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Unitarity-based Techniques for One-Loop Calculations in QCD

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We discuss new techniques developed in recent years for performing one-loop calculations in QCD, and present an example of results from the process $0 \to Vq\bar{q}gg$.

1. INTRODUCTION

Perturbative QCD, and jet physics in particular, have matured sufficiently that rather than being merely subjects of experimental studies, they are now tools in the search for new physics. This role can be seen in the search for the top quark, as well as in recent speculations about the implications of supposed high-E_T deviations of the inclusive-jet differential cross section at the Tevatron. One of the important challenges to both theorists and experimenters in coming years will be to hone jet physics as a tool in the quest for physics underlying the standard model. As such, it will be important to measurements of parameters of the theory or of non-perturbative quantities such as the parton distribution functions, as well as to searches for new physics at the LHC.

Jet production, or jet production in association with identified photons or electroweak vector bosons, appears likely to provide the best information on the gluon distribution in the proton, and may also provide useful information on the strong coupling α_s.

In order to make use of these final states, we need a wider variety of higher-order calculations of matrix elements. Indeed, as we shall review in the next section, next-to-leading order calculations are in a certain sense the *minimal* useful ones. On the other hand, these calculations are quite difficult with conventional Feynman rules. In following sections, we will discuss some of the techniques developed in recent years to simplify such calculations.

2. NEXT-TO-LEADING ORDER CALCULATIONS

While leading-order calculations often reproduce the shapes of distributions well, they suffer from several practical and conceptual problems whose resolution requires the use of next-to-leading order calculations. These problems are tied to the various logarithms that can arise in perturbation theory.

The first of these logarithms are 'UV' ones, connected with the renormalization scale. We are forced to introduce a renormalization scale μ in order to define the coupling, $\alpha_s(\mu)$; but physical quantities, such as cross sections or differential cross sections, should be independent of μ. When we compute such a quantity in perturbation theory, however, we necessarily truncate its expansion in α_s, and this introduces a spurious dependence on μ. This dependence is sig-

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nificant in real-world applications of perturbative QCD because the coupling is not that small, because it runs relatively quickly, and because we are interested in processes with a relatively large number of colored 'final'-state partons. Together, these effects can lead to anywhere from a 30% to a factor of 2–3 normalization uncertainty in predictions of experimentally-measured distributions. At leading order, the only dependence on μ comes from the resummation of logarithms in the running coupling $\alpha_s(\mu)$, and the scale choice is arbitrary. At next-to-leading order, however, the virtual corrections to the matrix element introduce another dependence on μ. This dependence can — and in practice, often does — reduce the over-all sensitivity of a prediction to variations in μ. (I should stress, however, that while varying μ by some preset amount, say a factor of two up and down from a typical scale, gives an indication of the sensitivity of or uncertainty in the calculation, it does *not* give an estimate of the error involved; for that one needs a next-to-next-to-leading order calculation.)

The other logarithms are the 'IR' ones, connected with the presence of soft and collinear radiation. Jets in a detector are not infinitely narrow pencil beams; they consist of a spray of hadrons spread over a finite segment. Experimental measurements of jet distributions and the like depend on resolution parameters, such as the jet cone size and minimum transverse energy. In a leading-order calculation, jets are modelled by lone partons. As a result, these predictions don't depend on these parameters (or have an incorrect dependence on them). In addition, the internal structure of a jet cannot be predicted at all.

At next-to-leading order, one necessarily includes contributions with additional real radiation, which either shows up inside one of the jets, or as soft radiation in the event. This introduces, for example, the required logarithmic dependence on the cone size ΔR.

At a more conceptual level, we must remember that jet differential cross sections are multi-scale quantities, involving not only a hard scale (say a jet transverse energy E_T) but also scales characterizing the resolution defining a jet (for example, $E_T\Delta R$ or $E_{T\min}$). As a result, the perturbative expansion is not one in α_s alone, but contains logarithms and logarithms-squared of ratios of scales. These logarithms, which arise from the infrared structure of gauge theories, might spoil the applicability of perturbation theory if they grow too large. Only a next-to-leading order calculation can tell us if we are safe.

3. ORGANIZING THE CALCULATION OF AMPLITUDES

A leading-order calculation of an n-jet process at a hadron collider such as the Tevatron or LHC requires knowledge of the parton distribution functions of the proton; of α_s; and of the tree-level matrix element for the $2 \to n$ parton process. A next-to-leading order calculation of the same process requires in addition the tree-level matrix element for the $2 \to n+1$ parton process, and the one-loop matrix element for the $2 \to n$ parton process. (In addition, it requires a general formalism such as that of refs. [1–4] for cancelling the infrared singularities analytically while allowing a numerical calculation of fully-differential observables.)

It is the calculation of the one-loop matrix element which presents the greatest difficulty in the traditional Feynman-diagram approach. This difficulty arises from the enormous number of diagrams, the large amount of vertex algebra in each diagram, and the complexity of loop integrals with many powers of the loop momentum in the numerator. A brute-force approach might easily lead to expressions thousands of times larger than an appropriate representation of a result.

The first step in developing a more efficient method for performing these calculations is to take advantage of the lessons from earlier developments in tree-level calculations, and from string-based methods for one-loop calculations. These include

- Color decomposition: one should write the amplitude as a sum over color factors multiplied by color-ordered subamplitudes, and calculate these latter coefficients. For a one-loop amplitude for $0 \to Vq\bar{q}g\cdots g$, the color decomposition takes the form

$$\mathcal{A}_{n+V}^{1\text{–loop}} = g^n \sum_{j=1}^{n-1} \sum_{\sigma \in S_{n-2}/S_{n-2;j}} \mathrm{Gr}_{n;j}^{q\overline{q}}(\sigma) \times A_{n+V;j}^{1\text{–loop}}(1_q, 2_{\overline{q}}; \sigma(3\cdots n); V) \quad (1)$$

where $\mathrm{Gr}_{n;1}^{q\overline{q}}(3\cdots n)$ is the leading-color trace, equal to the number of colors N_c times the tree-level color factor $(T^{a_3}\cdots T^{a_n})_{i_2}^{\bar{\imath}_1}$. The remaining color factors are subleading-color ones,

$$\mathrm{Gr}_{n;j>1}^{q\overline{q}}(3\cdots n) = \mathrm{Tr}(T^{a_3}\cdots T^{a_{j+1}}) \times (T^{a_{j+2}}\cdots T^{a_n})_{i_2}^{\bar{\imath}_1} \,. \quad (2)$$

- Spinor helicity method [5]: one should calculate helicity amplitudes (and square or compute interference terms only numerically), using spinor products $\langle i\, j\rangle$ and $[i\, j]$.
- Use of supersymmetric decompositions and supersymmetry identities, where applicable.
- Use of permutation identities [6] that allow rewriting subleading-color amplitudes as a sum of permutation of certain leading-color 'primitive' amplitudes. One should then calculate only primitive amplitudes, which correspond to leading-color amplitudes with a definite clockwise or counter-clockwise orientation of internal fermion lines, and in which the external fermion legs are not necessarily adjacent.

The reader will find a review of the techniques described below in ref. [7].

4. PRIMITIVE AMPLITUDES

In principle, one may imagine calculating primitive amplitudes using color-ordered Feynman rules. One can do better, calculating them using a combination of string-based rules and rules inspired by string theory. Can one do better yet? The unitarity-based technique, combined with judicious use of the collinear limits, provides an even better way of calculating these quantities.

In general, a one-loop amplitude has absorptive pieces; the basic idea behind the unitarity-based technique is that the cut-containing terms in the amplitude can be determined by dispersion relations from the dispersive parts, which are in turn related to a product of tree amplitudes. As I will review below, in supersymmetric theories, the basic technique in fact determines cut-free pieces as well.

In calculating loop diagrams, we use dimensional regularization for both the ultraviolet and infrared singularities; at one loop, it is possible to choose a variant (dimension reduction) that preserves supersymmetry manifestly. (The translation to the conventional scheme is detailed in a paper by Kunszt, Signer, and Trócsányi [8].)

5. INTEGRALS

Let us consider the calculation of an n-point one-loop amplitude in a gauge theory. The calculation necessarily involves up to n-point one-loop integrals; and given the powers of momentum in the three-gluon vertex, may involve up to m powers of the loop momentum in any m-point integral. Using either a combination of Passarino-Veltman [9] and van Neerven-Vermaseren [10] reduction techniques, or equivalent ones [11], we can rewrite any such integral (in $D = 4 - 2\epsilon$) as a linear combination of scalar box integrals, scalar three-mass triangles, scalar or tensor one- or two-mass triangles, and scalar or tensor bubble integrals. If we start with an amplitude where all lines are massless, the internal lines of all these integrals will also be massless; but the external legs will be sums of the original external momenta, and thus possibly massive. The scalar boxes we must consider in general will thus have anywhere from one to four external masses, for example. We will denote the set of integrals that appears in the computation of a n-point amplitude by $\mathcal{F}_n$. The amplitude can be written as a sum over integrals in this set, with coefficients c_i which are rational functions of the spinor products and Lorentz invariants.

6. UNIQUE RECONSTRUCTION OF AMPLITUDES

One of the basic tools in the method is the reconstruction theorem, which tells us under which conditions the knowledge of the cuts (in four dimensions) suffices to reconstruct the amplitude uniquely. It states that if, for every loop integral

$$\int \frac{d^D k}{(2\pi)^D} \frac{\mathrm{Poly}(k^\mu)}{k^2(k-p_1)^2\cdots(k-p_{n-1})^2} \tag{3}$$

which appears in the amplitude, the degree of the numerator polynomial is less than or equal to $n-2$ (less than or equal to 1 if $n=2$), then the amplitude is determined uniquely by its cuts in $D=4$. This does not imply that the amplitude is necessarily free of rational terms, but merely that in this case, the rational terms are always linked to the terms containing cuts.

Using the background-field method, and examining the effective action, one can show that the theorem applies to the complete amplitude in supersymmetric theories. In $N=4$ supersymmetric gauge theories, one can go further, and show that in the background-field method, the highest power of the loop momentum that can appear in the numerator of an n-point integral is $n-4$; this in turn implies that in these theories, the amplitude can be expressed as a sum of scalar box integrals with rational coefficients c_i (triangles and bubbles don't enter).

7. SEWING AMPLITUDES

The key point in the unitarity-based method is that we sew tree *amplitudes*, not tree diagrams. This allows us to take advantage of all the cancellations and reductions in numbers of terms that have already occurred in the process of computing the tree amplitudes.

A calculation using the unitarity-based method proceeds as follows. We want to compute the coefficients c_i of the integrals in the set $\mathcal{F}_n$. To this end, we consider in turn cuts in all possible channels. For a given channel, we form the cut in that channel, summing over all intermediate states; this gives rise to a phase-space integral of the form

$$\int d^D\mathrm{LIPS}(-\ell_1,\ell_2) \times A_L^{\mathrm{tree}}(-\ell_1,\ldots,\ell_2)A_R^{\mathrm{tree}}(-\ell_2,\ldots,\ell_1)\,, \tag{4}$$

where $\ell_{1,2}$ are the four-dimensional on-shell momenta crossing the cut, and where the $A_{L,R}^{\mathrm{tree}}$ are the color-ordered tree subamplitudes on the two sides of the cut. Using the Cutkosky rules, we can rewrite this expression as the absorptive or imaginary part of a loop amplitude,

$$\left[\int \frac{d^D\ell_1}{(2\pi)^D} A_L^{\mathrm{tree}} \frac{1}{\ell_2^2+i\varepsilon} A_R^{\mathrm{tree}} \frac{1}{\ell_1^2+i\varepsilon}\right]_{\mathrm{cut}}\,. \tag{5}$$

Using the power-counting theorem in those cases where it applies (or simply up to a rational ambiguity to be fixed later in cases where it does not apply), we can recover the real parts by dropping the subscript "cut". We then perform the usual reductions on the resulting loop integral, and extract the coefficient of any function in $\mathcal{F}_n$ containing a cut in the given channel. (Functions not containing a cut in the given channel should be dropped.) Finally, we reassemble the final answer by considering all channels.

Certain integrals, such as the box integrals, have cuts in more than one channel. Considering both channels provides us with a cross check — the coefficients as computed in both channels must agree — or alternatively we could reduce the amount of work we must do by considering only one channel. The latter choice is particularly appropriate when computing amplitudes in an $N=4$ supersymmetric gauge theory, since as noted above all amplitudes in this theory can be written in terms of scalar boxes.

8. RATIONAL TERMS

In nonsupersymmetric theories (such as QCD), there is a remaining rational part which cannot be determined in this manner. There are two approaches to determining these pieces: the use of collinear limits, and extending the cuts to $\mathcal{O}(\epsilon)$.

The collinear approach is based on the universal factorization of amplitudes in the collinear limit,

$$A_{n;1}^{1-\mathrm{loop}}(\cdots,a,b) \xrightarrow{a\parallel b}$$

$$\sum_{\lambda_\Sigma=\pm} \Big[C^{\rm tree}_{-\lambda_\Sigma}(a^{\lambda_a}, b^{\lambda_b}) A^{\rm 1-loop}_{n-1;1}(\cdots;\Sigma^{\lambda_\Sigma}) + C^{\rm 1-loop}_{-\lambda_\Sigma}(a^{\lambda_a}, b^{\lambda_b}) A^{\rm tree}_{n-1}(\cdots;\Sigma^{\lambda_\Sigma}) \Big] \tag{6}$$

where the splitting amplitudes C are universal functions depending only on the collinear momenta and their helicities λ, and on the momentum fraction z ($k_\Sigma = k_a + k_b$, $k_a = zk_\Sigma$). This limit is useful for deducing the rational pieces because the cut terms alone will not produce the desired collinear limit; one must add rational terms in order to obtain the correct limit. However, there is no proof that the terms deduced by this method are unique (though it is likely true for more than five external legs; for five external legs, there is an ambiguity arising from the existence of a term which contains no cuts but is collinear-finite). Such results thus need to be checked (for example, numerically) against results from another method.

Another possible method again uses the cuts, but at $\mathcal{O}(\epsilon)$ rather than just to $\mathcal{O}(\epsilon^0)$. The basic point is that amplitudes in a massless gauge theory have an over-all power of $(-s)^{-\epsilon}$, where s is an invariant. The "rational" pieces therefore do contain cuts at $\mathcal{O}(\epsilon)$, and can be deduced by sewing tree amplitudes, where the momenta crossing the cuts are taken to be on-shell in $(4-2\epsilon)$ dimensions rather than four dimensions. For scalars, this is equivalent to computing massive rather than massless amplitudes, followed by an appropriately weighted integration over the "mass" (really the (-2ϵ)-dimensional component of the momentum).

9. AN ALL-MULTIPLICITY RESULT

We had previously obtained[12], with Dave Dunbar, an explicit formula for the one-loop amplitude, in the $N=4$ supersymmetric gauge theory, with an *arbitrary* number of external gluons with the helicity configuration of the Parke-Taylor tree-level amplitudes [13,14], namely all but two gluons carrying the same helicity. (Let us take the majority to have positive helicity, the two opposite-helicity gluons to have negative helicity; exchanging these two amounts to a complex conjugation.) Here, we will outline the derivation of this result using the unitarity-based method. Such results demonstrate the power of the method, because they would require the computation of an infinite number of Feynman diagrams in the traditional approach.

This amplitude is uniquely determined by its cuts, and can be expressed entirely in terms of scalar box integrals. There are two kinds of cuts to consider, one where both opposite-helicity gluons are one the same side of the cut, the other where one such gluon appears on each side of the cut. The former cut receives contributions only from gluons crossing the cut, the latter from fermions and scalars as well; but it turns out that after summing over the $N=4$ supermultiplet (using supersymmetry Ward identities [15]), the structure of the second type of cut is similar to that of the first. It thus suffices to consider the first kind.

Let us suppose that the opposite-helicity gluons are on the left side of the cut (drawn vertically through the diagram). Then all the external legs on the right-hand side of the cut have positive helicity; as a result, the tree amplitude on the right-hand side will vanish unless both legs crossing the cut on the right have negative helicity. Since we adopt the convention that all momenta are directed inwards into an amplitude, the cut-crossing legs flip sign as we cross the cut to the left. We must flip their helicities as well, which tells us that all legs of the tree amplitude to the left of the cut have positive helicity, except the two external legs carrying negative helicity.

Thus both tree amplitudes on either side of the cut have the helicity configuration of the Parke-Taylor amplitudes [13,14], so that simple all-n formulæ are available for them. Most of the factors in these amplitudes are independent of the momenta crossing the cut, and so can be pulled out in front of the cut integral (4). Indeed, most of these factors reassemble into the tree amplitude itself, so that (up to a phase) we obtain

$$A_n^{\rm tree} \int d^D{\rm LIPS} \times \frac{\langle (m_1-1)\, m_1\rangle \, \langle \ell_1\, \ell_2\rangle^2 \, \langle m_2\, (m_2+1)\rangle}{\langle (m_1-1)\, \ell_1\rangle \, \langle \ell_1\, m_1\rangle \, \langle m_2\, \ell_2\rangle \, \langle \ell_2\, (m_2+1)\rangle} \tag{7}$$

where (m_1-1, m_1) and (m_2, m_2+1) are the pairs of legs adjacent to the cut. The four factors in the denominator give rise to four propagators, which along with the two propagators crossing the cut means that for an arbitrary number of external gluons, we need consider only a hexagon integral. Indeed, it turns out to be a rather special hexagon integral; rearranging the spinor products in its numerator, it decomposes into a sum of one- and two-mass scalar boxes. From this decomposition and the uniqueness theorem, we then obtain the complete one-loop $N = 4$ supersymmetric amplitude.

10. Z DECAY TO FOUR JETS

The decay of the Z into jets has been studied extensively at LEP and SLC. The three-jet to two-jet ratio is one of the better methods of determining $\alpha_s(M_Z)$, but at present this extraction of the running coupling is dominated by theoretical uncertainties. The resolution of these uncertainties requires a next-to-next-to-leading order (NNLO) calculation of three-jet production.

Such a calculation will contain as ingredients the two-loop corrections to $Z \to q\bar{q}g$, which have yet to be computed; the $Z \to q\bar{q}ggg$ and $Z \to q\bar{q}q'\bar{q}'g$ matrix elements at tree level, which are known [16]; and the one-loop corrections to $Z \to q\bar{q}gg$ and $Z \to q\bar{q}q'\bar{q}'$, which we are currently computing. (In addition, it will require an extension of one-loop techniques for cancellation of IR singularities, such as the slicing method of Giele and Glover [1], to NNLO.)

These one-loop corrections are also of interest outside of this NNLO context, as they can be used to produce a program calculating $Z \to 4$ jets at next-to-leading order. This process is the lowest-order one in which the gluon and quark color charges can be measured independently. A next-to-leading order calculation will allow reliable limits to be set, using LEP and SLC data, on other possible light colored fermions (such as light gluinos).

We present here the result for one of the leading-color subamplitudes of the $Z \to q\bar{q}gg$ process. It is convenient to tack on the amplitude for the Z production from two leptons, so that all external legs are massless; in the formulæ below, these legs will be k_5 and k_6. Furthermore, we will also include a factor of $1/s_{56}$ (corresponding to a photon propagator replacing the Z).

To express this result, we should first record the corresponding tree amplitude,

$$
\begin{aligned}
A_{4+V}^{\rm tree}(1_q^+, 2^+, 3^-, 4_{\bar{q}}^-; 5_\ell^-, 6_\ell^+) \;=\; & \\
i\Bigg[-\frac{[1\,2]\,\langle 1\,3\rangle\,\langle 4\,5\rangle\,\langle 6^+|(1+2)|3^+\rangle}{\langle 1\,2\rangle\, s_{23} t_{123} s_{56}} & \\
+\frac{\langle 3\,4\rangle\,[2\,4]\,[1\,6]\,\langle 5^-|(3+4)|2^-\rangle}{[3\,4]\, s_{23} t_{234} s_{56}} & \\
-\frac{\langle 5^-|(3+4)|2^-\rangle\,\langle 6^+|(1+2)|3^+\rangle}{\langle 1\,2\rangle\,[3\,4]\, s_{23} s_{56}}\Bigg]; & \qquad (8)
\end{aligned}
$$

define several functions (roughly corresponding to the various box and triangle integrals that appear)

$$
\begin{aligned}
&\mathrm{L}_0(r) = \frac{\ln(r)}{1-r}\,, \qquad \mathrm{L}_1(r) = \frac{\mathrm{L}_0(r)+1}{1-r}\,,\\
&\mathrm{Ls}_{-1}(r_1,r_2) = \mathrm{Li}_2(1-r_1) + \mathrm{Li}_2(1-r_2)\\
&\qquad + \ln r_1\, \ln r_2 - \frac{\pi^2}{6}\,,\\
&\mathrm{Ls}_{-1}^{2\mathrm{m}h}(s,t;m_1^2,m_2^2) = -\,\mathrm{Li}_2\left(1-\frac{m_1^2}{t}\right) \qquad (9)\\
&\qquad - \mathrm{Li}_2\left(1-\frac{m_2^2}{t}\right) - \frac{1}{2}\ln^2\left(\frac{-s}{-t}\right)\\
&\qquad + \frac{1}{2}\ln\left(\frac{-s}{-m_1^2}\right)\ln\left(\frac{-s}{-m_2^2}\right)\\
&\qquad + \left[\frac{1}{2}(s-m_1^2-m_2^2) + \frac{m_1^2 m_2^2}{t}\right]\\
&\qquad\quad \times I_3^{3\mathrm{m}}(s,m_1^2,m_2^2)\,.
\end{aligned}
$$

as well as a flip symmetry,

$$
\mathrm{flip} = \{1 \leftrightarrow 4,\ 2 \leftrightarrow 3,\ 5 \leftrightarrow 6;\ \langle\ \rangle \leftrightarrow [\ ]\}\,. \qquad (10)
$$

The one-loop amplitude is then

$$
A_{4+V;1}^{1-\mathrm{loop}} = V_a A_{4+V}^{\rm tree} + F_g + F_s \qquad (11)
$$

where

$$
\begin{aligned}
V_a = -\frac{1}{\epsilon^2}&\left[\left(\frac{\mu^2}{-s_{12}}\right)^\epsilon + \left(\frac{\mu^2}{-s_{23}}\right)^\epsilon + \left(\frac{\mu^2}{-s_{34}}\right)^\epsilon\right]\\
&-\frac{3}{2\epsilon}\left(\frac{\mu^2}{-s_{56}}\right)^\epsilon - \frac{7}{2}\,. \qquad (12)
\end{aligned}
$$

and where

$$F_g = B_1\,\mathrm{Ls}_{-1}\left(\frac{-s_{23}}{-t_{234}},\frac{-s_{34}}{-t_{234}}\right) + B_5\,\mathrm{Ls}_{-1}\left(\frac{-s_{12}}{-t_{123}},\frac{-s_{23}}{-t_{123}}\right)$$
$$+B_2\,\mathrm{Ls}_{-1}^{2\mathrm{m}h}(s_{34},t_{123};s_{56},s_{12}) + T\,I_3^{3\mathrm{m}}(s_{12},s_{34},s_{56})$$
$$+B_4\,\mathrm{Ls}_{-1}^{2\mathrm{m}h}(s_{12},t_{234};s_{56},s_{34})$$
$$-2\,\frac{\langle 1\,3\rangle\,\langle 6^+|(1+2)|3^+\rangle}{\langle 1\,2\rangle\,[5\,6]\,\langle 4^+|(2+3)|1^+\rangle} \tag{13}$$
$$\times\left[\frac{\langle 6^+|(2{+}3)1|2^-\rangle}{t_{123}}\frac{\mathrm{L}_0\left(\frac{-s_{23}}{-t_{123}}\right)}{t_{123}} + \frac{[6\,4]\langle 4\,3\rangle}{\langle 2\,3\rangle}\frac{\mathrm{L}_0\left(\frac{-s_{56}}{-t_{123}}\right)}{t_{123}}\right]$$
$$-2\,\frac{[4\,2]\,\langle 5^-|(3+4)|2^-\rangle}{[4\,3]\,\langle 6\,5\rangle\,\langle 1^-|(2+3)|4^-\rangle}$$
$$\times\left[\frac{\langle 5^-|(2{+}3)4|3^+\rangle}{t_{234}}\frac{\mathrm{L}_0\left(\frac{-s_{23}}{-t_{234}}\right)}{t_{234}} + \frac{\langle 5\,1\rangle[1\,2]}{[3\,2]}\frac{\mathrm{L}_0\left(\frac{-s_{56}}{-t_{234}}\right)}{t_{234}}\right],$$

and

$$F_{s1} = -\frac{1}{2}\,\frac{\langle 1\,3\rangle}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,[5\,6]\,t_{123}\,\langle 4^+|(2+3)|1^+\rangle}$$
$$\times\Bigg[\left(\langle 6^+|(2+3)12|3^+\rangle\right)^2\frac{\mathrm{L}_1\left(\frac{-t_{123}}{-s_{23}}\right)}{s_{23}^2}$$
$$+\left([6\,4]\,\langle 4\,3\rangle\,t_{123}\right)^2\frac{\mathrm{L}_1\left(\frac{-s_{56}}{-t_{123}}\right)}{t_{123}^2}\Bigg] \tag{14}$$
$$+\frac{1}{2}\,\frac{[6\,2]^2}{\langle 1\,2\rangle\,[2\,3]\,[3\,4]\,[5\,6]}$$
$$F_s = F_{s1} + \mathrm{flip}(F_{s1})\,.$$

In these expressions,

$$B_5 = \frac{\langle 1\,3\rangle\,\left(\langle 6^+|(1+2)|3^+\rangle\right)^2}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,[5\,6]\,t_{123}\,\langle 4^+|(2+3)|1^+\rangle}$$
$$+\frac{[1\,2]^3\langle 4\,5\rangle^2}{[2\,3]\,[1\,3]\,\langle 5\,6\rangle\,t_{123}\,\langle 1^+|(2+3)|4^+\rangle}\,.$$
$$B_2 = \frac{\langle 1\,3\rangle\,\left(\langle 6^+|(1+2)|3^+\rangle\right)^2}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,[5\,6]\,t_{123}\,\langle 4^+|(2+3)|1^+\rangle} \tag{15}$$
$$+\frac{[1\,2]^2\langle 4\,5\rangle^2\,\langle 2^+|(1+3)|4^+\rangle}{[2\,3]\,\langle 5\,6\rangle\,t_{123}\,\langle 1^+|(2+3)|4^+\rangle\,\langle 3^+|(1+2)|4^+\rangle}\,,$$
$$B_1 = \mathrm{flip}(B_5)\,,$$
$$B_4 = \mathrm{flip}(B_2)\,,$$

and

$$T_1 = \frac{t_{123}\,(s_{56}+s_{12}-s_{34}) - 2s_{12}s_{56}}{2t_{123}}\,B_2 \tag{16}$$
$$+\frac{1}{2}\,\frac{[1\,2]\left[\left(\langle 4^-|(1+2)(3+4)|5^+\rangle\right)^2 - s_{12}s_{34}\,\langle 4\,5\rangle^2\right]}{\langle 1\,2\rangle\,[3\,4]\,\langle 5\,6\rangle\,\langle 1^+|(2+3)|4^+\rangle\langle 3^+|(1+2)|4^+\rangle}$$
$$T = T_1 + \mathrm{flip}(T_1)\,.$$

This amplitude is one of three leading-color helicity amplitudes; in addition, five other primitive helicity amplitudes are needed to construct the subleading-color amplitudes for $Z \to$ four jets.

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