

SOME THOUGHTS ON DISCRETE PHYSICS AND THE RECONSTRUCTION OF QUANTUM MECHANICS*

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ABSTRACT

We explore how relativistic quantum mechanics might be reconstructed on a finite and discrete basis, starting from the observation that strong interactions break scale invariance at the pion Compton wavelength and provide an event horizon for space-time description. Regions of this size act coherently, and can only be explored by studying the momentum and quantum number distribution of particles produced in high energy collisions that attempt to probe them. This leads us to the concept of measurement accuracy bounded from below, from which we have derived finite and discrete replacements for the Maxwell and Dirac equations. We suggest how the Dirac equation could provide the sources and sinks for the Maxwell equations. A lower bound for mass measurement accuracy is proposed.

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1. INTRODUCTION

Classical physics, including both special and general relativity, accepts without question the idea of a continuum space-time as the proper framework within which to formulate the “laws” of physics. It is true that general relativity can be viewed as a program for *constructing* space-time — and also the proper way to construct a theory that satisfies “Mach’s Principle”^[1] — starting from the differential formulation of the equivalence principle, but the continuum Riemannian geometry is still basic. Quantum mechanics, including the relativistic quantum field theory, goes still further into the continuum by utilizing an infinite-dimensional Hilbert space as one basic concept in the formalism. Yet the very essence of quantum phenomena is the *discontinuity* of discrete events, which is a paradigm for the act of observation. This discreteness is reinforced by the ease with which quantized masses, charges and rational, conserved quantum numbers can be used to model laboratory experience. In the light of the failure of a century of efforts to reconcile quantum mechanics with relativity, and to produce a satisfactory theory of “quantum gravity”, it seems worth while to ask whether a fully finite, discrete and *constructive* approach in which particles, space and time all emerge from the combinatorics of more primitive elements might not provide a more successful conceptual starting point.

The employment of continuum mathematics in physics was justified so long as no experimental or observational phenomena had appeared which called into doubt the concepts of mass, length and time established by Newton. These physical dimensional concepts, like the Euclidean geometry in which they were embedded, were assumed to be *scale invariant*. But this scale invariance of classical physics came into question once three dimensional constants of *universal* significance had been identified and measured. Planck recognized this in 1907, but the three constants then clearly recognizable as universal — Newton’s gravitational constant G , the limiting velocity for information transfer c , and his own constant h — did not define mass, length and time units of obvious significance. The length scale is 25 orders of magnitude smaller than atomic dimensions and the time scale correspondingly short compared to the periodic times related to atomic transitions, while the mass scale of about a hundred-thousandth of a gram is correspondingly larger than atomic masses. These scales are now recognized as indeed critical to the formulation of any fundamental elementary particle theory, but the route to that understanding has been long and is still fraught with open questions. The success of the older quantum theory and then of quantum mechanics clearly established the ubiquity of h in atomic and nuclear phenomena, but the origin of the mass scale needed to understand laboratory phenomena in what has come to be called elementary particle physics remained mysterious.

To set the stage for our proposed resolution of this mystery, I offer a mnemonic given me and my fellow graduate students by Joe Weinberg in 1946 as an unexplained sequence. This useful set of facts relates atomic (Bohr radius $\hbar^2/m_e e^2 \approx 5 \times 10^{-9} \text{ cm}$) to QED (quantum electrodynamics, electron Compton wavelength $\hbar/m_e c \approx 3.9 \times 10^{-11} \text{ cm}$) to nuclear (classical electron radius $e^2/m_e c^2 \approx 2.8 \times 10^{-13} \text{ cm}$) sizes, namely

$$e^2/m_e c^2 = \alpha(\hbar/m_e c) = \alpha^2(\hbar^2/m_e e^2); \quad \alpha = e^2/\hbar c \approx 1/137 \quad (1.1)$$

That the electron Compton wave length should be 137 times smaller than the Bohr radius was, in a sense, explained by the Dyson in 1952^[2] because QED corrections to non-relativistic quantum mechanics — a power series in α carried out to 137 terms — necessarily exhaust the quantitative meaningfulness of this perturbative approach. But why the “classical electron radius” — the radius of a charged shell with charge e which has an electrostatic energy equal to the rest energy of an electron — should have anything at all to do with nuclear phenomena remains unexplained by conventional theorists to this day.

The route that I follow starts from the experimental fact that if one collides two elementary particles of the same mass at zero total momentum, but each with a momentum greater than mc , then there is a finite probability that a particle-antiparticle pair, each member of which has rest mass m will be produced. Consequently the standard quantum mechanical formalism breaks down and one must generalize to second quantized field theory, develop a finite particle number relativistic quantum mechanics, or take a still more radical tack. Qualitatively, the explanation I propose rests on the fact that, from an operationalist’s point of view, discussing distances so short that exploring them — directly or indirectly — *necessarily* requires the inclusion of more particulate degrees of freedom than occur in the initial state (eg. an electron and a positron in a high energy collider) because of the relativistic phenomenon of particle-antiparticle pair creation. Thus the concept of geometric structure in space-time needs to be replaced by the concept of particulate structure in momentum-energy space. Note that this structure can be just as complicated in its details as geometric structure, but making this into a clear picture requires the development of a finite particle number relativistic scattering theory — a task which is still in its infancy.

2. THE PION COMPTON WAVE LENGTH AS AN EVENT HORIZON

2.1 THE WICK-YUKAWA MECHANISM

One of the most profound papers in twentieth century physics is due to Gian Carlo Wick.^[3] He observed that when two particles approach closely enough together (Δr) for a short enough time (Δt) so that the uncertainty in their rest energy ($\Delta E \approx \hbar/\Delta t$) exceeds μc^2 , then it is possible for a particle of mass μ to be present. If the space-time volume is small enough so that the whole region acts coherently ($\Delta r < c\Delta t$), this “virtual” particle can transfer momentum between the two incident particles, causing them to scatter. Quantitatively

$$\Delta r < c\Delta t \leq \frac{c\hbar}{\Delta E} \approx \frac{c\hbar}{\mu c^2} = \frac{\hbar}{\mu c} \quad (2.1)$$

Thus, by combining special relativity with the quantum mechanical uncertainty principle for energy, Wick was able to explain on general grounds why the existence of a particle with mass energy of about 140 Mev would account for the observed range of the nuclear force of about $1.4 \times 10^{-13} cm$, consistent with Yukawa’s meson theory. Further, by scattering two nucleons with sufficient energy and momentum, these Yukawa particles could be expected to be produced copiously because of the known strength of the nuclear force between protons and/or neutrons. This prediction was verified in the next decade. Thus relativity plus quantum mechanics plus the existence of such a particle leads immediately to the starting point for a theory of strong interactions. But why the pion should have the mass it does remains a mystery for conventional physicists to this day.

2.2 GENERALIZATION OF THE DYSON-NOYES ARGUMENT TO STRONG INTERACTIONS

In my first paper on discrete physics^[4] I pointed out that if one has 137 electron-positron pairs within their own Compton wavelength, one will have enough electrostatic energy to form another pair. Quantitatively

$$\frac{Ne^2}{r} \geq \frac{Ne^2}{\hbar/2mc} = 2mc^2 \Rightarrow N = \frac{\hbar c}{e^2} \approx 137 \quad (2.2)$$

As it stands this is basically the same argument that Dyson (Ref.2) used to show that the QED perturbation series cannot be uniformly convergent. My immediate use of this argument was to show that the 137 arrived at by the Parker-Rhodes

construction of the combinatorial hierarchy can be interpreted as a counting number, but I also pointed out that this suggests that in some sense the pion could be thought of as a 137 electron-positron pairs, which gives the right mass for the pion ($274m_e c^2 \approx 140 \text{ Mev}$) and couples the system to both strong and weak interactions. This argument can be immediately generalized to gravitation for N_G particle pairs, each particle of mass m :

$$\frac{N_G G m^2}{r} \geq \frac{N_G G m^2}{\hbar/mc} = mc^2 \Rightarrow N_G = \frac{\hbar c}{G m^2} \quad (2.3)$$

which can be called the “quantum black hole” condition for any mass. Note that this is also an easy way to understand the meaning of the Planck mass $M_{Planck} = [\hbar c/G]^{1/2}$; when an assemblage of particles, each of mass m , is so large in number that, when concentrated within the Compton wavelength of any one of them their gravitational energy is enough create another one, the number of pairs in the assemblage is equal to the ratio of the square of the Planck mass to the square of the mass of any one of them. Usually this structure will be ephemeral, disappearing rapidly due to Hawking radiation. But if the assemblage has the quantum numbers of the lightest elementary fermion with those quantum numbers, and baryon number, lepton number and charge are absolutely conserved, the evaporation process must stop with that fermion.^[5,6]

For the case of an inverse square law interaction of arbitrary strength obtained by replacing the fine structure constant $\alpha = e^2/\hbar c$ by the strong interaction parameter $\alpha_s = g_s^2/\hbar c$, the generalized Dyson equation is

$$\frac{g_s^2}{r} = \frac{\hbar c \alpha_s}{\hbar/2mc} = 2mc^2 \Rightarrow \alpha_s = 1 \quad (2.4)$$

In particular, for quantum chromodynamics (QCD), the strong interaction parameter α_s is not fixed, but is a *running* coupling constant which becomes indefinitely large at low energy. Therefore there is some finite energy at which the strong interaction coupling constant passes the critical value one. The question is, at what particulate rest energy (mass) does this happen?

Our tentative hypothesis is that this happens at the pion rest energy $m_\pi c^2$, because then the fact that strong interactions set in at the pion Compton wavelength, as explained by the Wick-Yukawa mechanism, begins to become a quantitative explanation in terms of the QCD theory of strong interactions. If one looks at current experimental values in the region where α_s is known and runs from 0.11 to 0.4 as the energy decreases (cf. plot in the *Particle Physics Booklet*^[7]), one sees that our hypothesis that $\alpha_s(m_\pi) = 1$ is consistent with a reasonable extrapolation. Since

current theory allows no way to enter the regime we are talking about, what happens at this threshold is anybody's guess. Our speculation is that this threshold is the boundary or event horizon of a "black hole" for strong interactions, inside which the motion of the quarks is confined.^[6] This is also consistent with our model of the pion as 137 electron-positron pairs, as suggested by the original Noyes-Dyson argument given above.

3. Maxwell and Dirac Equations from Measurement Accuracy

3.1 THE FREE SPACE MAXWELL EQUATIONS

The Wick-Yukawa mechanism discussed in the last section shows us that the entire concept of individual particles in space-time must break down once we study systems that require us to talk about processes that occur at distances less than $1.4 \times 10^{-13} \text{ cm}$ with any significant probability. Yet this fact has not prevented conventional physicists from assuming that the classical concept of a continuum space-time is a reasonable theoretical framework to use down to distances of the Planck length ($\sim 10^{-33} \text{ cm}$). We will discuss elsewhere why, in our opinion, this is a fatal barrier to progress in fundamental physics.^[9]

In this sub-section we will make a first, crude approximation to the breaking of the length and time scale invariance of classical geometry and conventional physics by assuming that there is some finite and *fixed* lower bound on the accuracy to which we can measure distances, and call it Δx . For the moment we will simply assume that this is finite and fixed at a value large enough so that we need not consider quantum effects. We call this assumption *measurement accuracy bounded from below*. Because the *System International* has decided to define the meter as the distance light travels in a time of $1/299\,792\,458 \text{ second}$ (where 299 792 458 is an integer), this implies that there is also a shortest time interval which we can measure, i.e. $\Delta t = \Delta x/c$. Since velocity is the ratio of the distance a particle travels to the time it takes to move that distance, this discrete assumption introduces an ambiguity into the *measurement* of velocity: we can first measure the time at which a particle is at a particular position X and then define the velocity \dot{X} as the distance it moves in unit time interval Δt (which we take equal to one in the remainder of this section), or first measure the distance it moves in unit time interval, and then the position.

In a recent publication^[10] we formalize this observation by making the *hypothesis* that the difference between these two orders of measurement is a constant, i.e. $X\dot{X} - \dot{X}X = [X, \dot{X}] = \kappa$. We extend the calculus of finite differences to rigorously

include this possibility, creating a *discrete ordered calculus* or DOC. We relate this to the formalism of electromagnetism by relating the force per unit charge to the acceleration (force per unit mass) of a test-charge via the Lorentz force law. In 3-space we start from the assumption that

$$(i) \quad [X_i, X_j] = 0 \quad \forall_{ij}$$

$$(ii) \quad [X_i, \dot{X}_j] = \kappa \delta_{ij}$$

where κ is a constant and κ commutes with all expressions in DOC. We then show that

$$\text{div } H = 0$$

and

$$\frac{\partial H}{\partial t} + \nabla \times E = 0 .$$

I quote here our concluding remarks:

“As Dyson remarks,^[11] the other two Maxwell equations

$$\text{div } E = 4\pi\rho$$

$$\frac{\partial E}{\partial t} - \nabla \times H = 4\pi \vec{j}$$

can be taken to define the external charge and current densities ρ and j . *However* it is important to realize that our entire theory has applied only to a single trajectory. We can regard this trajectory (and its “particle”) as defining an electromagnetic field, *or* we can regard this particle as moving in an external field with these properties. We cannot have it both ways. The analysis so far in no way takes into account the self-interaction of this particle or its interactions with other particles and fields. Of course, our talk at this stage about the “trajectory” of a particle is an *analog* of a physical trajectory. The trajectory we talk about is in the space of $A \times A \times A$ where A denotes the non-commutative operator algebra that underlies the theory. An eventual interpretation of this theory in terms of trajectories in physical space is a possible consequence of further analysis of our formalism. It is beyond the scope of this preliminary paper.

“We feel that the foregoing analysis of the Feynman-Dyson derivation in a discrete context lays bare much of the beautiful structure of the electromagnetic formalism and its relation to a condition of discrete time. We hope to probe this structure more deeply in subsequent papers. ”

3.2 THE 1+1 DIRAC EQUATION

Our next joint paper^[12] takes a step toward understanding the source terms in the Maxwell equations. We still consider only the single trajectory, in this instance of a free particle, but break scale invariance in mass as well as in length and time by taking the shortest distance which the particle can move as its Compton wave length, \hbar/mc , and further assume that the only motion it can execute is a step of this length to the left or right at velocity c , i.e. we take $\Delta t = \hbar/mc^2$. This model, sometimes called the ‘‘Feynman Checkerboard’’, is given as an exercise in his book on the path integral formulation of quantum mechanics,^[13] but his objective was still the continuum limit, not a fully finite and discrete physics.

We introduce the light cone coordinates of a point (x, t) of space-time given by $[r, l] = [\frac{1}{2}(t + x), \frac{1}{2}(t - x)]$ and find that the Dirac algebra ($\alpha\beta + \beta\alpha = 1$; $\alpha^2 = 1 = \beta^2$) allows us to write the two representations of the 1+1 free particle

Dirac Equation for $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ which we consider, namely

$$RI : \quad \frac{\partial\psi_1}{\partial l} = \psi_2, \quad \frac{\partial\psi_2}{\partial r} = -\psi_1 \quad (3.1)$$

and

$$RII : \quad \frac{\partial\psi_1}{\partial l} = -i\psi_2, \quad \frac{\partial\psi_2}{\partial r} = -i\psi_1 \quad (3.2)$$

Note that the first form, where we follow a suggestion from Karmanov,^[14] has real solutions, suitable for use in bit-string physics, while the second yields a more conventional complex wave function.

Suppose that $f = f(x)$ is a function of a variable x . Let Δ be a fixed non-zero constant. The discrete derivative of f with respect to Δ is then defined by the equation

$$D_\Delta f(x) = \frac{f(x + \Delta) - f(x)}{\Delta} \quad (3.3)$$

Consider the function

$$x^{(n)} = x(x - \Delta)(x - 2\Delta) \cdots (x - (n - 1)\Delta) \quad (3.4)$$

Then it is easy to prove the Lemma:

$$D_\Delta x^{(n)} = nx^{(n-1)} \quad (3.5)$$

Note that as Δ approaches zero $x^{(n)}$ approaches x^n , the usual n^{th} power of x . Note

also that

$$\frac{x^{(n)}}{\Delta^n n!} = C_n^{x/\Delta} \quad (3.6)$$

where

$$C_n^z = \frac{z(z-1)\cdots(z-n+1)}{n!} \quad (3.7)$$

is a (generalized) binomial coefficient. Thus

$$\frac{x^{(n)}}{n!} = C_n^{x/\Delta} \quad (3.8)$$

With this formalism in hand, we can express functions whose combination will yield solutions to discrete versions of the RI and RII Dirac equations, by extending the discrete derivative to discrete partial derivatives in the obvious way. Define the following functions of r and l

$$\begin{aligned} \psi_R^\Delta(r, l) &= \sum_{k=0}^{\infty} (-1)^k \frac{r^{(k+1)}}{(k+1)!} \frac{l^{(k)}}{k!} \\ \psi_L^\Delta(r, l) &= \sum_{k=0}^{\infty} (-1)^k \frac{r^{(k)}}{k!} \frac{l^{(k+1)}}{(k+1)!} \\ \psi_0^\Delta(r, l) &= \sum_{k=0}^{\infty} (-1)^k \frac{r^{(k)}}{k!} \frac{l^{(k)}}{k!} \end{aligned} \quad (3.9)$$

We can now produce solutions to both the RI and the RII Dirac equations. We shall omit writing the Δ 's in those equations, since all these calculations take the same form independent of the choice of Δ . Of course for finite Δ and integral $r/\Delta, l/\Delta$ these series produce discrete calculus solutions to the equations.

Let

$$\begin{aligned} \psi_1 &= \psi_0 - \psi_L \\ \psi_2 &= \psi_0 + \psi_R \end{aligned} \quad (3.10)$$

It follows immediately that this gives a solution to the RI Dirac equation.

Similarly, if we let

$$\begin{aligned} \psi_1 &= \psi_0 - i\psi_L \\ \psi_2 &= \psi_0 - i\psi_R \end{aligned} \quad (3.11)$$

this gives a solution to the RII Dirac equation.

We then go on to consider the lattice path interpretations of these solutions, which have been discussed in the context of a more heuristic derivation elsewhere.^[15] That paper also indicates how we intend to go on from this discrete representation of a spin- $\frac{1}{2}$ wave function to a finite and discrete mass renormalization, the starting point for a new attack on the infinities in QED. Of course, we must also extent the model to 3+1 dimensions, which does not appear to be too difficult. The basic kinematic relation by which we go from discrete space-time to the discrete energy-momentum space needed for an S-Matrix type relativistic scattering theory coupling the particle trajectories described by the integers r, l to radiation is, of course,

$$\beta = \frac{v}{c} = \frac{r-l}{r+l}; \quad \frac{p^2}{m^2 c^2} = \frac{(r-l)^2}{4rl}; \quad \frac{E^2}{m^2 c^4} = \frac{(r+l)^2}{4rl} \quad (3.12)$$

Our next step will be to construct source and sink terms for the two remaining (discrete) Maxwell equations by imposing the requirement that the finite accelerations can occur only an integral number of deBroglie wavelengths apart.

4. To Be Continued . . .

We have no space here to forge the connection between this new approach and earlier results obtained by Bit-String Physics.^{[5], [16]} Our conjecture is that in order to make this connection rigorous, we will also have to bound mass-energy measurement from below by introducing a smallest rest-mass unit $\Delta m = m_{proton}^2 / M_{Planck} = m_{proton} / 1.3 \times 10^{19}$. This will require an analysis of finite particle number relativistic quantum scattering theory analagous to our treatment of the Maxwell and Dirac equations sketched above. Comparison of this approach with combinatorial physics^[17] will be made at the next meeting of the Alternative Natural Philosophy Association.^[18]

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