pi^0$ CP violation can occur only through the interference of two Feynman diagrams; one with p wave final states, and the other with s wave final states for

$\pi^{\pm} + \pi^0$. Since $\rho^{\pm}$ can have only p wave final states, Nelson et al. must have made a mistake somewhere. I first show that Nelson et al.'s result violates TCP and then show that the reason for this violation is that their assumption of arbitrary amplitude for different helicity amplitudes for the decay $\tau^{\pm} \to \nu_\tau + \rho^{\pm}$ violates rotational symmetry.

Let $A(h_{\rho^-}, h_\nu)$ be the helicity amplitude of $\tau^- \to \nu_\tau + \rho^-$ in the rest frame of τ^- and $B(h_{\rho^+}, h_{\overline{\nu}})$ be the corresponding amplitude for $\tau^+ \to \overline{\nu}_\tau + \rho^+$. If we ignore the final state interaction between ν and ρ we have

$$(TCP)\,|A(h_{\rho^-}, h_\nu)|(TCP)^{-1} = |B(-h_{\rho^+}, -h_{\overline{\nu}})|\ . \tag{30}$$

Nelson et al. proposed to measure

$$\frac{A(-1, -\frac{1}{2})}{A(0, -\frac{1}{2})} = r_a = |r_a| e^{i\beta_a} \tag{31}$$

and

$$\frac{B(1, \frac{1}{2})}{B(0, \frac{1}{2})} = r_b = |r_b| e^{i\beta_b}\ . \tag{32}$$

They suggested that $|r_a| \neq |r_b|$ and $\beta_a \neq \beta_b$ would indicate CP violation in $\tau^{\pm} \to \nu + \rho^{\pm}$. Equation (30) says that the first inequality will also violate TCP and we shall show later that the second iunequality does not have any physical significance. Hence they are not viable tests of CP. TCP is violated usually when any of the following three is violated: (1) Lorentz invariance, (2) hermiticity of Hamiltonian, and (3) spin and statistics.

In the following we show that $|r_a| \neq |r_b|$ defined in Eqs. (31) and (32) violates rotational symmetry that is part of the Lorentz symmetry required for CPT and thus CPT is violated in Nelson et al.'s papers.

For the left-handed and massless neutrino, the matrix element of $\tau^- \to \nu_\tau + \rho^-$ can be written *uniquely* and *covariantly* as

$$A(h_{\tau^-}, h_\nu, h_{\rho^-}) = g\overline{u}(p_\nu, h_\nu)\gamma_\mu(1 - \gamma_5)u(p_{\tau^-}, h_\tau)\epsilon_\mu(h_{\rho^-}) \tag{33}$$

and similarly the matrix element of $\tau^+ \to \overline{\nu}_\tau + \rho^+$ can be written as

$$A'(h_{\tau^+}, h_{\overline{\nu}}, h_{\rho^+}) = g'\overline{u}(p_{\tau^+}, h_{\tau^+})\gamma_\mu(1 - \gamma_5)v(p_{\overline{\nu}}, h_{\overline{\nu}})\epsilon_\mu(h_{\rho^+})\ . \tag{34}$$

If we ignore the final state interaction between ρ and ν, we have from hermiticity $g' = g^*$. If we assume further T invariance then g and g' must both be real and equal to each other. The imaginary part of g [or g'] that causes T violation cannot be measured unless there is another Feynman diagram B that is different in structure from A interfering with A. For if the structure is similar, we have $B = CA$ and thus

$$A^+B + B^+A = (C + C^*)A^+A \tag{35}$$

and the phase of g (or g') is unmeasurable.

Since Eq. (33) is unique for τ^- decaying into $\rho^- + \nu$, the diagram B cannot have a different structure from A, thus one cannot have T violation in $\tau^- \to \nu_\tau + \rho^-$. The diagram B must contain s wave for $\pi^- + \pi^0$ system in order to avoid the cancellation of phases shown in Eq. (35).

In the rest frame of ρ, ϵ_μ can have only three components ϵ_x, ϵ_y, and ϵ_z with $\epsilon_0 = 0$. We have thus

$$\epsilon_\mu \gamma_\mu = -\epsilon_z \gamma_z - \left(\frac{\epsilon_x + i\epsilon_y}{\sqrt{2}}\right)\left(\frac{\gamma_x - i\gamma_y}{\sqrt{2}}\right) - \left(\frac{\epsilon_x - i\epsilon_y}{\sqrt{2}}\right)\left(\frac{\gamma_x + i\gamma_y}{\sqrt{2}}\right) \ . \tag{36}$$

$\epsilon_z \gamma_z$ contributes to $A(-\frac{1}{2}, -\frac{1}{2}, 0)$ and the term $(\epsilon_x + i\epsilon_y)/\sqrt{2}\ (\gamma_x - i\gamma_z)/\sqrt{2}$ contributes to $A(+\frac{1}{2}, -\frac{1}{2}, 1)$. The term $(\gamma_x + i\gamma_y)/\sqrt{2}\ (\epsilon_x - i\epsilon_y)/\sqrt{2}$ gives zero because it projects out the right-handed neutrino. This shows that the rotational invariance demands

$$|r_a| = |r_b| \ .$$

Thus $|r_a| \neq |r_b|$ in Eqs. (31) and (32) violate rotational symmetry, hence this is not a viable test of CP! The values of r_a and r_b depend upon the Lorentz frame. Using explicit representation of spinors of u and v given in Ref. [20], one obtains in the rest frame of ρ:

$$r_a = \frac{A\left(\frac{1}{2}, -\frac{1}{2}, 1\right)}{A\left(-\frac{1}{2}, -\frac{1}{2}, 0\right)} = \frac{A'\left(-\frac{1}{2}, \frac{1}{2}, -1\right)}{A'\left(\frac{1}{2}, \frac{1}{2}, 0\right)} = r_b = -\sqrt{2}\ \frac{E_\tau + M_\tau - p_\tau}{E_\tau + M_\tau + p_\tau} \ . \tag{37}$$

We notice that the phase differences β_a and β_b are meaningless quantities because the states with different helicities are orthogonal to each other and thus they do not interfere, hence their phase difference can never be measured! Usually the phase difference depends upon the convention chosen and it is an irrelevant quantity. For example, the $-$ sign in Eq. (37) is due to my choice of $(\epsilon_x - i\epsilon_y)/\sqrt{2}$ to represent a ρ with negative helicity. Had I chosen $-(\epsilon_x - i\epsilon_y)/\sqrt{2}$, the negative sign in Eq. (37) would not be there. This shows that the phase convention between different helicity states should not have any observable consequence. Nelson et al.,'s statement that $\beta_a \neq 0$ or $\beta_b \neq 0$ implies a violation of T invariance in the absence of final state interaction is thus erroneous.

I would also like to make a comment on the method of stage-two spin-correlation (s2sc) advocated in Nelson et al.'s paper. The method consists of calculating the density matrix of ρ first and then calculating the decay distributions of π^- and π^0 from ρ^- decay using the helicity formalism. They claim using this method one can measure $|r_a|$, $|r_b|$, β_a, and β_b and thus test CP violation. The method must be all wrong because $|r_a| \neq |r_b|$ violates rotational symmetry and β_a and β_b are meaningless quantities as shown above. Let me show why it is wrong.

The matrix element for the process

$$\begin{array}{lll} \tau^- \to \nu_\tau + & \rho^- & \\ & \hookrightarrow & \pi^- + \pi^0 \end{array}$$

can be written covariantly using the Breit-Wigner ρ propagator:

$$g\, g_{\rho\pi}\overline{u}(p_2)\gamma_\mu(1-\gamma_5)u(p_1)\,\frac{g_{\mu\nu}-p_\mu p_\nu/p^2}{(p^2-m_\rho^2)+i\Gamma m_\rho}\,q_\nu \tag{38}$$

where $p = p_3 + p_4$ and $q = p_3 - p_4$ with $p_3 = p_{\pi^-}$ and $p_4 = p_{\pi^0}$, $p_1 = p_\tau$, and $p_2 = p_\nu$. The rest of the calculation follows the usual procedure of Feynman diagram calculation. There is no need to introduce the density matrix for ρ, there is no need to decompose Eq. (38) into helicity amplitudes, and the result is given essentially by the p wave part of Eq. (14), i.e. the coefficient of $|f_1|^2$ in Eq. (14). Since the only place one can produce CP violation in Eq. (38) is the weak coupling constant g being complex and since it is squared in the cross section, there is no way CP violation can be observed using Eq. (38), we need interference with X boson exchange to observe CP violation.

I would like to emphasize that the only way I know how to construct a T or CP violating theory without violating any sacred physics principle is to have a complex coupling constant somewhere in the theory. Hence all tests of T and CP should have direct bearing on how the proposed test can uncover this complex coupling constant. Had Nelson et al. done this, they would have discovered that the tests they have proposed have nothing to do with T or CP violation.

Another remark I would like to make is pedagogical: one should avoid doing the two-step calculation of obtaining the density matrix of the intermediate particle and then its decay as is done by Nelson et al. It should be treated in one step with a propagator like Eq. (38). This way of treating the production of an unstable particle and its subsequent decay was first done in Ref. [21].

We should also avoid unnecessary use of D functions and Wigner rotations occurring in the helicity formalism such as done in Nelson et al.'s paper. Most of the problems in high energy physics can be done covariantly. In Nelson et al.'s case, a problem that can be described covariantly in two lines turned into a ten page Physical Review nightmare with wrong results, because of the use of the two-step process and a noncovariant helicity formalism.

In Appendix C we show that $\pi^\pm\pi^0$ must be in $I = 2$ s state (Gell-Mann-Levy's σ article) in the scalar exchange diagram. The quark model of hadrons $\overline{d}u$ and $u\overline{d}$ can only be in $I = 1$ state and thus $I = 2$ is forbidden by isospin conservation in the strong interaction. Thus the decay mode, $\tau^\pm \to \nu_\tau + \pi^\pm + \pi^0$ discussed in Ref. [17], is not a very good candidate for discovering the CP violation despite of its large branching ratio. To investigate whether the vertex X_{ud} has an imaginary part the best candidate is probably the decay process, $\tau^\pm \to 3\pi + \nu$ that was discussed by Choi et al. [22]

I should mention that C. A. Nelson's paper was the first one to look into CP violation in τ decay. Even though it proved to be wrong, I am grateful for their effort because it interested me and caused me to look into the problem.

APPENDIX B—Can $X^{\pm}$ boson have spin 1?

The answer is no, because the axial and vector parts of the interaction do not interfere in the final state. Similar to the $W^{\pm}$ boson, the $W'^{\pm}$ boson can couple to quarks with either vector (V) or axial vector (A) current if its spin is 1. Vector coupling can have final states $J^P = 0^+$ and 1^-, whereas axial vector coupling can have $J^P = 0^-$ and 1^+. For example, the final state $K^{\pm} + \pi^0$, $J^P = 0^-$ and 1^+ are not possible, so only the vector current can participate for both W and W' and the rest of the final strong interactions are identical in two cases. The matrix element for W' must be proportional to that for W for any particular final state. Let M_0 be the CP conserving W exchange diagram and M_1 be the CP violating W' exchange diagram. Since for every hadronic final state only either V or A contributes, we have

$$M_1 = cM_0 \ . \tag{39}$$

In this case the interference between M_0 and M_1 is

$$M_0^+ M_1 + M_1^+ M_0 = (c + c^*) M_0^+ M_0 \tag{40}$$

and hence even if there is a CP violating complex phase in M_1, it will not be observable. Thus no CP violation is observable in the semileptonic decay of τ when X is a spin 1 particle. For pure leptonic decay, W' will not contribute to the CP violation as long as neutrinos in the W exchange have a definite helicity. In this case only the portion of the W' exchange diagram which is similar in structure to the W exchange diagram will interfere with the latter and again no complex CP violating phase in the W' exchange diagram could be detected for exactly the same reason as shown in Eq. (40). We conclude that we need to consider only spin 0 particle exchange interfering with the W exchange diagram to produce non-CKM-type CP violation.

In Ref. [16] it was shown that only spin 0 X exchange interfering with W exchange can produce CP violation in pure leptonic decay of τ.

APPENDIX C—Can 2π decay mode of τ violate CP?

The answer is yes, but the effect is small because of the quark model and isospin conservation.

The final state $\pi^{\pm} + \pi^0$ from spin 0 Xexchange must be in s state and isospin $I = 2$ because of statistics. In the quark model the scalar $X^{\pm}$ is coupled to $\overline{u}d$ or $\overline{d}u$ which has $I = 1$. Since we are dealing with strong interactions after $\overline{u}d$ and $\overline{d}u$ are formed, the final state must sill have $I = 1$. An $I = 2$ state cannot be obtained from an $I = 1$ if isospin is conserved. Isospin is violated by electromagnetic interactions and by mass difference between u and d quarks. The mass difference between $\pi^{\pm}$ and π^0 is caused mostly by the latter. Since the mass difference between $\pi^{\pm}$ and π^0

is about 3.5%, we expect isospin conservation is broken by a few percent. Thus the suppression due to the isospin conservation is about the same order of magnitude as the suppression due to the Cabibbo angle. In principle, the imaginary part of of the coupling constant X_{ud} can also be obtained from CP violation in the β decay of π^+: $\pi^+ \to \pi^0 + e^+ + \nu$ [9]. We need to investigate further in detail how the transversal polarization of a positron can be measured experimentally.

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