

## **Geometrical Wake of a Smooth Flat Collimator\***

G. V. Stupakov

*Stanford Linear Accelerator Center  
Stanford University, Stanford, CA 94309*

### **Abstract**

Transverse impedance of a flat smooth collimator with a gradually varying gap between the upper and lower walls is calculated. Numerical results are presented for the NLC-type collimators.

*Submitted for publication*

---

\*Work supported by Department of Energy contract DE-AC03-76SF00515

# 1. INTRODUCTION

In this paper we calculate the impedance of a flat smooth collimator schematically shown in Fig. 1. The collimator extends in the  $y$  direction from  $y = -h$  to  $y = h$  and is bounded by perfectly conducting walls. The beam propagates in the  $z$  direction. The collimator upper and lower walls are given by the equation  $x = \pm b(z)$ , where  $b(z)$  is a smooth function such that  $b'(z) \ll 1$ . Throughout this paper we assume that  $h \gg b(z)$ .

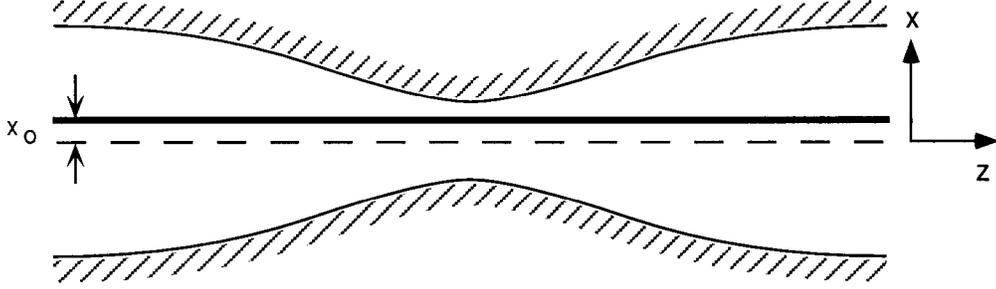


Fig. 1. Sketch of a smooth flat collimator. The collimator extends from  $y = -h$  to  $y = h$  in the  $y$  direction. A heavy line shows the trajectory of a driving particle.

For a smoothly varying wall and not very high frequency such that  $kb^2 < l$ , where  $k = \omega/c \approx \sigma_z^{-1}$ , and  $l$  is the length of the collimator, the energy loss of the beam due to the radiation in the collimator is small [1]. This results in the real part of the impedance being much smaller than its real part, and, in the first approximation, the real part can be neglected. In this approximation, the transverse impedance is purely imaginary and does not depend on the frequency. The latter allows us to greatly simplify its calculation considering only the limit  $\omega \rightarrow 0$  [1]. In this limit, the electric field can be found as a solution to electrostatic equations, and magnetic field satisfies magnetostatic equations with proper boundary conditions. For the electrostatic problem, one has to find the electric field of a charged wire of unit charge density stretched along the beam trajectory  $x = x_0, y = 0$ . The magnetostatic problem requires finding of the magnetic field generated by the same wire carrying a unit current. After the electric and magnetic fields are found, the transverse impedance in  $x$  direction can be calculated using the following formula

$$Z_x(x, y, x_0) = -\frac{i}{c} \int_{-\infty}^{\infty} [E_x(x, y, x_0; z) - H_y(x, y, x_0; z)] dz. \quad (1)$$

This impedance has a dimension of Ohm and depends on the location of the driving particle  $x_0$  and the coordinates of the test particle,  $x, y$ .

The wake  $w$  corresponding to a purely imaginary transverse impedance (1) is proportional to  $\delta(z)$ :

$$w(x, y, x_0, z) = icZ_x(x, y, x_0)\delta(z). \quad (2)$$

After the passage of the collimator the bunch will be deflected in the  $x$  direction by an angle  $Nr_e\kappa/\gamma$ , where  $N$  is the number of particles in the bunch,  $r_e$  is the classical electron radius (for electron/positron beam),  $\gamma$  is the relativistic factor, and  $\kappa = -c \operatorname{Im} Z_x / 2\sqrt{\pi}\sigma_z$ .

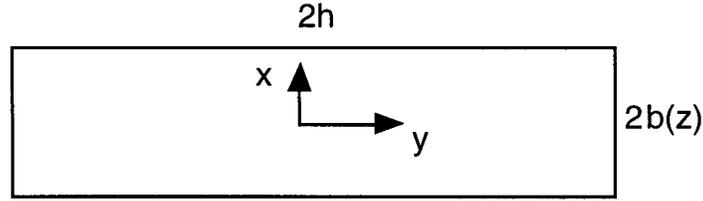


Fig. 2. Cross section of the collimator by a plane  $z = \text{const}$ .

## 2. ELECTROSTATIC PROBLEM

Since we assume that  $h \gg b(z)$ , in this section we set  $h \rightarrow \infty$  and consider a collimator that extends infinitely in  $\pm y$  directions. One can show that the effect of finite  $h$  in the electrostatic problem is exponentially small and can be neglected.

Let us introduce the electrostatic potential  $\varphi_e$  such that  $E_x = -\partial\varphi_e/\partial x$ . It satisfies the Poisson equation with the right hand side representing a linear charge with a unit charge density,

$$\Delta\varphi_e = -4\pi\delta(x - x_0)\delta(y), \quad (3)$$

with the boundary condition

$$\varphi_e|_{x=\pm b(z)} = 0. \quad (4)$$

Since the boundary  $b(z)$  is a slowly varying function of its argument, in the zero approximation, we can neglect the variation of the potential  $\varphi_e$  in the  $z$  direction. This assumption reduces Eq. (3) to

$$\Delta_{x,y}\varphi_e^{(0)} = -4\pi\delta(x - x_0)\delta(y), \quad (5)$$

where  $\Delta_{x,y} = \partial^2/\partial x^2 + \partial^2/\partial y^2$ , and the superscript 0 indicates the zero approximation to the potential. The solution to Eq. (5) with the boundary condition (4) is

$$\varphi_e^{(0)}(x, y, x_0; b(z)) = \ln \frac{\sinh^2 \frac{\pi y}{4b(z)} + \cos^2 \frac{\pi(x + x_0)}{4b(z)}}{\sinh^2 \frac{\pi y}{4b(z)} + \sin^2 \frac{\pi(x - x_0)}{4b(z)}}. \quad (6)$$

In the next approximation, the potential will be given by  $\varphi_e = \varphi_e^{(0)} + \varphi_e^{(1)}$ , where  $\varphi_e^{(1)}$  is a first order correction that satisfies the following equation

$$\Delta_{x,y}\varphi_e^{(1)} = -\frac{\partial^2\varphi_e^{(0)}}{\partial z^2}, \quad (7)$$

with the boundary condition  $\varphi_e^{(1)}|_{x=\pm b(z)} = 0$ . The solution to Eq. (7) can be found explicitly using  $\varphi_e^{(0)}$  as the Green function,

$$\varphi_e^{(1)}(x,y,x_0,z) = -\frac{1}{4\pi} \int_{-b(z)}^{b(z)} dx' \int_{-\infty}^{\infty} dy' \frac{\partial^2\varphi_e^{(0)}(x',y',x_0;b(z))}{\partial z'^2} \varphi_e^{(0)}(x,y-y',x';b(z)). \quad (8)$$

It is easy to show that the integral over  $y'$  in Eq. (8) converges on a scale of the order of  $|y'| \sim b$  which, by assumption, is much smaller than the half width of the collimator  $h$ . This observation justifies the limit  $h \rightarrow \infty$  assumed above.

### 3. MAGNETOSTATIC PROBLEM

A specific feature of the magnetostatic problem is that even though  $h \gg b(z)$ , we cannot set  $h \rightarrow \infty$  and consider an infinitely wide collimator. As we will see below, the contribution of the magnetic field into impedance has a term that is directly proportional to the width of the collimator  $h$ , and hence diverges in the limit  $h \rightarrow \infty$ . The physical reason for such a peculiarity is that the current generated in the walls of the collimator can flow along the circumference of the collimator cross section, as shown in Fig. 3, and generate the magnetic field  $H_y$  which is proportional to  $h$ .

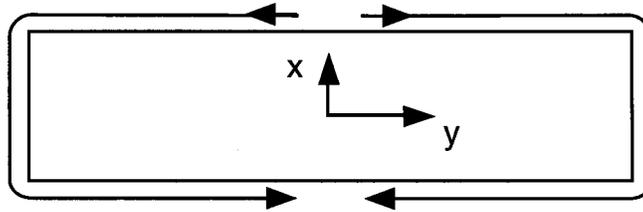


Fig. 3. Transverse currents flowing in the collimator walls.

It follows from the Maxwell equations that the  $y$  component of the magnetic field,  $H_y$ , of an infinitely thin current wire between perfectly conducting walls satisfies the following equation (see derivation in Appendix A)

$$\Delta H_y = 4\pi\delta'(x-x_0)\delta(y), \quad (9)$$

with the Neumann boundary condition at the upper and lower walls,  $\partial H_y / \partial n|_{x=\pm b(z)} = 0$ , and the Dirichlet boundary condition  $H_y = 0$  at the lateral walls  $y = \pm h$ . It is convenient to formulate the magnetic problem in a way analogous to the electric one by introducing the magnetic “potential”  $\varphi_m$  such that

$$H_y = -\frac{\partial\varphi_m}{\partial x_0}. \quad (10)$$

Note that the derivative in Eq. (10) is taken with respect to the location of the driving particle  $x_0$ , rather than  $x$ . The potential  $\varphi_m$  satisfies the Poisson equation

$$\Delta\varphi_m = 4\pi\delta(x-x_0)\delta(y) \quad (11)$$

with the boundary condition

$$\left.\frac{\partial\varphi_m}{\partial n}\right|_{x=\pm b(z)} = 0 \quad (12)$$

and

$$\varphi_m|_{y=\pm h} = 0 \quad (13)$$

Again, invoking a perturbation theory, in the zero approximation which neglects the  $z$ -dependence in the potential  $\varphi_m$ , we have

$$\Delta_{x,y}\varphi_m^{(0)} = 4\pi\delta(x-x_0)\delta(y). \quad (14)$$

The boundary condition (12) in this approximation takes the form

$$\left.\frac{\partial\varphi_m^{(0)}}{\partial x}\right|_{x=\pm b} = 0. \quad (15)$$

The solution to this problem valid in the limit  $h \gg b(z)$  is,

$$\begin{aligned} \varphi_m^{(0)}(x, y, x_0; b(z)) = & \ln \left[ 16 \left( \sinh^2 \frac{\pi y}{4b(z)} + \cos^2 \frac{\pi(x+x_0)}{4b(z)} \right) \right. \\ & \left. \times \left( \sinh^2 \frac{\pi y}{4b(z)} + \sin^2 \frac{\pi(x-x_0)}{4b(z)} \right) \right] - \frac{\pi h}{b(z)}. \end{aligned} \quad (16)$$

This function satisfies Eqs. (14) and (15), and is exponentially small at  $y = \pm h$ ,  $\varphi_m|_{y=\pm h} \sim \exp(-\pi h/4b)$ . For any practical purposes, the boundary condition (13) can be considered as satisfied as soon as  $h > 4b$ . Note that the presence of the second term on the right hand side of Eq. (16) does not allow us to consider the limit  $h \rightarrow \infty$  in the magnetostatic problem, as mentioned at the beginning of this section.

In the next approximation,  $\varphi_m = \varphi_m^{(0)} + \varphi_m^{(1)}$ , where  $\varphi_m^{(1)}$  satisfies

$$\Delta_{x,y}\varphi_m^{(1)} = -\frac{\partial^2\varphi_m^{(0)}}{\partial z^2}. \quad (17)$$

However, the boundary condition for the function  $\varphi_m^{(1)}$  differs from the zero approximation (12), because the normal to the wall  $\mathbf{n}$  in the first approximation is equal to  $\mathbf{n} = (\pm 1, 0, -b')$ , yielding the boundary condition

$$\left. \frac{\partial \varphi_m^{(1)}}{\partial x} \right|_{x=\pm b} = \pm b'(z) \left. \frac{\partial \varphi_m^{(0)}}{\partial z} \right|_{x=\pm b}. \quad (18)$$

The solution of the inhomogeneous equation (17) with the Neumann boundary condition (17) can be expressed in terms of the Green function  $\varphi_m^{(0)}$  [2]. Below, instead of presenting expressions for  $\varphi_m^{(1)}$ , we write down the derivative  $\partial \varphi_m^{(1)} / \partial x_0$  which (with a negative sign) is equal to the perturbation of the magnetic field in the first approximation:

$$\frac{\partial \varphi_m^{(1)}}{\partial x_0} = \frac{\partial \psi_m^{(1)}}{\partial x_0} + \frac{\partial \xi_m^{(1)}}{\partial x_0}, \quad (19)$$

where

$$\frac{\partial \psi_m^{(1)}}{\partial x_0}(x, y, x_0; b(z)) = -\frac{1}{4\pi} \int_{-b(z)}^{b(z)} dx' \int_{-h}^h dy' \frac{\partial^3 \varphi_m^{(0)}}{\partial z^2 \partial x_0}(x', y', x_0; b(z)) \varphi_m^{(0)}(x, y - y', x'; b(z)), \quad (20)$$

and

$$\begin{aligned} \frac{\partial \xi_m^{(1)}}{\partial x_0}(x, y, x_0; b(z)) = & -\frac{b'(z)}{4\pi} \int_{-h}^h dy' \left. \frac{\partial^2 \varphi_m^{(0)}}{\partial z \partial x_0}(x, y', x_0; b(z)) \right|_{x=b(z)} \varphi_m^{(0)}(x, y - y', b(z); b(z)) \\ & - \frac{b'(z)}{4\pi} \int_{-h}^h dy' \left. \frac{\partial^2 \varphi_m^{(0)}}{\partial z \partial x_0}(x, y', x_0; b(z)) \right|_{x=-b(z)} \varphi_m^{(0)}(x, y - y', -b(z); b(z)). \end{aligned} \quad (21)$$

#### 4. IMPEDANCE

In this section we calculate the integral in Eq. (1),

$$\int_{-\infty}^{\infty} (E_x - H_y) dz = - \int_{-\infty}^{\infty} \left( \frac{\partial \varphi_e^{(0)}}{\partial x} + \frac{\partial \varphi_e^{(1)}}{\partial x} - \frac{\partial \varphi_m^{(0)}}{\partial x_0} - \frac{\partial \varphi_m^{(1)}}{\partial x_0} \right) dz. \quad (22)$$

Note that from Eqs. (6) and (16) follows that

$$\partial \varphi_e^{(0)} / \partial x = \partial \varphi_m^{(0)} / \partial x_0, \quad (23)$$

which means that the zero order terms do not contribute to the impedance. This fact can be easily explained by that in the zero order theory the geometry of the collimator reduces to the rectangular pipe of a constant cross section, in which the wake of an ultrarelativistic beam is known to be zero. In the first approximation we have,

$$\int_{-\infty}^{\infty} (E_x - H_y) dz = - \int_{-\infty}^{\infty} \left( \frac{\partial \varphi_e^{(1)}}{\partial x} - \frac{\partial \psi_m^{(1)}}{\partial x_0} - \frac{\partial \xi_m^{(1)}}{\partial x_0} \right) dz. \quad (24)$$

Let us calculate the contribution of the first two terms in Eq. (24),

$$\begin{aligned} - \int_{-\infty}^{\infty} \left( \frac{\partial \varphi_e^{(1)}}{\partial x} - \frac{\partial \psi_m^{(1)}}{\partial x_0} \right) dz = \\ \frac{1}{4\pi} \int_{-\infty}^{\infty} dz \int_{-b(z)}^{b(z)} dx' \int_{-h}^h dy' \frac{\partial^2 \varphi_e^{(0)}}{\partial z^2} (x', y', x_0, z) \frac{\partial \varphi_e^{(0)}}{\partial x} (x, y - y', x', z) \\ + \frac{1}{4\pi} \int_{-\infty}^{\infty} dz \int_{-b(z)}^{b(z)} dx' \int_{-h}^h dy' \frac{\partial^3 \varphi_m^{(0)}}{\partial z^2 \partial x_0} (x', y', x_0, z) \varphi_m^{(0)} (x, y - y', x', z). \end{aligned} \quad (25)$$

This expression can be simplified by using Eq. (23) and integrating over  $x'$ :

$$\begin{aligned} - \int_{-\infty}^{\infty} \left( \frac{\partial \varphi_e^{(1)}}{\partial x} - \frac{\partial \psi_m^{(1)}}{\partial x_0} \right) dz = \\ \frac{1}{4\pi} \int_{-\infty}^{\infty} dz \int_{-b(z)}^{b(z)} dx' \int_{-h}^h dy' \frac{\partial^2 \varphi_e^{(0)}}{\partial z^2} (x', y', x_0; b(z)) \frac{\partial \varphi_m^{(0)}}{\partial x'} (x, y - y', x'; b(z)) \\ + \frac{1}{4\pi} \int_{-\infty}^{\infty} dz \int_{-b(z)}^{b(z)} dx' \int_{-\infty}^{\infty} dy' \frac{\partial^3 \varphi_e^{(0)}}{\partial z^2 \partial x'} (x', y', x_0; b(z)) \varphi_m^{(0)} (x, y - y', x'; b(z)) = \\ \frac{1}{4\pi} \int_{-\infty}^{\infty} dz \int_{-b(z)}^{b(z)} dx' \int_{-h}^h dy' \frac{\partial}{\partial x'} \left[ \frac{\partial^2 \varphi_e^{(0)}}{\partial z^2} (x', y', x_0; b(z)) \varphi_m^{(0)} (x, y - y', x'; b(z)) \right] = \\ \frac{1}{4\pi} \int_{-\infty}^{\infty} dz \int_{-h}^h dy' \left[ \frac{\partial^2 \varphi_e^{(0)}}{\partial z^2} (x', y', x_0; b(z)) \varphi_m^{(0)} (x, y - y', x'; b(z)) \right]_{x'=-b(z)}^{x'=b(z)}. \end{aligned} \quad (26)$$

Now we can integrate by parts over  $z$ ,

$$\begin{aligned} - \int_{-\infty}^{\infty} \left( \frac{\partial \varphi_e^{(1)}}{\partial x} - \frac{\partial \psi_m^{(1)}}{\partial x_0} \right) dz = \\ \frac{1}{4\pi} \int_{-\infty}^{\infty} dz \int_{-h}^h dy' \left[ \frac{\partial \varphi_e^{(0)}}{\partial z} (x', y', x_0; b(z)) \right]_{x'=b(z)} \frac{\partial \varphi_m^{(0)} (x, y - y', b(z); b(z))}{\partial z} - \\ \left[ \frac{\partial \varphi_e^{(0)}}{\partial z} (x', y', x_0; b(z)) \right]_{x'=-b(z)} \frac{\partial \varphi_m^{(0)} (x, y - y', -b(z); b(z))}{\partial z} \right]. \end{aligned} \quad (27)$$

It is necessary to emphasize that in Eq. (27) one has to first differentiate  $\varphi_e^{(0)}$  with respect to  $z$  and then substitute  $\pm b(z)$  for  $x'$ , however, for  $\varphi_m^{(0)}$ , one has to put first  $\pm b(z)$  for  $x'$  and then differentiate it with respect to  $z$ .

Returning to the contribution of the  $\xi_m^{(0)}$  term in Eq. (24), we find

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{\partial \xi_m^{(1)}}{\partial x_0} dz = & -\frac{1}{4\pi} \int_{-\infty}^{\infty} b'(z) dz \int_{-h}^h dy' \frac{\partial^2 \varphi_m^{(0)}}{\partial z \partial x_0} (x, y', x_0, b(z)) \Bigg|_{x=b(z)} \varphi_m^{(0)}(x, y-y', b(z); b(z)) \\ & -\frac{1}{4\pi} \int_{-\infty}^{\infty} b'(z) dz \int_{-h}^h dy' \frac{\partial^2 \varphi_m^{(0)}}{\partial z \partial x_0} (x, y', x_0, b(z)) \Bigg|_{x=-b(z)} \varphi_m^{(0)}(x, y-y', -b(z); b(z)). \end{aligned} \quad (28)$$

Eqs. (27) and (28) give the impedance of the collimator for arbitrary values of  $x$ ,  $y$  and  $x_0$ .

## 5. IMPEDANCE CLOSE TO THE AXIS

Near the axis when  $x, x_0 \ll b$  we can expand the general expressions of the previous section and carry out the integration over  $y$ . Omitting lengthy analytical calculations that were performed with the use of computer program Mathematica [3] we present here the result. We limit our consideration by the case  $y=0$  only, i.e. the case when both the driving and test particles are in the same vertical plane. In this case,

$$Z_x = \frac{-iZ_0}{4\pi} (Ax + Bx_0), \quad (29)$$

where

$$A = 2I_1, \quad B = -2I_1 + 2\pi h I_3, \quad (30)$$

and

$$I_1 = \int dz \frac{(b')^2}{b^2}, \quad I_2 = \int dz \frac{(b')^2}{b^3}. \quad (31)$$

For  $x = x_0$ , for a conventional definition of the transverse impedance  $Z_t$ , we find

$$Z_t \equiv \frac{1}{x_0} Z_x \Big|_{x=x_0} = \frac{-iZ_0}{2} h I_3. \quad (32)$$

Note that the integral  $I_1$  also appears in Yokoya's theory of a smooth axisymmetric collimator [4], where  $b(z)$  plays a role of the pipe radius. Comparison of our result with [4] shows that the impedance of a flat collimator is much larger (by factor of  $h/b$ ) than the impedance of a cylindrical collimator of a radius  $b(z)$ .

## 6. NLC-TYPE COLLIMATOR

We apply the results obtained above to collimators considered in the design of the Next Linear Collider [5]. The geometry of a typical collimator is shown in Fig. 4 with the following parameters:  $b = 0.5$  cm,  $g = 0.1$  cm,  $l = 40$  cm, and the width of the collimator  $h = 0.7$  cm. The applicability condition of the theory developed in the previous sections require the function  $b(z)$  to be smooth together with its first two derivatives. Strictly speaking, this requirement does not hold for the profile shown in Fig. 4 where  $b'(z)$  is not continuous at the entrance and the exit of the taper. In our calculations we assumed that in reality the angles of the collimator will be rounded in such a way that the smoothness condition is satisfied.

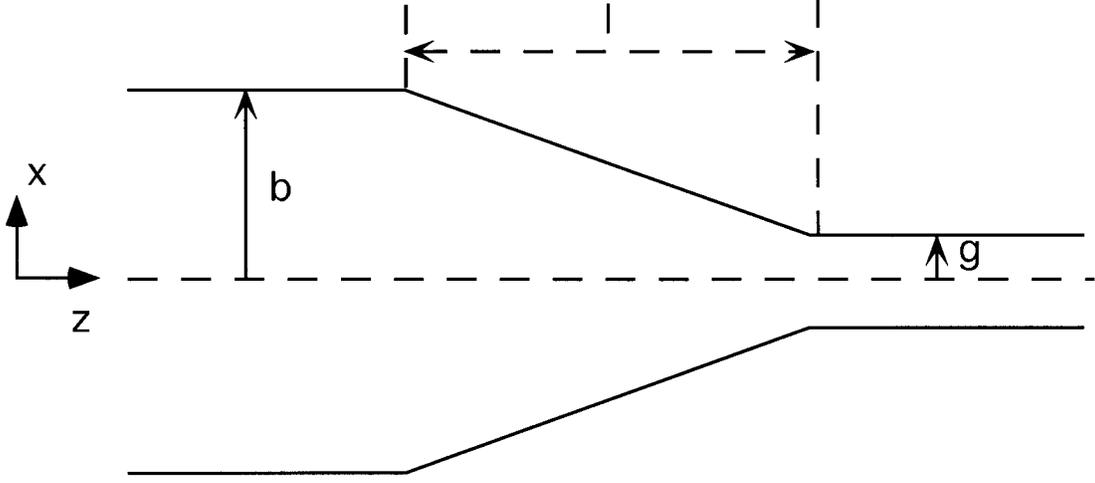


Fig. 4. Schematic of the NLC collimator.

The integrals (31) for this collimator are equal:  $I_1 = 0.08 \text{ cm}^{-1}$  and  $I_2 = 0.48 \text{ cm}^{-2}$ , which gives the transverse impedance near the axis,

$$-\text{Im} Z_t = 6.3 \times 10^3 \text{ Ohm/m} . \quad (33)$$

With a simple computer code we also calculated the impedance for large values of  $x$  and  $x_0$ . The results of the calculations are shown in Figs. 5-7. Note that in the case of  $x = x_0$ , (see Fig. 7), the impedance grows faster when  $x$  approaches  $g$ . It can be shown from general expressions (27) and (28) that in the limit  $g - x \rightarrow 0$  and  $x = x_0, y = 0$ , the impedance asymptotically equals to

$$Z_x = \frac{-iZ_0}{\pi} \int \frac{(b')^2}{b(z) - x} dz. \quad (34)$$

For the collimator shown in Fig. 4 with a linear dependence of  $b$  of  $z$ , this formula predicts a logarithmic divergence of  $Z_x$  when  $x \rightarrow g$ . The indication of this divergence is seen in Fig. 7 and a faster growth of the impedance for small values of  $g - x$ .

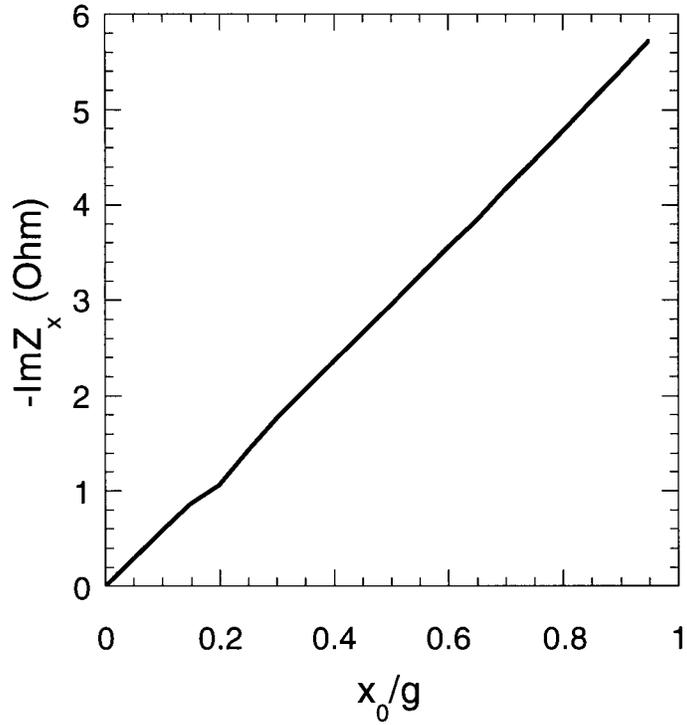


Fig. 5. Impedance of NLC collimator as a function of  $x_0$  for  $x = 0$  and  $y = 0$ .

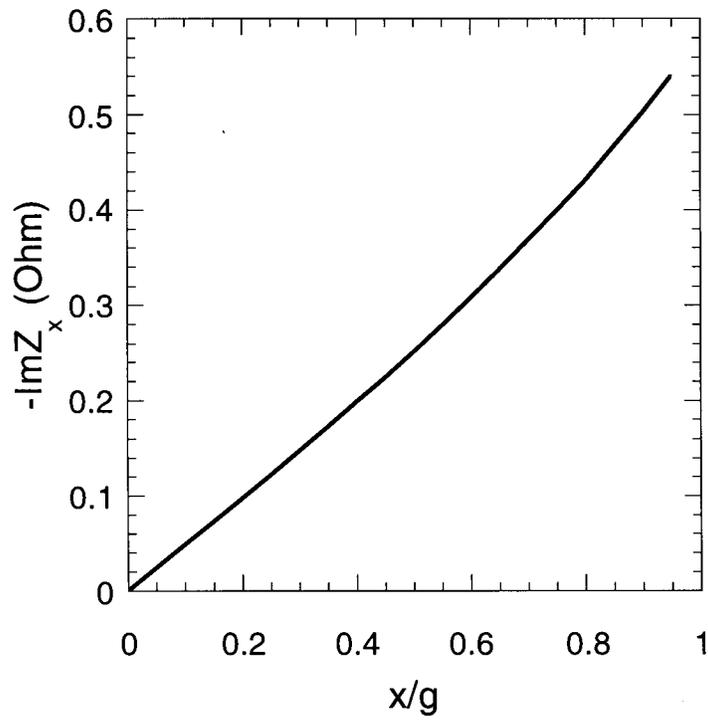


Fig. 6. Impedance of NLC collimator as a function of  $x$  for  $x_0 = 0$  and  $y = 0$ .

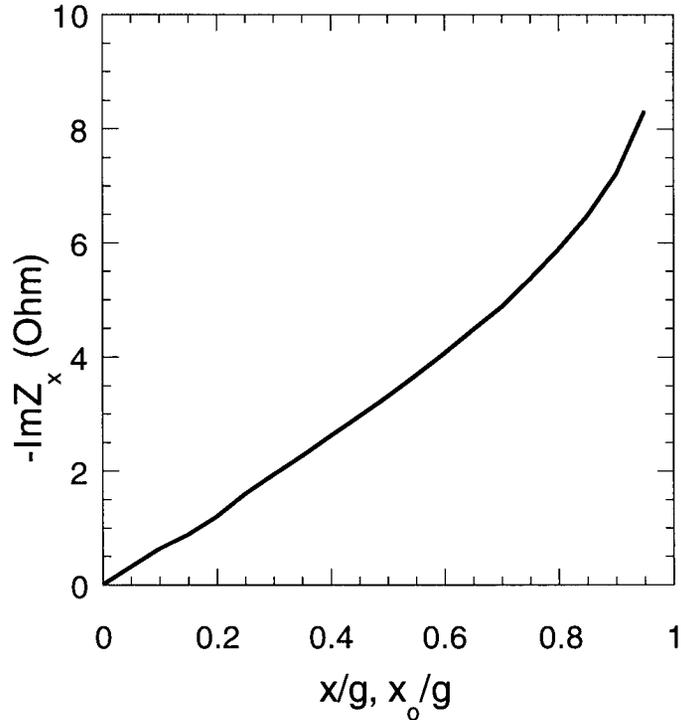


Fig. 7. Impedance of NLC collimator as a function of  $x$  for  $x_0 = x$  and  $y = 0$ .

### VIII. ACKNOWLEDGMENT

I would like to thank K. Bane, A. Chao, S. Heifets, J. Irwin, and R. Warnock for useful discussions.

### References

1. G.V. Stupakov. Geometrical Wake of a Smooth Taper. SLAC-PUB-7086, 1995; submitted to *Particle Accelerators*.
2. P. M. Morse and H. Feshbach. *Methods of Theoretical Physics*. McGraw-Hill, Inc., Chapter 7.
3. S. Wolfram. *Mathematical: a System for Doing Mathematics by Computer*. Addison-Wesley, Redwood City CA, 1991.
4. K. Yokoya. Impedance of Slowly Tapered Structures. CERN SL/90-88 (AP) (1990).
5. NLC Zero Design Review Workbook, SLAC, unpublished.

## APPENDIX A

In this appendix we derive Eq. (9) with proper boundary conditions for a flat collimator shown in Figs. 1 and 2. Applying the operator  $\nabla \times$  to the Maxwell equation

$$\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{j} \quad (\text{A1})$$

with the current in the  $z$ -direction given by  $\mathbf{j} = \mathbf{e}_z j_z = \mathbf{e}_z c \delta(x - x_0) \delta(y)$ , one finds

$$\nabla \times \nabla \times \mathbf{H} = -\Delta \mathbf{H} = \frac{4\pi}{c} \nabla \times \mathbf{j} = -\frac{4\pi}{c} \mathbf{e}_y \frac{\partial j_z}{\partial x} = -4\pi \mathbf{e}_y \delta'(x - x_0) \delta(y), \quad (\text{A2})$$

where  $\mathbf{e}_y$  and  $\mathbf{e}_z$  are the unit vectors in the  $y$  and  $z$  directions, respectively, and  $\delta'$  denotes the derivative of the delta function. The  $y$  component of Eq. (A2) gives Eq. (9).

Boundary conditions for  $H_y$  follow from the requirement  $H_n = 0$  at the perfectly conducting walls, where the subscript  $n$  indicated a component normal to the wall. For the upper and bottom walls of the collimator we have  $H_n = H_x \sin \alpha + H_z \cos \alpha$ , where  $\alpha$  is the angle between the wall and the  $x$ -axis. Differentiating  $H_n$  with respect to  $y$ , we obtain

$$\frac{\partial H_n}{\partial y} = \frac{\partial H_x}{\partial y} \sin \alpha + \frac{\partial H_z}{\partial y} \cos \alpha = \frac{\partial H_y}{\partial x} \sin \alpha + \frac{\partial H_y}{\partial z} \cos \alpha = \frac{\partial H_y}{\partial n} = 0. \quad (\text{A3})$$

In this equation we have used the relations  $\partial H_x / \partial y = \partial H_y / \partial x$  and  $\partial H_z / \partial y = \partial H_y / \partial z$  which follow from the equation  $\nabla \times \mathbf{H} = 0$  in the region free of current. For the lateral walls of the collimator,  $y = \pm h$ , we have

$$H_n|_{y=\pm h} = 0. \quad (\text{A4})$$