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Beam Position Monitor Engineering*

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Abstract

The design of beam position monitors often involves challenging system design choices. Position transducers must be robust, accurate, and generate adequate position signal without unduly disturbing the beam. Electronics must be reliable and affordable, usually while meeting tough requirements on precision, accuracy, and dynamic range. These requirements may be difficult to achieve simultaneously, leading the designer into interesting opportunities for optimization or compromise. Some useful techniques and tools are shown. Both finite element analysis and analytic techniques will be used to investigate quasi-static aspects of electromagnetic fields such as the impedance of and the coupling of beam to striplines or buttons. Finite-element tools will be used to understand dynamic aspects of the electromagnetic fields of beams, such as wakefields and transmission-line and cavity effects in vacuum-to-air feedthroughs. Mathematical modeling of electrical signals through a processing chain will be demonstrated, in particular to illuminate areas where neither a pure time-domain nor a pure frequency-domain analysis is obviously advantageous. Emphasis will be on calculational techniques, in particular on using both time-domain and frequency domain approaches to the applicable parts of interesting problems.

INTRODUCTION

We will work through a beam position monitor system from transducer to digitization; starting with a simple case, expressed in a simplified model. Then we will analyze a tougher problem, again using simple mathematical models. Finally we will apply more complicated analysis to understand some details of the system.

Signal Modeling

A common difficulty is choosing a particular approach to the problem at hand; is a frequency domain or a time domain approach more suitable. Table 1 contains some suggestions of when to best use either the time domain or frequency domain.

The approach used here is to describe time-domain phenomena as discrete quantities sampled at a uniform rate in time and then to transform back and forth

from time to frequency domain as needed to take advantages of the best features of each representation. For example let a voltage $V(t)$ be represented by N samples V_j sampled at times $t_j = t_{\max} \frac{j}{N}$. The discrete Fourier transform and its inverse are given by

$$fV_k = \frac{1}{N} \sum_j V_j e^{-2\pi \frac{jk}{N}} \quad (\text{FFT}) \qquad V_j = \sum_k fV_k e^{2\pi \frac{jk}{N}} \quad (\text{FFT}^{-1})$$

Table 1. Time domain versus frequency domain

Frequency Domain appropriate when:	Time Domain appropriate when:
Periodic processes	Single shot
High Q, low bandwidth	Low Q, high bandwidth
Frequency dependent parameters <ul style="list-style-type: none"> • Complex impedances • Filter response 	Amplitude dependent parameters, e.g. limits of linear range of components <ul style="list-style-type: none"> • Saturation (P_{1dB}) • Slew rates • Damage thresholds (V_{max})
Linear phenomena	Non-linear phenomena <ul style="list-style-type: none"> • mixers • diodes
Discrete frequency phenomena <ul style="list-style-type: none"> • oscillators 	Discrete-time operations <ul style="list-style-type: none"> • Sample & Hold • Digitization

ANALYZE A SIMPLE BPM

Let's analyze a button BPM in some ring with stored beam, bunched at frequency f_b . We wish to calculate the signal, noise, and position sensitivities. With this information we can establish a noise figure budget needed to achieve some required resolution. This is an obvious case for frequency domain analysis. (1,2)

We estimate the intrinsic resolution from the ratio of signal to thermal noise. First we calculate the signal. The image charge on a button is given by:

$$Q(\omega) = \frac{ButtonArea}{DuctCircumference} \cdot \rho(\omega)$$

where $\rho(\omega)$ is the linear charge density, which we have assumed varies slowly on the scale of the button size. Then the image current out of the button is given by:

$$I_{img} = \frac{dQ}{dt} = \frac{ButtonArea}{DuctCircumference} \cdot \frac{d\rho}{dt}$$

We have made the (usually excellent) approximation here that the beam acts as a perfect current source in generating image currents. Expressing the linear charge density in terms of the beam current:

$$\rho(\omega) = \frac{I(\omega)}{\beta c} \quad \text{so} \quad \frac{d\rho}{dt} = \frac{1}{\beta c} \cdot \frac{dI}{dt} = \frac{i\omega}{\beta c} \cdot I(\omega)$$

$$I_{img} = \frac{\pi\alpha^2}{2\pi b} \frac{i\omega}{\beta c} I(\omega)$$

The button voltage is the product of the current out of the button and the impedance seen by this current. The dominant pieces of this impedance are the impedance of the cable shunted by the reactance of some parasitic capacitance C_b between the button and the walls of the beam duct.

$$Z = Z_{coax} \parallel \frac{1}{i\omega C_b} \quad Z \rightarrow Z(\omega)$$

$$V_b = Z \cdot I_{img} = \frac{\pi\alpha^2}{2\pi b} \frac{\omega}{\beta c} Z(\omega) I(\omega)$$

Later we will find it useful to rearrange this so the button voltage appears as the product of beam current times something that looks like an impedance, which we will call a "transfer impedance" Z_t .

$$V_b = Z_t(\omega) I(\omega) \quad \text{where in this case} \quad Z_t = \frac{\pi\alpha^2}{2\pi b} \frac{\omega}{\beta c} Z(\omega)$$

For an average current I_{avg} circulating with angular frequency $\omega_b = 2\pi f_b$ the beam current is given by:

$$I(t) = I_{avg} \cdot \left[1 + 2 \sum_m A_m \cos(m\omega_b t) \right]$$

So in frequency space the current consists of a line spectrum where the amplitude of the m^{th} harmonic is given by

$$I_m = I_{avg} \cdot \begin{cases} 1 & \text{for } m = 0 \\ 2A_m & \text{for } m > 0 \end{cases}$$

The coefficients A_m are determined by the shape of the bunch and are near unity for frequencies well below the inverse of the bunch length. Now we pick a processing frequency $f_0 = m\omega_b / 2\pi$ corresponding to the m^{th} revolution harmonic and find the signal voltage:

$$V_b = \frac{\pi a^2 Z}{2\pi b \beta c} \cdot \frac{dI}{dt} = \frac{\pi a^2 Z}{b \beta c} 2A_m f_0 I_{avg}$$

To make our example more concrete, we specify a few of the parameters.

Table 2. Example parameters for a narrow-band beam position monitor

Parameter	Symbol	Value
Duct radius	b	3 cm
Beam current	I_{avg}	10 mA
Bunch frequency	f_b	500 MHz
Button radius	a	5 mm
Coax impedance	Z	50 Ω
Beam velocity	β	1
Measurement bandwidth	B	1 MHz
Required resolution	σ_x	10 μm
Processing harmonic	m	1

The signal power is

$$P_s = \frac{1}{2} \frac{V_b^2}{Z} = \frac{2\pi^2 a^4}{b^2 \beta^2 c^2} Z A_m^2 f_0^2 I_{avg}^2 = 0.19 \mu W = -37 dBm$$

The noise power in this bandwidth is:

$$P_n = k_B T Z B = -114 dBm \quad (T=300 K, B = 1 MHz)$$

$$SNR = \frac{P_s}{P_n} = 77 dB$$

For small displacements from the center of the beam duct, the beam position in terms of voltages on hypothetical left and right buttons is given by:

$$X = \frac{b}{2} \cdot \frac{V_L - V_R}{V_L + V_R} \quad (\text{Difference-over-Sum algorithm})$$

Translating the voltage noise into a position error

$$\sigma_x = \frac{b}{2} \cdot \frac{\sqrt{2} \sigma_V}{2V} = \frac{b}{2\sqrt{2}} \cdot \frac{1}{\sqrt{SNR}} = 1.5 \mu m$$

This assumes the parameters in Table 1 and noiseless, lossless processing. Assuming a required resolution of $\sigma_x = 10 \mu m$, we have a noise and loss budget

$$F_n = \frac{10 \mu m}{1.5 \mu m} = 6.7 = 16 dB$$

which we can allocate to losses and electronics noise.

A MORE INTERESTING CASE...

The first example was readily handled in the frequency domain. Now let's look at a system which must respond to a single bunch, a few bunches, or continuous train of beam bunches. In this case we'll need to choose the most convenient point of view for each aspect of the problem. We'll follow the signal from the beam pipe to the ADC. We'll start by analyzing the response of a button BPM to a single beam bunch. Essential parameters for this example are listed in Table 2. The tool we use to do all of the calculations, transformations, and plotting is Mathcad.

Table 3. Example parameters for a wide-band beam position monitors, taken (loosely) from the PEP-II straight section BPM's.

Parameter	Symbol	Value
Duct radius	b	4.4 cm
Bunch charge	Q	$8 \cdot 10^8$ electrons
Bunch frequency	f_b	238 MHz
Button radius	a	7.5 mm
Coax cable impedance	Z	50 Ω
Beam velocity	β	1
Bunch shape	Gaussian	
Bunch length (rms)	σ_z	1 cm
Processing harmonic	m	4

Again we start at the button, but this time we calculate the image charge as a function of time. At any given time, the image charge on the button is found by integrating the button angular coverage over the longitudinal beam charge distribution:

$$Q_{img}(t) = \int \rho(z) \cdot \frac{width(z)}{2\pi b} \cdot dz$$

where z is the distance coordinate in the beam direction and the button width as a function of z is given by

$$width(z) = 2\sqrt{a^2 - z^2}$$

for a round button of radius a . Since the beam charge density propagates down the beam pipe at $v = c$ the time dependent image charge

$$Q_{img}(t) = \int \rho(z - ct) \cdot \frac{width(z)}{2\pi b} \cdot dz$$

This is just the convolution of the charge density and the button shape so we can evaluate by multiplying the Fourier transforms and transforming back:

$$Q_{img}(t) = FFT^{-1}\left(FFT\left(\frac{width(ct)}{2\pi b}\right) \cdot FFT(\rho(ct))\right)$$

Image current is the time derivative of the image charge, which is easy to do by transforming to the frequency domain, multiplying by $i\omega$, and transforming back to the time domain.

$$I_{img} = \frac{dQ_{img}}{dt} = FFT^{-1}(i\omega \cdot FFT(Q_{img}))$$

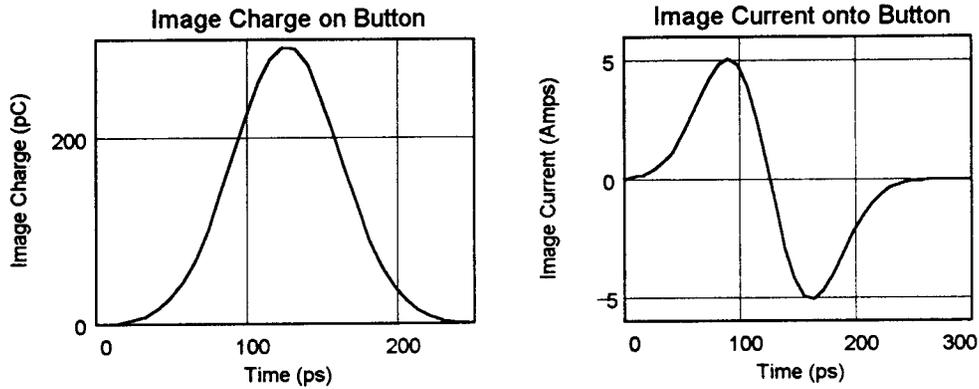


Figure 1. Image charge and current vs. time

The impedance seen by the button is that of the coax shunted by the button capacitance. Expressed as a function of frequency the impedance is given by

$$Z_b = (Z_0^{-1} + i\omega C_b)^{-1} \quad \text{therefore} \quad V_b = FFT^{-1}(Z_b \cdot FFT(I_{img}))$$

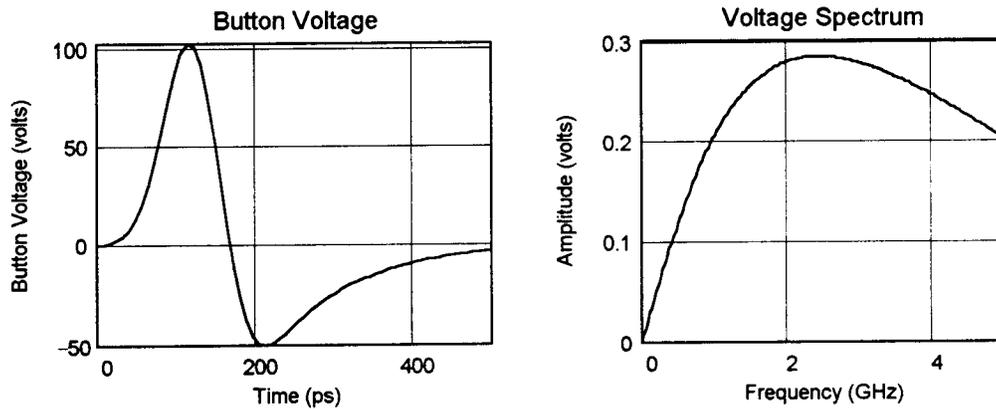


Figure 2. Signal voltage on a button and its frequency spectrum

A coaxial cable with skin effect losses has a frequency response given by (3)

$$f_{coax}(f) = e^{-\sqrt{\frac{f}{f_e}}}$$

Here f_e is the frequency at which the amplitude is attenuated by a factor of e . For example a cable of length 40 m with a loss of 6 dB/100ft will have $f_e=1.2$ GHz. Convoluting this response with the button voltage yields the voltage at the other end of the cable.

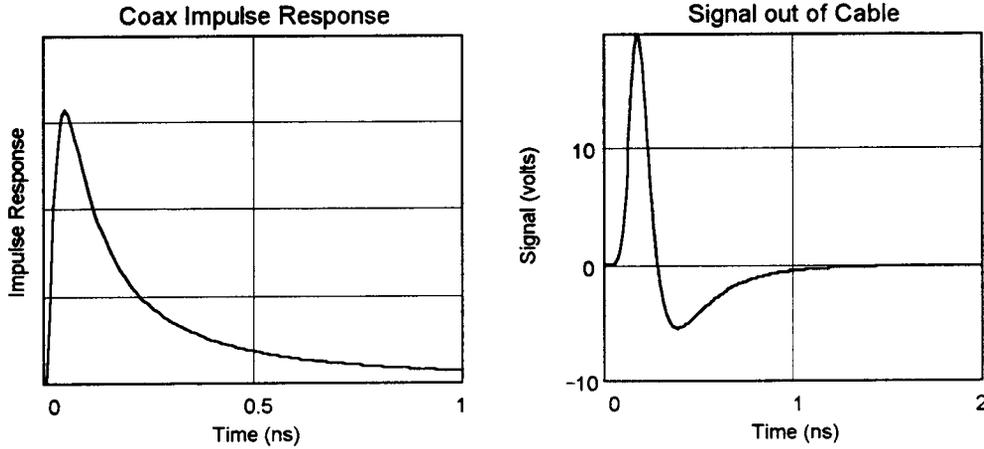


Figure 3. Impulse response of a coaxial cable and the beam signal at the far end.

This signal is passed through a two-pole Bessel band-pass filter, specified by its Laplace transform (conveniently supplied by MATLAB), a polynomial in $s=i\omega$ in the Laplace transform sense

$$Bessel(s) = \frac{0.102 \cdot s^2}{s^4 + 0.554 \cdot s^3 + 71.61 \cdot s + 1278} \quad fBessel(f) = Bessel(2\pi f)$$

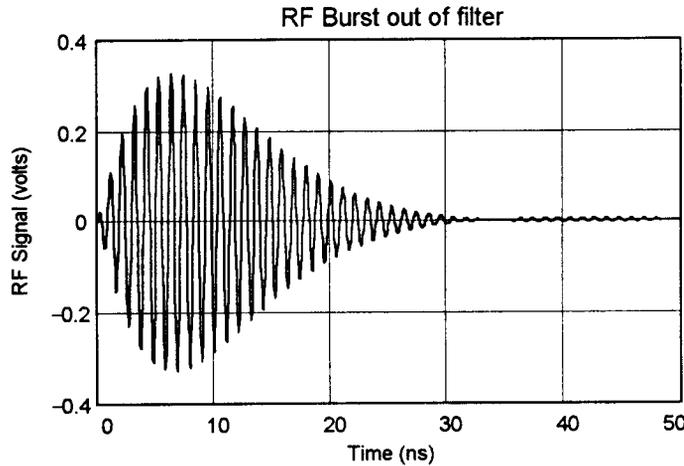


Figure 4. RF burst out of bandpass filter.

The ringing signal can now be demodulated. We choose synchronous detection. This is an obvious place to model via a time-domain approach, since demodulation is an inherently non-linear phenomenon. We'll do it by multiplying the ringing signal with a pure sine function whose phase is chosen to maximize the demodulated signal. Then we apply a low-pass filter, again with a Bessel characteristic which leaves only the baseband signal.

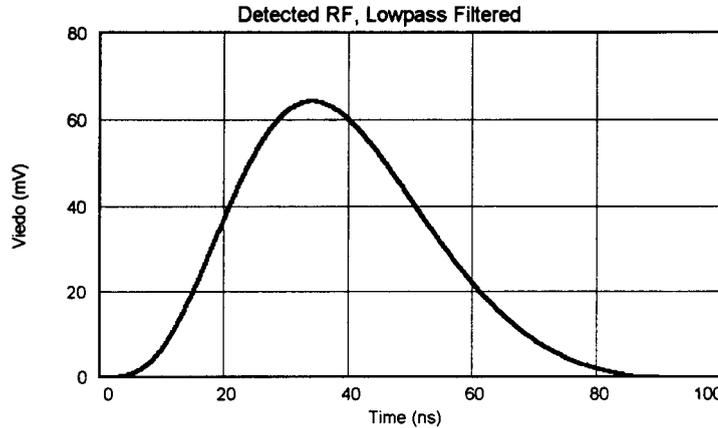


Figure 5. Demodulated RF after 3-pole Bessel lowpass filter

But what happens if these bunches occur every $t_b = 4.2$ ns? We send a finite train through our simulation by convoluting the button signals with a finite-comb response. In the frequency domain, a sixteen element comb of delta functions separated by time δ looks like:

$$fComb(\omega) = \sum_{n=1}^{16} e^{-i\omega n\delta}$$

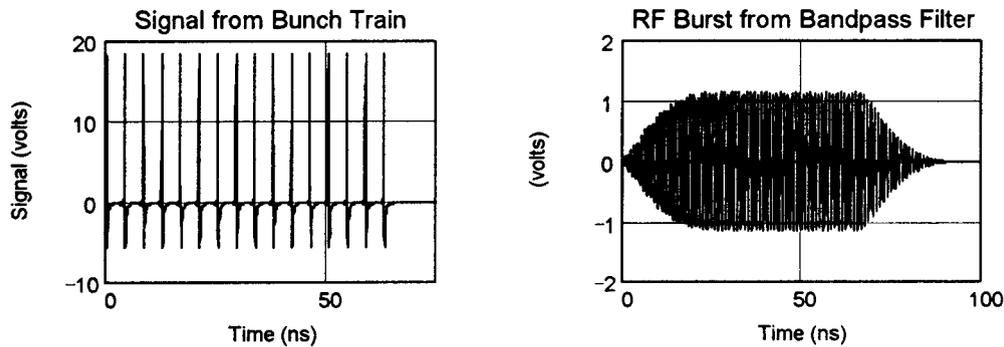


Figure 6. Button and bandpass filter response to a train of 16 beam bunches.

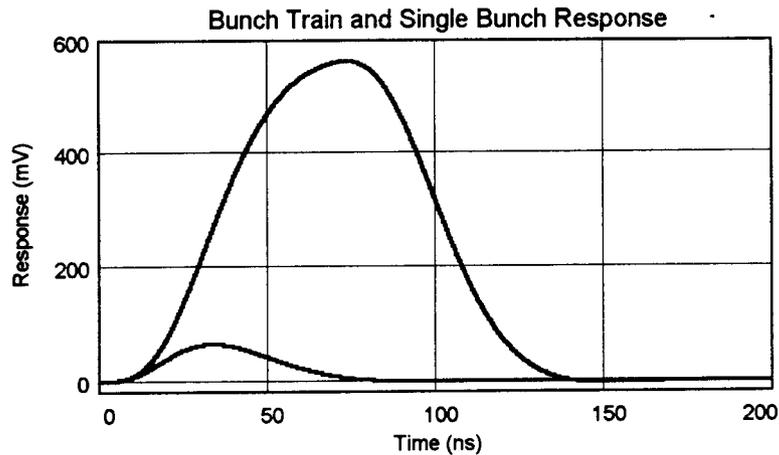


Figure 7. Demodulated RF for 16 bunch train (upper trace) and single bunch (lower trace).

Now we calculate position resolution from the peak signal and noise. Using the previous formula to convert signal amplitude to position resolution we get, for the single bunch case

$$\sigma_x = \frac{b}{2} \cdot \frac{\sqrt{2}\sigma_v}{2V} = \frac{b}{2\sqrt{2}} \cdot \frac{2\mu V}{64mV} = 0.5\mu m$$

The intrinsic resolution is ten times better for multibunch case. This is for thermal noise only; add electronic noise figure and system losses to this to get real resolution.

We've generated a reasonable first approximation to a BPM system. There are limitations to the accuracy of this model; we've assumed a round beam pipe, taken a low-frequency approximation to the button response (good to a few GHz), and of course we only get out what we've thought to put in the model.

DUCT-BUTTON COUPLING

Determination of signal amplitude and position sensitivity of a position monitor requires knowledge of the coupling of the beam to the transducer, whether button or stripline, as a function of beam position. For relativistic beams, two-dimensional electrostatic calculations give sufficient estimates of coupling for most cases. Analytic calculations are good for simple cases, such as round pipes. Usually beam

ducts are more complicated structures, often requiring numerical techniques to estimate coupling. Typical tools are POISSON, ANSYS, Electro, and even spread sheets like Excel using the relaxation method to solve Poisson's Equation.

We have used conformal mapping to solve for the field in non-trivial beam ducts, in particular elliptical and octagonal ducts. This gives (formally) analytic solutions, although one must still evaluate the resulting expressions by numerical techniques. Figure 8 shows the results of the calculation of electric field and equipotential contours for a beam displaced by 1 cm in the PEP-II high energy ring arc beam duct. We have used this technique to find the optimal location for the BPM buttons, to find the beam coupling, and its dependence on beam position.

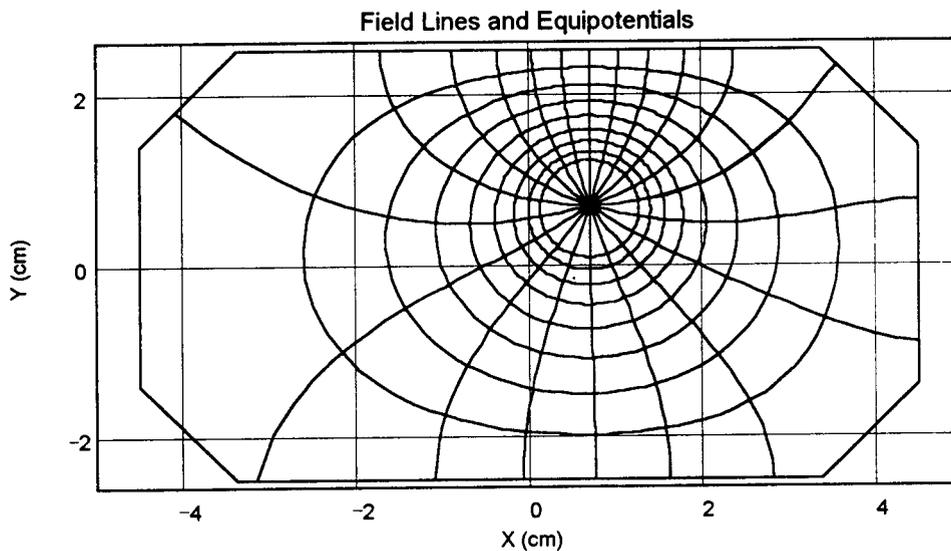


Figure 8. Field lines and equipotential contours found by conformal mapping.

THREE-DIMENSIONAL FIELD CALCULATION

Now we analyze the consequences of the real three dimensional geometry of the beam duct, the buttons, and the vacuum feedthroughs. The goal is twofold: to look for the effects of the actual geometry on the coupling of the beam to the button, which we have so far modeled only in two dimensions, and to estimate the wakefield impedances presented by the position monitor to the circulating beam. The tool used is MAFIA, a finite-difference 3-D electromagnetic field solver. A detailed accounting of this analysis is presented in reference 4. The beam duct for the PEP-II high energy ring arcs, whose cross section through the BPM buttons is shown in Fig. 9, is modeled in three dimensions as shown in Fig. 10. Since we are interested in wakefields for a few consecutive beam bunches separated by 2.1 ns, a section of pipe up to 5 meters long is modeled.

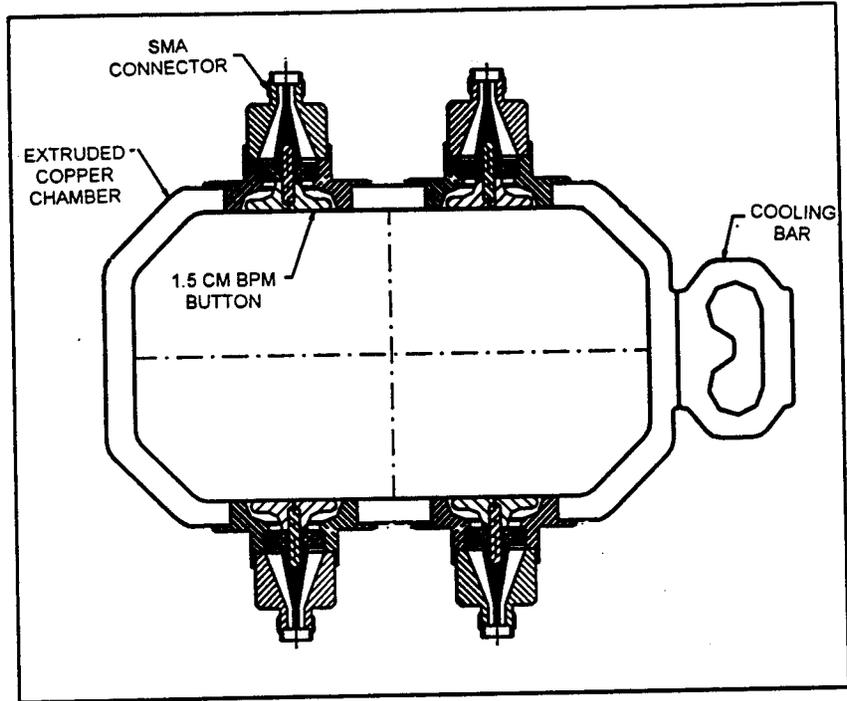
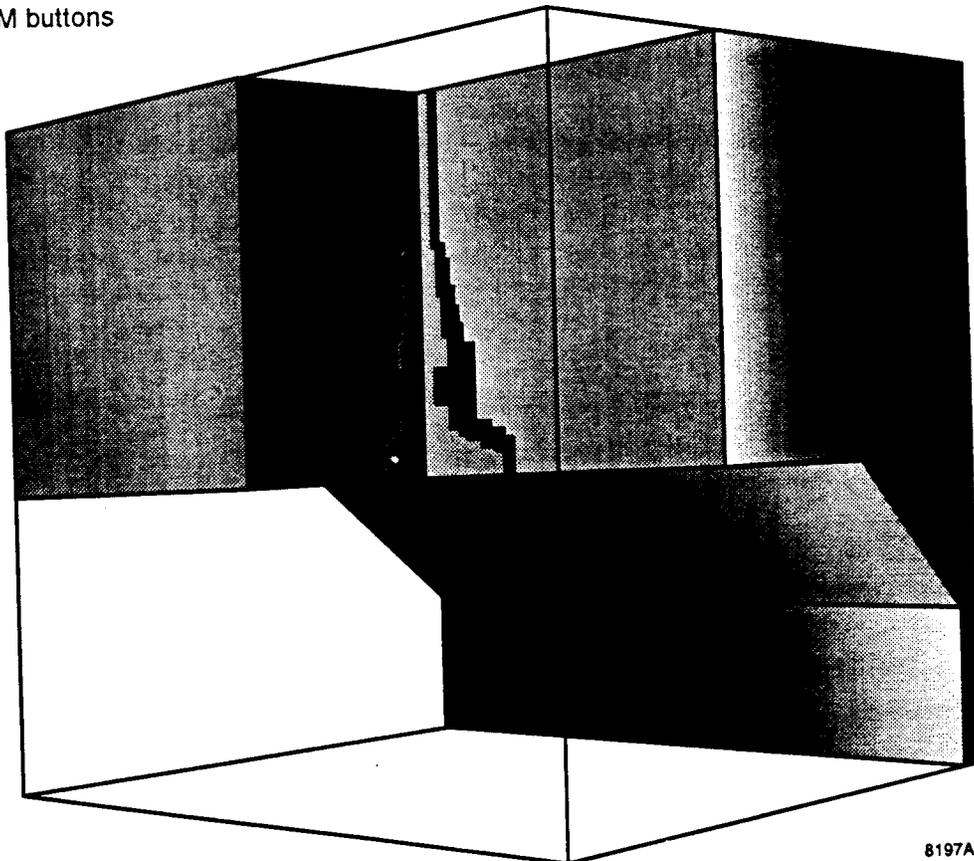


Figure 9. PEP-II high energy ring arc vacuum chamber cross section taken through the BPM buttons



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Figure 10. Geometry of beam duct and BPM buttons as described in MAFIA.

Wakefield impedances

We also need to know the effect of the buttons on the beam. The beam induces fields on the position monitor buttons; that's how we deduce beam position. These fields act back on the beam, in particular on subsequent bunches. Narrow-band resonances in this response can lead to coupled-bunch instabilities. The longitudinal electric field left in the PEP-II beam pipe after passage of a single beam bunch is shown in Fig. 11. It's frequency spectrum is shown in Fig. 12.

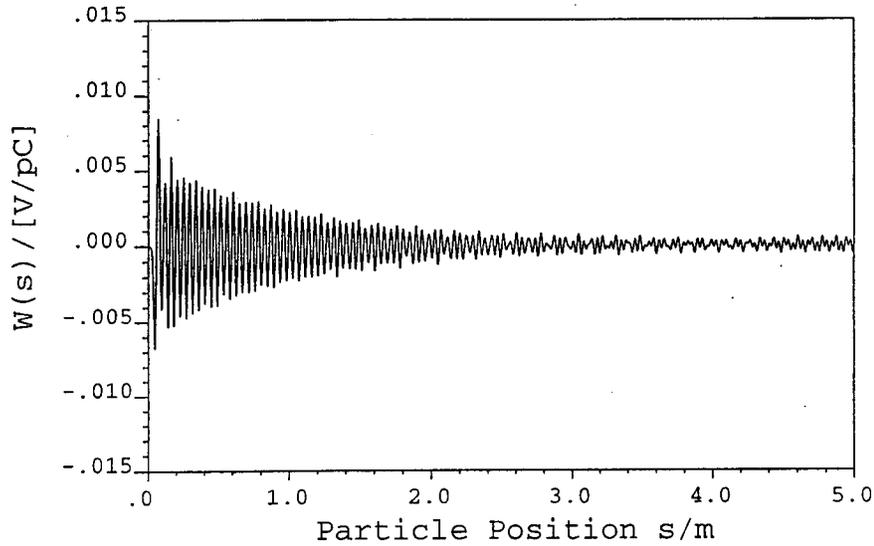


Figure 11. Longitudinal wakefield. E_z vs. distance behind bunch.

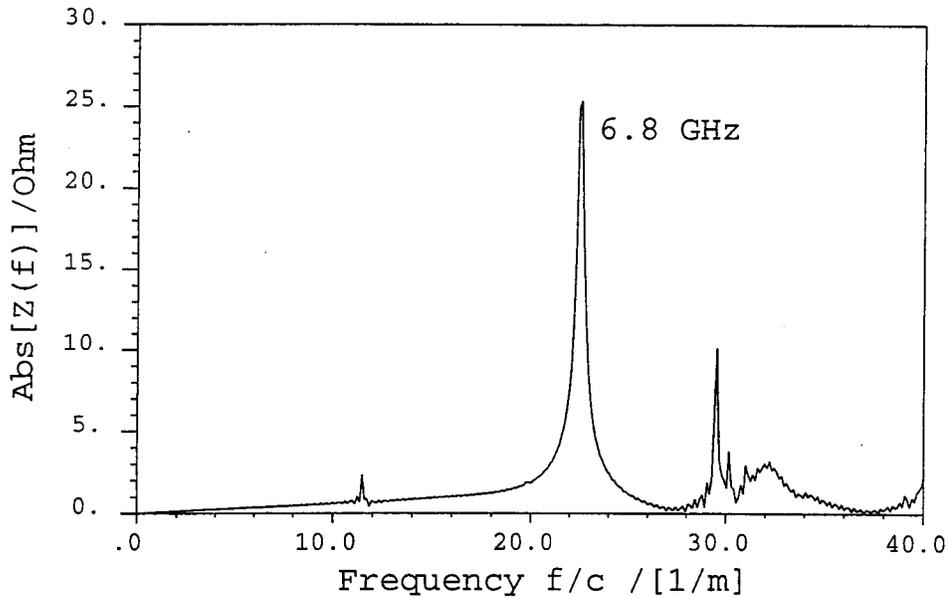


Figure 12. Spectrum of longitudinal wake.

A resonance is apparent at 6.8 GHz. This is due to a TE_{11} -like resonance localized in the gap between the edge of the button and the beam duct wall. This gap, running around the edge of the button, is like a slotline waveguide, so that modes propagating around the button are in resonance when their wavelength is approximately the button circumference. Figure 13 shows a snapshot of the electric field around the edge of the button after passage of a beam bunch, as calculated by MAFIA. The response shown in Fig. 12 meets the PEP-II impedance budget. However the initial design for the PEP-II BPM button called for buttons with a 2 cm diameter. This analysis showed that the TE_{11} resonance would have been intolerable. Reducing the button diameter to 1.5 cm eliminated the problem; the coupling of the beam to the resonance is reduced by roughly the area of the button, and the resonance is moved up to the 6.8 GHz shown in Fig. 12, by which frequency the bunch power spectrum has fallen drastically. Of course the position signal amplitude is also reduced along with the button area, hence the change could only be made along with a comparable reduction of the noise budget for the processing system, made possible by the introduction of a low-noise preamplifier in the electronics.

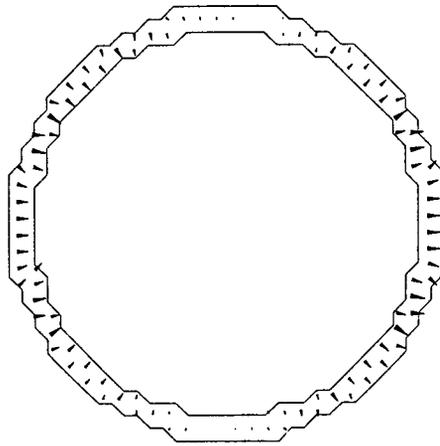


Figure 13. Calculated electric field around the edge of the BPM button .

Transfer Impedance

We have previously calculated the coupling from beam to button assuming cylindrical symmetry for the beam pipe, plus ideal transmission lines from the buttons out through the vacuum wall. We then used MAFIA to calculate the fields induced by the beam in the coaxial cable leading to the electronics, incorporating the full three-dimensional geometry of the beam pipe, buttons, vacuum feedthrough, and transition to the coaxial cable. In particular we want to know how the beam couples to TEM modes propagating up the coaxial cable to the

processing electronics. The three-dimensional model includes a short stub of coax attached to the button, properly terminated at the far end. MAFIA projects the propagating TEM modes out of the calculated fields. Figure 14 shows the coax voltage versus time calculated in this manner. The frequency spectrum is shown in Fig. 15. We extract the transfer impedance from this plot; at the signal processing frequency of 952 MHz, the transfer impedance is 0.65Ω , in good agreement with the estimates based on two dimensional approximations.

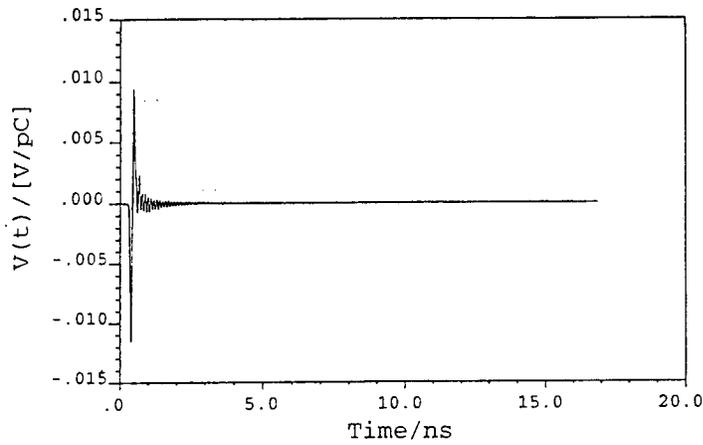


Figure 14. Calculated beam signal fom button.

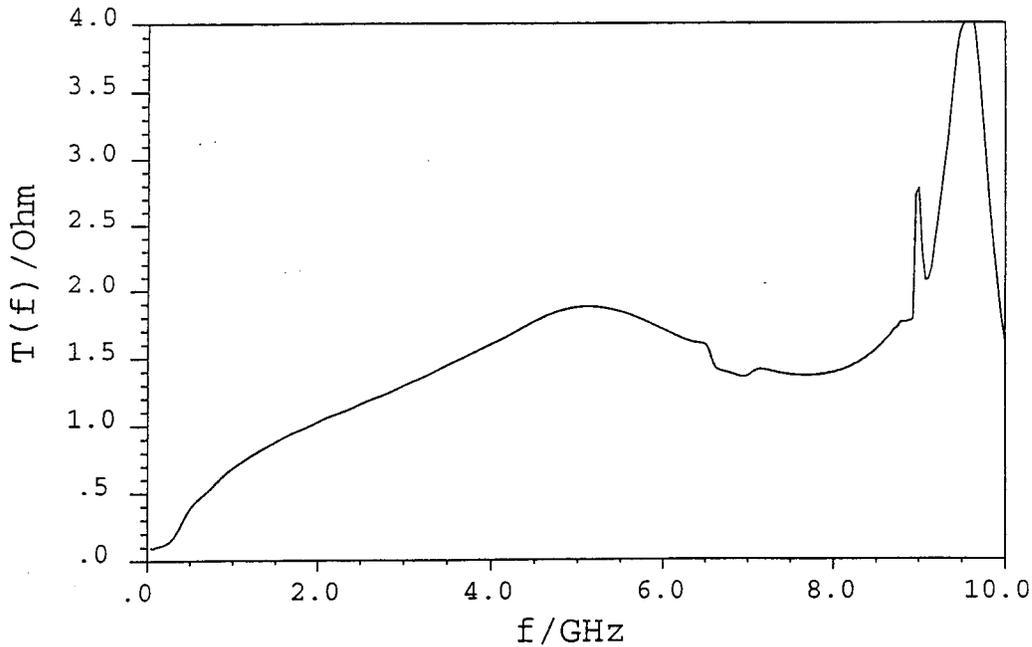


Figure 15. Transfer impedance of BPM

CONCLUSIONS

We have shown a few tools which allow the calculation of the performance of a beam position monitor system. We started with simple ways of estimating signals in simplified cases. Then we addressed ways of incorporating more detail in the models. We concluded with a full three-dimensional analysis of time-dependent electromagnetic fields.

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