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QCD: Questions, Challenges, and Dilemmas *

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Abstract

An introduction to some outstanding issues in QCD is presented, with emphasis on work by Diakonov and co-workers on the influence of the instanton vacuum on low-energy QCD observables. This includes the calculation of input valence-parton distributions for deep-inelastic scattering.

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1 Introduction

Quantum electrodynamics (QED) is the correct theory of electromagnetism.

Quantum Chromodynamics (QCD) is the correct theory of the strong force.

These bold, bald statements are slightly unscientific. Nevertheless they are not far from the truth, in the sense that to challenge either is a very serious enterprise, and one that is likely not to bear fruit unless the challenge is an especially incisive one.

It is remarkable to me that in the short span of two decades QCD has attained a degree of credibility competitive with QED: the truth and the degree of falsity of the lead sentences above are at a comparable level for the two theories. In fact we know that QED at short distances does break down. The noble photon becomes the offspring of an ugly, unaesthetic $U(1)$ gauge boson and the neutral $SU(2)$ electroweak boson. Nothing like that fate appears to await the gluons, at least this side of the GUT scale.

Both QED and QCD live in the family of gauge theories and are structurally similar. Their Lagrangian densities both are $E^2 - B^2$. Both require the gauge-invariant substitution $p \to p - eA$. The Heavy-Quark Effective Theory of QCD has its counterpart in the Heavy-Nucleon Effective Theory of QED, responsible for the nonrelativistic limit of electrodynamics, which contains the foundations of condensed matter theory, chemistry, biology, and more.

Both QED and QCD have their Feynman-diagram perturbation-theory processes, leading to incisive precision tests—which work. Their coupling constants run and are seen to run. QED and QCD are very well "tested".

But just as nonperturbative QED contains very interesting phenomena, as mentioned above, nonperturbative QCD is a most interesting portion of that theory as well. To me it is *the* most interesting and most important portion of QCD to address, despite the evident difficulty in doing so. The lectures in this school emphasize the doable, perturbation-theory based piece of QCD, because that is where most of the work is occurring. In this introduction I have decided to try to highlight the opposite extreme, with emphasis on material not covered in the other lectures, as well as on the troubles, not successes. I will omit some other unconventional QCD topics which I regard as especially relevant to future high-energy collider experimentation, because they are covered in another talk given to the Snowmass workshop earlier this summer.[1]

2 Questions

These are rather random, just to set the tone. First some easy ones:

Q1. Does the force between quarks get weaker at short distances?

A1. You had better answer no. The force follows an approximate inverse-square law, with a coefficient which at short distances very slowly gets smaller (asymptotic freedom). Please don't accuse me of nit-picking. It may be acceptable for us to use sloppy language to each other but it is definitely very wrong when trying to explain QCD to the outside world at the Scientific American level.

Say it right! When you do, it becomes perfectly clear why there are so many high-p_t jets in hadron-hadron collisions, jets that justify the livelihood of so many experimentalists and theorists. The forces between quarks get so incredibly strong that 500 GeV partons which collide head-on can make the right-angle turn at rates high enough to be detected.

Q2: In idealized QCD, with light quarks omitted, does the force between quarks grow as their separation becomes very large?

A2: Again, no. It's the potential energy that grows linearly.

Q3: Is the QCD strong force CP-conserving?

A3: In general, no. There is a CP-violating term $E.B$ in the Lagrangian which is allowed and admits observable effects like a nonvanishing neutron electric dipole moment. Renormalization effects make the coefficient of the CP-violating term formally divergent, but the actual coefficient is very small, less than 10^{-9}. What to do remains an unsolved problem, probably not mentioned again in this school.

Q4: Do instantons matter?

A4: Yes. These will not appear in other lectures but will be mentioned in this one later on. They impact on, among other things, the CP violation issue mentioned above.

Q5: Does old fashioned pre-QCD S-matrix theory have anything to do with QCD?

A5: Yes. While there seems to be a feeling that quarks, QCD and parton ideology have rendered that body of work obsolete, this is not true. The S-matrix techniques were built from general principles (analyticity, unitarity, microscopic causality, crossing symmetry, spectrum, $\cdots$) which are rigorously true in QCD.[2] Much can still be salvaged from these ideas in describing the nonperturbative, confining, low-energy limit of QCD. It is still something worth learning, and I fear that it is taught less and less, much being eventually lost and having to be someday rediscovered afresh.

Q6: Does Regge-Pole theory have anything to do with QCD?

A6: This question is a special case of the previous one, with the same answer, but with very clear implications, for example, in main-line QCD structure-function phenomenology. Nonsinglet deep-inelastic structure functions in the limit of small x should be describable by exchange of well-established Regge-trajectories like the ρ or ω. These Reggeons are very well-established experimentally and precisely parametrized. There is much less uncertainty in the theoretical underpinnings of the asymptotic limit of nonsinglet structure functions than there is in the related world of soft and hard Pomeron physics, to be described by Al Mueller in this school.[3] Nevertheless there is very little work going on to understand this problem in the context of QCD, perturbative or otherwise.[4] It is becoming of special current interest because of the experimental situation regarding the small-x behavior of the polarized structure functions.[5]

Q7: Is the boundary between what is legally calculable from perturbation theory and what is not well defined?

A7: I believe not. Furthermore it seems to be crossed more and more indiscriminately as time goes on. Many calculations treat initial and final quarks and gluons as on-shell, asymptotic states. This is illegal; there is no S-matrix for quark and gluon interactions. At a less fundamental level, some perturbative-QCD-inspired models for hadronization push shamelessly into regions of parameter space (small momenta, large distance scales) which are indefensible. While boldness in this regard is in itself no vice, an uncritical attitude is. It is not enough to say "It agrees with data, therefore it makes sense and is a prediction of the perturbative theory."

Q8: Will these questions ever end?

A8: Yes, right now.

3 Challenges

The basic challenges in understanding QCD can be seen very clearly in a space-time description: it is how to link the phenomena at short distances with phenomena at large distances. The simplest case is the static limit, with all light quark degrees of freedom left out. The short-distance limit is that of onium physics—a Coulomb-like interaction between heavy quarks with a weak coupling constant. This is under very good theoretical control. As the heavy quarks are pulled apart there emerges a linear potential between them, something described quite well via the lattice calculations.[6] The microscopic picture is believed to be that there is a color-electric flux tube of

smallish diameter between quark and antiquark in this limit. However the dynamics creating it is the essence of the problem of confinement and not "understood" well. And if light quarks are included, long flux tubes invariably break and are terminated by constituent quarks or antiquarks. Pull apart bottomonium and you get a B-meson and anti-B-meson. A B-meson is (by definition!) a constituent quark plus a heavy spectator b-quark which can be treated perturbatively. Therefore the B-meson dynamics is an especially simple way in principle (alas, not so much in experimental practice) of learning about the properties of single, "isolated", constituent quarks.[7]

Challenges for "pure" QCD with light quarks excluded include the understanding of the glueball spectrum,[8] as well as the details of the flux tube. When the light quarks are introduced, there are major changes to deal with: the glueballs mix with the myriad of ordinary meson excitations of $q - \overline{q}$ pairs, perhaps toward the limit of total extinction. Flux tubes break, but the microscopic description is obscure. Perhaps the flux-tube concept is likewise driven to the edge of extinction.

Another very basic challenge for the static picture is the nature of chiral symmetry breaking. Because the bare masses of up and down quarks are so small, the QCD Lagrangian has an almost exact $SU(2)_L \times SU(2)_R = O(4)$ chiral symmetry. These $SU(2)$'s describe independent isospin rotations of left- and right-handed up and down quarks. There is a vacuum condensate $\langle 0|\sigma|0\rangle \neq 0$, with σ the fourth component of an internal-symmetry four-vector $(\sigma, \vec{\pi})$ built from the quark densities. The situation is very analogous to the Higgs sector of electroweak theory. In QCD the spontaneous symmetry breakdown leads to nearly massless Goldstone bosons (the pions) as well as the 300–400 MeV of constituent-quark mass. So in the large distance limit (momentum scales smaller than 500–1000 MeV), the QCD dynamics is best described by an effective chiral Lagrangian containing the σ, π, and constituent-quark degrees of freedom (plus some glue) rather than the partonic quark-gluon degrees of freedom which form the basis of perturbative-QCD phenomenology.[9]

It is an extremely basic question to relate this long-distance chiral description to the short-distance Lagrangian. The boundary between large and short distances needs to be sharpened and quantified. And the connection of this chiral-symmetry breaking phenomenon to confinement needs elucidation. So far the main clue comes from the lattice: the chiral phase transition and deconfinement phase transition in finite-temperature QCD are indistinguishable so far.

I have devoted the final section of this talk to a description of a specific attack on the above questions by Diakonov and his co-workers. I am no expert in this topic.

But their work strikes me as a promising attack on the question at an impressively fundamental level, work which respects a variety of fundamental principles. Right or wrong, I think it is well worth careful attention and study.

Much closer to most of the material contained in the lectures at this school is what goes on in QCD in the high-energy limit. Again we may look at this in space-time. But for high-energy collision dynamics the important action is in the neighborhood of the light cone. Near the past light cone, there is perturbative "evolution"; it is here where each incoming hadron is replaced, in parton-model ideology, by an incoherent beam of incident partons which eventually scatter off a similar "beam" of partons in the other projectile. Near the future light cone there occur perturbative branching processes which create the multijet structure of typical QCD final states. Further into the interior of the future light cone, things get messy because the partons must find their way into final-state hadrons without violating the nonperturbative demand of perfect confinement; never must a single quark escape into an isolated final state. Finally, deep inside the future light cone there may also be dynamics: some of us speculate that this region contains a vacuum state with a rotated value of its order parameter (disoriented chiral condensate) which decays into coherent states of pions with curious properties.[10] It is conceivable that there could be other mechanisms of particle production from this region of spacetime as well. This need not happen, but if it does it is novel physics not contained in existing event generators.

The time scales for evolution of the final state in high energy collisions is very large, proportional to the energies involved. The time scale for hadronization of leading particles in a jet, in reference frames where the nearest neighboring jet is 90° away (The correct way, in fact, to define what is and is not in jets is to do it in such frames.[11]), is proportional to the transverse momentum or transverse energy of the jet. Thus there is a direct correspondence between the configuration-space and momentum-space description of jets: the production angles are of course the same, while the (large) p_t's and (large) jet-hadronization time-scales T are in direct proportion.

Indeed since one can simultaneously describe the gross properties of jet contents in both momentum space and space-time, it is clear that the description must be macroscopic, quasi-classical in nature. The vital region for phenomenology is the region of space-time where the real observed hadrons are produced. In QCD this is typically a fractal surface, because there can be jets within jets within jets .... Recall that in the absence of QCD jet phenomena, hadrons are produced with rather

uniform density in the lego plot. Since the lego plot area is proportional to $\log s$, the multiplicity should rise with s in a similar way. When additional jets populate the lego plot, they increase the phase space area by an amount equal to $\log p_t$ (or $\log T$) per jet. The jets themselves can contain additional jets in their (extended) phase space, leading to a branching structure and fractality by the time all jet evolution is accounted for. The hadron multiplicity is then proportional to the total area of this extended phase space, which thereby acquires fractal properties,[12] and the multiplicity growth with s becomes more rapid.

It is of course a challenge to provide a sharp description of all this. And the situation is in fact quite good. There is the phenomenon of "preconfinement", which is a perturbative mechanism which keeps color and anticolor close together (most of the time) in momentum space as the branching scale becomes soft and hadronization is invoked.[13] The Monte-Carlo programs which employ QCD branching mechanisms work well, and subtle, QCD-specific phenomena like the "string effect" are predicted and seen.[14] Nevertheless some of the other claimed successes are consequences of phase space and little more. And a purely perturbatively based approach cannot be complete, because confinement is neglected and confinement is important. For example, much is made of "local parton-hadron duality" which is the statement that the perturbatively computed momentum-space densities of "produced" soft partons matches smoothly to the corresponding densities of produced hadrons. This principle is reasonable almost always, especially when the densities are not small. But now and then the phase space densities of produced partons will fluctuate to small values, and some nonperturbative mechanism (*e.g.* flux tubes) must intervene. For example the Z occasionally will decay into two pions and nothing else. Local parton-hadron duality asserts that with comparable probability the Z will evolve into two final-state partons and nothing else, *e.g.* a q and $\overline{q}$; no gluons choose to be emitted. But this is a disaster because at the hadronization time of the quarks they are 50 to 100 fermis apart. The local duality should apply to space-time as well as momentum space, and there is a clear problem with simple causality. One cannot be satisfied with a theory of hadronization which accounts for confinement only most of the time.

Finally there are challenges even within the perturbative sector. These need only be mentioned here briefly, because they will get a lot of attention in the other lectures. It turns out that despite the phenomenon of asymptotic freedom, the interaction of partons at extreme cms energies and a fixed small distance scale (say, of impact parameter) is supposed to grow as a power of energy, perhaps almost linearly with

cms energy.[3] Thus at extremely high values of s/t, the parton-parton interactions might become strong, with a breakdown of perturbation theory and lots of diffractive phenomena. This is focusing much-needed attention on diffractive phenomena, especially short-distance, high-p_t diffractive processes. The buzz words are hard diffraction, soft and hard Pomerons, BFKL Pomerons, etc. It is an exciting new field, as the proceedings of this school exhibit.[3,15,16]

4 Dilemmas

It is the challenges facing QCD that makes its investigation so much fun. But with the challenges come the dilemmas, which can sometimes make the investigations frustrating. What follows is a rather random potpourri of dilemmas:

On the experimental side, many of the greatest challenges lie in the nonperturbative sector: low energy spectroscopy (*e.g.* of glueballs) and collision dynamics, as well as the problems of hadronization and soft diffraction at high energies. Unfortunately these problems nowadays have little sex appeal, and the interest in—and resources for doing—low energy spectroscopy and soft-collision dynamics is simply insufficient. For example a low-energy full-acceptance spectrometer with modern capability would not be costly in comparison with most modern detectors, and could by itself augment the spectroscopy data base—much of which was established long ago via bubble chamber techniques—by orders of magnitude. There is not even an initiative anywhere for doing this. I am informed by Bill Dunwoodie that there actually was a proposal not so long ago for a full-acceptance spectrometer[17] at the proposed Canadian facility KAON. But it did not survive the death of KAON itself and is now abandoned. What a pity!

I also bemoan the lack of interest in full-acceptance, large-cross-section physics at hadron-hadron collider energies. The bemoanings are made in my Snowmass talk[1] and elsewhere,[18] and will not be repeated here.

Most of the challenges for theorists mentioned in the previous section are low energy or soft phenomena which go beyond perturbation theory. And there are not too many good options for theorists under those circumstances. Lattice QCD is a very powerful way of going beyond perturbation theory, but it is very difficult to apply to high-energy collision dynamics.

A basic dilemma at higher energies is the problem of hadronization, where as already mentioned there is a fuzzy boundary between what is perturbative and what

is not. The techniques for creating a precise understanding are still lacking.

Finally there is the problem of QCD vacuum structure. Understanding the QCD vacua (there are many of them) is the key to the question of confinement and is important for the phenomenology of the effective chiral theory valid in the low energy limit. Again the available techniques are limited; nevertheless the problem is being attacked and progress is being made. The next section on the recent work of Diakonov and his co-workers is evidence of some of the best of this work and, right or wrong, is an exemplar of the kind of thing that is sorely needed to really make major new inroads into the full understanding of QCD.

5 Diakonov *et al.*: Instantons and their Consequences

The starting point of Diakonov's work[19] was to study the influence of instantons on low-energy QCD. The instanton, something not easy to explain even at length,[21] no less in a short summary like this, is a classical solution of the (Euclidean) QCD field equations, which physically is related to the mixing (via a tunneling mechanism) of various QCD vacua which differ by gauge transformations of nontrivial topology. All this was discovered twenty or so years ago,[22] but at that time infrared divergences in the calculations[23] made quantitative consequences near impossible to attain. More recently Shuryak determined phenomenologically the properties that the instanton "fluid" should have in order to be consistent with known data.[24] Diakonov and Petrov[20] then performed variational calculations which supported the Shuryak picture. Since then there have appeared some lattice calculations of instanton effects which, although still somewhat controversial, appear to lend support as well.[25] The net result is that by now there is a credible picture of what the instanton effects are. The immediate ones are the solution of the $U(1)$ problem (why the η' meson is so heavy) and the existence of a gluon condensate (seen in QCD sum rules, yet another piece of QCD not discussed at this school[26]).

All this, however, is still rather abstract; it is not clear what more these abstruse considerations have to say to the experimentalists in the trenches. However the next steps taken are more directly related to QCD phenomenology. The most relevant features, in my opinion, are as follows:

1. The quark-parton degrees of freedom are influenced by the presence of the

instantons, and they get a "constituent-quark" mass as a consequence of having to propagate through the instantons.

2. The above mechanism leads naturally to spontaneous breaking of the strong-interaction chiral symmetry.

3. Therefore there must be the Goldstone degrees of freedom (almost-massless pions) in the spectrum as well as the constituent quarks.

4. The low-energy chiral effective Lagrangian can be constructed. The lowest order terms are universal (model independent) in form, depending only on symmetry considerations. However, higher order terms are also present and can be estimated. The magnitudes of these terms are in agreement with what is needed for the phenomenology.

5. The Goldstone pions can be shown to be composites of the constituent quarks. This is an improvement on the scheme put forward by Manohar and Georgi[27] some time ago. They argued for the chiral constituent-quark-plus-pion picture based on the success of the additive quark model. They had, however, an awkward time in understanding whether their Goldstone pion is the same as, or distinct from, the ${}^1S_1$ partner of the ${}^3S_1$ ρ. In the Diakonov instanton picture they are not distinct.

6. Diakonov *et al.* in addition put forward an interesting model of baryons, which is a variant of the somewhat popular Skyrmion picture, and to my eye an improvement.[28] They assume that the pion cloud surrounding the three constituent quarks of the baryon has a nontrivial "hedgehog" topology, as originally suggested by Skyrme long long ago[29] (see Eq. (14) below). Then it can be shown that in such an external field there will be one and only one quark bound state with energy in the gap between the continua starting at $E = +m$ and $E = -m$. This state can be populated with one quark of each color to make objects with the quantum numbers of the nucleons. In the large N_c limit the combined wave functions of these quarks can be treated *a la* Thomas-Fermi atomic theory as a source of the "hedgehog" pion field, leading to a self-consistent semiclassical description of the nucleon. To recover quantum mechanics, in particular the classification of the energy levels, the "cranking-model" techniques of nuclear theory can be employed to give a reasonable description.[30] So this picture has a quite good formal justification in the large N_c limit.

7. Finally, with this picture of the nucleon, they calculate[31] the distributions of the "primordial" partons within the nucleon, namely the leading twist parton distributions at a low value of $Q^2 \approx 0.5\ GeV^2$, which when evolved to higher Q^2 via the DGLAP evolution equations give the leading twist contributions to the structure functions. Their results agree reasonably with the Gluck, Reya, Vogt[32] primordial parton distribution functions which are input by hand in order to reproduce deep-inelastic scattering data. But more important in my opinion is the way Diakonov *et al.* can maintain the internal consistency of the formalism. The validity of a variety of current algebra sum rules is established. This is highly nontrivial, because relativistic effects are very important, and valence antiquark distributions must be present; they are created by the back reaction of the nucleon valence quarks on the pionic "hedgehog" sea. The techniques which are employed provide valuable lessons for all bag-model descriptions of hadrons.

There remains a major missing link: an understanding of confinement. The effects of gluons in this low-energy limit are formally of "higher order". In one sense this is good, because in the low energy constituent-quark spectroscopic world the gluon degrees of freedom do not seem to play a central role. On the other hand their effects cannot be omitted, because confinement depends on them. Probably some of the effective quark mass is accounted by something akin to color flux-tube energy, and the dynamical effects of fluctuations about an average value are not too important. I think the ideal arena for studying this problem is that of heavy-flavor mesons and baryons, where the source of color is static and understood (a stationary heavy b-quark, or Wilson line) and only how the color finds its way into the single constituent quark degree of freedom of the B-meson, or alternatively into the "Skyrmionic" quark-baryon wave function, needs to be solved. Some work has been done, but more is needed.[33]

While there may well be reason to exhibit skepticism regarding the whole Diakonov program, I still want to emphasize that this kind of work is at the most important forefront of QCD. It links the confining world to the perturbative sector. Most of the known nonperturbative QCD phenomena are involved, and the work touches upon the edge of some of the best perturbative phenomenology which exists, namely the information on deep-inelastic structure functions. The level of attack is much deeper than mere phenomenological model building. It deserves, I believe, close attention and constructive criticism.

6 Some more details on the Diakonov program

The preceding description was very general in nature, and what follows is a slightly more technical version of some of the same material. It is far from definitive, if for no other reason than the limited competence of yours truly. However there are recent lecture notes[19] to consult for a more detailed and authoritative version.

6.1 What about these instantons?

As already mentioned, an instanton is a solution of the QCD classical field equations in Euclidean space-time with finite action. It contains a "topological knot" and is localized in space-time. It also has a size parameter which can take any value in principle. The immediate function of these instantons is to create couplings, via tunneling, between different Minkowski-space QCD vacua, vacua which differ from each other by a gauge transformation which also contains a "topological knot". Because of these nonperturbative tunneling couplings, the many initially degenerate QCD vacua, which can be classified in terms of the number of gauge knots they contain, are coupled together and must be diagonalized, leading to the so-called θ-vacua, which are the true energy eigenfunctions of the vacuous QCD theory.

When theorists initially attempted to estimate the magnitude of these effects they were thwarted by the presence of large numbers of large instantons, whose effects were not under control. Shuryak, working phenomenologically, argued that if instantons with sizes larger than about 0.3 fermis (or a momentum scale $\approx 600\ MeV$) were suppressed, instanton-induced phenomenology could be understood. Furthermore, were this true, the instanton "liquid" in Euclidean space-time would be dilute, in the sense that the mean separation R between instantons would be 2 to 3 times larger than the important instanton size ρ. As already mentioned, Diakonov and Petrov,[20] using variational techniques, found a candidate mechanism for this to happen, namely medium-range instanton-antiinstanton repulsion.

The bottom line is that the effects of large instantons are arguably damped out at a known scale, with a bonus of a small parameter (the instanton packing fraction in Euclidean space-time) in the formalism. This then becomes the working hypothesis for going further. It is not rigorously established but is credible.

6.2 How do the instantons induce chiral symmetry breaking?

The next step is to introduce the quarks and calculate their influence. The equation of motion of quarks in a classical instanton field (again in Euclidean space time) also shows a remarkable feature—the existence of "zero-mode" solutions of the Dirac equation of the quark in the presence of the instanton (with zero eigenvalue of the Euclidean Dirac operator) and which are localized around the instanton. Just as for the instanton itself, the implication of these solutions for physics is subtle and deep. For example they influence the presence (or absence) of CP violation in the strong interactions. The vital buzzword here is "spectral flow": a filled negative-energy level (now in Minkowski space) in the negative energy sea can, because of the knotty gauge potentials, be pushed above zero (in the chiral limit of massless quarks), while other empty positive energy levels with different quantum numbers can be pushed into the negative energy sea.[34] The net result is that there can be net pair-creation induced, with the pair not necessarily having vacuum quantum numbers. All this activity is quite sufficient to create the mechanism of spontaneous symmetry breakdown.

In the calculations which argue for spontaneous symmetry breaking, it is necessary to include the mixings of zero modes associated with different instantons, something rather nontrivial. What follows are a few equations for theorists and well-educated experimentalists to give a flavor of what is done. The information about all this kind of thing is to be extracted from the Euclidean partition function

$$\begin{aligned} Z &\sim \int \mathcal{D}A\, e^{-\frac{1}{g^2}\int d^4x\, F^2} \int \mathcal{D}\psi\, \not{\mathcal{D}}\overline{\psi} e^{\int d^4x \overline{\psi}(\not{\nabla} - g\not{A} - m)\psi} \\ &\equiv \langle \det(\not{\nabla} - g\not{A} - m)\rangle \Rightarrow \left\langle e^{\frac{1}{2}\sum_n \ell n(\lambda_n^2 + m^2)} \right\rangle \end{aligned} \tag{1}$$

where in the second line the Gaussian integral over fermionic quark fields is performed, and where

$$\lambda_n = n_{th} \text{ eigenvalue of } (\not{\nabla} - g\not{A}) \tag{2}$$

and m the small quark-parton mass of a few MeV.

There is one zero eigenvalue per instanton per quark flavor in the dilute-instanton approximation. But when the effect of the overlapping of zero modes from separate instantons is taken into account, the zero eigenvalues repel. The typical values become

$$\langle \lambda \rangle \sim \frac{\rho^2}{R^3} \ . \tag{3}$$

Now by definition the chiral order parameter is

$$\begin{aligned} V\left\langle\overline{\psi}\psi\right\rangle &\simeq \frac{\partial}{\partial m}\,\ell n\, Z\,\Big|_{m=0} \\ &= \frac{\partial}{\partial m}\left\langle\frac{1}{2}\int d\lambda\,\nu(\lambda)\,\ell n\,(\lambda^2+m^2)\right\rangle \\ &= \left\langle\int d\lambda\,\nu(\lambda)\left(\frac{m}{\lambda^2+m^2}\right)\right\rangle_{m\to 0} \end{aligned} \tag{4}$$

where we go to continuum normalization via

$$\sum_n \Leftrightarrow \int d\lambda\,\nu(\lambda) \tag{5}$$

and V is the (Euclidean) space-time volume.

This shows that chiral symmetry breaking will occur provided the density of zero modes $\nu(\lambda)$ at $\lambda = 0$ is nonvanishing. But this is what is estimated to occur,

$$\left\langle\overline{\psi}\psi\right\rangle \sim \frac{\nu(0)}{V} = \frac{N}{\langle\lambda\rangle\, V} \sim \left(\frac{R^2}{\rho}\right)\left(\frac{1}{R}\right)^4 = \frac{1}{\rho R^2}\ . \tag{6}$$

The factor ρ/R^2 for $\langle\lambda\rangle$ occurs because

$$\langle\lambda\rangle^2 \Rightarrow \left\langle\lambda^2\right\rangle = N\left\langle\lambda^2\right\rangle_0 = N\int\frac{d^4R}{V}\left(\frac{\rho^2}{R^3}\right)^2 = \frac{N\rho^2}{V} = \frac{\rho^2}{R^4} \tag{7}$$

with $\langle\lambda^2\rangle_0$ the contribution to λ^2 from one instanton. Note that this happens because we have added the contributions to splittings from all the neighboring instantons in quadrature. This rough argument, due to Diakonov, actually can be refined, so that the conclusion is quite robust.

The parameters of the constituent quarks can be estimated from the instanton parameters, which are

$$\text{instanton size:} \qquad \overline{\rho} \approx 0.3\, f \tag{8}$$

$$\text{instanton spacing:} \qquad R \approx 1\, f\ . \tag{9}$$

This means that the fraction of Euclidean space-time occupied by instantons is

$$\pi^2\left(\frac{\rho}{R}\right)^4 \approx 0.1\ . \tag{10}$$

The quark mass, in order of magnitude, turns out to be

$$M_Q \sim \frac{\rho}{R^2} = \frac{1}{\rho}\left(\frac{\rho}{R}\right)^2 \tag{11}$$

and the careful calculations produce a reasonable value of constituent quark mass of 350–400 MeV.

The pion decay constant F_π can also be estimated

$$F_\pi \cong \frac{\text{const}}{\rho} \cdot \left(\frac{\rho}{R}\right)^2 \sqrt{\ell n \, \frac{R}{\rho}} \approx 100 \; MeV \; . \tag{12}$$

In the very low momentum limit, the constituent quark degrees of freedom can be integrated out of the partition function, leaving a chiral effective action of the form

$$Z = e^{iS(\pi)} = \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \; \exp i \int d^4x \left(\overline{\psi}_L \left[i \not{\nabla} - M \, e^{i\vec{\tau}\cdot\vec{\pi}} \right] \psi_R \; + \text{h.c.} \right) \; . \tag{13}$$

There is also a "gap equation" relating how the pionic degrees of freedom are related to the quarks, but I have had difficulty dredging the details out of the easily available literature.

6.3 How does this lead to a model of the nucleon?

Thus far it has been sufficient to look at the theory in Euclidean space-time, a clear indicator that phenomenology is somewhat distant. The reason for the success is that the theory has been about the vacuum properties much more than about excitations of the vacuum, where Minkowski-space description is essential. (If the energy of the system is zero, then its analytic continuation to imaginary energies does not change too many things.) Nevertheless the Euclidean analysis has led to an effective action, which can be continued to Minkowski space-time and used for dynamics.

The model of the nucleon is built from this action via the Skyrme ansatz for the pion "condensate":

$$U \equiv e^{i\vec{\tau}\cdot\vec{\pi}} = e^{i\vec{\tau}\cdot\hat{r}f(r)} \tag{14}$$

with

$$f(0) = \pi \qquad f(\infty) = 0 \; . \tag{15}$$

Because

$$U(0) = -1 \tag{16}$$

and

$$U(\infty) = +1 \tag{17}$$

the pion field contains the "topological knot"; U cannot be continuously deformed to the unit matrix.

Now the Dirac equation is solved in this pion field

$$i \not\nabla - MU(r) = 0 \tag{18}$$

and, as already advertised, one bound state is found to exist with $|E| < M$. The bound-state wave function is then determined by calculating the summed energy of the negative-energy Dirac sea and the bound state contribution as a function of the trial function $f(r)$, and then minimizing with respect to the choice of f. The resulting structure is classical, and the quantum structure is built by using the "cranking model", *i.e.* projecting the constructions on eigenfunctions of rotations and translations. The nucleon and Δ masses can be calculated; the nucleon mass is somewhat on the high side (1200 MeV or so), although there are several candidate apologies for this situation. With this model, a variety of nucleon static properties are calculated with reasonable success.

6.4 What implications does this have for deep-inelastic structure functions?

An especially interesting application of the model is in the construction of the primordial parton distributions, defined as follows:[35]

$$\left.\begin{array}{rl} q(x) & x > 0 \\ -\overline{q}(-x) & x < 0 \end{array}\right\} = \frac{1}{4\pi} \int_{-\infty}^{\infty} dt\, e^{ixMt} \left\langle P \,|\, \psi^\dagger(0)(1+\gamma^0\gamma^2)\,\psi(y) \,|\, P \right\rangle \ . \tag{19}$$

with

$$y = (t, -t, 0, 0) \ . \tag{20}$$

This is to be interpreted as the input parton distributions at the highest value of scale allowed by the effective chiral theory, namely the scale associated with the typical instanton size, 600 MeV, or $Q^2 \approx 0.4\ GeV^2$. Note that it is defined in the nucleon rest frame, but when boosted to an infinite-momentum frame becomes the usual correlation function defining the parton distributions.

Note that the definition in Eq. (19) admits the introduction by necessity of valence antiquark distributions. And, as mentioned earlier, the contribution of the discrete level by itself leads to negative-definite valence antiquark distributions. It is necessary to calculate the (distorted) negative-energy continuum contributions before obtaining sensible results. When this is carefully done, the antiquark distributions happily are positive definite. Some of these are shown in Figs. 1–5. In particular,

in Fig. 3, which exhibits the flavor singlet antiquark distributions, is sketched the negative contribution of the discrete level, as well as the summed result.

A variety of deep-inelastic sum-rules are also tested, and shown to be in principle (as well as numerically) satisfied. These include the sum rules for baryon number, momentum (at this level all momentum is carried by quarks), isospin, and flavor-nonsinglet polarized distributions. Also, the Gottfried sum, which measures the flavor non-singlet antiquark distribution, is calculated and has nonvanishing right-hand side, with the sign needed to account for the data. The argumentation for these results goes deep into the basic structure of the model, and the consistency is very satisfying.

It would be a great advance if the description of mesons, for which there is no Skyrmionic topological starting point, could be carried to the same level of sophistication. Are mesons really so different from baryons? I think the best candidate for study is the B-meson. If progress can be made there, it may also shed light on the confinement issue, which so far has remained beyond the scope of these methods.

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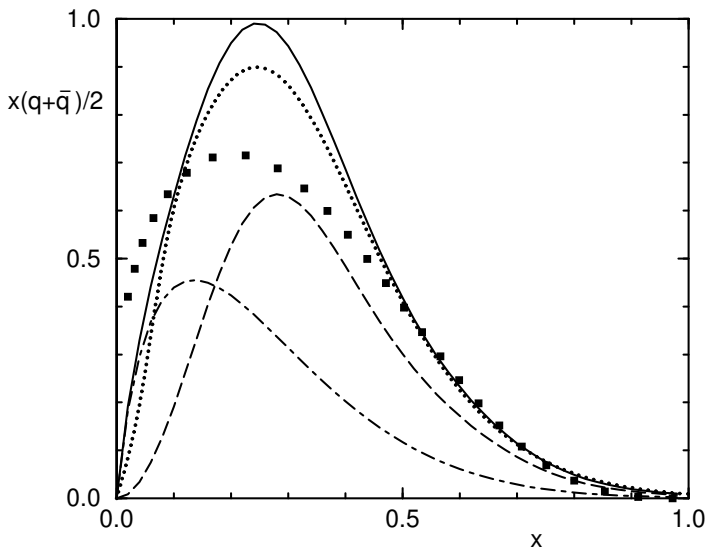

Figure 1: The singlet unpolarized distribution, $x[u(x)+d(x)+\overline{u}(x)+\overline{d}(x)]/2$. Dashed line: regularized contribution from the discrete level; dash-dotted line: contribution from the Dirac continuum; solid line: the total distribution, namely the sum of the dashed and dash-dotted curves, dotted line: the exact total distribution; squares: the parametrization of Ref. 32.

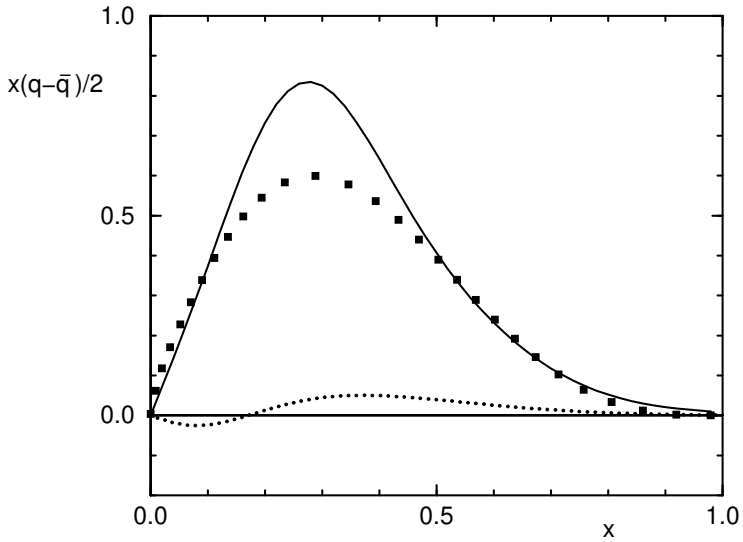

Figure 2: The baryon number distribution, $x[u(x)+d(x)-\overline{u}(x)-\overline{d}(x)]/2$. Solid line: distribution from the unregularized discrete level; dotted line: exact Dirac continuum contribution; squares: the parametrization of Ref. 32.

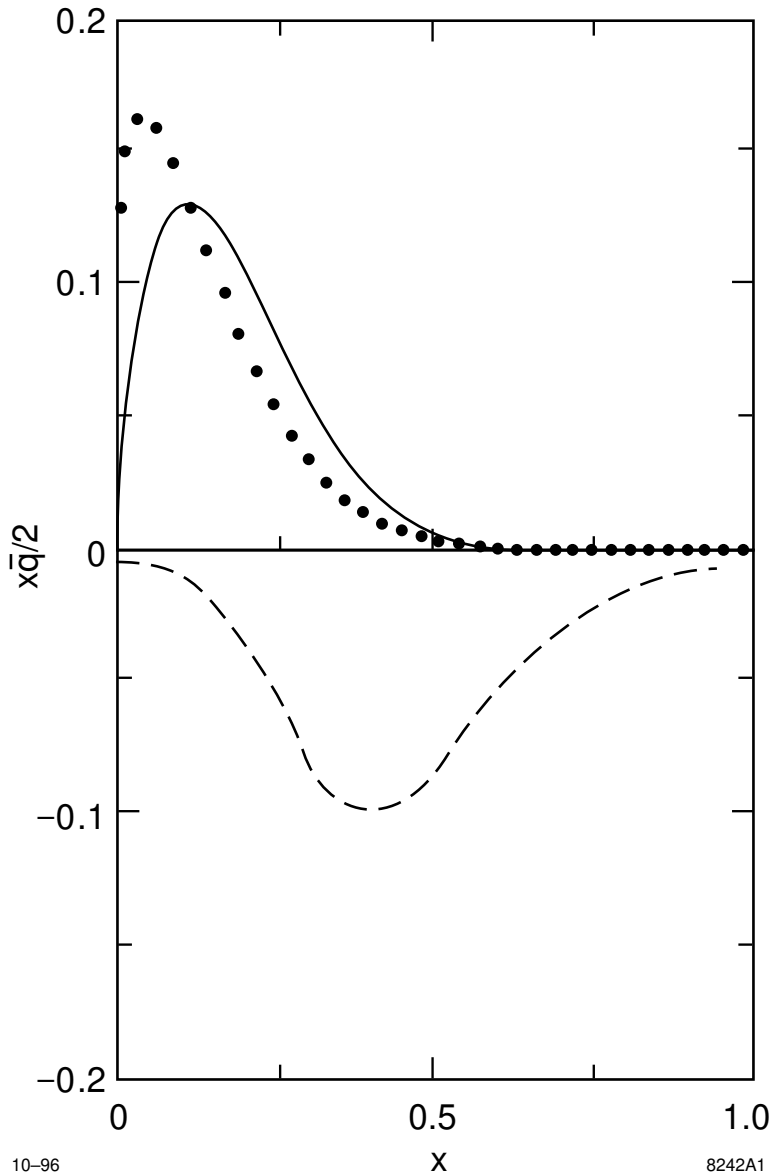

Figure 3: The antiquark distribution, $x[\overline{u}(x) + \overline{d}(x)]/2$. Solid line: theory; squares: the parametrization of Ref. 32; dashed line, contribution from the discrete level only.

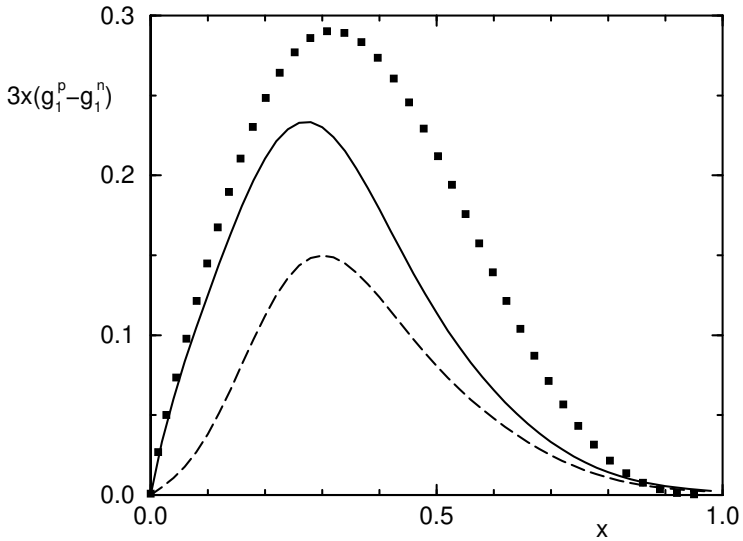

Figure 4: The isovector polarized distribution, $x[\Delta u(x) - \Delta d(x) + \Delta\overline{u}(x) - \Delta\overline{d}(x)]/2$. Dashed line: regularized contribution from the discrete level; solid line: the sum of the contributions from the discrete level and from the continuum; squares: the parametrization of Ref. 32; dashed line: contribution from the discrete level only.

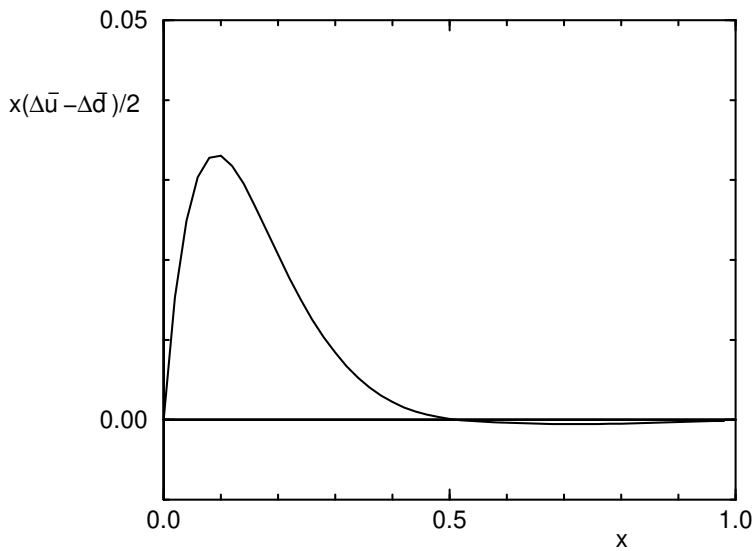

Figure 5: The isovector polarized distribution of antiquarks, $x[\Delta\overline{u}(x) - \Delta\overline{d}(x)]/2$. Reference 32 assumes this quantity to be zero.