

Differences between Monte Carlo models for DIS at small- x and the relation to BFKL dynamics *

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Abstract

The differences between two standard Monte Carlo models, LEPTO and ARIADNE, for deep inelastic scattering at small- x is analysed in detail. It is shown that the difference arises from a ‘unorthodox’ suppression factor used in ARIADNE which replaces the normal ratio of parton densities. This gives rise to a factor that qualitatively is similar to what one would expect from BFKL dynamics for some observables like the energy flow and forward jets but not for the 2+1 jet cross-section. It is also discussed how one could use the 2+1 jet cross-section as a probe for BFKL dynamics.

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Deep inelastic electron proton scattering at small- x (with x being the ordinary Bjorken- x) has drawn much theoretical and experimental interest the last couple of years. Since the observation [1,2] of the predicted [3] rise of the deep inelastic structure function F_2 at small- x , the main question has been whether the ‘unconventional’ so called BFKL [4] dynamics can be observed or whether the interactions can be described by ordinary leading $\log(Q^2)$ dynamics in perturbative QCD as given by the DGLAP [5] evolution equation. From the inclusive measurement of F_2 it seems not possible to draw any such conclusions based on the presently available data and it is also theoretically questionable whether it will be at all possible as long as one only considers one observable [6].

Other observables that have been proposed to see effects of BFKL dynamics are the transverse energy flow [7] and forward jets [8]. One problem in looking for possible effects of BFKL dynamics is that so far there has only been analytical calculations on parton level and the hadronisation effects are difficult to estimate. In this respect the forward jets analysis is more promising since the hadronisation corrections are expected to be small for a jet cross-section. However, the forward jets are close to the target remnant jet which makes it difficult to disentangle the two.

So far there exists no Monte Carlo (MC) model based on the BFKL equation even though there has recently been significant progress in creating such a model [9] based on the CCFM equations [10] which interpolate smoothly between the BFKL and DGLAP equations. In the mean time the ARIADNE [11] MC which is based on the colour dipole model for DIS [12] has often been used to estimate the possible effects of BFKL dynamics. The argument for this has been that it also contains parton emissions which are non-ordered in transverse momentum. However, the present paper will show that the difference between ARIADNE and ordinary DGLAP evolution is mainly due to a ‘unorthodox’ suppression factor used for additional emissions from dipoles connected with the proton remnant. Comparing the results from the ARIADNE MC with data has also shown rather good agreement for the transverse energy flows and forward jets whereas a pure DGLAP Monte Carlo fails. There are, however, large hadronisation uncertainties and it has been shown that the LEPTO [13] MC model based on leading order (LO) QCD matrix-elements and leading logarithmic corrections in the form of parton showers according to the DGLAP evolution equation supplemented with non-perturbative hadronisation effects can also describe the data on transverse energy flow [14].

An observable which at first sight might seem more or less uncorrelated to the question of whether one observes BFKL dynamics or not is the 2+1 jet cross-section. Since this is a genuinely hard process it should be well described by conventional perturbative QCD as long as the more complicated forward region is excluded. However this is not the case. As argued in this paper the 2+1 jet cross-section should exhibit features of BFKL dynamics when the mass \hat{s} of the jet-system becomes much larger than the momentum transfer \hat{t} from the incoming parton. In other words one should be able to observe effects of BFKL dynamics as the rapidity difference ($\Delta y = \ln(\hat{s}/|\hat{t}|)$) between the two jets increases. This is just the same kind of mechanism as in dijet production in hadron collisions as proposed by Mueller and Navelet [15]. There has also been a proposal [16] to use the 2+1 jet cross-section for small jet-systems as a probe of BFKL dynamics since this would essentially probe the x -distribution of gluons at small x where effects of BFKL dynamics should be possible to observe. A complication with this proposal is that the gluon density is not an observable

and the constraints from data on what it should be in a DGLAP evolution scenario are small especially if one takes into the account the theoretical uncertainties in defining the gluon density. There are also experimental difficulties in finding jets for small mass jet-systems.

At large values of photon virtuality Q^2 (typically $Q^2 > 100 \text{ GeV}^2$) the measured jet cross-section has been shown to be in good agreement [17–20] with the Monte Carlo model LEPTO and next-to-leading order (NLO) perturbative QCD calculations using programs such as DISJET [21] and PROJET [22]. In addition, the ARIADNE MC also describes the y_{cut} dependence (where y_{cut} is the cut used in the JADE jet-definition, $m_{ij}^2 \leq y_{cut} W^2$) of the data with a similar accuracy to the one found for LEPTO whereas the Q^2 -dependence is not so well described [18,19]. These cross-section measurements for large Q^2 have also been used for the extraction of α_s and the observation of its running [19,20].

However, if one instead looks at the small- x and small Q^2 region, where $x = Q^2/2P \cdot q$ with q and P being the photon and proton four momenta respectively, but still avoiding the more complicated forward region, the picture does not seem to be so clear anymore. There is still fairly good agreement between LEPTO and data [18,19] but the two MC models give quite different results. As an example Table I gives the 2+1 jet cross-section on matrix-element level as calculated with LEPTO 6.5, ARIADNE 4.08 and MEPJET 1.1 [23] which is a NLO perturbative QCD calculation. The kinematical region used is: $0.0001 < x < 0.001$, $0.05 < y_B < 0.7$ ($y_B = q \cdot P/k \cdot P$ with k being the momentum of the incoming lepton), $5.0 < Q^2 < 70.0 \text{ GeV}^2$ and the MRS A parton distributions [24] have been used. The jets have been defined using the cone-algorithm in the hadronic center of mass system ($hcms$) with a cone-size of $R = \sqrt{\Delta\eta^2 + \Delta\phi^2} = 1$ and requiring the jets to have transverse momenta $p_{\perp} > 5 \text{ GeV}$ and $0.1 < z < 0.9$ (where $z = P \cdot j_i / P \cdot q$ with j_i being the jet-momenta) to avoid the forward region. This gives jets which have pseudo-rapidities in the photon-hemisphere in the $hcms$ and $-2 < \eta < 2$ in lab, thus avoiding the forward region. The cut in z also decreases the effects from parton showers on the 2+1 jet cross-section to less than 10 percent [19].

TABLE I. Jet cross-sections for $0.0001 < x < 0.001$, $0.05 < y_B < 0.7$ and $5.0 < Q^2 < 70.0 \text{ GeV}^2$.

Program	μ_F^2, μ_R^2	$\sigma [nb]$
ARIADNE	Q^2, p_{\perp}^2	6.38
LEPTO	Q^2	1.52
MEPJET LO	Q^2	1.49
MEPJET LO	$Q^2, (\Sigma p_{\perp}/2)^2$	1.19
MEPJET LO	$(\Sigma p_{\perp}/2)^2$	1.45
MEPJET NLO	Q^2	0.89 ± 0.06
MEPJET NLO	$(\Sigma p_{\perp}/2)^2$	0.95 ± 0.04

The first thing to note is that the 2+1 jet cross-section predicted by the ARIADNE MC is about four times larger than the one from LEPTO. Comparing with the LO result from MEPJET using Q^2 as renormalisation and factorisation scale shows good agreement with LEPTO whereas the LO result from MEPJET with the same factorisation and renormalisation scales as in ARIADNE differs a factor five from the ARIADNE result. Clearly the LO 2+1 jet cross-section in ARIADNE does not agree with the result from LO perturbative QCD. The

second thing to note is that the NLO corrections are quite large and that they in fact are negative such that the NLO cross-section is smaller than the LO one. Of course the question of choosing an appropriate renormalisation scale and factorisation scheme and scale has to be addressed if one wants to compare the NLO cross-sections with data but for the present purposes, i.e. to see that the NLO-corrections cannot increase the cross-section by a large factor it is not necessary to pursue these theoretical uncertainties further (for a discussion of the renormalisation scale uncertainty see for example [25]).

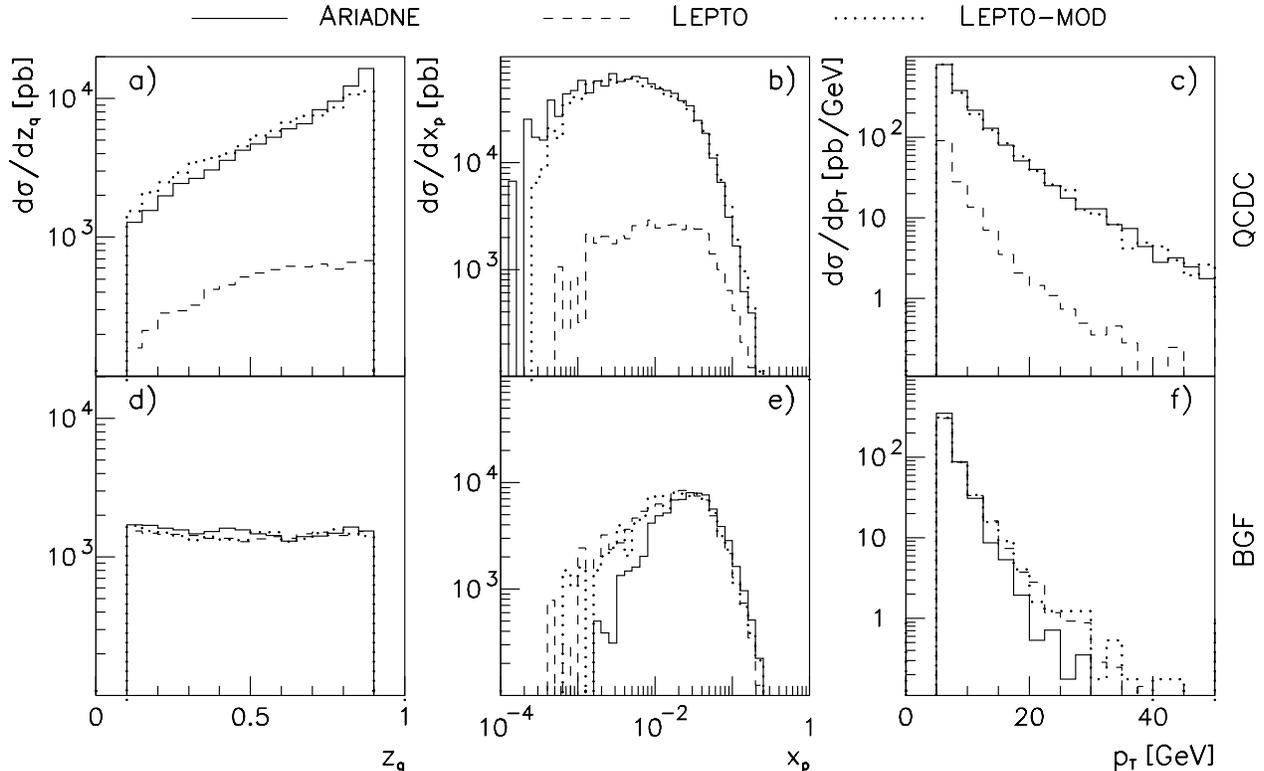


FIG. 1. The z_q , x_p and p_\perp distributions from the LO matrix-elements in ARIADNE 4.08 (full line), LEPTO 6.5 (dashed line) and the modified version of LEPTO (dotted line) for the two basic processes QCD-compton (QCDC) and Boson Gluon Fusion (BGF).

To see in more detail what the difference between ARIADNE and LO perturbative QCD is (as represented by LEPTO), it is instructive to look at the z , x_p and p_\perp distributions which are given in Fig. 1. Here $x_p = x/\xi$ where ξ is the longitudinal momentum share of the parton entering the hard interaction. For clarity the cross-section has been divided into two different parts, QCD-compton (QCDC) and Boson Gluon Fusion (BGF), depending on the underlying hard process as illustrated in Fig. 2. As can be seen from Fig. 1, the BGF part agrees well between the two programs taking into account the different renormalisation scales, but for the QCDC part the difference is dramatic and requires some other explanation.

In fact, this difference should come as no surprise. The colour dipole model for DIS does not use the pure LO perturbative QCD matrix-element for the QCDC process but rather

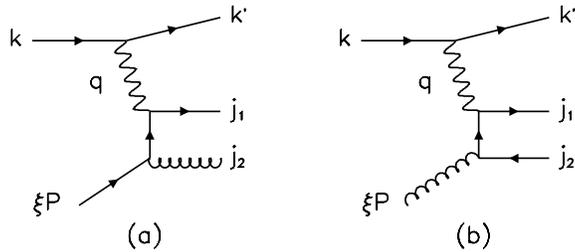


FIG. 2. Illustration of the two leading order 2+1 jet processes, QCD-compton (a) and Boson Gluon Fusion (b).

an approximation. For a given x and Q^2 the cross-section can (assuming single photon exchange) be written as,

$$\frac{d\sigma_{QCDC}(x, Q^2, z, x_p)}{\sigma_{tot}(x, Q^2)} = \frac{C_F \alpha_s}{2\pi} \frac{f_q(\xi, Q^2)}{f_q(x, Q^2)} g(x, Q^2, z, x_p) \frac{dx_p}{x_p} dz \quad (1)$$

where f_q is the sum of the quark densities in the proton and $g(x, Q^2, z, x_p)$ is a simple function which is not needed for the present discussion. In the colour dipole model for DIS the ratio of the parton densities is replaced by a suppression factor due to the extended proton remnant,

$$\frac{f_q(\xi, Q^2)}{f_q(x, Q^2)} \rightarrow \Theta \left(\frac{W}{e^y + (p_\perp/\mu)^a e^{-y}} - p_\perp \right). \quad (2)$$

where y is the rapidity of the emitted parton in the $hcms$ and W is the mass of the hadronic system ($W^2 = (P + q)^2$). This limits the transverse momentum p_\perp of the emitted gluon which corresponds to that only a fraction $(\mu/p_\perp)^a$ of the proton remnant takes part in the emission. Normally one uses $a = 1$ but other choices are also possible. Now, for small- x the suppression factor used in the colour dipole model for DIS starts to differ substantially from the ratio of the parton densities. In the region of interest x is very small and ξ is moderately small (typically $10^{-2} - 10^{-1}$) so that this ratio goes essentially as $x_p^{1+\lambda}$ if one assumes that $f_q(x) \propto x^{-1-\lambda}$. For recent parton distributions λ is in the order of 0.2 to 0.3 so this gives a strong suppression of small x_p in the LO perturbative QCD formula.

To see that it really is this suppression factor that gives the difference a "toy-model" version of LEPTO has been constructed. In this modified version of LEPTO the QCDC matrix-element has been multiplied by a phenomenological factor x_p^{-b} where $b = d - 0.05 \ln 1/x_p$ with d being a function of x . The value of d was obtained by a fit to the ratio of $d\sigma_{QCDC}/dx_p$ from ARIADNE and LEPTO for fixed x and Q^2 . The resulting powers turn out to be easily parameterised by $d = 0.2(1 - \log_{10} x)$ in the region of interest ($10^{-4} < x < 10^{-3}$). As can be seen in Fig. 1 this "toy-model" reproduces the distributions from ARIADNE to a very good approximation, especially the p_\perp -distribution. Thus, the difference between the QCDC cross-section in LEPTO and ARIADNE can be explained by a factor x_p^{-b} which results from using the suppression factor $\Theta \left(\frac{W}{e^y + (p_\perp/\mu)^a e^{-y}} - p_\perp \right)$ instead of $f_q(\xi, Q^2)/f_q(x, Q^2)$. As an

aside, it is interesting to note that for the BGF-part of the jet cross-section, the suppression factor used in ARIADNE is indeed the corresponding ratio of the parton densities.

The important thing to note is that all emissions in ARIADNE, where the proton remnant is part of the dipole, are treated in the same way as the first emission in the QCDC matrix-element. To be more precise the probability for an extra emission, when the proton remnant is part of the dipole, is given by [26]

$$dP \propto \frac{4\alpha_s}{3\pi} \Theta \left(\frac{W}{e^y + (p_\perp/\mu)^a e^{-y}} - p_\perp \right) \frac{dp_\perp^2}{p_\perp^2} dy. \quad (3)$$

This means that the ratio of parton densities $f_a(x_a, Q^2)/f_b(x_b, Q^2)$ does not enter as they would in a traditional backwards evolution initial state parton shower where,

$$dP_{a \rightarrow bc} \propto \frac{4\alpha_s}{3\pi} \frac{f_a(x_a, Q^2)}{f_b(x_b, Q^2)} \frac{dp_\perp^2}{p_\perp^2} dy. \quad (4)$$

Instead the boundary condition present from the proton is taken into account by the extra cut-off in transverse momenta for emitted gluons due to the extendedness of the proton remnant. A direct comparison with a traditional initial state parton shower is complicated by the fact that the emissions in the colour dipole model for DIS are not ordered in x . But even so, it is evident that the difference in the suppression factor gives a large effect for the rest of the emissions just as for the QCDC matrix-element. There are two main effects. First of all the probability for an emission is increased and secondly the emissions become harder, in the sense that they get higher p_\perp , just in the same way as the QCDC part of the 2+1 jet cross-section increases and the p_\perp spectrum becomes harder (see Fig. 1c).

To estimate the magnitude of the effects on the transverse energy flow due to the ‘unorthodox’ suppression factor the modified version of LEPTO with the increased QCDC cross-section has been used to calculate the transverse energy flow. The results are shown in Fig. 3 where data from H1 [27] are compared with ARIADNE, LEPTO and the modified version of LEPTO. One should here note that to be able to compare with more or less pure DGLAP dynamics the soft colour interactions and the new seaquark treatment in LEPTO have been shut off for both LEPTO versions (see [14] for more details).

As can be seen from Fig. 3 the modified version of LEPTO interpolates nicely between the ARIADNE result in the photon hemisphere where the first emission is important and the DGLAP-LEPTO result in the proton hemisphere where higher order emissions are important. In other words, the modified matrix-element increases the transverse energy flow to the same level as in Ariadne in the photon hemisphere, whereas in the proton hemisphere there is no difference between the two versions of LEPTO since the same initial state parton shower is used. It would also be possible to modify the initial state parton shower in LEPTO in the same way as the matrix-element (essentially by multiplying the splitting function $P_{q \rightarrow qG}(z)$ with a factor z^{-b}) which would lead to an increased transverse energy flow in the proton hemisphere. However, already from the result based on the modified QCDC matrix-element one can conclude that the difference in transverse energy flow between DGLAP-LEPTO and ARIADNE is mainly due to the ‘unorthodox’ suppression factor used in ARIADNE instead of the ratio of parton densities and not due to the difference in p_\perp -ordering.

The following question then arises. Has this difference in suppression factor anything to do with BFKL dynamics? As it turns out, the answer depends on which observable one

is interested in. Starting with the 2+1 jet cross-section one first notes that the relevant scales for the onset of BFKL dynamics are the momentum transfer \hat{t} from the incoming gluon and the mass \hat{s} of the jet-system. One expects effects of BFKL dynamics when the rapidity difference $\Delta y = \ln(\hat{s}/|\hat{t}|)$ becomes large so that $\alpha_s \Delta y \sim \mathcal{O}(1)$ which indicates that resummation is necessary. In the *hcms* the relevant ratio becomes $|\hat{t}|/\hat{s} \simeq p_\perp^2/\hat{s} = z(1-z)$. This is quite different from the increase with decreasing $x_p = Q^2/(Q^2 + \hat{s})$ given by ARIADNE. One also notes that typical ξ values are of the order of $10^{-2} - 10^{-1}$ which is not very small and therefore one does not expect any effects from the BFKL dynamics in the parton densities.

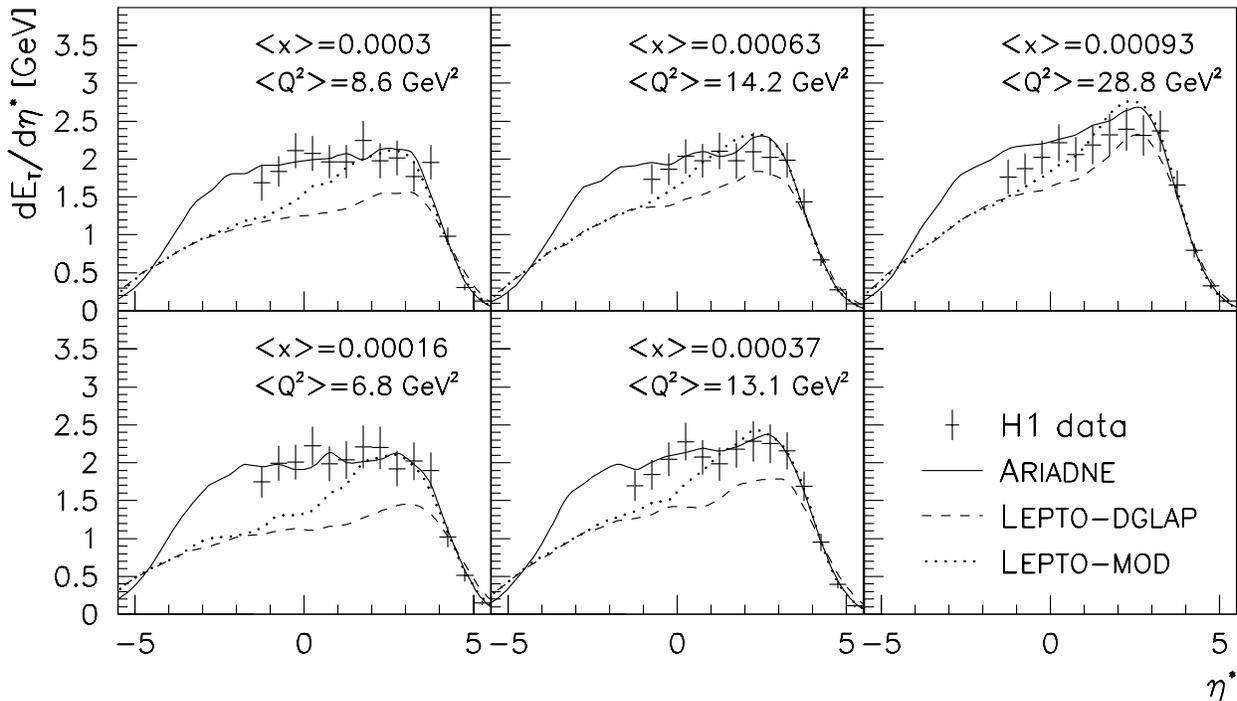


FIG. 3. The transverse energy flows for different x and Q^2 values in the hadronic cms comparing ARIADNE (full line), LEPTO with pure DGLAP (dashed line) and the modified version of LEPTO (dotted line) with data from H1 [27]. (The plot has been generated using the HzTool package [28]).

The dominating diagram for 2+1 jet production with enhancement from the resummation of soft gluons according to the BFKL prescription is depicted in Fig. 4 where the two partons from the quark box form one jet. In addition to this process there is also a similar one where the incoming and outgoing gluon is replaced by a quark. At sufficiently large rapidity difference Δy the contribution from this kind of diagrams should be dominating thanks to the t-channel gluon exchange. This is the same kind of diagram that has been analysed for the forward jet production [8] but with the important difference that two jets are required. So instead of having k_\perp of the forward jet being of the same order as Q for the BFKL dynamics to be valid, two jets with the same p_\perp are required. In a sense this is closer to the original proposal by Mueller and Navelet [15] to look for BFKL effects in dijet-production

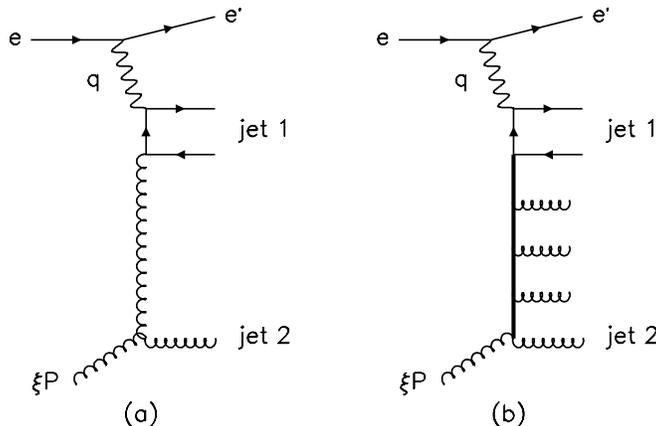


FIG. 4. Diagrams with enhancement from BFKL dynamics in the large Δy limit: (a) lowest order diagram (b) all orders diagram with the reggeized gluon indicated with a thick line. In addition there is a similar diagram where the incoming and outgoing gluon is replaced by a quark.

in hadron collisions.

In a BFKL calculation of the 2+1 jet cross-section one would take the large $\Delta y = \ln(\hat{s}/|\hat{t}|)$ limit of the 2+1 jet cross-section and reggeize the t-channel gluon which would give a cross-section of the type [29]

$$\frac{d\sigma}{d\xi d\Delta y} \propto K_{BFKL} \frac{d\hat{\sigma}}{d\xi d\Delta y} \Big|_{large \Delta y} \left[\xi f_g(\xi, \mu^2) + \frac{C_F}{N_C} \xi f_q(\xi, \mu^2) \right], \quad (5)$$

where the effective parton density [30], $f_g(\xi, \mu^2) + \frac{C_F}{N_C} f_q(\xi, \mu^2)$, reflects the domination of t-channel gluon exchange for large Δy . The factorisation scale μ is arbitrary but should be of the order of transverse momentum cut-off for the jets, $p_{\perp, \min}$.

In a leading order calculation the enhancement factor is equal to one so by forming the ratio between the BFKL calculation and the fixed order calculation for fixed ξ one isolates the K_{BFKL} -factor. Thus, as a probe for BFKL dynamics, it should be possible to use the 2+1 jet cross-section as a function of the rapidity difference Δy between the jets in the $hcms$ (for fixed ξ) and see whether the data start to deviate exponentially from the NLO fixed order perturbative QCD predictions when Δy becomes large. To be able to do a quantitative analysis one would of course need to actually calculate the proposed cross-section.

Integrating over \hat{t} from $-p_{\perp, \min}^2$ the asymptotic form of the enhancement factor will be [29]

$$K_{BFKL} = \frac{\exp(\alpha_s C_A \Delta y 4 \ln 2 / \pi)}{\sqrt{7 \alpha_s C_A \zeta_3 \Delta y / 2}}. \quad (6)$$

Using the standard value for $\alpha_s C_A 4 \ln 2 / \pi = 0.5$ this gives,

$$K_{BFKL} \propto \exp(\alpha_s C_A \Delta y 4 \ln 2 / \pi) = \left(\frac{\hat{s}}{p_{\perp, \min}^2} \right)^{0.5}. \quad (7)$$

For forward jets $-\hat{t} \simeq Q^2$ is required and thus $\Delta y \simeq \ln(1/x_p)$ such that $K_{BFKL} \propto \left(\frac{1}{x_p}\right)^{0.5}$ which also is consistent with the results from [31].

Returning to the cross-section for the QCDC process used in ARIADNE, it can be written in the following way,

$$\begin{aligned} \frac{d\sigma_{QCDC}(x, Q^2, z, x_p)}{\sigma_{tot}(x, Q^2)} &= \frac{C_F \alpha_s}{2\pi} \frac{f_q(\xi, Q^2)}{f_q(x, Q^2)} \frac{\Theta\left(\frac{W}{e^{y+(p_\perp/\mu)^{\alpha_e-y}} - p_\perp}\right)}{\frac{f_q(\xi, Q^2)}{f_q(x, Q^2)}} g(x, Q^2, z, x_p) \frac{dx_p}{x_p} dz \\ &\simeq \frac{C_F \alpha_s}{2\pi} \frac{f_q(\xi, Q^2)}{f_q(x, Q^2)} \left(\frac{1}{x_p}\right)^b g(x, Q^2, z, x_p) \frac{dx_p}{x_p} dz \end{aligned} \quad (8)$$

with b being in the order of $0.5 - 1.0$ for small x . Qualitatively this gives a K -factor x_p^{-b} which is similar to the one in the forward jet BFKL calculation [31]. This also explains why ARIADNE agrees quite well with data on forward jets [27] just in the same way as the forward jet BFKL calculation [31] agrees quite well with data.

Even though there is some qualitative agreement between the colour dipole model and a BFKL calculation for forward jets there are quantitative differences which are important: *(i)* the colour dipole model has an enhancement already for the first emission whereas the BFKL enhancement only comes in at higher order as depicted in Fig. 4, *(ii)* the factor giving the enhancement from the BFKL dynamics multiplies the cross-section both for incoming gluon and incoming quark, *(iii)* the BFKL enhancement is in the hard cross-section whereas in the colour dipole model the enhancement comes from replacing the ratio of parton densities with a cut-off in transverse momentum for the emitted gluon. It should also be stressed that for the 2+1 jet cross-section the ARIADNE prediction gives an even larger enhancement of the cross-section than one would expect from BFKL dynamics.

In summary this paper has shown the underlying dynamical reason for the differences in 2+1 jet cross-sections, forward jet cross-sections and transverse energy flow between the DGLAP based LEPTO Monte Carlo and the ARIADNE Monte Carlo based on the colour dipole model for DIS. The dominating difference is the ‘unorthodox’ suppression factor used instead of the ratio of parton densities and not the p_\perp -ordering. The difference can essentially be parameterised by a factor x_p^{-b} , where b is in the order of $0.5 - 1.0$ for small x , which multiplies the cross-section for additional emissions in ARIADNE. For the forward jet production and the transverse energy flow this factor x_p^{-b} resembles qualitatively what one gets from a BFKL calculation where $b = \alpha_s C_A 4 \ln 2 / \pi \simeq 0.5$. However for the 2+1 jet production one rather expects an enhancement from BFKL dynamics as a function of the rapidity difference $\Delta y = \ln(\hat{s}/|\hat{t}|)$ between the jets. Thus, it should be possible to use the 2+1 jet cross-section for fixed ξ and its dependence on the rapidity difference as a probe for BFKL dynamics.

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